GGS CMT, Kharar

Lecture Notes

Power System 2
Subject code – BTEE-601-18

6th Semester
Electrical Engineering

POWER SYSTEM STABILITY

5.1 INTRODUCTION

Power system stability of modern large inter-connected systems is a major problem for secure operation of the system. Recent major black-outs across the globe caused by system instability, even in very sophisticated and secure systems, illustrate the problems facing secure operation of power systems. Earlier, stability was defined as the ability of a system to return to normal or stable operation after having been subjected to some form of disturbance. This fundamentally refers to the ability of the system to remain in synchronism. However, modern power systems operate under complex interconnections, controls and extremely stressed conditions. Further, with increased automation and use of electronic equipment, the quality of power has gained utmost importance, shifting focus on to concepts of voltage stability, frequency stability, inter-area oscillations etc.

The IEEE/CIGRE Joint Task Force on stability terms and conditions have proposed the following definition in 2004: "*Power System stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded, so that practically the entire system remains intact*".

The Power System is an extremely non-linear and dynamic system, with operating parameters continuously varying. Stability is hence, a function of the initial operating condition and the nature of the disturbance. Power systems are continually subjected to small disturbances in the form of load changes. The system must be in a position to be able to adjust to the changing conditions and operate satisfactorily. The system must also withstand large disturbances, which may even cause structural changes due to isolation of some faulted elements.

A power system may be stable for a particular (large) disturbance and unstable for another disturbance. It is impossible to design a system which is stable under all disturbances. The power system is generally designed to be stable under those disturbances which have a high degree of occurrence. The response to a disturbance is extremely complex and involves practically all the equipment of the power system. For example, a short circuit leading to a line isolation by circuit breakers will cause variations in the power flows, network bus voltages and generators rotor speeds. The voltage variations will actuate the voltage regulators in the system and generator speed variations will actuate the prime mover governors; voltage and frequency variations will affect the system loads. In stable systems, practically all generators and loads remain connected, even though parts of the system may be isolated to preserve bulk operations. On the other hand, an unstable system condition could lead to cascading outages and a shutdown of a major portion of the power system.

5.2 CLASSIFICATION OF POWER SYSTEM STABILITY

The high complexity of stability problems has led to a meaningful classification of the power system stability into various categories. The classification takes into account the main system variable in which instability can be observed, the size of the disturbance and the time span to be considered for assessing stability.

5.2.1 ROTOR ANGLE STABILITY

Rotor angle stability refers to the ability of the synchronous machines of an interconnected power system to remain in synchronism after being subjected to a disturbance. Instability results in some generators accelerating (decelerating) and losing synchronism with other generators. Rotor angle stability depends on the ability of each synchronous machine to maintain equilibrium between electromagnetic torque and mechanical torque. Under steady state, there is equilibrium between the input mechanical torque and output electromagnetic torque of each generator, and its speed remains a constant. Under a disturbance, this equilibrium is upset and the generators accelerate/decelerate according to the mechanics of a rotating body. Rotor angle stability is further categorized as follows:

Small single (or small disturbance) rotor angle stability: It is the ability of the power system to maintain synchronism under small disturbances. In this case, the system equation can be linearized around the initial operating point and the stability depends only on the operating point and not on the disturbance. Instability may result in

(i) A non oscillatory or a periodic increase of rotor angle

(ii) Increasing amplitude of rotor oscillations due to insufficient damping.

The first form of instability is largely eliminated by modern fast acting voltage regulators and the second form of instability is more common. The time frame of small signal stability is of the order of 10-20 seconds after a disturbance.

Large-signal rotor angle stability or transient stability: This refers to the ability of the power system to maintain synchronism under large disturbances, such as short circuit, line outages etc. The system response involves large excursions of the generator rotor angles. Transient stability depends on both the initial operating point and the disturbance parameters like location, type, magnitude etc. Instability is normally in the form of a periodic angular separation. The time frame of interest is 3-5 seconds after disturbance.

The term dynamic stability was earlier used to denote the steady-state stability in the presence of automatic controls (especially excitation controls) as opposed to manual controls. Since all generators are equipped with automatic controllers today, dynamic stability has lost relevance and the Task Force has recommended against its usage.

5.2.2 VOLTAGE STABILITY

Voltage stability refers to the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance. It depends on the ability of the system to maintain equilibrium between load demand and load supply. Instability results in a progressive fall or rise of voltages of some buses, which could lead to loss of load in an area or tripping of transmission lines, leading to cascading outages. This may eventually lead to loss of synchronism of some generators.

The cause of voltage instability is usually the loads. A run-down situation causing voltage instability occurs when load dynamics attempt to restore power consumption beyond the capability of the transmission network. Voltage stability is also threatened when a disturbance increases the reactive power demand beyond the sustainable capacity of the available reactive power resources. Voltage stability is categorized into the following sub-categories:

Small – disturbance voltage stability: It refers to the system's ability to maintain steady voltages when subjected to small perturbations such as incremental changes in load. This is primarily influenced by the load characteristics and the controls at a given point of time.

Large disturbance voltage stability: It refers to the systems ability to maintain steady voltages following large disturbances; It requires computation of the non-linear response of the power system to include interaction between various devices like motors, transformer tap changers and field current limiters. Short term voltage stability involves dynamics of fast acting load components and period of interest is in the order of several seconds. Long term voltage stability involves slower acting equipment like tap-changing transformers and generator current limiters. Instability is due to loss of long-term equilibrium.

5.2.3 FREQUENCY STABILITY

Frequency stability refers to the ability of a power system to maintain steady frequency following a severe disturbance, causing considerable imbalance between generation and load. Instability occurs in the form of sustained frequency swings leading to tripping of generating units or loads. During frequency swings, devices such as under frequency load shedding, generator controls and protection equipment get activated in a few seconds. However, devices such as prime mover energy supply systems and load voltage regulators respond after a few minutes. Hence, frequency stability could be a short-term or a long-term phenomenon.

5.3 MECHANICS OF ROTATORY MOTION

Since a synchronous machine is a rotating body, the laws of mechanics of rotating bodies are applicable to it. In rotation we first define the fundamental quantities. The angle θ_m is defined, with respect to a circular arc with its center at the vertex of the angle, as the ratio of the arc length *s* to radius *r*.

$$\theta_m = \frac{s}{r} \tag{5.1}$$

The unit is radian. Angular velocity ω_m is defined as

$$\omega_m = \frac{d\theta_m}{dt} \tag{5.2}$$

and angular acceleration as

$$\alpha = \frac{d\omega_m}{dt} = \frac{d^2\theta_m}{dt^2} \tag{5.3}$$

The torque on a body due to a tangential force F at a distance *r* from axis of rotation is given by $T = r\,F$ (5.4)

The total torque is the summation of infinitesimal forces, given by

$$T = \int r\,dF \tag{5.5}$$

The unit of torque is N-m. When torque is applied to a body, the body experiences angular acceleration. Each particle experiences a tangential acceleration $a = r\alpha$, where r is the distance of the particle from axis of rotation. The tangential force required to accelerate a particle of mass *dm* is

$$dF = a\,dm = r\,\alpha\,dm \tag{5.6}$$

The torque required for the particle is

$$dT = r\,dF = r^2\,\alpha\,dm \tag{5.7}$$

and that required for the whole body is given by

$$T = \alpha \int r^2 dm = I\,\alpha \tag{5.8}$$

Here, $I = \int r^2 dm$ (5.9)

It is called the moment of inertia of the body. The unit is $Kg - m^2$. If the torque is assumed to be the result of a number of tangential forces F, which act at different points of the body

$$T = \sum r\,F$$

Now each force acts through a distance, $ds = r\,d\theta_m$ and the work done is $\sum F \cdot ds$ i.e.,

$$dW = \sum F\, r\, d\theta_m = d\theta_m\, T$$

$$W = \int T\, d\theta_m \tag{5.10}$$

and $$T = \frac{dW}{d\theta_m} \tag{5.11}$$

Thus the unit of torque may also be Joule per radian. The power is defined as rate of doing work. Using (5.11)

$$P = \frac{dW}{dt} = \frac{T\, d\theta_m}{dt} = T\omega_m \tag{5.12}$$

The angular momentum M is defined as

$$M = I\, \omega_m \tag{5.13}$$

And the kinetic energy is given by

$$KE = \frac{1}{2} I\omega_m^2 = \frac{1}{2} M\, \omega_m \tag{5.14}$$

From (5.14) we can see that the unit of M has to be J-sec/rad.

5.4 SWING EQUATION:

The laws of rotation developed in section.3 are applicable to the synchronous machine. From(.5.8)

$$I\alpha = T$$

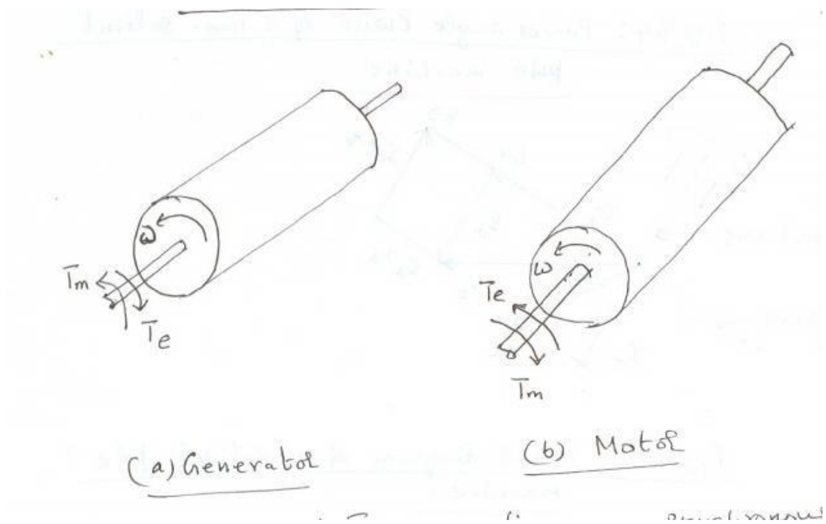
or $$\frac{I\, d^2\theta_m}{dt^2} = T \tag{5.15}$$

Here T is the net torque of all torques acting on the machine, which includes the shaft torque (due to prime mover of a generator or load on a motor), torque due to rotational losses (friction, windage and core loss) and electromagnetic torque.

Let T_m = shaft torque or mechanical torque corrected for rotational losses

T_e = Electromagnetic or electrical torque

For a generator T_m tends to accelerate the rotor in positive direction of rotation as shown in Fig 5.1. It also shows the corresponding torque for a motor with respect to the direction of rotation.



(a) Generator **(b) Motor**

Fig. 5.1 Torque acting on a synchronous machine

The accelerating torque for a generator is given by:

$$T_a = T_m - T_e \tag{5.16}$$

Under steady-state operation of the generator, T_m is equal to T_e and the accelerating torque is zero. There is no acceleration or deceleration of the rotor masses and the machines run at a constant synchronous speed. In the stability analysis in the following sections, T_m is assumed to be a constant since the prime movers (steam turbines or hydro turbines) do no act during the short time period in which rotor dynamics are of interest in the stability studies.

Now (5.15) has to be solved to determine θ_m as a function of time. Since θ_m is measured with respect to a stationary reference axis on the stator, it is the measure of the absolute rotor angle and increases continuously with time even at constant synchronous speed. Since machine acceleration /deceleration is always measured relative to synchronous speed, the rotor angle is measured with respect to a synchronously rotating reference axis. Let

$$\delta_m = \theta_m - \omega_{sm} t \tag{5.17}$$

where ω_{sm} is the synchronous speed in mechanical rad/s and δ_m is the angular displacement in mechanical radians. Taking the derivative of (5.17) we get

$$\frac{d\delta_m}{dt} = \frac{d\theta_m}{dt} - \omega_{sm}$$

$$\frac{d^2\delta_m}{dt^2} = \frac{d^2\theta_m}{dt^2} \tag{5.18}$$

Substituting (5.18) in (5.15) we get

$$I\frac{d^2\delta_m}{dt^2} = T_a = T_m - T_e \quad \text{N-m} \tag{5.19}$$

Multiplying by ω_m on both sides we get

$$\omega_m I\frac{d^2\delta_m}{dt^2} = \omega_m (T_m - T_e) \quad \text{N-m} \tag{5.20}$$

From (5.12) and (5.13), we can write

$$M\frac{d^2\delta_m}{dt^2} = P_m - P_a \qquad W \tag{5.21}$$

where M is the angular momentum, also called inertia constant, P_m is shaft power input less rotational losses, P_e is electrical power output corrected for losses and P_a is the acceleration power. M depends on the angular velocity ω_m, and hence is strictly not a constant, because ω_m deviates from the synchronous speed during and after a disturbance. However, under stable conditions ω_m does not vary considerably and M can be treated as a constant. (21) is called the *"Swing equation"*. The constant M depends on the rating of the machine and varies widely with the size and type of the machine. Another constant called H constant (also referred to as inertia constant) is defined as

$$H = \frac{\begin{array}{c}\text{stored kinetic energy in mega joules}\\ \text{at sychronous speed}\end{array}}{\text{Machine rating in MVA}} \; MJ/MVA \tag{5.22}$$

H falls within a narrow range and typical values are given in Table 5.1. If the rating of the machine is G MVA, from (5.22) the stored kinetic energy is GH Mega Joules. From (5.14)

$$\text{GH} = \frac{1}{2} M\omega_{sm} \quad \text{MJ} \tag{5.23}$$

or

$$\text{M} = \frac{2GH}{\omega_{sm}} \quad \text{MJ-s/mech rad} \tag{5.24}$$

The swing equation (5.21) is written as

$$\frac{2H}{\omega_{sm}} \frac{d^2\delta_m}{dt^2} = \frac{P_a}{G} = \frac{P_m - P_e}{G} \tag{5.25}$$

In (5.25) δ_m is expressed in mechanical radians and ω_{sm} in mechanical radians per second (the subscript *m* indicates mechanical units). If δ and ω have consistent units (both are mechanical or electrical units) (5.25) can be written as

$$\frac{2H}{\omega_s} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad \text{pu} \tag{5.26}$$

Here ω_s is the synchronous speed in electrical rad/s ($\omega_s = \left(\frac{p}{2}\right)\omega_{sm}$) and P_a is acceleration power in per unit on same base as H. For a system with an electrical frequency f Hz, (5.26) becomes

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad \text{pu} \tag{5.27}$$

when δ is in electrical radians and

$$\frac{H}{180 f} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad \text{pu} \tag{5.28}$$

when δ is in electrical degrees. Equations (5.27) and (5.28) also represent the swing equation. It can be seen that the swing equation is a second order differential equation which can be written as two first order differential equations:

$$\frac{2H}{\omega_s} \frac{d\omega}{dt} = P_m - P_e \quad \text{pu} \tag{5.29}$$

$$\frac{d\delta}{dt} = \omega - \omega_s \tag{5.30}$$

In which ω, ω_s and δ are in electrical units. In deriving the swing equation, damping has been neglected.

Table 5.1 H constants of synchronous machines

Type of machine		H (MJ/MVA)
Turbine generator condensing	1800 rpm	9 – 6
	3600 rpm	7 – 4
Non condensing	3600 rpm	4 – 3
Water wheel generator		
	Slow speed < 200 rpm	2 – 3
	High speed > 200 rpm	2 – 4
Synchronous condenser	Large	1.25 } 25% less for hydrogen cooled
	Small	1.0
Synchronous motor with load varying from 1.0 to 5.0		2.0

In defining the inertia constant H, the MVA base used is the rating of the machine. In a multi machine system, swing equation has to be solved for each machine, in which case, a common MVA base for the system has to chosen. The constant H of each machine must be consistent with the system base.

Let G_{mach} = Machine MVA rating (base)

G_{system} = System MVA base

In (5.25), H is computed on the machine rating $G = G_{mach}$

Multiplying (5.25) by $\dfrac{G_{mach}}{G_{system}}$ on both sides we get

$$\left(\frac{G_{mach}}{G_{system}}\right)\frac{2H}{\omega_{sm}}\frac{d^2\delta_m}{dt^2} = \frac{P_m - P_e}{G_{mach}}\left(\frac{G_{mach}}{G_{system}}\right) \tag{5.31}$$

$$\frac{2H_{system}}{\omega_{sm}}\frac{d^2\delta_m}{dt^2} = P_m - P_e \quad \text{pu (on system base)}$$

where $H_{system} = H\,\dfrac{G_{mach}}{G_{system}}$ (5.32)

In the stability analysis of a multi machine system, computation is considerably reduced if the number of swing equations to be solved is reduced. Machines within a plant normally swing together after a disturbance. Such machines are called coherent machines and can be replaced by a single equivalent machine, whose dynamics reflects the dynamics of the plant. The concept is best understood by considering a two machine system.

5.4.1 SWING EQUATION OF TWO COHERENT MACHINES

The swing equations for two machines on a common system base are:

$$\frac{2H_1}{\omega_s}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1} \text{ pu} \tag{5.33}$$

$$\frac{2H_2}{\omega_s}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2} \text{ pu} \tag{5.34}$$

Now $\delta_1 = \delta_2 = \delta$ (since they swing together). Adding (5.33) and (5.34) we get

$$\frac{2H_{eq}}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ pu} \tag{5.35}$$

Where $H_{eq} = H_1 + H_2$

$$P_m = P_{m1} + P_{m2}$$

$$P_e = P_{e1} + P_{e2}$$

The relation (5.35) represents the dynamics of the single equivalent machine.

5.4.2 SWING EQUATION OF TWO NON – COHERENT MACHINES

For any two non – coherent machines also (5.33) and (5.34) are valid. Subtracting (5.34) from (33) we get

$$\frac{2}{\omega_s}\frac{d^2\delta_1}{dt^2} - \frac{2}{\omega_s}\frac{d^2\delta_2}{dt^2} = \frac{P_{m1} - P_{e1}}{H_1} - \frac{P_{m2} - P_{e2}}{H_2} \tag{5.36}$$

Multiplying both sides by $\frac{H_1 H_2}{H_1 + H_2}$ we get

$$\frac{2}{\omega_s}\left(\frac{H_1 H_2}{H_1+H_2}\right)\frac{d^2(\delta_1-\delta_2)}{dt^2}=\frac{P_{m1}H_2-P_{m2}H_1}{H_1+H_2}-\frac{P_{e1}H_2-p_{e2}H_1}{H_1+H_2}$$

i.e

$$\frac{2}{\omega_s}H_{12}\frac{d^2\delta_{12}}{dt^2}=P_{m12}-P_{e12} \tag{5.37}$$

where $\delta_{12}=\delta_1-\delta_2$, the relative angle of the two machines

$$H_{12}=\frac{H_1 H_2}{H_1+H_2}$$

$$P_{m12}=\frac{p_{m1}H_2-p_{m2}H_1}{H_1+H_2}$$

$$P_{e12}=\frac{p_{e1}H_2-p_{e2}H_1}{H_1+H_2}$$

From (5.37) it is obvious that the swing of a machine is associated with dynamics of other machines in the system. To be stable, the angular differences between all the machines must decrease after the disturbance. In many cases, when the system loses stability, the machines split into two coherent groups, swinging against each other. Each coherent group of machines can be replaced by a single equivalent machine and the relative swing of the two equivalent machines solved using an equation similar to (5.37), from which stability can be assessed.

The acceleration power is given by $P_a = P_m - P_e$. Hence, under the condition that P_m is a constant, an accelerating machine should have a power characteristic, which would increase P_e as δ increases.

This would reduce P_a and hence the acceleration and help in maintaining stability. If on the other hand, P_e decreases when δ increases, P_a would further increase which is detrimental to stability. Therefore, $\frac{\partial P}{\partial \delta}$ must be positive for a stable system. Thus the power-angle relationship plays a crucial role in stability.

5.5 POWER–ANGLE EQUATION:

In solving the swing equation, certain assumptions are normally made

(i) Mechanical power input P_m is a constant during the period of interest, immediately after the disturbance

(ii) Rotor speed changes are insignificant.

(iii) Effect of voltage regulating loop during the transient is neglected i.e the excitation is assumed to be a constant.

As discussed in section 4, the power–angle relationship plays a vital role in the solution of the swing equation.

5.5.1 POWER–ANGLE EQUATION FOR A NON–SALIENT POLE MACHINE:

The simplest model for the synchronous generator is that of a constant voltage behind an impedance. This model is called the classical model and can be used for cylindrical rotor (non–salient pole) machines. Practically all high–speed turbo alternators are of cylindrical rotor construction, where the physical air gap around the periphery of the rotor is uniform. This type of generator has approximately equal magnetic reluctance, regardless of the angular position of the rotor, with respect to the armature mmf. The phasor diagram of the voltages and currents at constant speed and excitation is shown in Fig. 5.2.

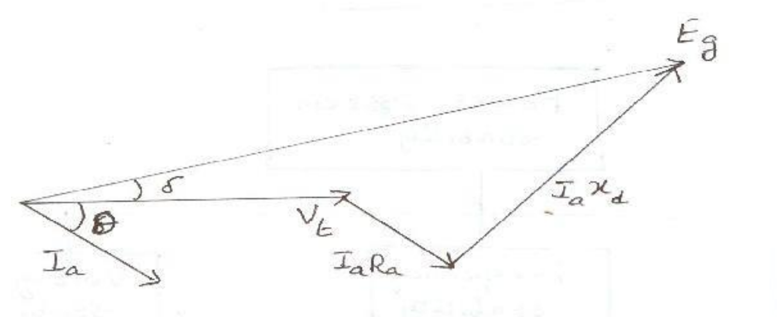


Fig 5.2 Phasor diagram of a non–salient pole synchronous generator

E_g = Generator internal emf.

V_t = Terminal voltage

θ = Power factor angle

I_a = Armature current

R_a = Armature resistance

x_d = Direct axis reactance

The power output of the generator is given by the real part of $E_g\, I_a^*$.

$$I_a = \frac{E_g \angle\delta \; - V_t \;\angle 0°}{R_a \;+\; jx_d} \tag{5.38}$$

Neglecting R_a,

$$I_a = \frac{E_g \;\angle\delta \;\; - V_t \;\angle 0°}{j\, x_d}$$

$$P = \mathbf{R}\left\{ \left(E_g \;\angle\delta\right) \left(\frac{E_g \;\;\angle 90° - \delta}{x_d} - \frac{V_t \;\angle 90°}{x_d} \right)^{*} \right\}$$

$$= \frac{E_g^{\;2} \;\cos 90°}{x_d} - \frac{E_g\, V_t \;\cos\left(90° + \delta\right)}{x_d}$$

$$= \frac{E_g\, V_t \;\sin\delta}{x_d} \tag{5.39}$$

(Note- **R** stands for real part of). The graphical representation of (9.39) is called the power angle curve and it is as shown in Fig 5.3.

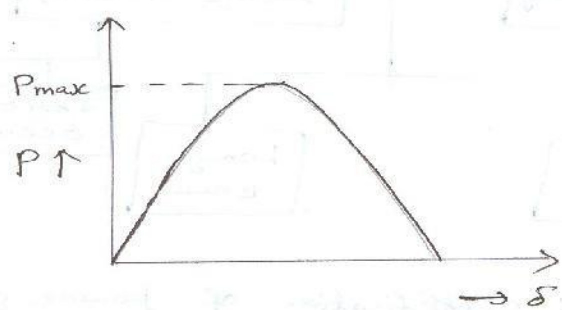


Fig 5.3 Power angle curve of a non – salient pole machine

The maximum power that can be transferred for a particular excitation is given by $\dfrac{E_g\, V_t}{x_d}$ at $\delta = 90^o$.

5.5.2 POWER ANGLE EQUATION FOR A SALIENT POLE MACHINE:

Here because of the salient poles, the reluctance of the magnetic circuit in which flows the flux produced by an armature mmf in line with the quadrature axis is higher than that of the magnetic circuit in which flows the flux produced by the armature mmf in line with the direct axis. These two components of armature mmf are proportional to the corresponding components of armature current. The component of armature current producing an mmf acting in line with direct axis is called the direct component, I_d. The component of armature current producing an mmf acting in line with the quadrature axis is called the quadrature axis component, I_q. The phasor diagram is shown in Fig 5, with same terminology as Fig 5.4 and R_a neglected.

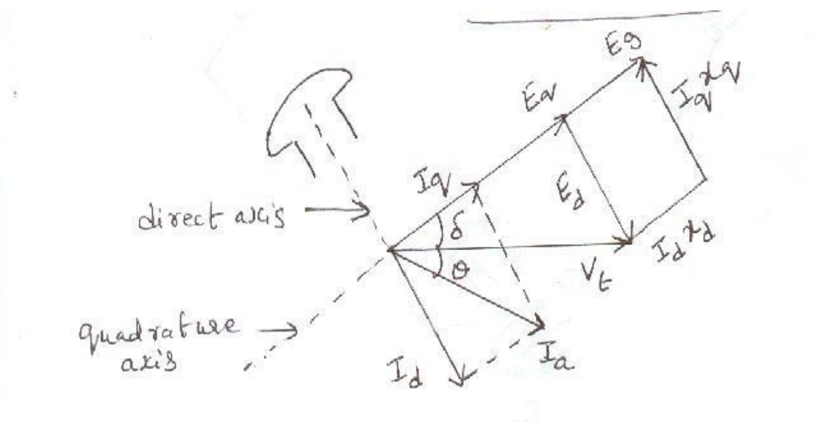


Fig 5.4 Phasor diagram of a salient pole machine

Power output $\quad P = V_t I_a \cos\theta$

$$= E_d I_d + E_q I_q \tag{5.40}$$

From Fig 5.4, $\quad E_d = V_t \sin\delta \;;\quad E_q = V_t \cos\delta$

$$I_d = \frac{E_g - E_q}{x_d} = I_a \sin(\delta + \theta)$$

$$I_q = \frac{E_d}{x_q} = I_a \cos(\delta + \theta) \tag{5.41}$$

Substituting (5.41) in (5.40), we obtain

$$P = \frac{E_g V_t \sin\delta}{x_d} + \frac{V_t^2 (x_d - x_q)\sin 2\delta}{2 x_d x_q} \tag{5.42}$$

the relation (5.42) gives the steady state power angle relationship for a salient pole machine. The second term does not depend on the excitation and is called the reluctance power component. This component makes the maximum power greater than in the classical model. However, the angle at which the maximum power occurs is less than 90°.

5.6 POWER ANGLE RELATIONSHIP IN A SMIB SYSTEM:

Without loss of generality, many important conclusions on stability can be arrived at by considering the simple case of a Single Machine Infinite Bus (SMIB), where a generator supplies power to an infinite bus. The concept of an infinite bus arises from the fact that if we connect a generator to a much larger power system, it is reasonable to assume that the voltage and frequency of the larger system will not be affected by control of the generator parameters. Hence, the external system can be approximated by an infinite bus,

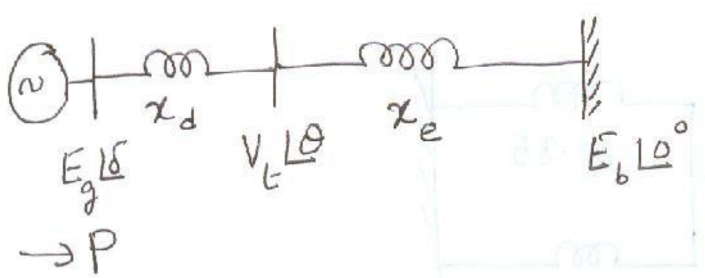


Fig. 5.5 SMIB System

In Fig. 5.5, the infinite bus voltage is taken as reference and δ is the angle between E_g and E_b. The generator is assumed to be connected to the infinite bus through a lossless line of reactance x_e. The power transferred (using classical model) is given by

$$P = \frac{E_g E_b}{x_d + x_e} \sin\delta \qquad (5.43)$$

and using salient pole model,

$$P = \frac{E_g E_b}{x_d + x_e} \sin\delta + \frac{E_b^2 (x_d - x_q)}{2(x_d + x_e)(x_q + x_e)} \sin 2\delta \qquad (5.44)$$

An important measure of performance is the *steady state stability limit*, which is defined as *the maximum power that can be transmitted in steady state without loss of synchronism*, to the receiving end. If transient analysis is required, respective transient quantities namely E'_g, x'_d and x'_q are used in (5.43) and (5.44) to calculate the power output.

5.7 TRANSIENT STABILITY

Transient stability is the ability of the system to remain stable under large disturbances like short circuits, line outages, generation or load loss etc. The evaluation of the transient stability is required offline for planning, design etc. and online for load management, emergency control and security assessment. Transient stability analysis deals with actual solution of the nonlinear differential equations describing the dynamics of the machines and their controls and interfacing it with the algebraic equations describing the interconnections through the transmission network.

Since the disturbance is large, linearized analysis of the swing equation (which describes the rotor dynamics) is not possible. Further, the fault may cause structural changes in the network, because of which the power angle curve prior to fault, during the fault and post fault may be different (See example 9.8). Due to these reasons, a general stability criteria for transient stability cannot be established, as was done in the case of steady state stability (namely $P_S > 0$). Stability can be established, for a given fault, by actual solution of the swing equation. The time taken for the fault to be cleared (by the circuit breakers) is called the *clearing time*. If the fault is cleared fast enough, the probability of the system remaining stable after the clearance is more. If the fault persists for a longer time, likelihood of instability is increased.

Critical clearing time is the maximum time available for clearing the fault, before the system loses stability. Modern circuit breakers are equipped with auto reclosure facility, wherein the breaker automatically recloses after two sequential openings. If the fault still persists, the breakers open permanently. Since most faults are transient, the first reclosure is in general successful. Hence, transient stability has been greatly enhanced by auto closure breakers.

Some common assumptions made during transient stability studies are as follows:

1. Transmission line and synchronous machine resistances are neglected. Since resistance introduces a damping term in the swing equation, this gives pessimistic results.
2. Effect of damper windings is neglected which again gives pessimistic results.
3. Variations in rotor speed are neglected.
4. Mechanical input to the generator is assumed constant. The governor control loop is neglected. This also leads to pessimistic results.
5. The generator is modeled as a constant voltage source behind a transient reactance, neglecting the voltage regulator action.
6. Loads are modeled as constant admittances and absorbed into the bus admittance matrix.

The above assumptions, vastly simplify the equations. A digital computer program for transient stability analysis can easily include more detailed generator models and effect of controls, the discussion of which is beyond the scope of present treatment. Studies on the transient stability of an SMIB system, can shed light on some important aspects of stability of larger systems. One of the important methods for studying the transient stability of an SMIB system is the application of equal-area criterion.

5. 8 EQUAL- AREA CRITERION

Transient stability assessment of an SMIB system is possible without resorting to actual solution of the swing equation, by a method known as equal–area criterion. In a SMIB system, if the system is unstable after a fault is cleared, $\delta(t)$ increases indefinitely with time, till the machine loses synchronism. In contrast, in a stable system, $\delta(t)$ reaches a maximum and then starts reducing as shown in Fig.5.6.

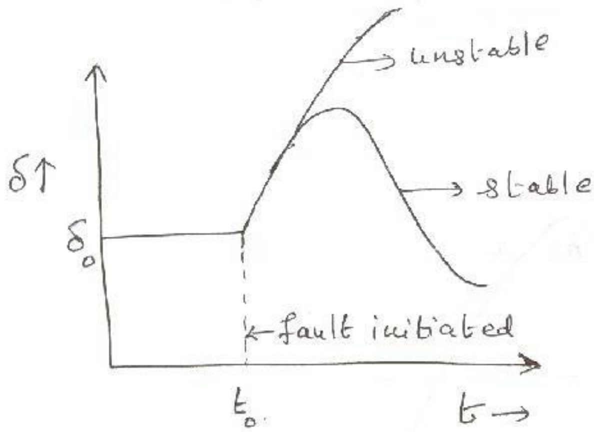


Fig.5.6 Swing Curve (δ V_S t) for stable and unstable system

Mathematically stated,

$$\frac{d\delta(t)}{dt} = 0$$

some time after the fault is cleared in a stable system and $\frac{d\delta}{dt} > 0$, for a long time after the fault is cleared in an unstable system.

Consider the swing equation (21)

$$M\frac{d^2\delta}{dt^2} = P_m - P_e = P_a$$

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M}$$

Multiplying both sides by $2\frac{d\delta}{dt}$, we get

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = 2\frac{d\delta}{dt}\cdot\frac{P_a}{M}$$

This may be written as

$$\frac{d}{dt}\left[\left(\frac{d\delta}{dt}\right)^2\right] = 2\frac{d\delta}{dt}\frac{P_a}{M}$$

Integrating both sides we get

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_o}^{\delta} P_a\, d\delta$$

$$\text{or } \frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int_{\delta_o}^{\delta} P_a\, d\delta} \qquad (5.45)$$

For stability $\frac{d\delta}{dt} = 0$, some time after fault is cleared. This means

$$\int_{\delta_o}^{\delta} P_a\, d\delta = 0 \qquad (5.46)$$

The integral gives the area under the $P_a - \delta$ curve. The condition for stability can be, thus stated as follows: A SMIB system is stable if the area under the $P_a - \delta$ curve, becomes zero at some value of δ. This means that the accelerating (positive) area under $P_a - \delta$ curve, must equal the decelerating (negative) area under $P_a - \delta$ curve. Application of equal area criterion for several disturbances is discussed next.

5.9 SUDDEN CHANGE IN MECHANICAL INPUT

Consider the SMIB system shown in Fig. 5.7.

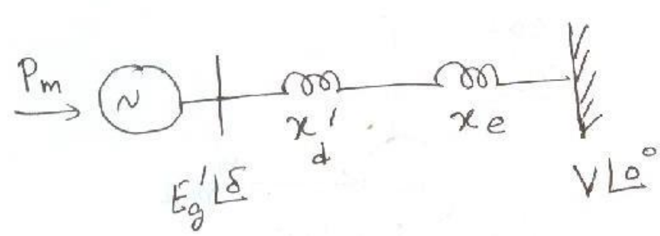


Fig.5.7 SMIB System

The electrical power transferred is given by

$$P_e = P_{max} \sin\delta$$

$$P_{max} = \frac{E_g' V}{x_d' + x_e}$$

Under steady state $P_m = P_e$. Let the machine be initially operating at a steady state angle δ_o, at synchronous speed ω_s, with a mechanical input P_{mo}, as shown in Fig.5.8 (point *a*).

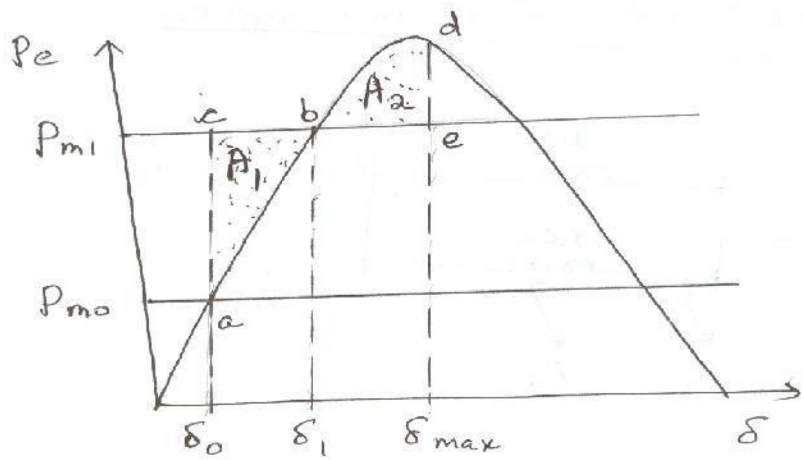


Fig.5.8 Equal area criterion–sudden change in mechanical input

If there is a sudden step increase in input power to P_{m1} the accelerating power is positive (since $P_{m1} > P_{mo}$) and power angle δ increases. With increase in δ, the electrical power P_e increases, the accelerating power decreases, till at $\delta = \delta_1$, the electrical power matches the new input P_{m1}. The area A_1, during acceleration is given by

$$A_1 = \int_{\delta_0}^{\delta_1} (P_{m1} - P_e)\, d\delta$$

$$= P_{m1}(\delta_1 - \delta_0) - P_{max}(\cos\delta_0 - \cos\delta_1) \tag{5.47}$$

At *b*, even though the accelerating power is zero, the rotor is running above synchronous speed. Hence, δ and P_e increase beyond *b*, wherein $P_e < P_{m1}$ and the rotor is subjected to deceleration. The rotor decelerates and speed starts dropping, till at point *d*, the machine reaches synchronous speed and $\delta = \delta_{max}$. The area A_2, during deceleration is given by

$$A_2 = \int_{\delta_1}^{\delta_{max}} (P_e - P_{m1})\, d\delta = P_{max}(\cos\delta_1 - \cos\delta_{max}) - P_{m1}(\delta_{max} - \delta_1) \tag{5.48}$$

By equal area criterion $A_1 = A_2$. The rotor would then oscillate between δ_0 and δ_{max} at its natural frequency. However, damping forces will reduce subsequent swings and the machine finally settles down to the new steady state value δ_1 (at point *b*). Stability can be maintained only if area A_2 at least equal to A_1, can be located above P_{m1}. The limiting case is shown in Fig.5.9, where A_2 is just equal to A_1.

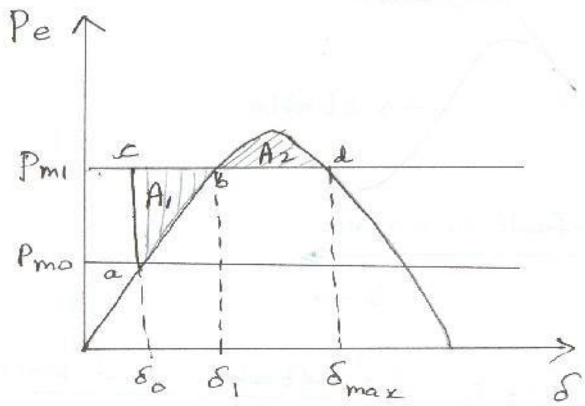


Fig.5.9 Maximum increase in mechanical power

Here δ_{max} is at the intersection of P_e and P_{m1}. If the machine does not reach synchronous speed at *d*, then beyond *d*, P_e decreases with increase in δ, causing δ to increase indefinitely. Applying equal area criterion to Fig.5.9 we get

$A_1 = A_2$.

From (5.47) and (5.48) we get

$$P_{m1}(\delta_{max} - \delta_0) = P_{max}(\cos\delta_0 - \cos\delta_{max})$$

Substituting $P_{m1} = P_{max}\sin\delta_{max}$, we get

$$(\delta_{max} - \delta_0)\sin\delta_{max} + \cos\delta_{max} = \cos\delta_0 \qquad (5.49)$$

Equation (5.49) is a non-linear equation in δ_{max} and can be solved by trial and error or by using any numerical method for solution of non-linear algebraic equation (like Newton-Raphson, bisection etc). From solution of δ_{max}, P_{m1} can be calculated. $P_{m1} - P_{mo}$ will give the maximum possible increase in mechanical input before the machine looses stability.

5.10 NUMERICAL EXAMPLES

Example 1: A 50Hz, 4 pole turbo alternator rated 150 MVA, 11 kV has an inertia constant of 9 MJ / MVA. Find the (a) stored energy at synchronous speed (b) the rotor acceleration if the input mechanical power is raised to 100 MW when the electrical load is 75 MW, (c) the speed at the end of 10 cycles if acceleration is assumed constant at the initial value.

Solution:

(a) Stored energy = GH = 150 × 9 = 1350 MJ

(b) $P_a = P_m - P_e = 100 - 75 = 25$ MW

$$M = \frac{GH}{180f} = \frac{1350}{180 \times 50} = 0.15 \text{ MJ} - \text{s}/^{o}\text{e}$$

$$0.15\frac{d^2\delta}{dt^2} = 25$$

$$\text{Acceleration } \alpha = \frac{d^2\delta}{dt^2} = \frac{25}{0.15} = 166.6 \ ^{o}\text{e/s}^2$$

$$= 166.6 \times \frac{2}{P} \ ^{o}\text{m/s}^2$$

$$= 166.6 \times \frac{2}{P} \times \frac{1}{360} \ rps/\text{s}$$

$$= 166.6 \times \frac{2}{P} \times \frac{1}{360} \times 60 \text{ rpm/s}$$

$$= 13.88 \text{ rpm/s}$$

* Note ^{o}e = electrical degree; ^{o}m = mechanical degree; P=number of poles.

(c) $10 \text{ cycles} = \frac{10}{50} = 0.2$ s

$$N_S = \text{Synchronous speed} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Rotor speed at end of 10 cycles = $N_S + \alpha \times 0.2 = 1500 + 13.88 \times 0.2 = 1502.776$ rpm.

Example 2: Two 50 Hz generating units operate in parallel within the same plant, with the following ratings: Unit 1: 500 MVA, 0.8 pf, 13.2 kV, 3600 rpm: H = 4 MJ/MVA; Unit 2: 1000 MVA, 0.9 pf, 13.8 kV, 1800 rpm: H = 5 MJ/MVA. Calculate the equivalent H constant on a base of 100 MVA.

Solution:

$$H_{1system} = H_{1mach} \times \frac{G_{1mach}}{G_{system}} = 4 \times \frac{500}{100} = 20 \text{ MJ/MVA}$$

$$H_{2system} = H_{2mach} \times \frac{G_{2mach}}{G_{system}} = 5 \times \frac{1000}{100} = 50 \text{ MJ/MVA}$$

$$H_{eq} = H_1 + H_2 = 20 + 50 = 70 \text{ MJ/MVA}$$

This is the equivalent inertia constant on a base of 100 MVA and can be used when the two machines swing coherently.

Example 3: Obtain the power angle relationship and the generator internal emf for (i) classical model (ii) salient pole model with following data: x_d = 1.0 pu : x_q = 0.6 pu : V_t = 1.0 pu : I_a = 1.0 pu at upf

Solution:

(i) **Classical model**: The phasor diagram is shown in Fig P3.

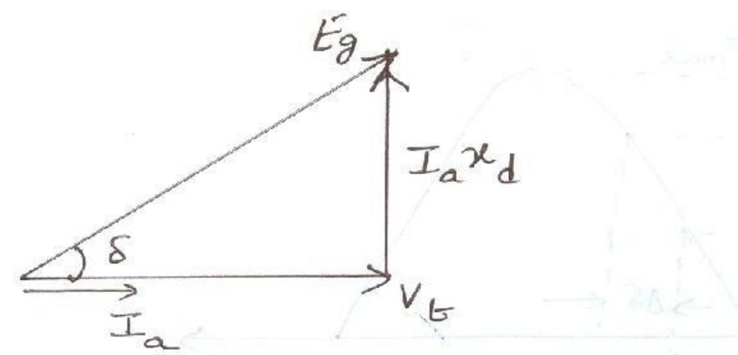


Fig.P3 Example 3, case(i)

$$\left|E_g\right| = \sqrt{V_t^2 + (I_a x_d)^2} = \sqrt{(1.0)^2 + (1.0 \times 1.0)^2} = 1.414$$

$$\delta = \tan^{-1}\frac{I_a x_d}{V_t} = \tan^{-1}\frac{1.0}{1.0} = 45°$$

$$\therefore \; E_g = 1.414 \; \angle 45° .$$

If the excitation is held constant so that $\left|E_g\right|$ = 1.414, then power output

$$P = \frac{1.414 \times 1.0 \sin\delta}{1.0} = 1.414 \sin\delta$$

(ii) **Salient pole:** From Fig (5), we get using (41a) to (41d)

$$E_g = E_q + I_d x_d = V_t \cos\delta + I_d x_d$$

$$= V_t \cos\delta + I_a \sin\delta \, x_d$$

(* $\theta = 0^0$, since we are considering upf)

Substituting given values we get

$$E_g = \cos\delta + \sin\delta.$$

Again from Fig (9.5) we have

$$E_d = V_t \sin\delta = I_q x_q$$

$$\therefore\ V_t \sin\delta - I_q x_q = 0$$

$$V_t \sin\delta - I_a \cos\delta\, x_q = 0$$

Substituting the given values we get

$$0 = \sin\delta - 0.6\cos\delta$$

We thus have two simultaneous equations.

$$E_g = \cos\delta + \sin\delta$$

$$0 = \sin\delta - 0.6\cos\delta$$

Solving we get $\delta = 30.96^o$

$$E_g = 1.372 \text{ pu}$$

If the excitation is held constant, then from (42)

$$P = 1.372 \sin\delta + 0.333 \sin 2\delta$$

Example 4: Determine the steady state stability limit of the system shown in Fig 8, if V_t = 1.0 pu and the reactances are in pu.

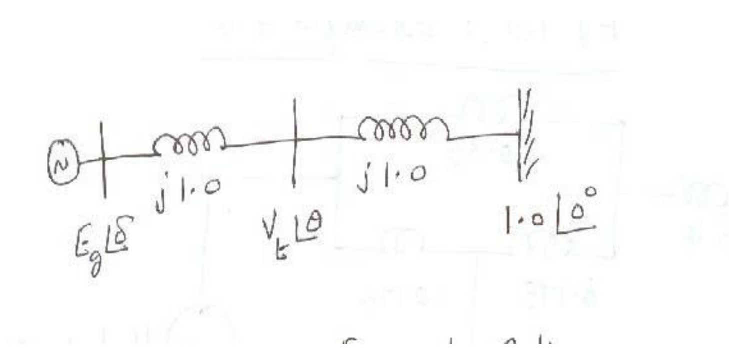


Fig. P4 Example 4

Solution:

$$\text{Current I} = \frac{V_t\angle\theta - 1.0\angle0^\circ}{j1.0} = \frac{1.0\angle\theta - 1.0\angle0^\circ}{j1.0}$$

$$E_g\angle\delta = V_t\angle\theta + j1.0(I)$$

$$= 1\angle\theta + \frac{j1.0(1.0\angle\theta - 1.0\angle 0^\circ)}{j1.0}$$

$$= \cos\theta + j\sin\theta + \cos\theta + j\sin\theta - 1.0$$

$$= 2\cos\theta - 1 + j\,2\sin\theta$$

When maximum power is transferred $\delta = 90^o$; which means real part of E = 0

$$\therefore\ 2\cos\theta - 1 = 0$$

$$\theta = \cos^{-1} 0.5 = 60^o$$

$$|E_g| = 2\times\sin 60^o = 1.732$$

$$E_g = 1.732\angle 90^\circ \text{ (for maximum power)}$$

$$\text{Steady state stability limit} = \frac{1.732\times 1.0}{1.0 + 1.0} = 0.866 \text{ pu}$$

Example 5: A 50 Hz synchronous generator having an internal voltage 1.2 pu, H = 5.2 MJ/MVA and a reactance of 0.4 pu is connected to an infinite bus through a double circuit line, each line of reactance 0.35 pu. The generator is delivering 0.8pu power and the infinite bus voltage is 1.0 pu. Determine: maximum power transfer, Steady state operating angle, and Natural frequency of oscillation if damping is neglected.

Solution: The one line diagram is shown in Fig P5.

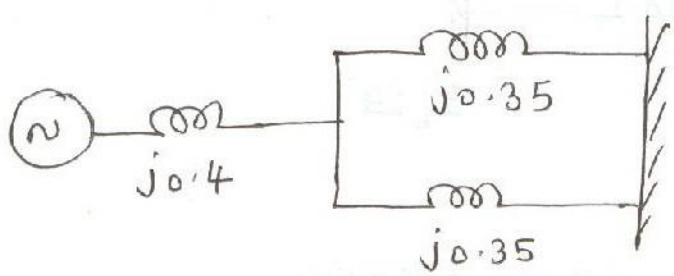


Fig . P5 Example 6

$$\text{(a) } X = 0.4 + \frac{0.35}{2} = 0.575 \text{ pu}$$

$$P_{max} = \frac{E_g E_b}{X} = \frac{1.2 \times 1.0}{0.575} = 2.087 \text{ pu}$$

(b) $P_e = P_{max \sin} \delta_o$

$$\therefore \delta_o = \sin^{-1} \frac{P_e}{P_{max}} = \sin^{-1}\left(\frac{0.8}{2.087}\right) = 22.54° .$$

(c) $P_s = P_{max} \cos \delta_o = 2.087 \cos (22.54^o)$

$= 1.927$ MW (pu)/ elec rad.

$$M \text{ (pu)} = \frac{H}{\Pi f} = \frac{5.2}{\Pi \times 50} = 0.0331 \ s^2 / elec \quad rad$$

$$\text{Without damping s} = \pm j \sqrt{\frac{P_S}{M}} = \pm j \sqrt{\frac{1.927}{0.0331}}$$

$= \pm$ j 7.63 rad/sec = 1.21 Hz

Natural frequency of oscillation $\omega_n = 1.21$ Hz.

Example 6: In example .6, if the damping is 0.14 and there is a minor disturbance of $\Delta \delta = 0.15$ rad from the initial operating point, determine: (a) ω_n (b) ξ (c) ω_d (d) setting time and (e) expression for δ.

Solution:

(a) $$\omega_n = \sqrt{\frac{P_S}{M}} = \sqrt{\frac{1.927}{0.0331}} = 7.63 \text{ rad/sec} = 1.21 \text{ Hz}$$

(b) $$\xi = \frac{D}{2}\sqrt{\frac{1}{M P_S}} = \frac{0.14}{2}\sqrt{\frac{1}{0.0331 \times 1.927}} = 0.277$$

(c) $$\omega_d = \omega_n \sqrt{1-\xi^2} = 7.63\sqrt{1-(0.277)^2} = 7.33 \text{ rad/sec} = 1.16 \text{ Hz}$$

(d) $$\text{Setting time} = 4\tau = 4 \frac{1}{\xi\omega_n} = 4 \times \frac{1}{0.277 \times 7.63} = 1.892 \text{ s}$$

(e) $\Delta \delta_o = 0.15$ rad $= 8.59^o$

$$\theta = \cos^{-1} \xi = \cos^{-1} 0.277 = 73.9^o$$

$$\delta = \delta_o + \frac{\Delta \delta_o}{\sqrt{1-\xi^2}} e^{-\xi \omega_n t} \sin(\omega_d t + \theta)$$

$$= 22.54^o + \frac{8.59}{\sqrt{1 - 0.277^2}} e^{-0.277 \times 7.63 t} \sin\left(7.33t + 73.9^o\right)$$

$= 22.54^o + 8.94\, e^{-2.11t} \sin(7.33t + 73.9^o)$

The variation of delta with respect to time is shown below. It can be observed that the angle reaches the steady state value of 22.54^o after the initial transient. It should be noted that the magnitudes of the swings decrease in a stable system with damping.

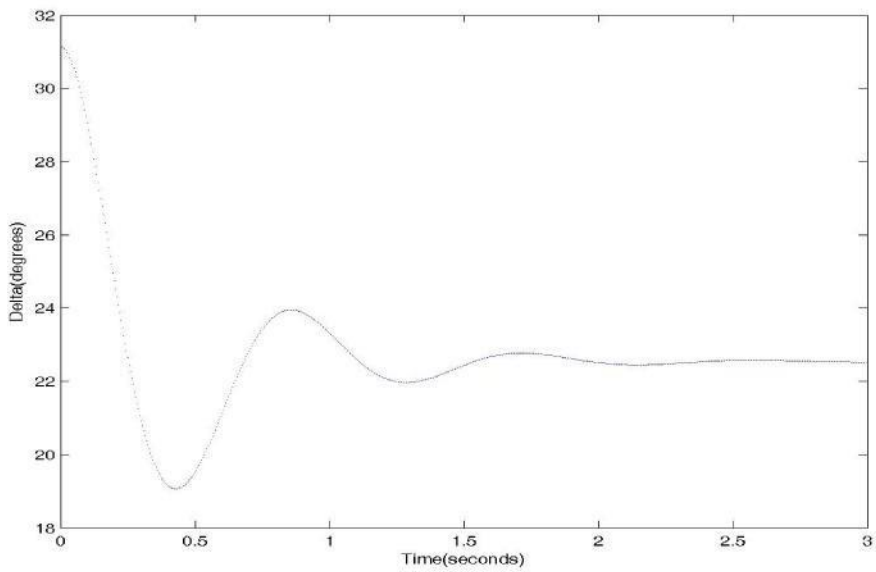


Fig.P6 Swing Curve for example 7

<u>Example 7</u>: In example 6, find the power angle relationship

(i) For the given network

(ii) If a short circuit occurs in the middle of a line

(iii) If fault is cleared by line outage

Assume the generator to be supplying 1.0 pu power initially.

<u>Solution</u>:

(i) From example 6, $P_{max} = 2.087$, $P_e = 2.087 \sin \delta$.

(ii) If a short circuit occurs in the middle of the line, the network equivalent can be draw as shown in Fig. 12a.

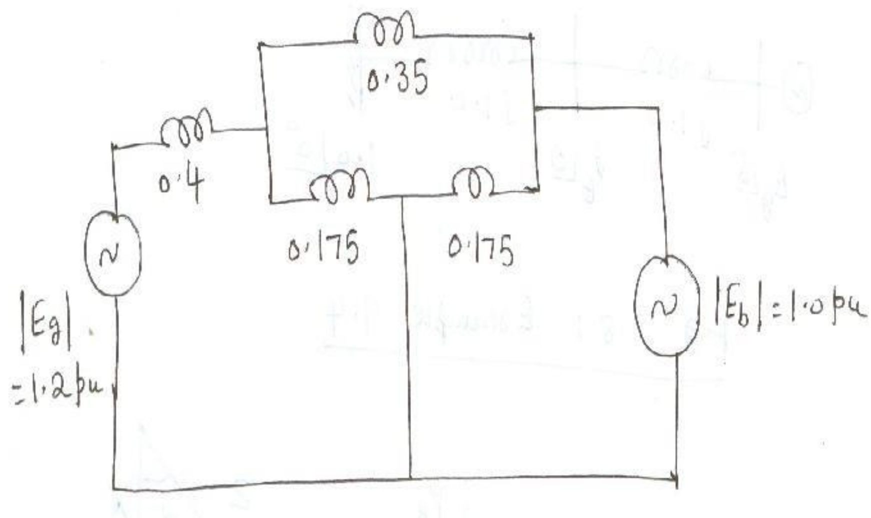
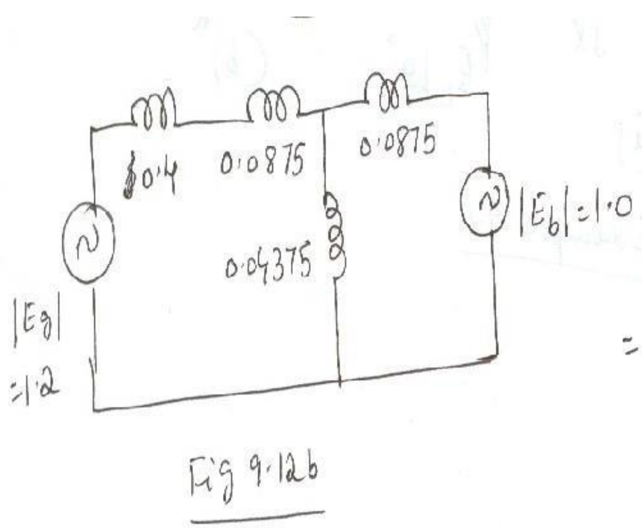


Fig.P7a Short circuit in middle of line

The network is reduced by converting the delta to star and again the resulting star to delta as shown in Fig P7a, P7b and P7c.



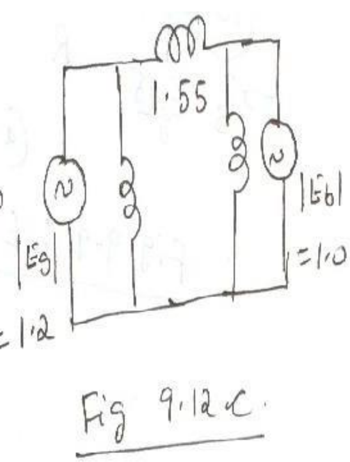


Fig.P7b **Fig.P7c**

The transfer reactance is 1.55 pu. Hence,

$$P_{max} = \frac{1.2 \times 1.0}{1.55} = 0.744 \;;\quad P_e = 0.744 \sin\delta$$

(iii) When there is a line outage

$$X = 0.4 + 0.35 = 0.75$$

$$P_{max} = \frac{1.2 \times 1.0}{0.75} = 1.6$$

$$P_e = 1.6 \sin\delta$$

Example 8: A generator supplies active power of 1.0 pu to an infinite bus, through a lossless line of reactance $x_e = 0.6$ pu. The reactance of the generator and the connecting transformer is 0.3 pu. The transient internal voltage of the generator is 1.12 pu and infinite bus voltage is 1.0 pu. Find the maximum increase in mechanical power that will not cause instability.

Solution:

$$P_{max} = \frac{1.12 \times 1.0}{0.9} = 1.244 \text{ pu}$$

$$P_{mo} = P_{eo} = 1.0 = P_{max} \sin\delta_o = 1.244 \sin\delta_o$$

$$\therefore\ \delta_o = \sin^{-1}\frac{1.0}{1.244} = 53.47^o = 0.933 \text{ rad.}$$

The above can be solved by N–R method since it is of the form $f(\delta_{max}) = K$. Applying N–R method, at any iteration 'r', we get

$$\Delta\delta_{max}^{(r)} = \frac{K - f\left(\delta_{max}^{(r)}\right)}{\dfrac{df}{d\delta_{max}^{(r)}}}$$

$$\frac{df}{d\delta_{max}^{(r)}} = \left(\delta_{max}^{(r)} - \delta_o\right)\cos\delta_{max}^{(r)}$$

(This is the derivative evaluated at a value of $\delta = \delta_{max}^{(r)}$) $\quad \delta_{max}^{(r+1)} = \delta_{max}^{(r)} + \Delta\delta_{max}^{(r)}$

Starting from an initial guess of δ_{max} between $\frac{\pi}{2}$ to π, the above equations are solved iteratively till $\Delta\,\delta_{max}^{(r)} \le \in$. Here $K = \cos\delta_o = 0.595$. The computations are shown in table P8, starting from an initial guess $\delta_{max}^{(1)} = 1.745$ rad.

Table P8

Interaction r	$\delta_{\max}^{(r)}$	$\frac{df}{d\delta_{\max}^{(r)}}$	$f\left(\delta_{\max}^{(r)}\right)$	$\Delta\delta_{\max}^{(r)}$	$\delta_{\max}^{(r+1)}$
1	1.745	– 0.1407	0.626	0.22	1.965
2	1.965	– 0.396	0.568	– 0.068	1.897
3	1.897	– 0.309	0.592	– 0.0097	1.887
4	1.887	– 0.2963	0.596	– 0.0033	1.883

Since $\Delta\delta_{\max}^{(r)}$ is sufficient by small, we can take

δ_{max} = 1.883 rad = 107.88°

$\delta_1 = 180 - \delta_{\max} = 72.1^{o}$

$P_{m1} = P_{max} \sin \delta_{max}$ = 1.183

Maximum step increase permissible is $P_{m1} - P_{mo}$, = 1.183 – 1.0 = 0.183 pu

Example 9: Transform a two machine system to an equivalent SMIB system and show how equal area criterion is applicable to it.

Solution: Consider the two machine system show in Fig.P9.

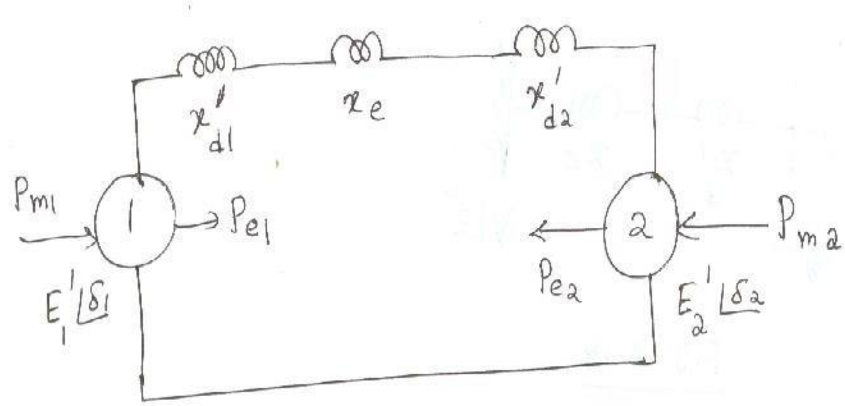


Fig.P9 Two machine system under steady state (neglecting losses)

$P_{m1} = -P_{m2} = P_m\,;\;\; P_{e1} = -P_{e2} = P_e$

The swing equations are

$$\frac{d^2\delta_1}{dt^2} = \frac{P_{m1} - P_{e1}}{M_1} = \frac{P_m - P_e}{M_1}$$

$$\frac{d^2\delta_2}{dt^2} = \frac{P_{m2} - P_{e2}}{M_2} = \frac{P_e - P_m}{M_2}$$

Simplifying, we get

$$\frac{d^2(\delta_1 - \delta_2)}{dt^2} = \frac{M_1 + M_2}{M_1 M_2}(P_m - P_e)$$

or $$M_{eq}\frac{d^2\delta}{dt^2} = P_m - P_e$$

where $M_{eq} = \dfrac{M_1 M_2}{M_1 + M_2}$

$\delta = \delta_1 - \delta_2$

$$P_e = \frac{E_1' E_2'}{x_{d1}' + x_e + x_{d2}'}\sin\delta$$

This relation is identical to that of an SMIB system in form and can be used to determine the relative swing $(\delta_1 - \delta_2)$ between the two machines to assess the stability.
