Stairs

"A set of steps leading from surface of a building to another surface, typically inside the building is termed as Stairs."

Parts of Stairs :

1. Step :

It is a portion of stair which permits ascent & descent.

2. Tread :

It is the upper horizontal portion of step upon which the feet is placed.

3. Riser :

The vertical portion between each tread on the stair.

4. Handrail :

A handrail is a rail that is designed to be grasped by the hand so as to provide stability or support

5. Baluster :

It is vertical member of wood or metal supporting the handrail.

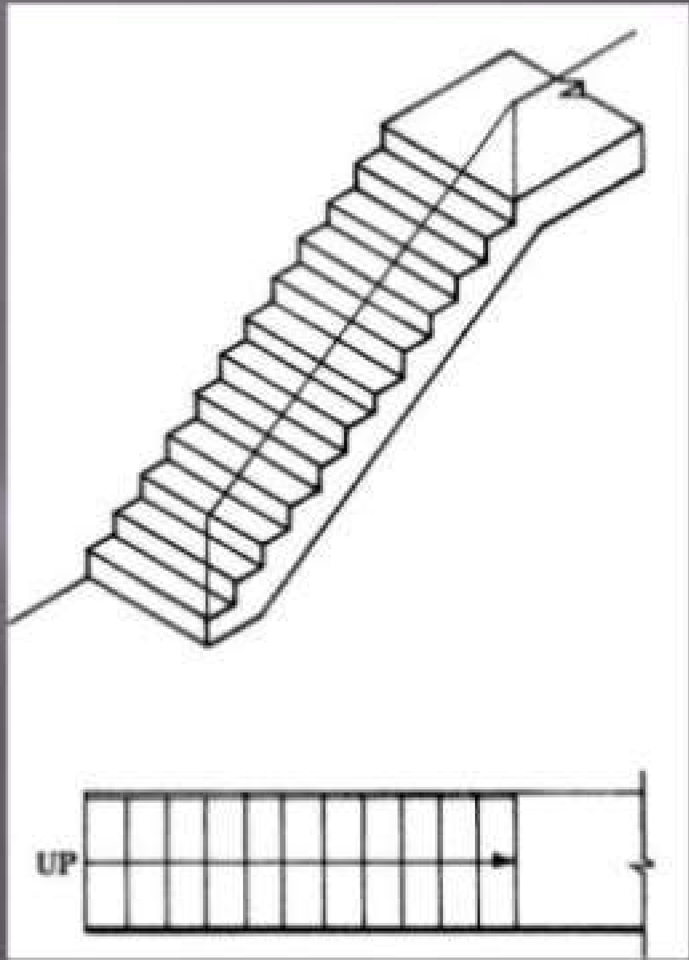
Types of Stairs :

1. *Straight Stair*
2. *Dog legged Stair*
3. *Quarter turn Stair*
4. *Open newel Stair*
5. *Three quarter turn Stair*
6. *Bifurcated Stair*
7. *Geometrical Stair*
8. *Circular Stair*

1. Straight Stair :

- *These are the stairs along which there is no change in direction on any flight.*

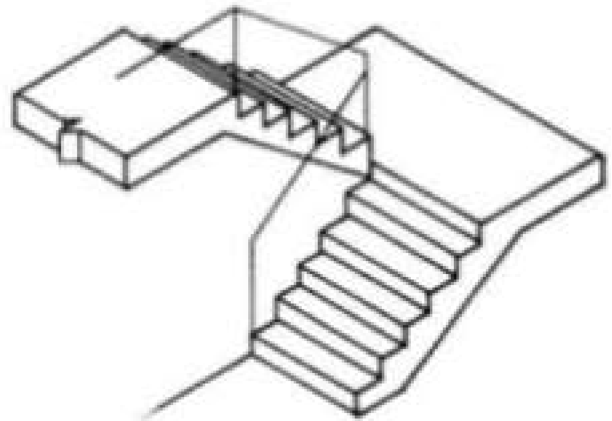
- *It is used where stair case hall is long and narrow*

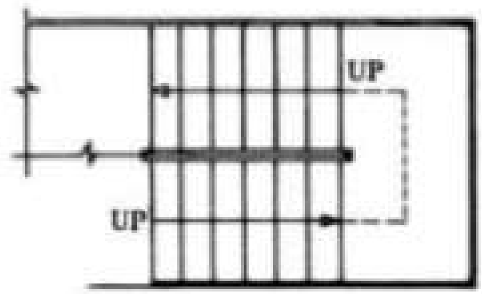


2. Dog legged Stair :

- It consists of two straight flights of steps with abrupt turn between them. A level landing is placed across the two flights at the change of direction.

- This type of stair is useful where the width of the staircase hall is just sufficient to accommodate two width of stair.

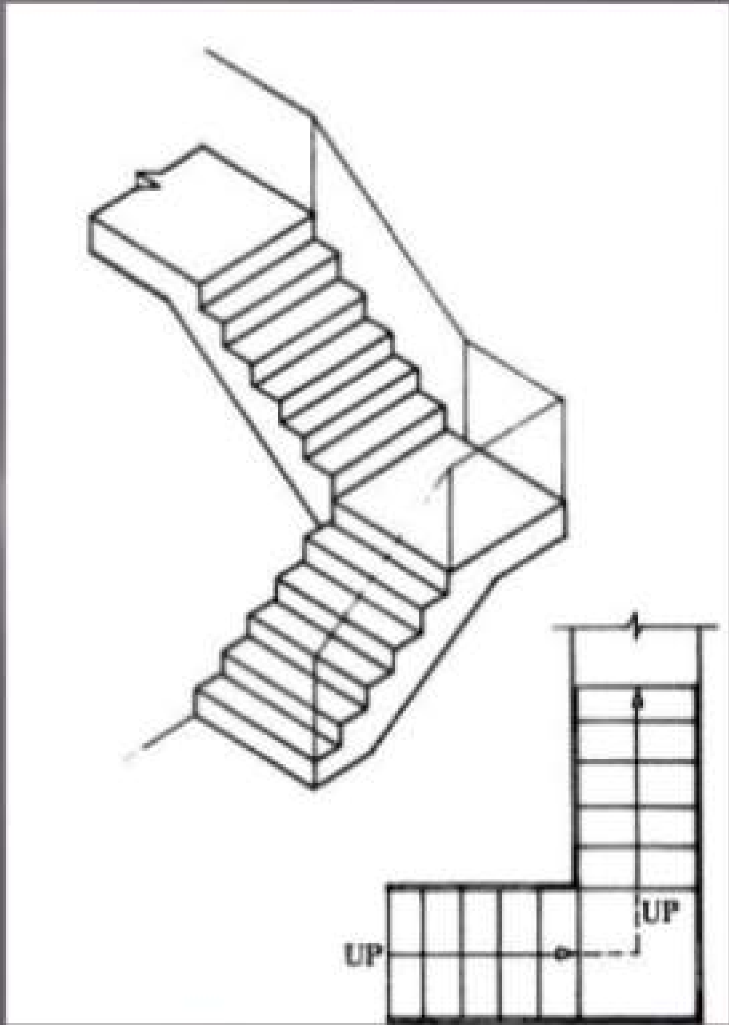




3. Quarter turn Stair :

- *A stair turning through one right angle is known as quarter turn stair.*

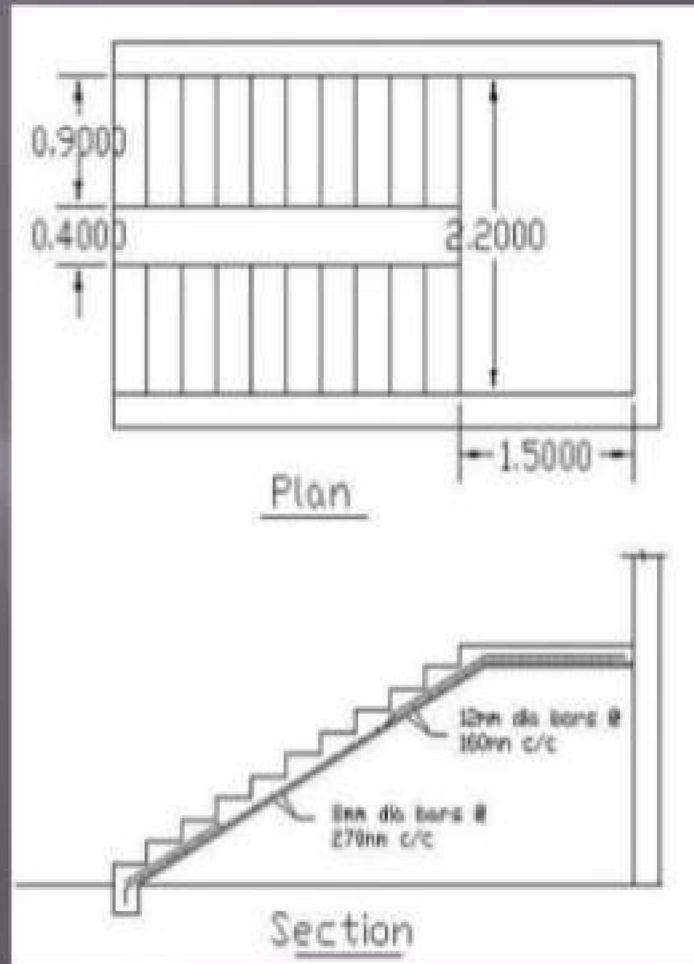
- *The change in direction can be affected by either introducing a landing or by providing winders.*



4. Open Newel Stair :

- *In these type of stair there is a well or opening between the flights in plan.*

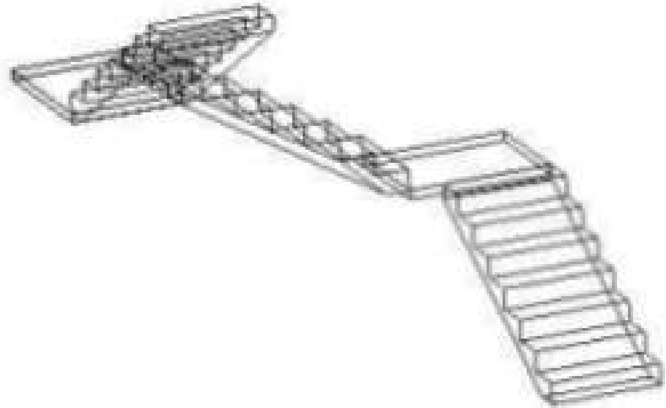
- *This well may be rectangular or of any geometrical shape and it can be used for fixing lift.*

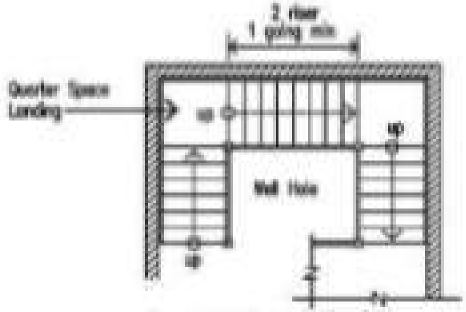


5. Three Quarter Turn Stair :

- *A stair turning through right angles (270 degree) is known as three quarter turn stair.*

- *In this case an open well is formed.*





Open Well Newel Stair with Quarter Space Landing

6. Bifurcated Stair :

- *These stair are so arranged that there is a wide flight at the start which is further sub-divided into two narrow flights at the mid-landing.*

- *The two narrow flight starts from either side of the mid-landing.*

- *These stairs are suitable for modern sub building.*

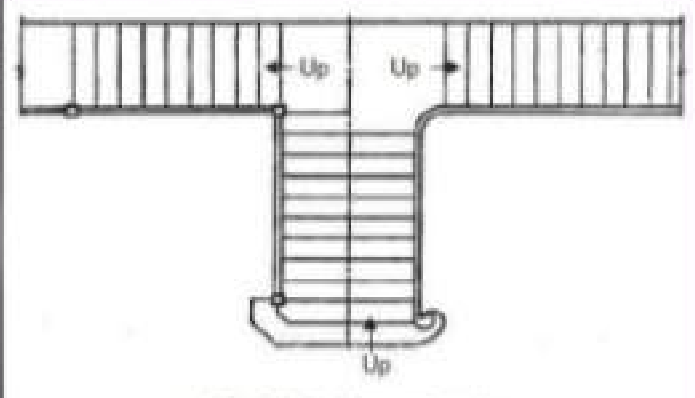


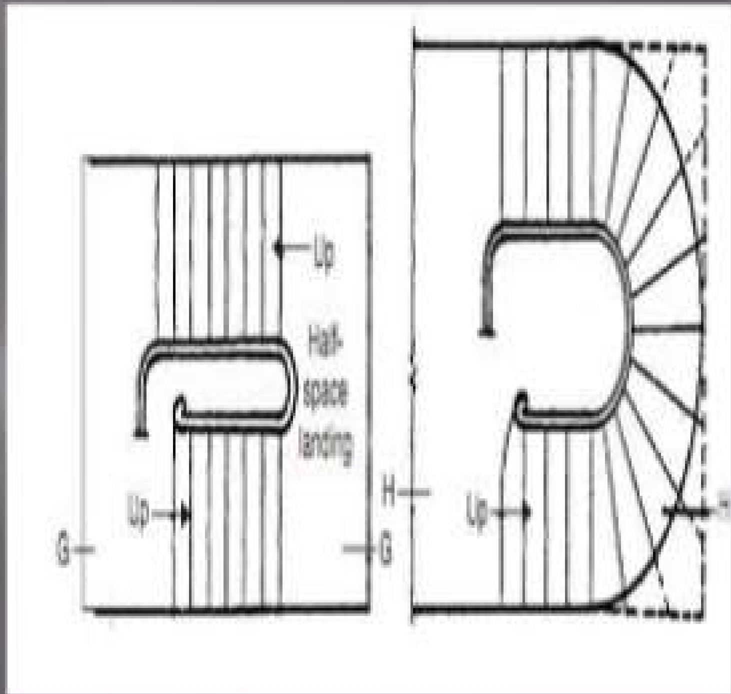
Fig. 8.40. Bifurcated stairs

7. Geometrical Stair :

- *These stairs have no newel post and are of any geometrical shape.*

- *The change in direction is achieved through winders.*

- *The stairs require more skill for its construction and are weaker than open newel stair.*



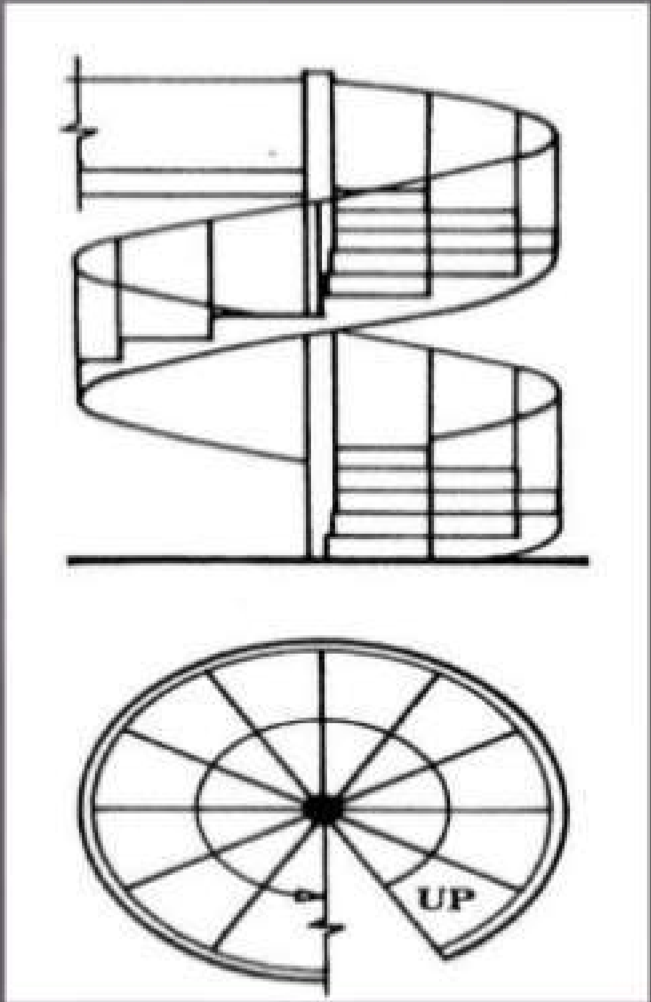
8. Spiral or Circular Stair :

It is known as spiral stair.

When viewed from top it appears to follow a circle with a single centre of curvature.

The spiral stairs are provided where space available is limited and traffic is low.

These stairs can be constructed in R.C.C., Steel or Stone



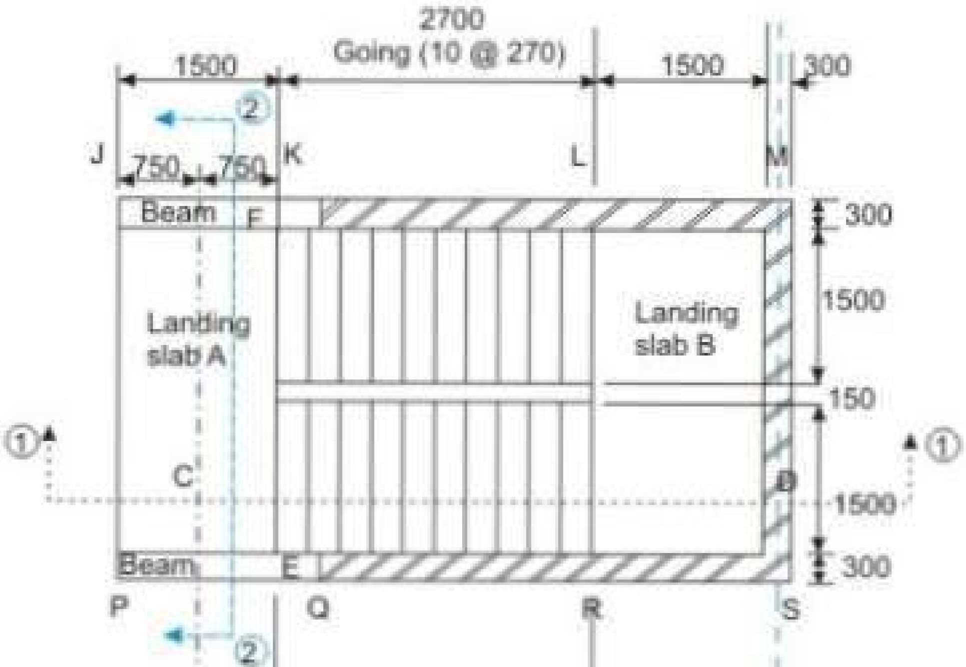


Fig. 9.20.10: Example 9.1

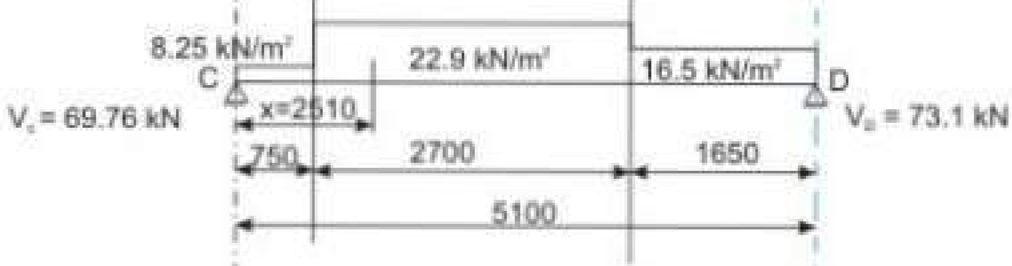


Fig. 9.20.11: Calculation of loads, sec 1-1 of Example 9.1, (Fig. 9.20.10)

Example 9.1:

Design the waist-slab type of the staircase of Fig.9.20.10. Landing slab A is supported on beams along JK and PQ, while the waist-slab and landing slab B are spanning longitudinally as shown in Fig.9.20.10. The finish loads and live loads are 1 kN/m^2 and 5 kN/m^2, respectively. Use riser R = 160 mm, trade T = 270 mm, concrete grade = M 20 and steel grade = Fe 415.

Solution:

With R = 160 mm and T = 270 mm, the inclined length of each step = $\{(160)^2 + (270)^2\}^{½}$ = 313.85 mm.

(A) Design of going and landing slab B

Step 1: Effective span and depth of slab

The effective span (cls. 33.1b and c) = 750 + 2700 + 1500 + 150 = 5100 mm. The depth of waist slab = 5100/20 = 255 mm. Let us assume total depth of 250 mm and effective depth = 250 – 20 – 6 = 224 mm (assuming cover = 20 mm and diameter of main reinforcing bar = 12 mm). The depth of landing slab is assumed as 200 mm and effective depth = 200 – 20 – 6 = 174 mm.

Step 2: Calculation of loads (Fig.9.20.11, sec. 1-1)

(i) Loads on going (on projected plan area)

(a) Self-weight of waist-slab = 25(0.25)(313.85)/270 = 7.265 kN/m^2

(b) Self-weight of steps = 25(0.5)(0.16) = 2.0 kN/m^2

(c) Finishes (given) = 1.0 kN/m^2

(d) Live loads (given) = 5.0 kN/m^2

Total = 15.265 kN/m^2

Total factored loads = 1.5(15.265) = 22.9 kN/m^2

(ii) Loads on landing slab A (50% of estimated loads)

(a) Self-weight of landing slab = 25(0.2) = 5 kN/m^2

(b) Finishes (given) = 1 kN/m^2

(c) Live loads (given) = 5 kN/m^2

Total = 11 kN/m^2

Factored loads on landing slab A = 0.5(1.5)(11) = 8.25 kN/m^2

(iii) Factored loads on landing slab B = (1.5)(11) = 16.5 kN/m^2

The loads are drawn in Fig.9.20.11.

Step 3: Bending moment and shear force (Fig. 9.20.11)

Total loads for 1.5 m width of flight = 1.5{8.25(0.75) + 22.9(2.7) + 16.5(1.65)}

= 142.86 kN

V_C = 1.5{8.25(0.75)(5.1 – 0.375) + 22.9(2.7)(5.1 – 0.75 – 1.35)

+ 16.5(1.65)(1.65)(0.5)}/5.1 = 69.76 kN

V_D = 142.86 – 69.76 = 73.1 kN

The distance *x* from the left where shear force is zero is obtained from:

x = {69.76 – 1.5(8.25)(0.75) + 1.5(22.9)(0.75)}/(1.5)(22.9) = 2.51 m

The maximum bending moment at *x* = 2.51 m is

= 69.76(2.51) – (1.5)(8.25)(0.75)(2.51 – 0.375)

- (1.5)(22.9)(2.51 – 0.75)(2.51 – 0.75)(0.5) = 102.08 kNm.

For the landing slab B, the bending moment at a distance of 1.65 m from D

= 73.1(1.65) – 1.5(16.5)(1.65)(1.65)(0.5) = 86.92 kNm

Step 4: Checking of depth of slab

From the maximum moment, we get *d* = $\{102080/2(2.76)\}^{½}$ = 135.98 mm < 224 mm for waist-slab and < 174 mm for landing slabs. Hence, both the depths of 250 mm and 200 mm for waist-slab and landing slab are more than adequate for bending.

For the waist-slab, τ_v = 73100/1500(224) = 0.217 N/mm^2. For the waist-slab of depth 250 mm, *k* = 1.1 (cl. 40.2.1.1 of IS 456) and from Table 19 of IS 456, τ_c = 1.1(0.28) = 0.308 N/mm^2. Table 20 of IS 456, $\tau_{c\max}$ = 2.8 N/mm^2. Since $\tau_v < \tau_c < \tau_{c\max}$, the depth of waist-slab as 250 mm is safe for shear.

For the landing slab, τ_v = 73100/1500(174) = 0.28 N/mm^2. For the landing slab of depth 200 mm, *k* = 1.2 (cl. 40.2.1.1 of IS 456) and from Table 19 of IS 456, τ_c = 1.2(0.28) = 0.336 N/mm^2 and from Table 20 of IS 456, $\tau_{c\max}$ = 2.8 N/mm^2. Here also $\tau_v < \tau_c < \tau_{c\max}$, so the depth of landing slab as 200 mm is safe for shear.

Step 5: Determination of areas of steel reinforcement

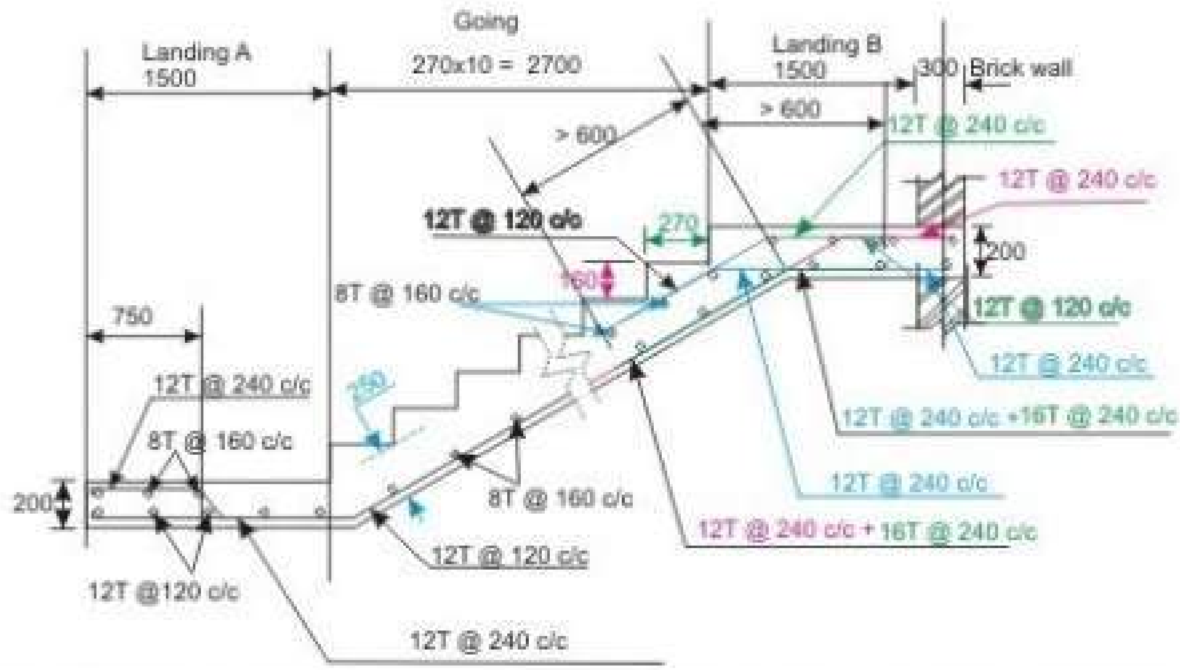


Fig. 9.20.12: Reinforcing bars of Example 9.1, sec 1-1 of Fig. 9.20.10

(i) Waist-slab: M_u/bd^2 = 102080/(1.5)224(224) = 1.356 N/mm^2. Table 2 of SP-16 gives p = 0.411.

The area of steel = 0.411(1000)(224)/(100) = 920.64 mm^2. Provide 12 mm diameter @ 120 mm c/c (= 942 mm^2/m).

(ii) Landing slab B: M_u/bd^2 at a distance of 1.65 m from V_D (Fig. 9.20.11) = 86920/(1.5)(174)(174) = 1.91 N/mm^2. Table 2 of SP-16 gives: p = 0.606. The area of steel = 0.606(1000) (174)/100 = 1054 mm^2/m. Provide 16 mm diameter @ 240 mm c/c and 12 mm dia. @ 240 mm c/c (1309 mm^2) at the bottom of landing slab B of which 16 mm bars will be terminated at a distance of 500 mm from the end and will continue up to a distance of 1000 mm at the bottom of waist slab (Fig. 9.20.12).

Distribution steel: The same distribution steel is provided for both the slabs as calculated for the waist-slab. The amount is = 0.12(250) (1000)/100 = 300 mm^2/m. Provide 8 mm diameter @ 160 mm c/c (= 314 mm^2/m).

Step 6: Checking of development length and diameter of main bars

Development length of 12 mm diameter bars = 47(12) = 564 mm, say 600 mm and the same of 16 mm dia. Bars = 47(16) = 752 mm, say 800 mm.

(i) For waist-slab

M_1 for 12 mm diameter @ 120 mm c/c (= 942 mm^2) = 942(102.08)/920.64 = 104.44 kNm. With V (shear force) = 73.1 kN, the diameter of main bars $\leq$ {1.3(104440)/73.1}/47 $\leq$ 39.5 mm. Hence, 12 mm diameter is o.k.

(ii) For landing-slab B

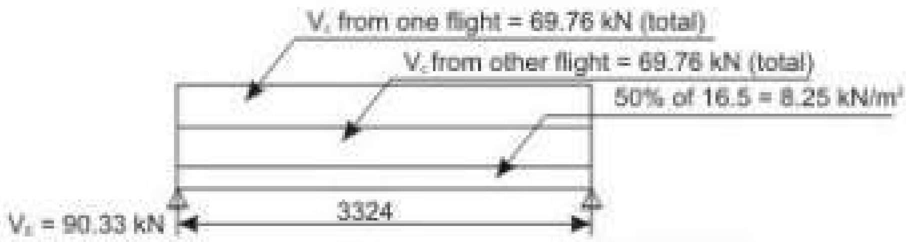
M_1 for 16 mm diameter @ 120 mm c/c (= 1675 mm^2) = 1675(102.08)/1650.88 = 103.57 kNm. With V (shear force) = 73.1 kN, the diameter of main bars $\leq$ {1.3(103570)/73.1}/47 = 39.18 mm. Hence, 16 mm diameter is o.k.

The reinforcing bars are shown in Fig.9.20.12 (sec. 1-1).

(B) Design of landing slab A

Step 1: Effective span and depth of slab

The effective span is lesser of (i) (1500 + 1500 + 150 + 174), and (ii) (1500 + 1500 + 150 + 300) = 3324 mm. The depth of landing slab = 3324/20 = 166 mm, < 200 mm already assumed. So, the depth is 200 mm.



Module 11

Foundations - Theory and Design

Lesson 28

Foundations - Theory

Instructional Objectives:

At the end of this lesson, the student should be able to:

- explain the two major and other requirements of the design of foundation,
- identify five points indicating the differences between the design of foundation and the design of other elements of the superstructure,
- differentiate between footing and foundation,
- differentiate between shallow and deep foundations,
- identify the situations when a combined footing shall be used,
- explain the safe bearing capacity of soil mentioning the difference between gross and net safe bearing capacities,
- determine the minimum depth of foundation,
- determine the critical sections of bending moment and shear in isolated footings,
- draw the distributions of pressure of soil below the footing for concentric and eccentric loads with $e \leq L/6$ and $e > L/6$,
- determine the soil pressure in a foundation which is unsymmetrical.

11.28.1 Introduction

Till now we discussed the different structural elements viz. beams, slabs, staircases and columns, which are placed above the ground level and are known as superstructure. The superstructure is placed on the top of the foundation structure, designated as substructure as they are placed below the ground level. The elements of the superstructure transfer the loads and moments to its adjacent element below it and finally all loads and moments come to the foundation structure, which in turn, transfers them to the underlying soil or rock. Thus, the foundation structure effectively supports the superstructure. However, all types of soil get compressed significantly and cause the structure to settle. Accordingly, the major requirements of the design of foundation structures are the two as given below (see cl.34.1 of IS 456):

1. Foundation structures should be able to sustain the applied loads, moments, forces and induced reactions without exceeding the safe bearing capacity of the soil.

2. The settlement of the structure should be as uniform as possible and it should be within the tolerable limits. It is well known from the structural analysis that differential settlement of supports causes additional moments in statically indeterminate structures. Therefore, avoiding the differential settlement is considered as more important than maintaining uniform overall settlement of the structure.

In addition to the two major requirements mentioned above, the foundation structure should provide adequate safety for maintaining the stability of structure due to either overturning and/or sliding (see cl.20 of IS 456). It is to be noted that this part of the structure is constructed at the first stage before other components (columns / beams etc.) are taken up. So, in a project, foundation design and details are completed before designs of other components are undertaken.

However, it is worth mentioning that the design of foundation structures is somewhat different from the design of other elements of superstructure due to the reasons given below. Therefore, foundation structures need special attention of the designers.

1. Foundation structures undergo soil-structure interaction. Therefore, the behaviour of foundation structures depends on the properties of structural materials and soil. Determination of properties of soil of different types itself is a specialized topic of geotechnical engineering. Understanding the interacting behaviour is also difficult. Hence, the different assumptions and simplifications adopted for the design need scrutiny. In fact, for the design of foundations of important structures and for difficult soil conditions, geotechnical experts should be consulted for the proper soil investigation to determine the properties of soil, strata wise and its settlement criteria.

2. Accurate estimations of all types of loads, moments and forces are needed for the present as well as for future expansion, if applicable. It is very important as the foundation structure, once completed, is difficult to strengthen in future.

3. Foundation structures, though remain underground involving very little architectural aesthetics, have to be housed within the property line which may cause additional forces and moments due to the eccentricity of foundation.

4. Foundation structures are in direct contact with the soil and may be affected due to harmful chemicals and minerals present in the soil and fluctuations of water table when it is very near to the foundation. Moreover,

periodic inspection and maintenance are practically impossible for the foundation structures.

5. Foundation structures, while constructing, may affect the adjoining structure forming cracks to total collapse, particularly during the driving of piles etc.

However, wide ranges of types of foundation structures are available. It is very important to select the appropriate type depending on the type of structure, condition of the soil at the location of construction, other surrounding structures and several other practical aspects as mentioned above.

11.28.2 Types of Foundation Structures

Foundations are mainly of two types: (i) shallow and (ii) deep foundations. The two different types are explained below:

(A) Shallow foundations

Shallow foundations are used when the soil has sufficient strength within a short depth below the ground level. They need sufficient plan area to transfer the heavy loads to the base soil. These heavy loads are sustained by the reinforced concrete columns or walls (either of bricks or reinforced concrete) of much less areas of cross-section due to high strength of bricks or reinforced concrete when compared to that of soil. The strength of the soil, expressed as the safe bearing capacity of the soil as discussed in sec.11.28.3, is normally supplied by the geotechnical experts to the structural engineer. Shallow foundations are also designated as footings. The different types of shallow foundations or footings are discussed below.

1. Plain concrete pedestal footings

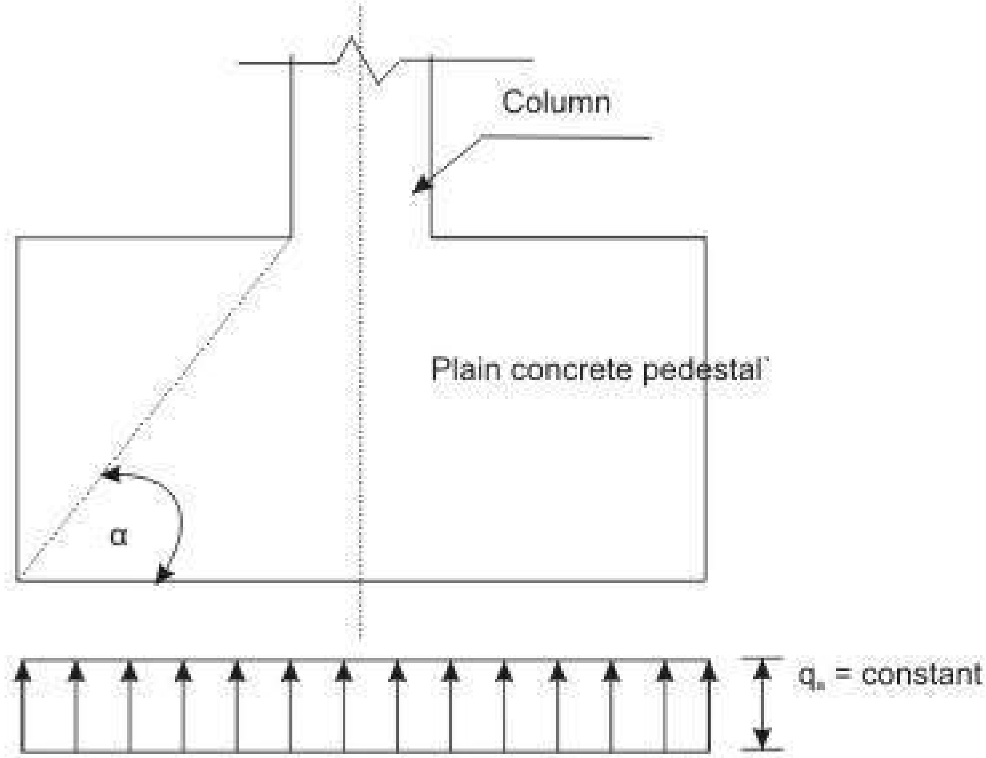


Fig. 11.28.1: Plain concrete pedestal

Plain concrete pedestal footings (Fig.11.28.1) are very economical for columns of small loads or pedestals without any longitudinal tension steel (see cls.34.1.2 and 34.1.3 of IS 456). In Fig.11.28.1, the angle α between the plane passing through the bottom edge of the pedestal and the corresponding junction edge of the column with pedestal and the horizontal plane shall be determined from Eq. 11.3.

2. Isolated footings

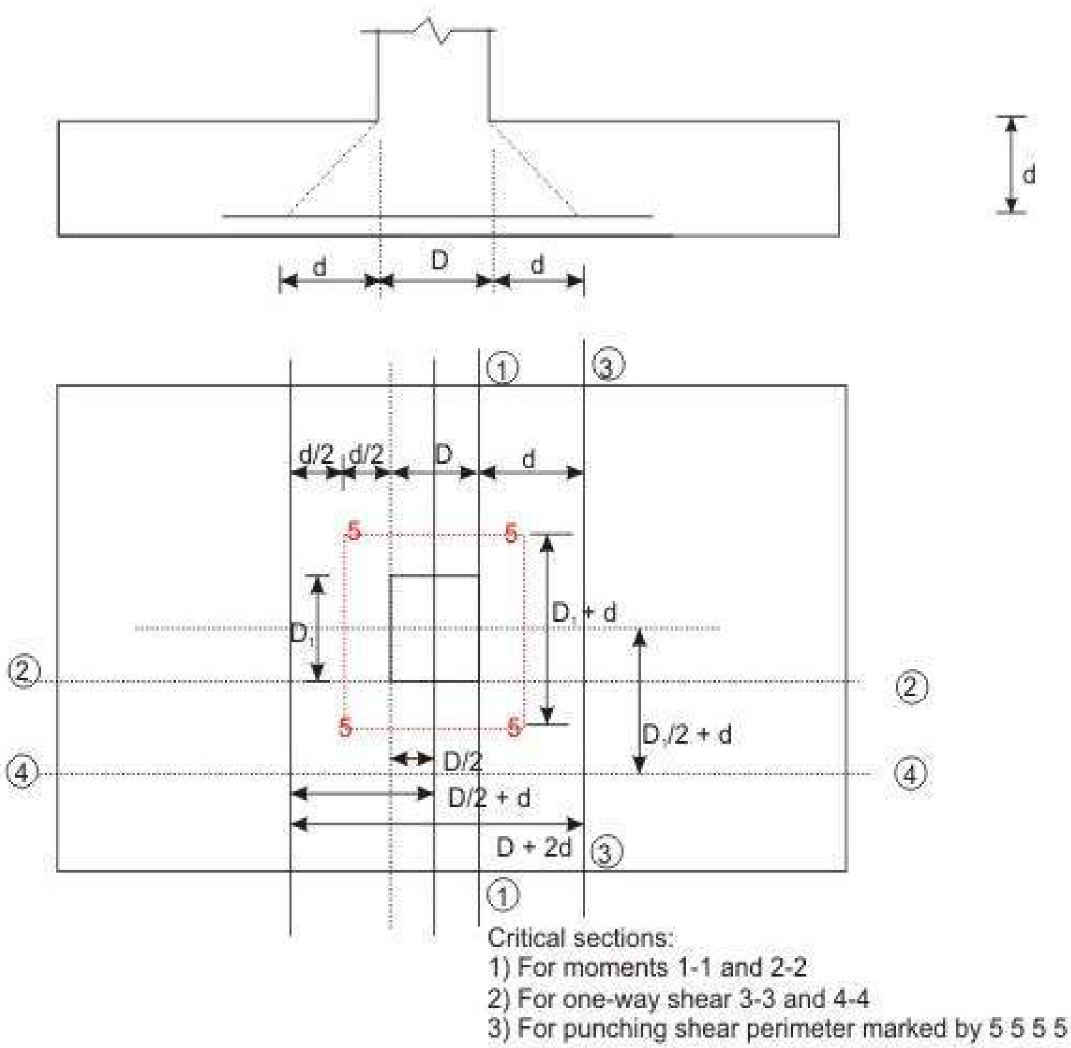
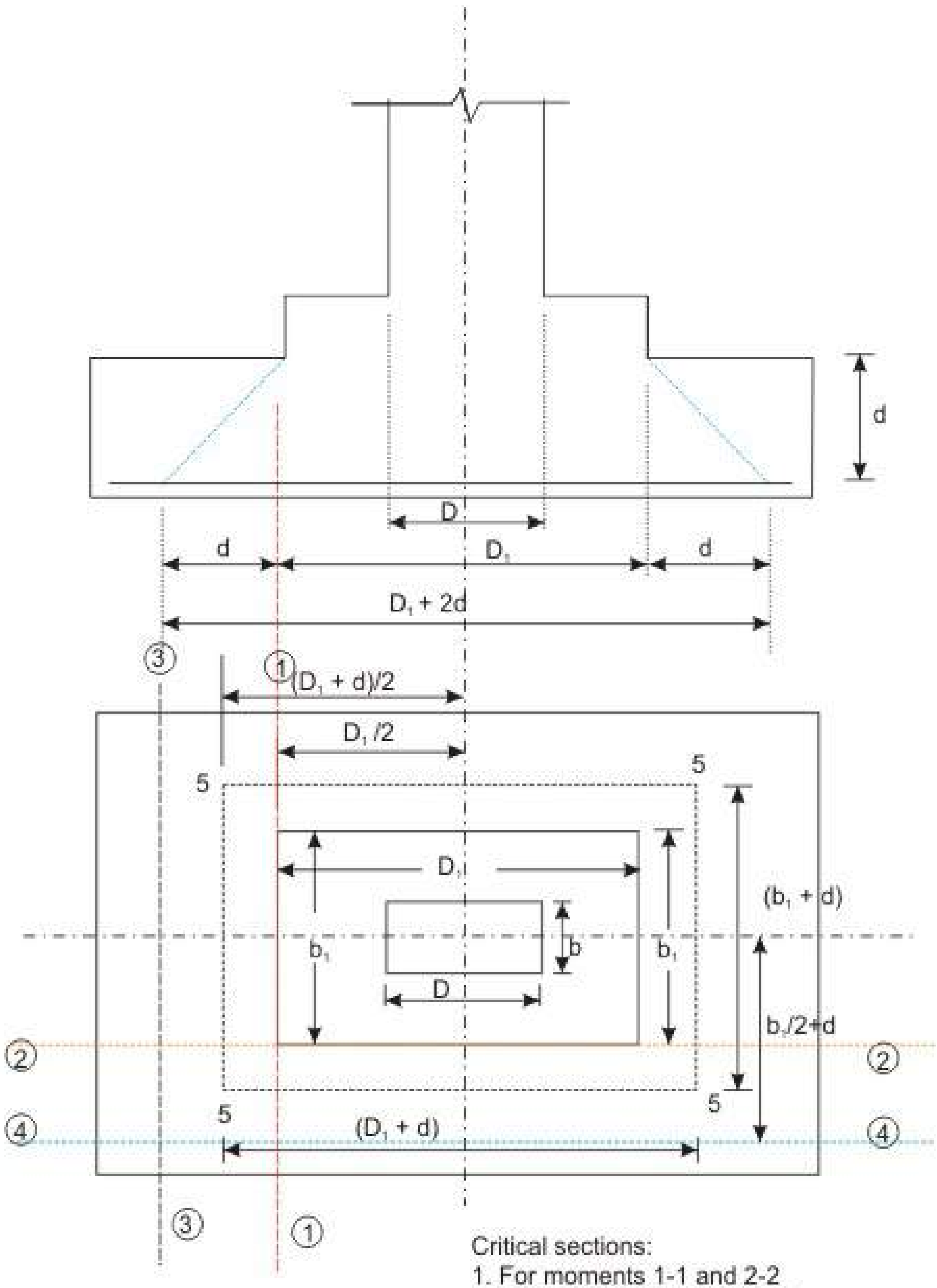


Fig. 11.28.2: Uniform and rectangular footing



Critical sections:
1. For moments 1-1 and 2-2
2. One way shear 3-3 and 4-4
3. Two-way punching shear marked by 5555

Fig. 11.28.3: Stepped and rectangular footing

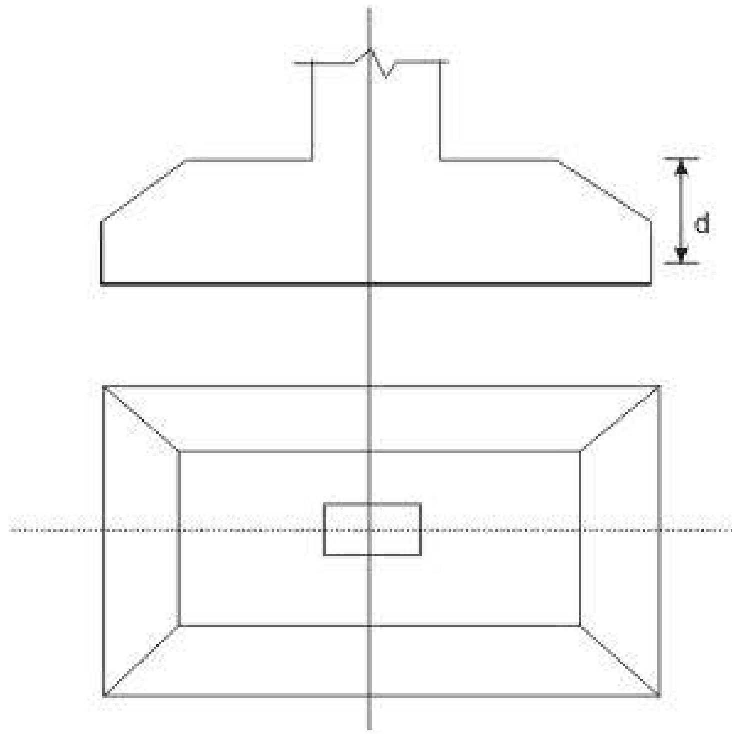


Fig. 11.28.4: Sloped and rectangular footing

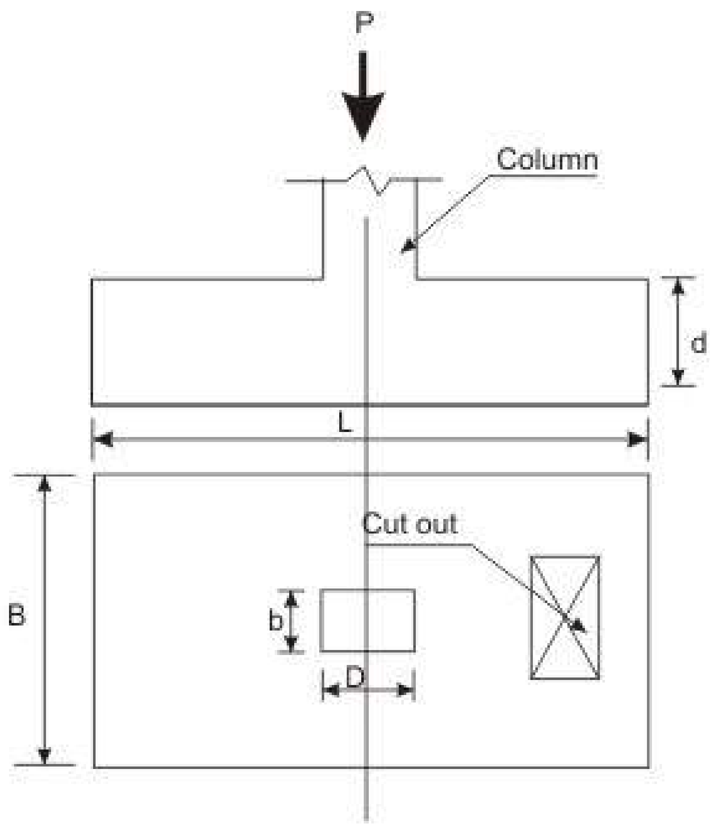


Fig. 11.28.5: Unsymmetrical footing about x axis

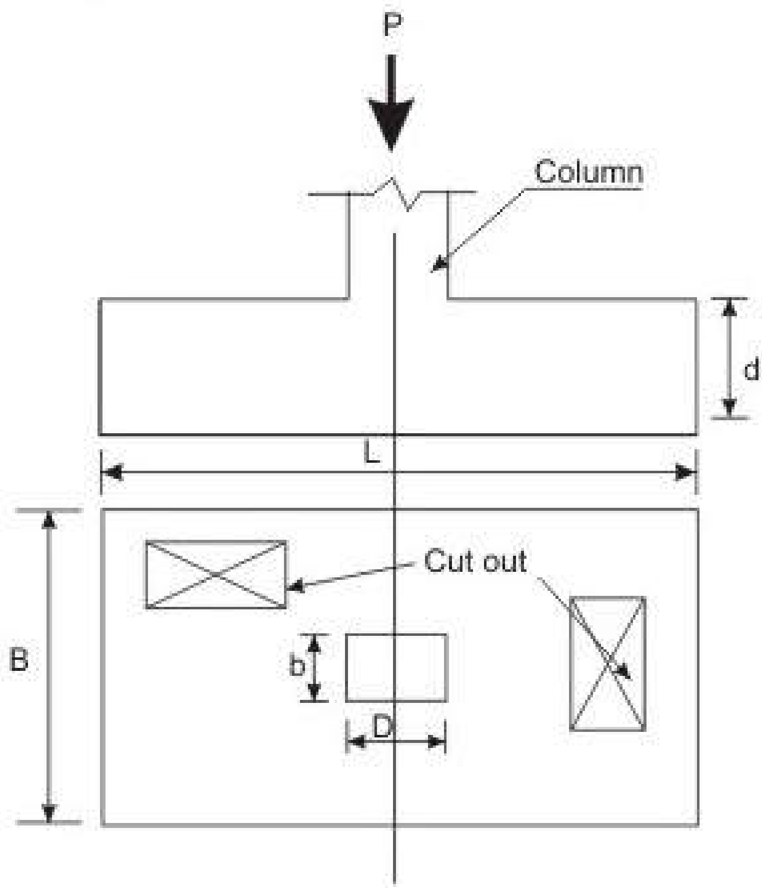


Fig. 11.28.6: Unsymmetrical footing about both axes

These footings are for individual columns having the same plan forms of square, rectangular or circular as that of the column, preferably maintaining the proportions and symmetry so that the resultants of the applied forces and reactions coincide. These footings, shown in Figs.11.27.2 to 11.27.4, consist of a slab of uniform thickness, stepped or sloped. Though sloped footings are economical in respect of the material, the additional cost of formwork does not offset the cost of the saved material. Therefore, stepped footings are more economical than the sloped ones. The adjoining soil below footings generates upward pressure which bends the slab due to cantilever action. Hence, adequate tensile reinforcement should be provided at the bottom of the slab (tension face). Clause 34.1.1 of IS 456 stipulates that the sloped or stepped footings, designed as a unit, should be constructed to ensure the integrated action. Moreover, the effective cross-section in compression of sloped and stepped footings shall be limited by the area above the neutral plane. Though symmetrical footings are desirable, sometimes situation compels for unsymmetrical isolated footings (Eccentric footings or footings with cut outs) either about one or both the axes (Figs.11.28.5 and 6).

3. Combined footings

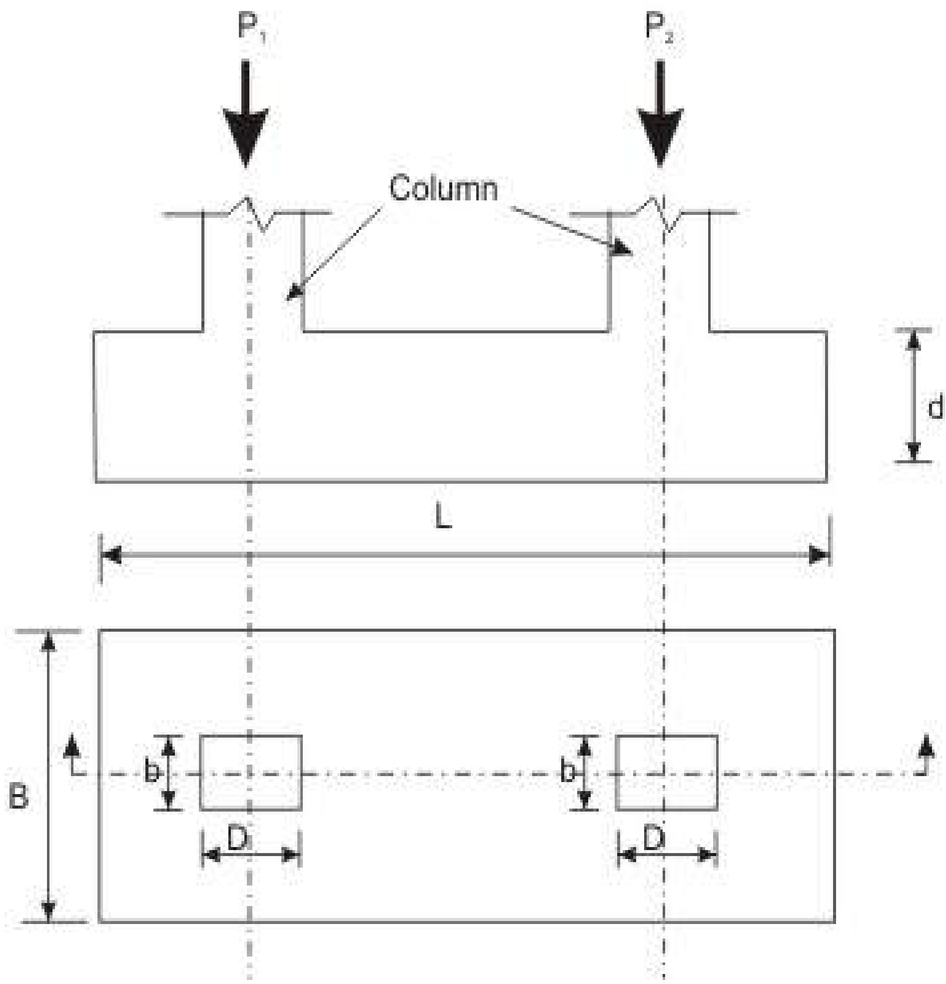


Fig. 11.28.7: Combined footing without a central beam

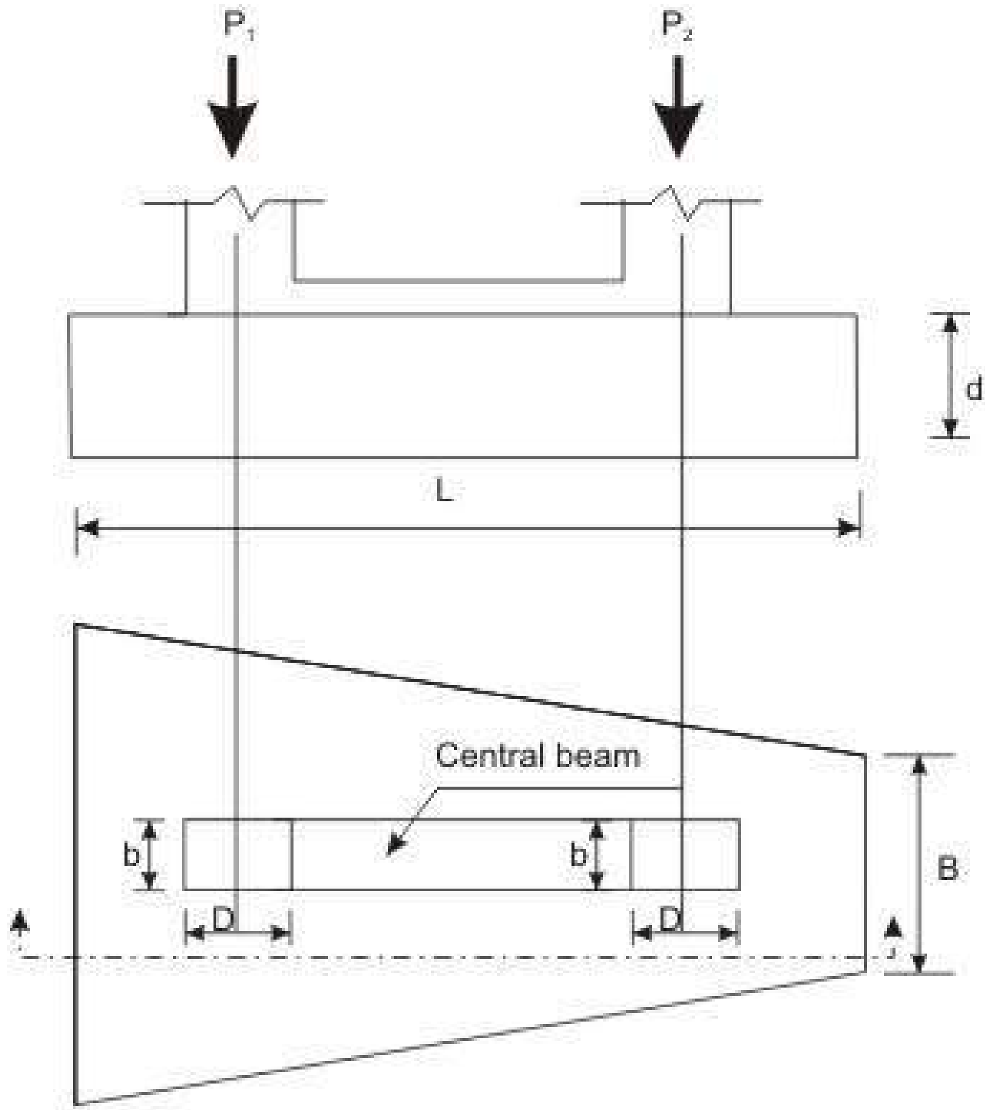


Fig. 11.28.8: Combined footing with a central beam

When the spacing of the adjacent columns is so close that separate isolated footings are not possible due to the overlapping areas of the footings or inadequate clear space between the two areas of the footings, combined footings are the solution combining two or more columns. Combined footing normally means a footing combining two columns. Such footings are either rectangular or trapezoidal in plan forms with or without a beam joining the two columns, as shown in Figs.11.28.7 and 11.28.8.

4. Strap footings

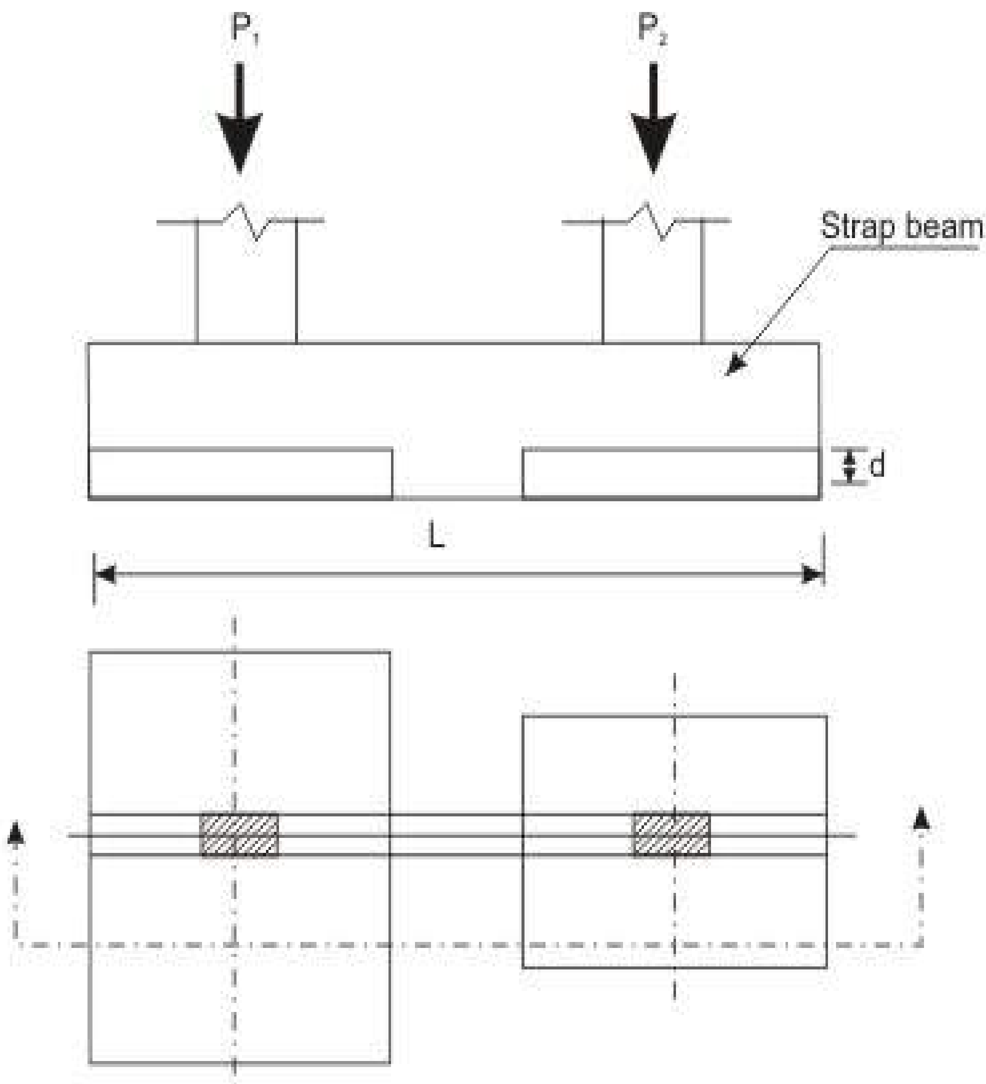


Fig. 11.28.9: Strap footing

When two isolated footings are combined by a beam with a view to sharing the loads of both the columns by the footings, the footing is known as strap footing (Fig.11.28.9). The connecting beam is designated as strap beam. These footings are required if the loads are heavy on columns and the areas of foundation are not overlapping with each other.

5. Strip foundation or wall footings

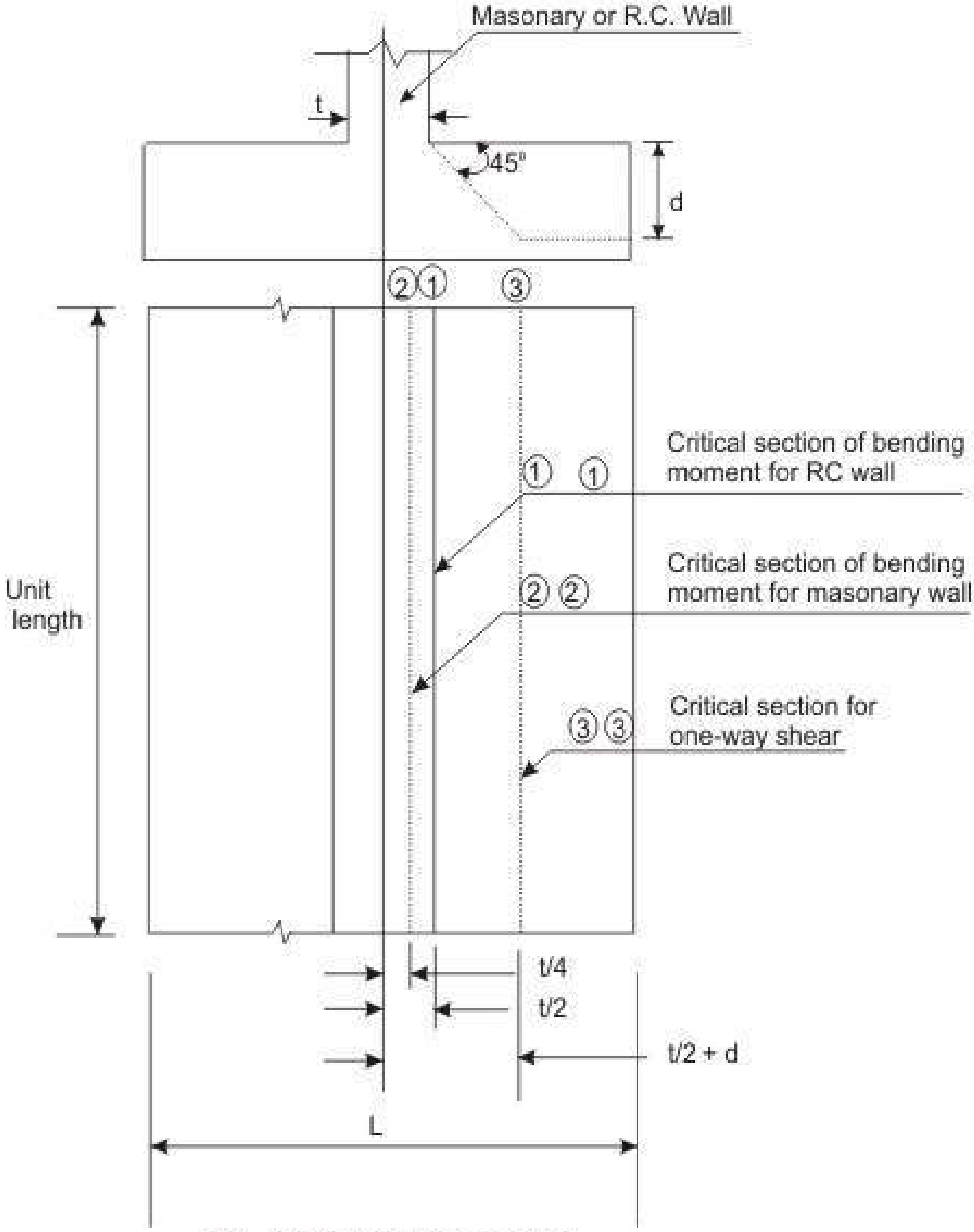


Fig. 11.28.10: Wall footing

These are in long strips especially for load bearing masonry walls or reinforced concrete walls (Figs.11.28.10). However, for load bearing masonry walls, it is common to have stepped masonry foundations. The strip footings distribute the loads from the wall to a wider area and usually bend in transverse direction. Accordingly, they are reinforced in the transverse direction mainly, while nominal distribution steel is provided along the longitudinal direction.

6. Raft or mat foundation

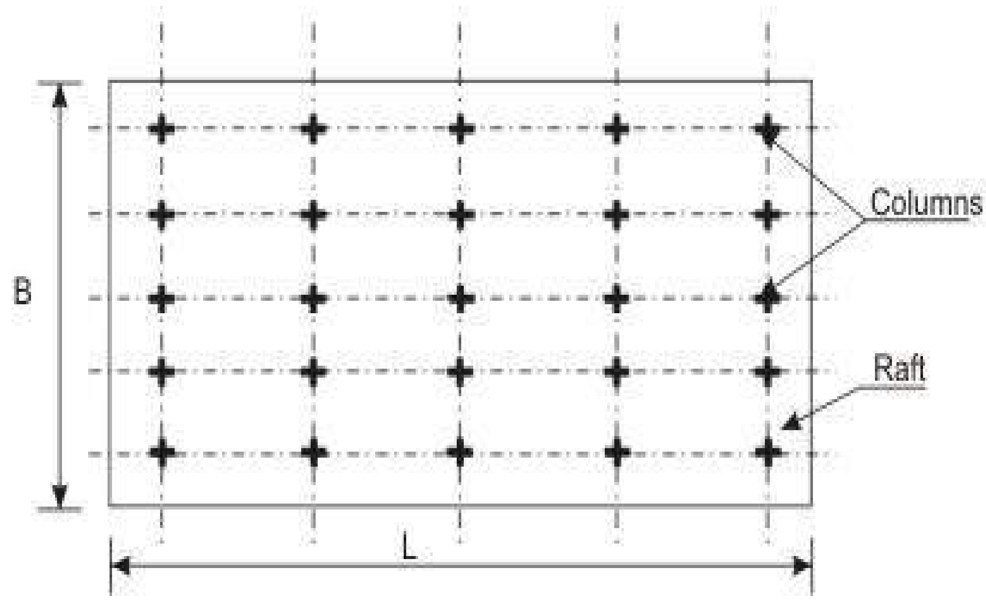


Fig. 11.28.11: Raft footing

These are special cases of combined footing where all the columns of the building are having a common foundation (Fig.11.28.11). Normally, for buildings with heavy loads or when the soil condition is poor, raft foundations are very much useful to control differential settlement and transfer the loads not exceeding the bearing capacity of the soil due to integral action of the raft foundation. This is a threshold situation for shallow footing beyond which deep foundations have to be adopted.

(B) Deep foundations

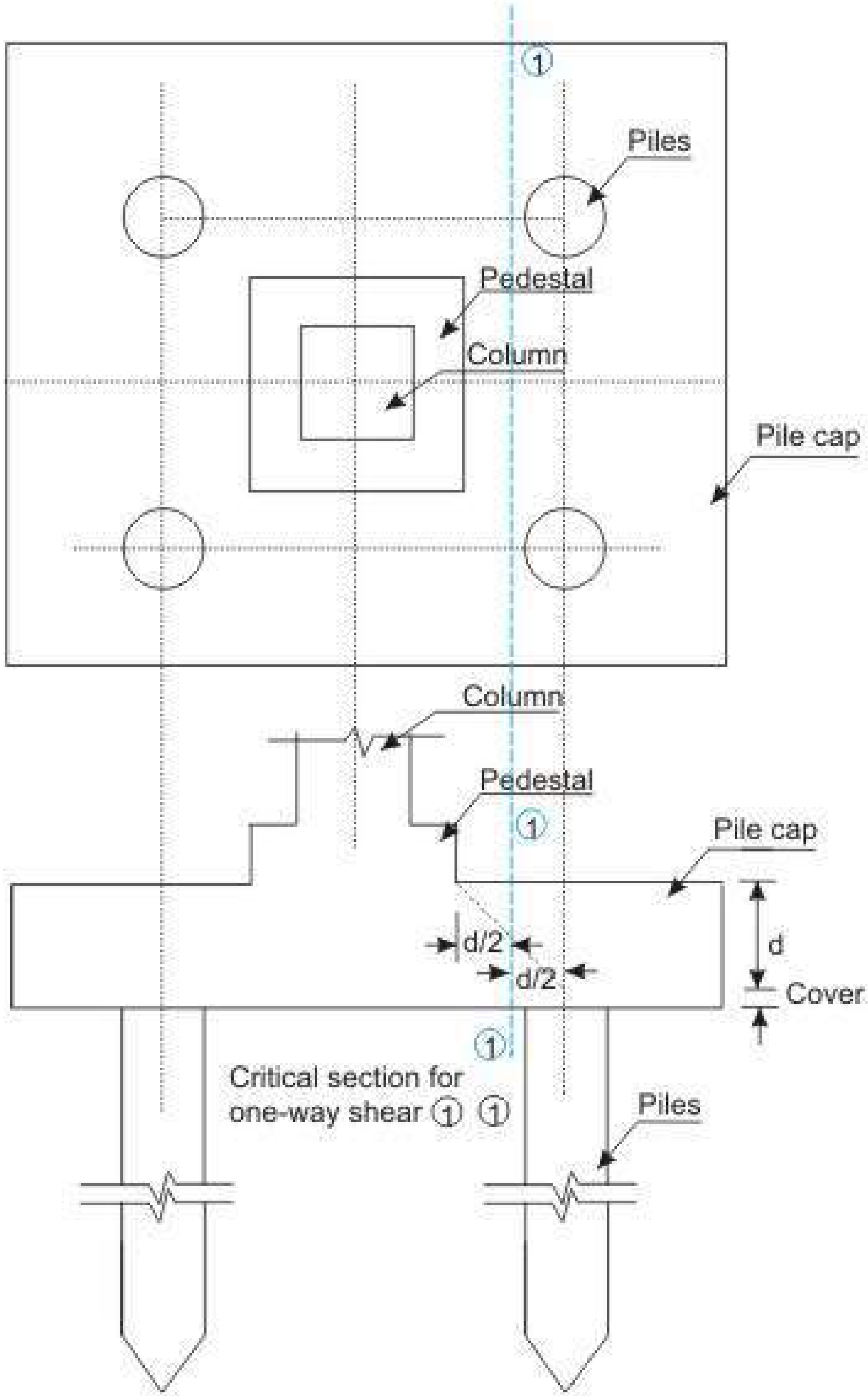


Fig. 11.28.12: Pile foundation

As mentioned earlier, the shallow foundations need more plan areas due to the low strength of soil compared to that of masonry or reinforced concrete. However, shallow foundations are selected when the soil has moderately good strength, except the raft foundation which is good in poor condition of soil also. Raft foundations are under the category of shallow foundation as they have comparatively shallow depth than that of deep foundation. It is worth mentioning that the depth of raft foundation is much larger than those of other types of shallow foundations.

However, for poor condition of soil near to the surface, the bearing capacity is very less and foundation needed in such situation is the pile foundation (Figs.11.28.12). Piles are, in fact, small diameter columns which are driven or cast into the ground by suitable means. Precast piles are driven and cast-in-situ are cast. These piles support the structure by the skin friction between the pile surface and the surrounding soil and end bearing force, if such resistance is available to provide the bearing force. Accordingly, they are designated as frictional and end bearing piles. They are normally provided in a group with a pile cap at the top through which the loads of the superstructure are transferred to the piles.

Piles are very useful in marshy land where other types of foundation are impossible to construct. The length of the pile which is driven into the ground depends on the availability of hard soil/rock or the actual load test. Another advantage of the pile foundations is that they can resist uplift also in the same manner as they take the compression forces just by the skin friction in the opposite direction.

However, driving of pile is not an easy job and needs equipment and specially trained persons or agencies. Moreover, one has to select pile foundation in such a situation where the adjacent buildings are not likely to be damaged due to the driving of piles. The choice of driven or bored piles, in this regard, is critical.

Exhaustive designs of all types of foundations mentioned above are beyond the scope of this course. Accordingly, this module is restricted to the design of some of the shallow footings, frequently used for normal low rise buildings only.

11.28.3 Safe Bearing Capacity of Soil

The safe bearing capacity q_c of soil is the permissible soil pressure considering safety factors in the range of 2 to 6 depending on the type of soil, approximations and assumptions and uncertainties. This is applicable under service load condition and, therefore, the partial safety factors λ_f for different load combinations are to be taken from those under limit state of serviceability (vide Table 18 of IS 456 or Table 2.1 of Lesson 3). Normally, the acceptable value of q_c is supplied by the geotechnical consultant to the structural engineer after proper soil investigations. The safe bearing stress on soil is also related to corresponding permissible displacement / settlement.

Gross and net bearing capacities are the two terms used in the design. Gross bearing capacity is the total safe bearing pressure just below the footing due to the load of the superstructure, self weight of the footing and the weight of earth lying over the footing. On the other hand, net bearing capacity is the net pressure in excess of the existing overburden pressure. Thus, we can write

Net bearing capacity = Gross bearing capacity - Pressure due to overburden soil (11.1)

While calculating the maximum soil pressure q, we should consider all the loads of superstructure along with the weight of foundation and the weight of the backfill. During preliminary calculations, however, the weight of the foundation and backfill may be taken as 10 to 15 per cent of the total axial load on the footing, subjected to verification afterwards.

11.28.4 Depth of Foundation

All types of foundation should have a minimum depth of 50 cm as per IS 1080-1962. This minimum depth is required to ensure the availability of soil having the safe bearing capacity assumed in the design. Moreover, the foundation should be placed well below the level which will not be affected by seasonal change of weather to cause swelling and shrinking of the soil. Further, frost also may endanger the foundation if placed at a very shallow depth. Rankine formula gives a preliminary estimate of the minimum depth of foundation and is expressed as

$$d = (q_c/\lambda)\{(1 - \sin\phi)/(1 + \sin\phi)\}^2 \quad (11.2)$$

where d = minimum depth of foundation

q_c = gross bearing capacity of soil

λ = density of soil

ϕ = angle of repose of soil

Though Rankine formula considers three major soil properties q_c, λ and ϕ, it does not consider the load applied to the foundation. However, this may be a guideline for an initial estimate of the minimum depth which shall be checked subsequently for other requirements of the design.

11.28.5 Design Considerations

(a) Minimum nominal cover (cl. 26.4.2.2 of IS 456)

The minimum nominal cover for the footings should be more than that of other structural elements of the superstructure as the footings are in direct contact with the soil. Clause 26.4.2.2 of IS 456 prescribes a minimum cover of 50 mm for footings. However, the actual cover may be even more depending on the presence of harmful chemicals or minerals, water table etc.

(b) Thickness at the edge of footings (cls. 34.1.2 and 34.1.3 of IS 456)

The minimum thickness at the edge of reinforced and plain concrete footings shall be at least 150 mm for footings on soils and at least 300 mm above the top of piles for footings on piles, as per the stipulation in cl.34.1.2 of IS 456.

For plain concrete pedestals, the angle α (see Fig.11.28.1) between the plane passing through the bottom edge of the pedestal and the corresponding junction edge of the column with pedestal and the horizontal plane shall be determined from the following expression (cl.34.1.3 of IS 456)

$$\tan\alpha \leq 0.9\{(100\, q_a/f_{ck}) + 1\}^{1/2} \qquad (11.3)$$

where q_a = calculated maximum bearing pressure at the base of pedestal in N/mm^2, and

f_{ck} = characteristic strength of concrete at 28 days in N/mm^2.

(c) Bending moments (cl. 34.2 of IS 456)

1. It may be necessary to compute the bending moment at several sections of the footing depending on the type of footing, nature of loads and the distribution of pressure at the base of the footing. However, bending moment at any section shall be determined taking all forces acting over the entire area on one side of the section of the footing, which is obtained by passing a vertical plane at that section extending across the footing (cl.34.2.3.1 of IS 456).

2. The critical section of maximum bending moment for the purpose of designing an isolated concrete footing which supports a column, pedestal or wall shall be:

(i) at the face of the column, pedestal or wall for footing supporting a concrete column, pedestal or reinforced concrete wall, (Figs.11.28.2, 3 and 10), and

(ii) halfway between the centre-line and the edge of the wall, for footing under masonry wall (Fig.11.28.10). This is stipulated in cl.34.2.3.2 of IS 456.

The maximum moment at the critical section shall be determined as mentioned in 1 above.

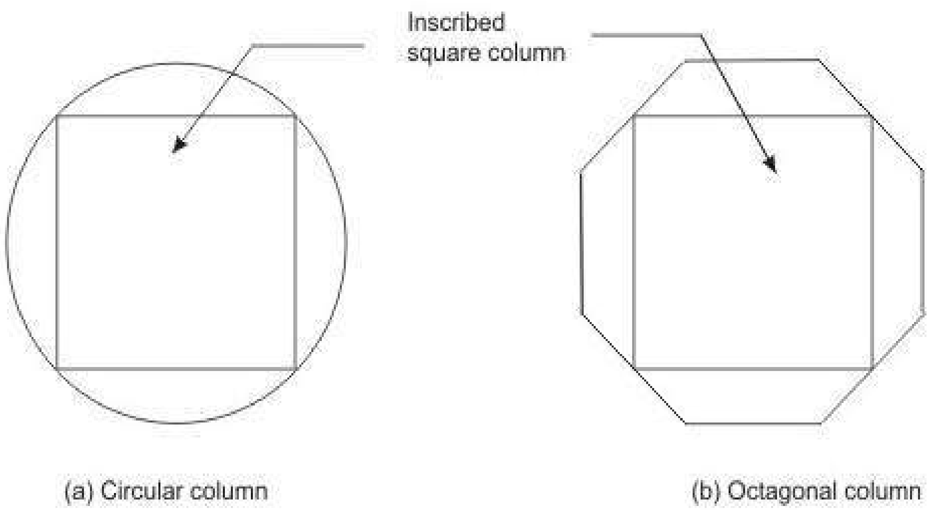


Fig. 11.28.13: Equivalent square columns (cl 34.2.2 of IS 456:2000)

For round or octagonal concrete column or pedestal, the face of the column or pedestal shall be taken as the side of a square inscribed within the perimeter of the round or octagonal column or pedestal (see cl.34.2.2 of IS 456 and Figs.11.28.13a and b).

(d) Shear force (cl. 31.6 and 34.2.4 of IS 456)

Footing slabs shall be checked in one-way or two-way shears depending on the nature of bending. If the slab bends primarily in one-way, the footing slab shall be checked in one-way vertical shear. On the other hand, when the bending is primarily two-way, the footing slab shall be checked in two-way shear or punching shear. The respective critical sections and design shear strengths are given below:

1. One-way shear (cl. 34.2.4 of IS 456)

One-way shear has to be checked across the full width of the base slab on a vertical section located from the face of the column, pedestal or wall at a distance equal to (Figs.11.28.2, 3 and 10):

(i) effective depth of the footing slab in case of footing slab on soil, and

(ii) half the effective depth of the footing slab if the footing slab is on piles (Fig.11.28.12).

The design shear strength of concrete without shear reinforcement is given in Table 19 of cl.40.2 of IS 456.

2. Two-way or punching shear (cls.31.6 and 34.2.4)

Two-way or punching shear shall be checked around the column on a perimeter half the effective depth of the footing slab away from the face of the column or pedestal (Figs.11.28.2 and 3).

The permissible shear stress, when shear reinforcement is not provided, shall not exceed $k_s \tau_c$, where $k_s = (0.5 + \beta_c)$, but not greater than one, β_c being the ratio of short side to long side of the column, and $\tau_c = 0.25(f_{ck})^{1/2}$ in limit state method of design, as stipulated in cl.31.6.3 of IS 456.

Normally, the thickness of the base slab is governed by shear. Hence, the necessary thickness of the slab has to be provided to avoid shear reinforcement.

(e) Bond (cl.34.2.4.3 of IS 456)

The critical section for checking the development length in a footing slab shall be the same planes as those of bending moments in part (c) of this section. Moreover, development length shall be checked at all other sections where they change abruptly. The critical sections for checking the development length are given in cl.34.2.4.3 of IS 456, which further recommends to check the anchorage requirements if the reinforcement is curtailed, which shall be done in accordance with cl.26.2.3 of IS 456.

(f) Tensile reinforcement (cl.34.3 of IS 456)

The distribution of the total tensile reinforcement, calculated in accordance with the moment at critical sections, as specified in part (c) of this section, shall be done as given below for one-way and two-way footing slabs separately.

(i) In one-way reinforced footing slabs like wall footings, the reinforcement shall be distributed uniformly across the full width of the footing i.e., perpendicular to the direction of wall. Nominal distribution reinforcement shall be provided as per cl. 34.5 of IS 456 along the length of the wall to take care of the secondary moment, differential settlement, shrinkage and temperature effects.

(ii) In two-way reinforced square footing slabs, the reinforcement extending in each direction shall be distributed uniformly across the full width/length of the footing.

(iii) In two-way reinforced rectangular footing slabs, the reinforcement in the long direction shall be distributed uniformly across the full width of the footing slab. In the short direction, a central band equal to the width of the footing shall be marked along the length of the footing, where the portion of the reinforcement shall be determined as given in the equation below. This portion of the reinforcement shall be distributed across the central band:

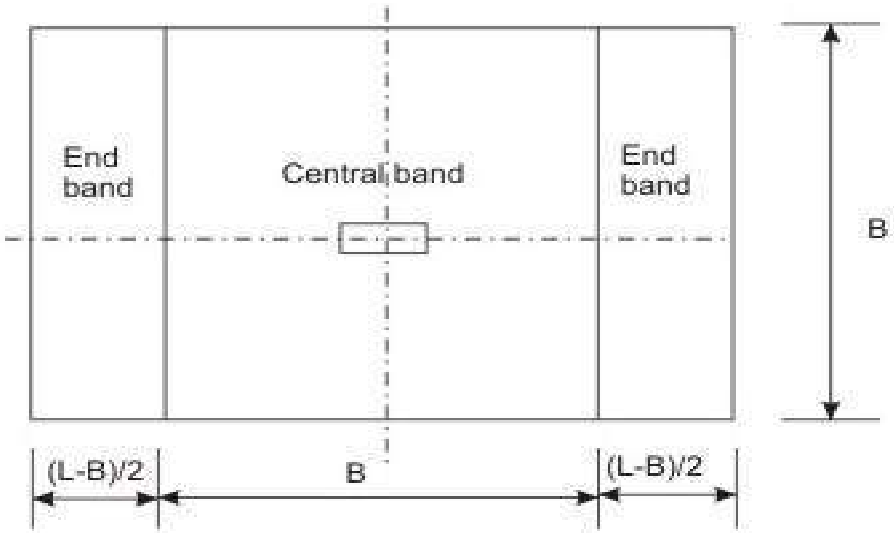


Fig. 11.28.14: Bands for reinforcement in a rectangular footing

Reinforcement in the central band = $\{2/(\beta+1)\}$ (Total reinforcement in the short direction)
(11.4)

where β is the ratio of longer dimension to shorter dimension of the footing slab (Fig.11.28.14).

Each of the two end bands shall be provided with half of the remaining reinforcement, distributed uniformly across the respective end band.

(g) Transfer of load at the base of column (cl.34.4 of IS 456)

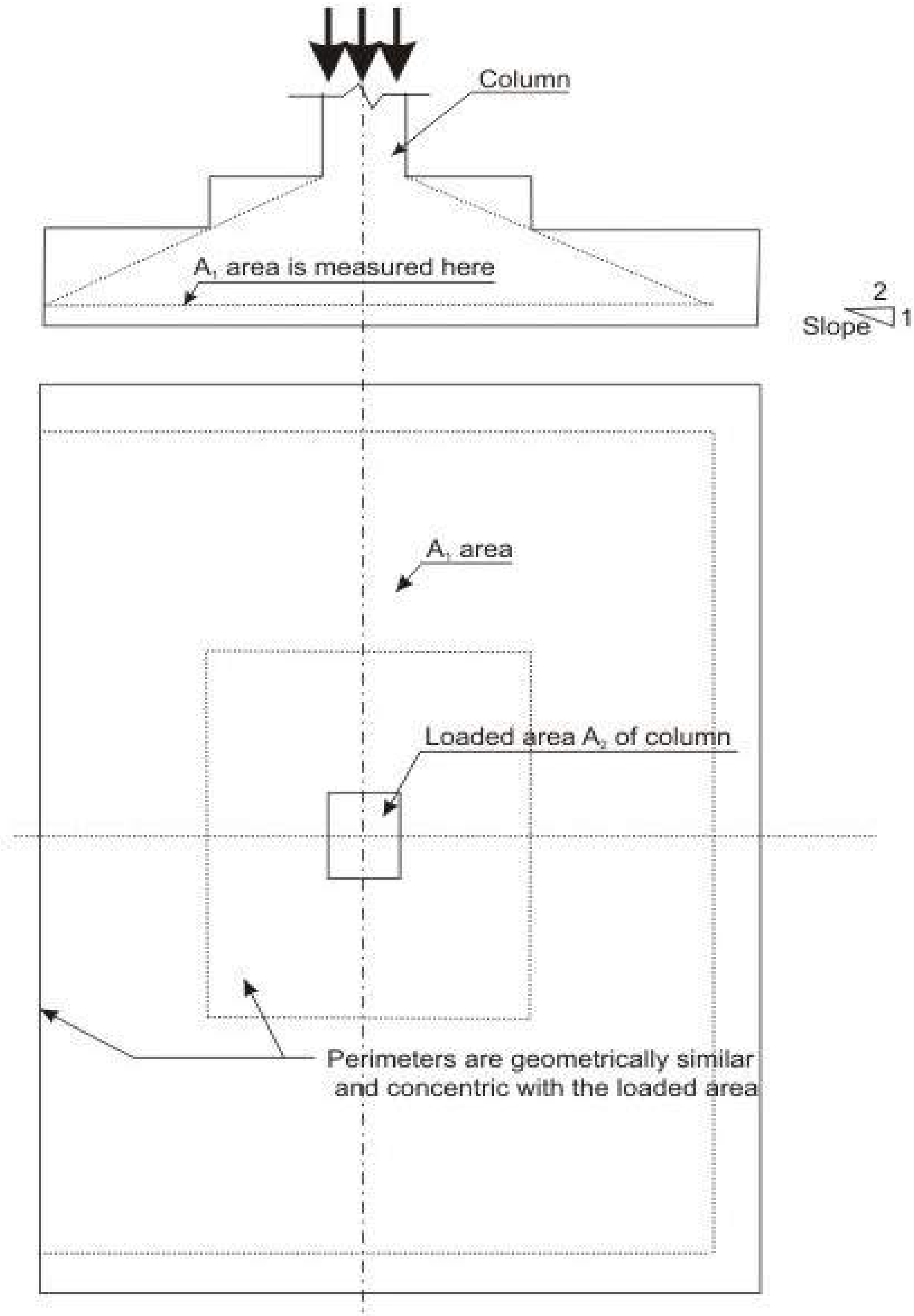


Fig. 11.28.15: Bearing area in sloped or stepped footing

All forces and moments acting at the base of the column must be transferred to the pedestal, if any, and then from the base of the pedestal to the footing, (or directly from the base of the column to the footing if there is no pedestal) by compression in concrete and steel and tension in steel. Compression forces are transferred through direct bearing while tension forces are transferred through developed reinforcement. The permissible bearing stresses on full area of concrete shall be taken as given below from cl.34.4 of IS 456:

$$\sigma_{br} = 0.25 f_{ck}, \text{ in working stress method, and} \tag{11.5}$$

$$\sigma_{br} = 0.45 f_{ck}, \text{ in limit state method} \tag{11.6}$$

It has been mentioned in sec. 10.26.5 of Lesson 26 that the stress of concrete is taken as $0.45f_{ck}$ while designing the column. Since the area of footing is much larger, this bearing stress of concrete in column may be increased considering the dispersion of the concentrated load of column to footing. Accordingly, the permissible bearing stress of concrete in footing is given by (cl.34.4 of IS 456):

$$\sigma_{br} = 0.45 f_{ck} (A_1/A_2)^{1/2} \tag{11.7}$$

with a condition that

$$(A_1/A_2)^{1/2} \leq 2.0 \tag{11.8}$$

where A_1 = maximum supporting area of footing for bearing which is geometrically similar to and concentric with the loaded area A_2, as shown in Fig.11.28.15

A_2 = loaded area at the base of the column.

The above clause further stipulates that in sloped or stepped footings, A_1 may be taken as the area of the lower base of the largest frustum of a pyramid or cone contained wholly within the footing and having for its upper base, the area actually loaded and having side slope of one vertical to two horizontal, as shown in Fig.11.28.15.

If the permissible bearing stress on concrete in column or in footing is exceeded, reinforcement shall be provided for developing the excess force (cl.34.4.1 of IS 456), either by extending the longitudinal bars of columns into the footing (cl.34.4.2 of IS 456) or by providing dowels as stipulated in cl.34.4.3 of IS 456 and given below:

(i) Sufficient development length of the reinforcement shall be provided to transfer the compression or tension to the supporting member in accordance with cl.26.2 of IS 456, when transfer of force is accomplished by reinforcement of column (cl.34.4.2 of IS 456).

(ii) Minimum area of extended longitudinal bars or dowels shall be 0.5 per cent of the cross-sectional area of the supported column or pedestal (cl.34.4.3 of IS 456).

(iii) A minimum of four bars shall be provided (cl.34.4.3 of IS 456).

(iv) The diameter of dowels shall not exceed the diameter of column bars by more than 3 mm.

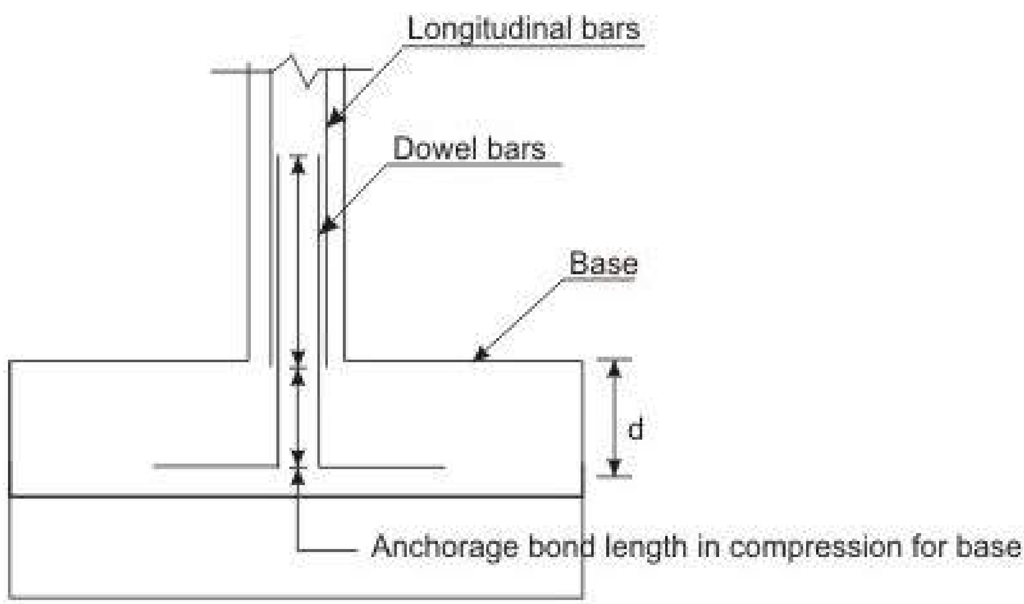


Fig. 11.28.16: Anchorage length of dowels

(v) Column bars of diameter larger than 36 mm, in compression only can be doweled at the footings with bars of smaller size of the necessary area. The dowel shall extend into the column, a distance equal to the development length of the column bar and into the footing, a distance equal to the development length of the dowel, as stipulated in cl.34.4.4 of IS 456 and as shown in Fig.11.28.16.

(h) Nominal reinforcement (cl. 34.5 of IS 456)

1. Clause 34.5.1 of IS 456 stipulates the minimum reinforcement and spacing of the bars in footing slabs as per the requirements of solid slab (cls.26.5.2.1 and 26.3.3b(2) of IS 456, respectively).

2. The nominal reinforcement for concrete sections of thickness greater than 1 m shall be 360 mm^2 per metre length in each direction on each face, as stipulated in cl.34.5.2 of IS 456. The clause further specifies that this provision does not supersede the requirement of minimum tensile reinforcement based on the depth of section.

11.28.6 Distribution of Base Pressure

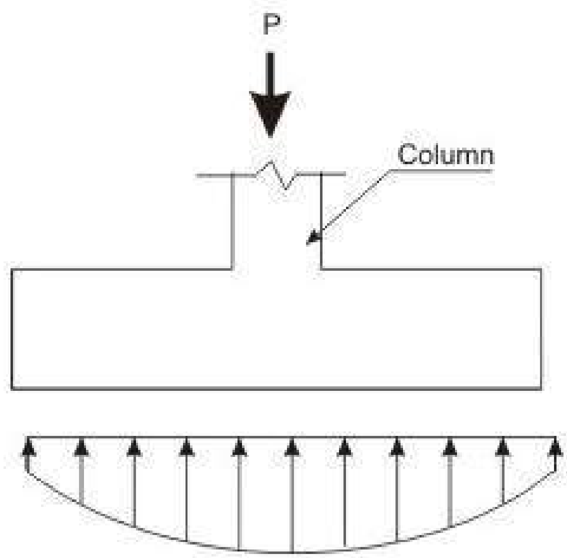


Fig. 11.28.17: Pressure distribution in sandy soil

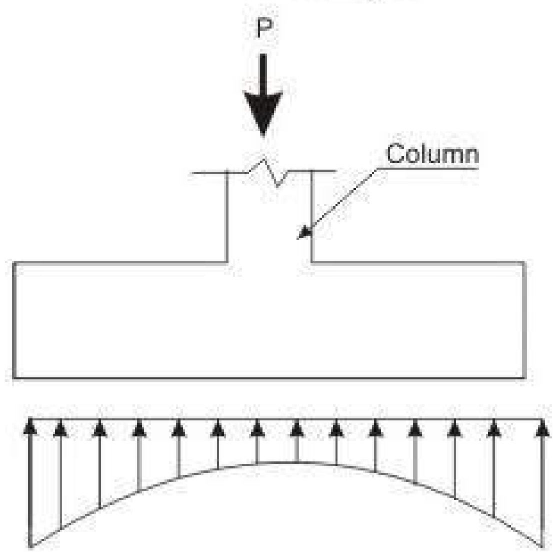


Fig. 11.28.18: Pressure distribution in clayey soil

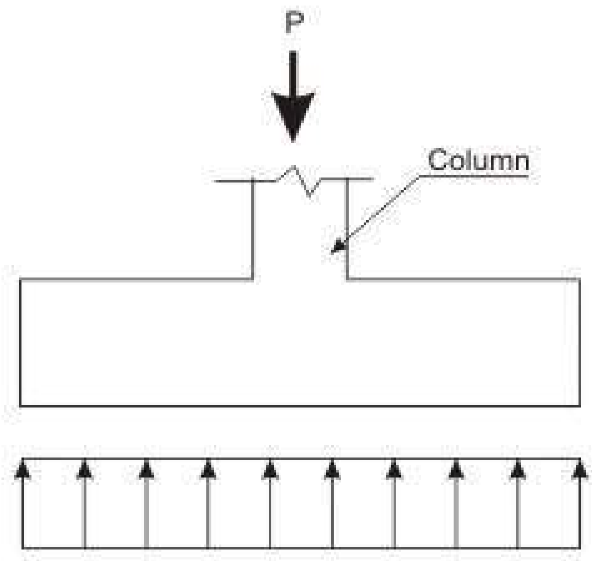


Fig. 11.28.19: Assuming uniform pressure in the design

The foundation, assumed to act as a rigid body, is in equilibrium under the action of applied forces and moments from the superstructure and the reactions from the stresses in the soil. The distribution of base pressure is different for different types of soil. Typical distributions of pressure, for actual foundations, in sandy and clayey soils are shown in Figs.11.28.17 and 18, respectively. However, for the sake of simplicity the footing is assumed to be a perfectly rigid body, the soil is assumed to behave elastically and the distributions of stress and stain are linear in the soil just below the base of the foundation, as shown in Fig.11.28.19. Accordingly, the foundation shall be designed for the applied loads, moments and induced reactions keeping in mind that the safe bearing capacity of the soil is within the prescribed limit. It is worth mentioning that the soil bearing capacity is in the serviceable limit state and the foundation structure shall be designed as per the limit state of collapse, checking for other limit states as well to ensure an adequate degree of safety and serviceability.

In the following, the distributions of base pressure are explained for (i) concentrically loaded footings, (ii) eccentrically loaded footings and (iii) unsymmetrical (about both the axes) footings.

(i) Concentrically loaded footings

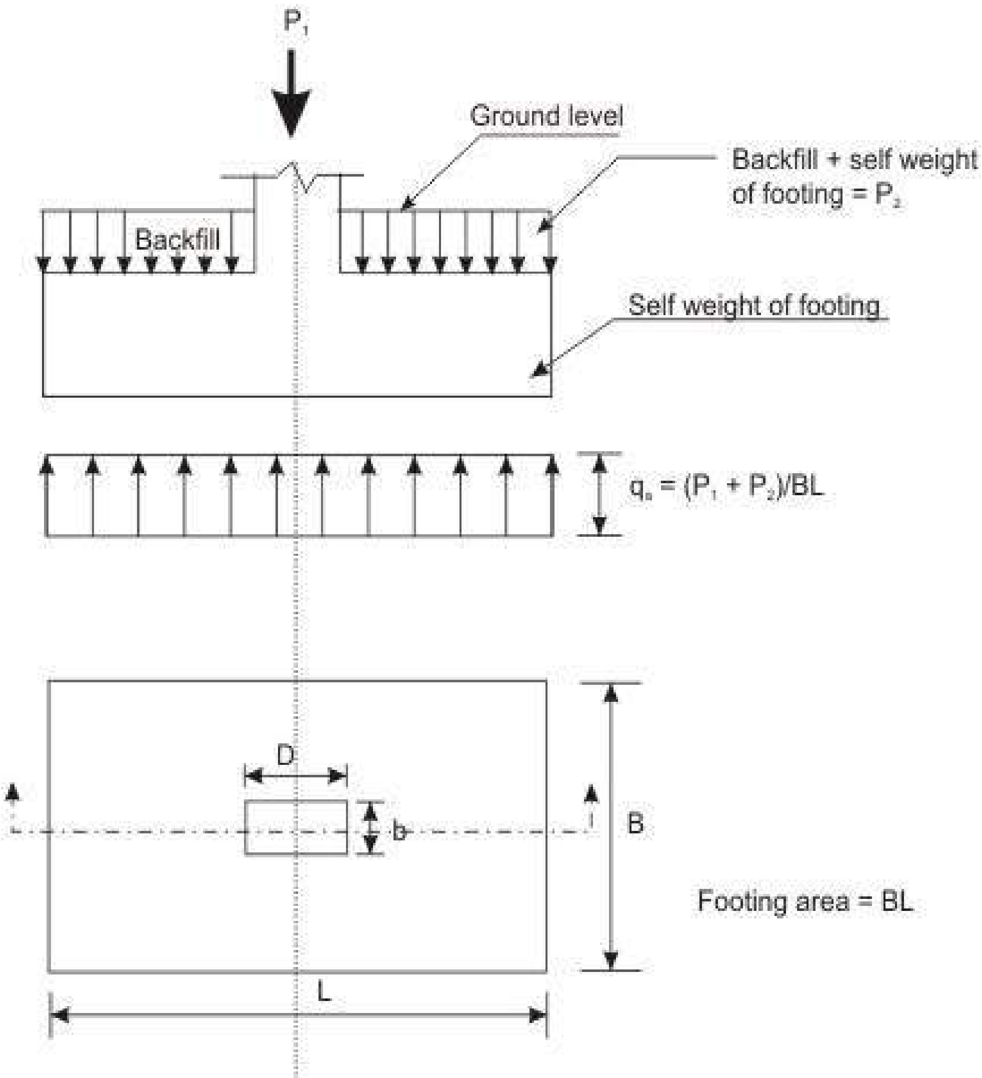


Fig. 11.28.20: Isolated footing subjected to concentric loading

Figure 11.28.20 shows rectangular footing symmetrically loaded with service load P_1 from the superstructure and P_2 from the backfill including the weight of the footing. The assumed uniformly distributed soil pressure at the base of magnitude q is obtained from:

$$q = (P_1 + P_2)/A \tag{11.9}$$

where A is the area of the base of the footing.

In the design problem, however, A is to be determined from the condition that the actual gross intensity of soil pressure does not exceed q_c, the bearing capacity of the soil, a known given data. Thus, we can write from Eq.11.9:

$$A = (P_1 + P_2)/q_c \tag{11.10}$$

From the known value of *A*, the dimensions *B* and *L* are determined such that the maximum bending moment in each of the two adjacent projections is equal, i.e., the ratio of the dimensions *B* and *L* of the footing shall be in the same order of the ratio of width *b* and depth *D* of the column.

(ii) Eccentrically loaded footings

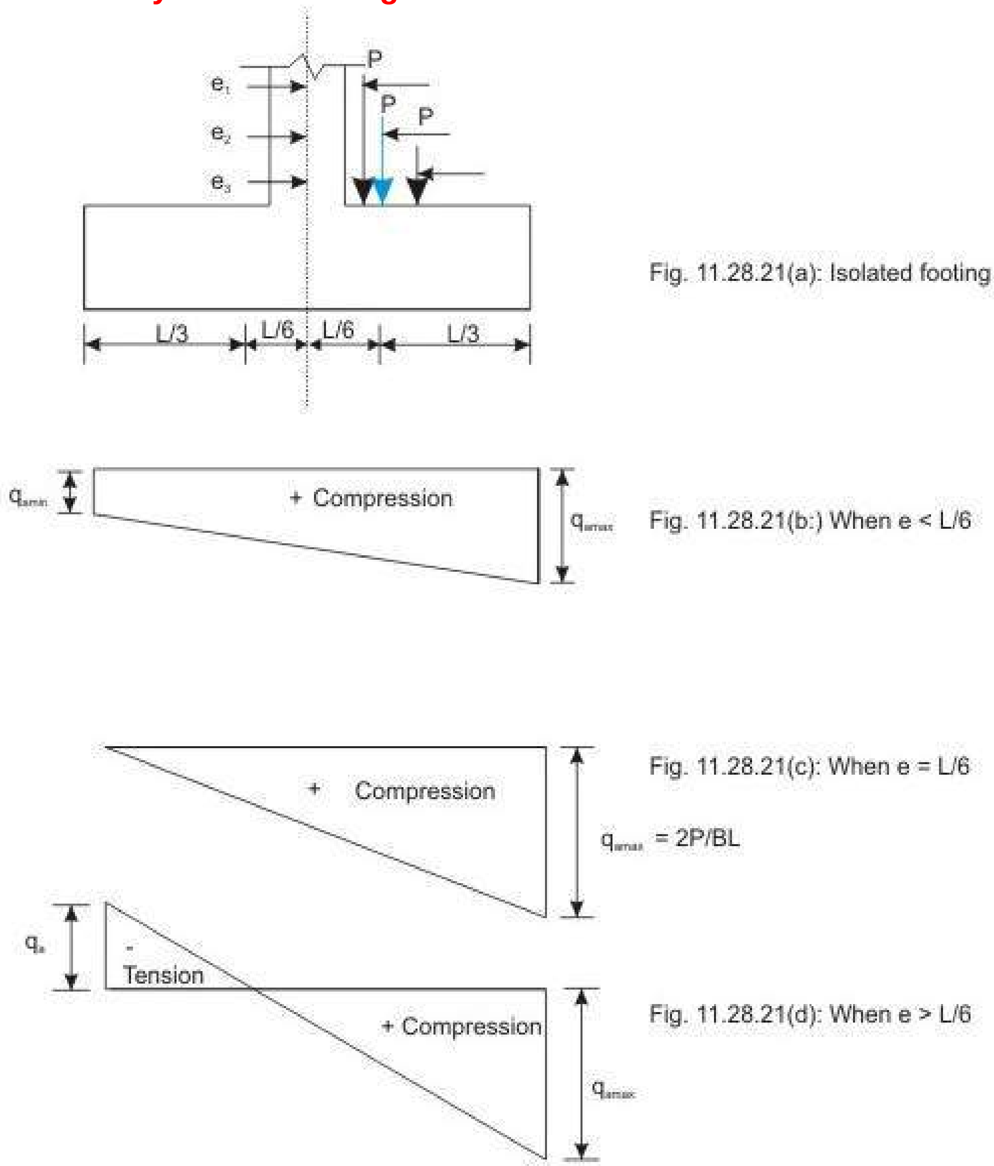


Fig. 11.28.21: Isolated footing subjected to different eccentric loadings

In most of the practical situations, a column transfers axial load *P* and moment *M* to the footing, which can be represented as eccentrically loaded footing when a load *P* is subjected to an eccentricity $e = M/P$. This eccentricity may also be there, either alone or in combined mode, when

- the column transfers a vertical load at a distance of *e* from the centroidal axis of the footing, and

- the column or the pedestal transfers a lateral load above the level of foundation, in addition to vertical loads.

Accordingly, the distribution of pressure may be of any one of the three types, depending on the magnitude of the eccentricity of the load, as shown in Figs.11.28.21b to d. The general expression of q_a, the intensity of soil pressure at a distance of y from the origin is:

$$q_a = P/A \pm (Pe/I_x)y \quad (11.11)$$

We would consider a rectangular footing symmetric to the column. Substituting the values of $A = BL$, $I_x = BL^3/12$ and $y = L/2$, we get the values of q_a at the left edge.

$$q_a \text{ at the left edge } = (P/BL)\{1 - (6e/L)\} \quad (11.12)$$

It is evident from Eq.11.12, that the three cases are possible:

(A) when $e < L/6$, q_a at the left edge is compression (+),

(B) when $e = L/6$, q_a at the left edge is zero, and

(C) when $e > L/6$, q_a at the left edge is tension (-).

The three cases are shown in Figs.11.28.21b to d, respectively. It is to be noted that similar three cases are also possible when eccentricity of the load is negative resulting the values of q_a at the right edge as compression, zero or tension. Evidently, these soil reactions, in compression and tension, should be permissible and attainable.

Case (A): when $|e| \leq L/6$

Figures 11.28.21b and c show these two cases, when $|e| < L/6$ or $|e| = L/6$, respectively. It is seen that the entire area of the footing is in compression having minimum and maximum values of q at the two edges with a linear and non-uniform variation. The values of q are obtained from Eq.11.11.

In the limiting case i.e., when $|e| = L/6$, the value of q_a is zero at one edge and the other edge is having $q_a = 2P/BL$ (compression) with a linear variation. Similarly, when $e = 0$, the footing is subjected to uniform constant pressure of P/BL. Thus, when $|e| = L/6$, the maximum pressure under one edge of the footing is twice of the uniform pressure when $e = 0$.

In a more general case, as in the case of footing for the corner column of a building, the load may have biaxial eccentricities. The general expression of q_a at a location of (x,y) of the footing, when the load is having biaxial eccentricities of e_x and e_y is,

$$q_a = P/A \pm P\,e_x y/I_x \pm P\,e_y x/I_y \quad (11.13)$$

Similarly, it can be shown that the rectangular footing of width B and length L will have no tension when the two eccentricities are such that

$$6e_x/L + 6e_y/B \leq 1 \quad (11.14)$$

Case (B): when | *e* | > *L*/6

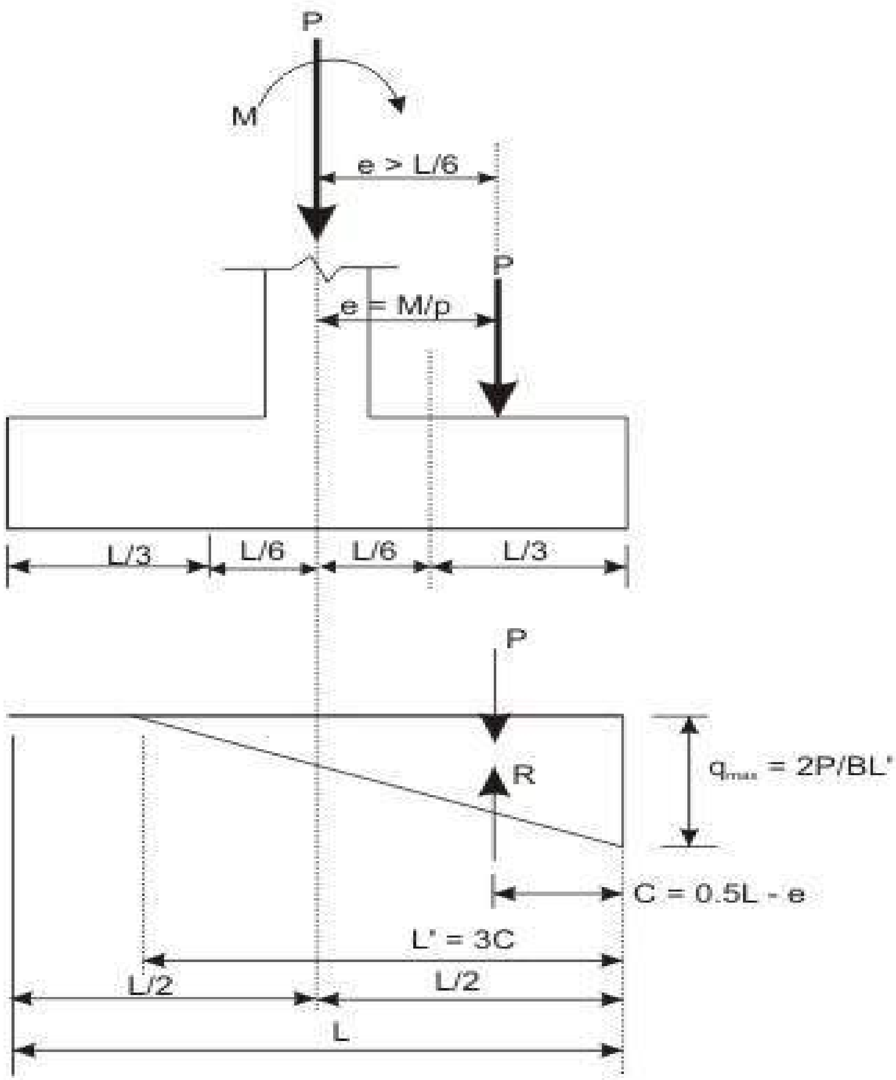


Fig. 11.28.22: Eccentrically loaded isolated footing (e > L/6)

The eccentricity of the load more than $L/6$ results in development of tensile stresses in part of the soil. Stability, in such case, is ensured by either anchoring or weight of overburden preventing uplift. However, it is to ensure that maximum compressive pressure on the other face is within the limit and sufficient factor of safety is available against over turning. Accordingly, the maximum pressure in such a case can be determined considering the soil under compression part only. Further, assuming the line of action of the eccentric load coincides with that of resultant soil pressure (Fig.11.28.22) we have:

$$q_{max} = P/L'B + 12P(0.5\,C)(1.5\,C)/BL' = 2P/L'B \qquad (11.15)$$

where $L' = 3C \qquad (11.16)$

(iii) Unsymmetrical footings

It may be necessary to provide some cutouts in the foundation to reduce the uplift pressure or otherwise. The footing in such cases becomes unsymmetrical about both the axes. It is possible to determine the soil pressure distribution using the structural mechanics principle as given below.

$$q_a(x,y) = P/A \pm \{(M_yI_x - M_xI_{xy})(x)/(I_xI_y - I_{xy}^2)\} + \{(M_xI_y - M_yI_{xy})(y)/(I_xI_y - I_{xy}^2)\} \qquad (11.17)$$

where M_x = moment about x axis,

M_y = moment about y axis,

I_x = moment of inertia about x axis,

I_y = moment of inertia about y axis,

I_{xy} = product of inertia

11.28.7 Practice Questions and Problems with Answers

Q.1: **(A)** What are the two essential requirements of the design of foundation?

(B) Mention five points indicating the differences between the design of foundation and the design of other elements of superstructure.

A.1: See sec. 11.28.1.

Q.2: Draw sketches of different shallow foundations.

A.2: Figure Nos. 11.28.1 to 11.

Q.3: Explain the difference between gross and net safe bearing capacities of soil. Which one is used for the design of foundation?

A.3: See sec. 11.28.3.

Q.4: How would you determine the minimum depth of foundation?

A.4: See sec.11.28.4.

Q.5: What are the critical sections of determining the bending moment in isolated footing?

A.5: See part (c)2 of sec.11.28.5.

Q.6: Explain the one-way and two-way shears of foundation slabs.

A.6: See part (d) of sec.11.28.5.

Q.7: Draw the actual distributions of base pressures of soil below the footing in sandy and clayey soils. Draw the assumed distribution of base pressure below the footing.

A.7: Figure Nos. 11.28.17 and 18.

Q.8: Draw the distributions of pressure in a footing for concentric and eccentric loadings ($e \leq L/6$ and $e > L/6$).

A.8: Figure Nos. 11.28.20 and 21.

Q.9: How would you determine the pressure at any point (x,y) of a foundation which is unsymmetrical?

A.9: See part (iii) of sec.11.28.6.

11.28.8 References

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11.28.9 Test 28 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

TQ.1: (A) What are the two essential requirements of the design of foundation? (5 marks)

(B) Mention five points indicating the differences between the design of foundation and the design of other elements of superstructure. (5 marks)

A.TQ.1: See sec. 11.28.1.

TQ.2: How would you determine the minimum depth of foundation? (10 marks)

A.TQ.2: See sec.11.28.4.

TQ.3: What are the critical sections of determining the bending moment in isolated footing? (10 marks)

A.TQ.3: See part (c)2 of sec.11.28.5.

TQ.4: Explain the one-way and two-way shears of foundation slabs. (10 marks)

A.TQ.4: See part (d) of sec.11.28.5.

TQ.5: Draw the distributions of pressure in a footing for concentric and eccentric loadings ($e \le L/6$ and $e > L/6$). (10 marks)

A.TQ.5: Figure Nos. 11.28.20 and 21.

10.26.11 Summary of this Lesson

This lesson explains the two major and other requirements of the design of foundation structures. Various types of shallow foundations and pile foundation are discussed explaining the distribution of pressure in isolated footings loaded concentrically and eccentrically with $e \le L/6$ and $e > L/6$. The gross and net safe bearing capacities are explained. The equation for determining the minimum depth of the foundation is given. Various design considerations in respect of minimum nominal cover, thickness at the edge of footing, bending moment, shear force, bond, tensile reinforcement, transfer of load at the base of the column, and minimum distribution reinforcement are discussed, mentioning the codal requirements. The actual and the assumed distributions of base pressure are discussed. The distributions of base pressure for concentric and eccentric loads with eccentricity $\le L/6$ and $> L/6$ are explained. Determination of bearing pressure of soil for unsymmetrical footing is also discussed.

All the discussions are relevant in understanding the load carrying mechanism of the foundation and the behaviour of soil. These understandings are essential in designing the foundation structures which is taken up in the next lesson.

Design.28.4 A square column 400 mm × 400 mm carries an axial load of 1500 kN. Design the column and a square footing for the column. The safe bearing capacity of the soil is 150 mm². Use M 20 concrete and Fe 250 steel.

Solution. Design of the column

Load on the column $W = 1500\ kN.$

Factored load $P_u = 1.5 \times 1500 = 2250\ kN.$

Overall area of the column section $= 500 \times 500 = 250000\ mm^2$

Area of Steel $= A_{sc}$

Area of concrete $= A_c = 250000 - A_{sc}$

Ultimate load $P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc} = 2250 \times 10^3$

$0.4 \times 20\,(250000 - A_{sc}) + 0.67 \times 250\, A_{sc} = 2250 \times 10^3$

$A_{sc} = 1567.4\ mm^2$

Provide 8 bars of 16 mm ϕ (1608 mm²)

Lateral Ties

Diameter of lateral ties shall not be less than.

(i) $\frac{1}{4}$ diameter of longitudinal bars $= \frac{16}{4} = 4\ mm,$

(ii) 5 *mm.*

Provide 6mm ϕ ties

Pitch of Lateral ties

The pitch of lateral ties shall not exceed the following.

(i) Least lateral dimension of the column = 400 *mm.*

(ii) 16 times the diameter of the longitudinal bars $= 16 \times 16 = 256\ mm.$

(iii) 48 times the diameter of ties $= 48 \times 6 = 288\ mm.$

Provide 6 mm ϕ ties @ 250 mm c/c.

Design of the Foundation.

Load on the column = 1500 *kN*

Approximate weight of footing = 150 *kN*

Total load = 1650 *kN.*

Safe bearing capacity of the soil $= 150\ kN/m^2$

∴ Area of foundation $= \frac{1650}{150} = 11\ m^2$

∴ Side of footing $= \sqrt{11} = 3.32\ m$

Provide 3.40 m × 3.40 m

Net upward pressure intensity $= p = \frac{1500 \times 10^3}{3.40 \times 3.40} = 129758\ N/m^2$

Depth from B.M. Consideration

Critical section for bending moment is shown in Fig.28.12 Projection beyond the critical section $\frac{3400 - 400}{2}$ $= 1500\ mm$

Maximum B.M. $= M = 129758 \times 3.40 \times 1.5 \times \frac{1.5}{2} = 496324.35\ Nm$

Factored moment

$M_u = 1.5 \times 496324.35 = 744486.53\ Nm$

Equating $M_{u,lim}$ to M_u

$0.149 f_{ck} b d^2 = 0.149 \times 20 \times 400\, d^2 = 744486.53 \times 10^3$

$d = 790.3\ mm$

Providing 12 mm ϕ bars at a clear cover of 60 mm.

Effective cover to the upper layer of bars

$= 60 + 12 + 6 = 78\ mm$

Overall depth required $= 790.3 + 78 = 868.3\ mm$

The overall depth may be by increased by 30% to limit the shear stresses

$\therefore$ Overall depth $= 1.3 \times 868.3 = 1128.8$ say 1130 mm

Depth from punching shear consideration

Punching load = Column load – Reaction on the column area

$= 1500 \times 10^3 - 129758 \times 0.40^2 = 1479238.7\ N$

Factored punching load $= 1.5 \times 1479238.7 = 2218858.1\ N$

Equating punching shear resistance to factored punching load,

$$4 \times 400 \times D \times 1.8 = 2218858.1$$

$$\therefore \quad D = 770\ mm$$

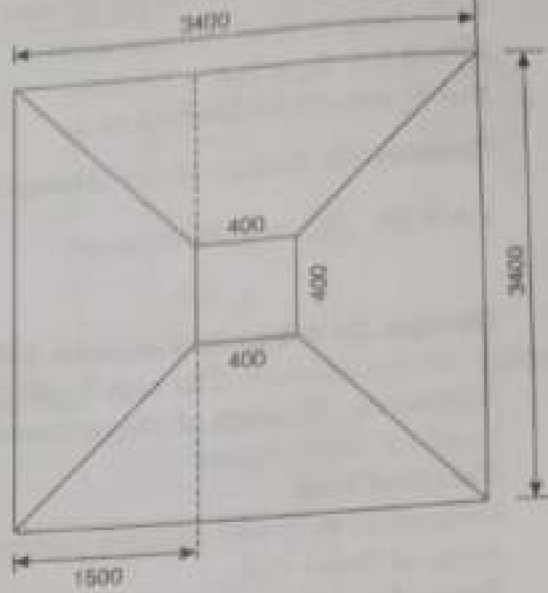


Fig. 28.12

Hence let us provide an overall depth of 1130 mm as determined earlier.

Actual effective depth $d = 1130 - 78 = 1052\ mm$

$$\frac{M_u}{bd^2} = \frac{744486.53 \times 1000}{400 \times 1052^2} = 1.682$$

Percentage of steel required

$$p_t = 50\left[\frac{1-\sqrt{1-\frac{4.6}{20}\times 1.682}}{\frac{250}{20}}\right] = 0.868\%$$

$$\therefore \quad A_{st} = \frac{0.868}{100} \times 400 \times 1052 = 3653\ mm^2$$

Provide 33 bars of 12 mm ϕ (3729 mm²)

Provide also 33 bars of 12 mm ϕ in the other principal direction also.

Check for Shear

(i) ***Check for one-way shear*** The critical section for one-way shear is considered at a distance equal to the effective depth from the face of the column.

Let the depth of the footing be reduced to 400 mm at the edges.

$\therefore$ Overall depth at the critical section.

$$= D' = 1130 - \frac{(1130 - 400)}{1500} \times 1052 = 618.03\ mm$$

Effective depth at the critical section $= d' = 618.03 - 78 = 540.03\ mm$

Shear force at this critical section

$= 129758 \times 3.40 \times 0.448 = 197647.4\ N$

Factoned shear $V_u = 1.5 \times 197647.4 = 296471.1\ N$

Width of the footing of the top at this critical section

$= b' = b + 2d = 400 + 2 \times 1052 = 2504\ mm$

Nominal shear stress at this critical section

$$\tau_u = \frac{V_u}{b'd'} = \frac{296471.1}{2504 \times 540.03} = 0.22\ N/mm^2$$

Area of steel provided

$A_{st} = 33 \times 113 = 3729\ mm^2$

Percentage of steel provided $= \dfrac{3729 \times 100}{2504 \times 540.03} = 0.28\%$

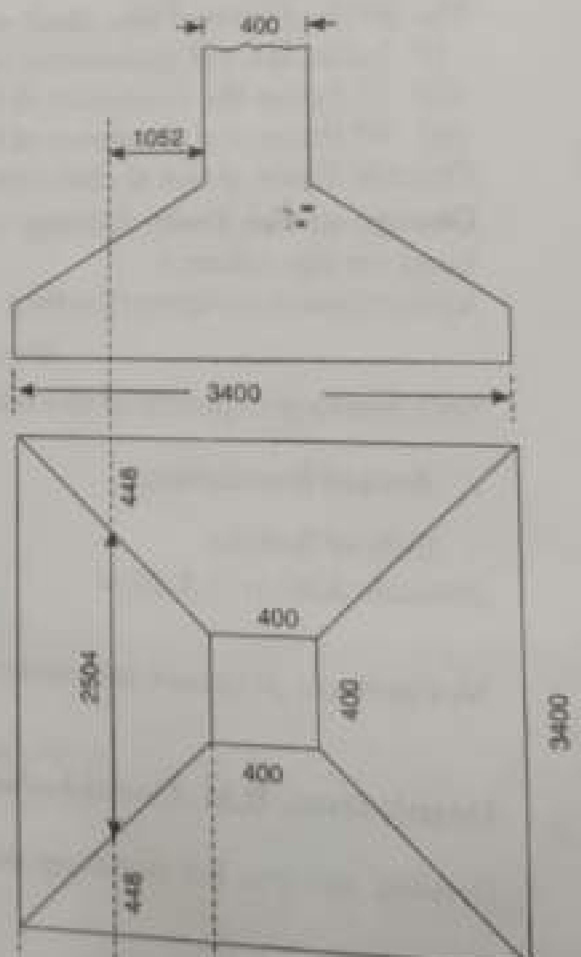


Fig. 28.13.

Design shear strength corresponding to the above percentage of steel $= \tau_c = 0.38\ N/mm^2$

$$\tau_v < \tau_v$$

(ii) Check for two way shear

The critical section for two-way shear is taken at the periphery surrounding the column at a distance of half the effective depth of the footing from the face of the column.

Overall depth of the footing at a distance $\frac{d}{2} = \frac{1052}{2} = 526\ mm$ from the column face.

$$D' = 1130 - \frac{(1130 - 400)}{1500} \times 526 = 874\ mm$$

Effective depth of this critical section

$$d' = 874 - 78 = 796\ mm$$

Critical perimeter $= b' = 4\ (400 + 1052) = 5808\ mm$

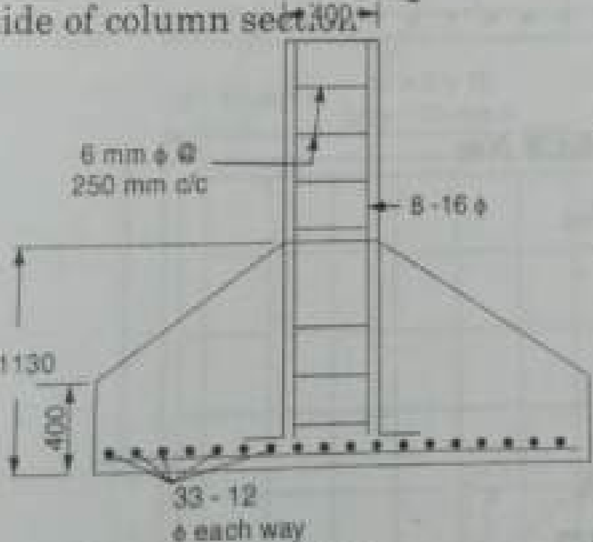
Shear force at the critical section

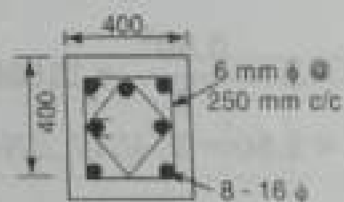
$= V = 129758\ (3.40^2 - 1.452)^2 = 1226433.2\ N$

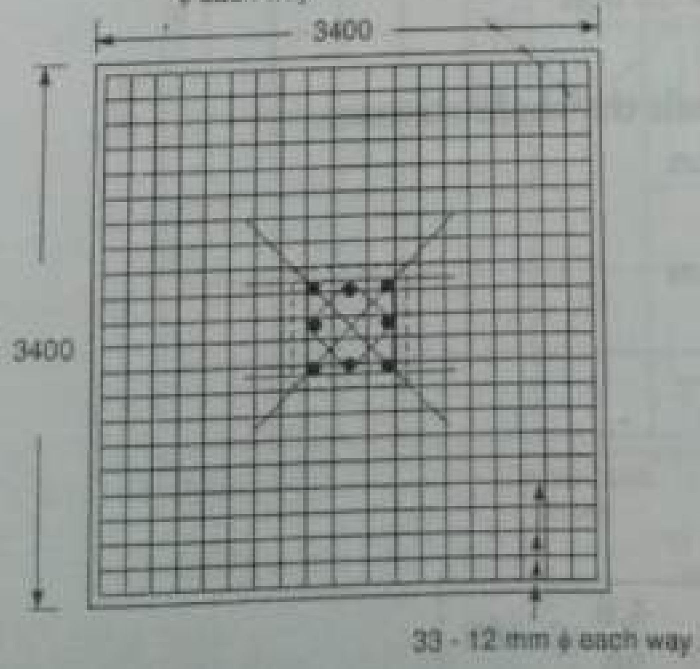
Factored Shear $V_u = 1.5 \times 1226433.2 = 1839649.8\ N$

$$\text{Nominal shear stress } \tau_v = \frac{1839649.8}{5808 \times 796} = 0.40\ N/mm^2$$

$$\beta_c = \frac{\text{Short side of column section}}{\text{Long side of column section}} = 1$$







$K_s = 0.5 + \beta_c = 0.5 + 1 = 1.5$

Permissible design shear stress $= K_s \times 0.25$

$$= 1 \times 0.25\ \sqrt{20} = 1.12\ N/mm^2$$

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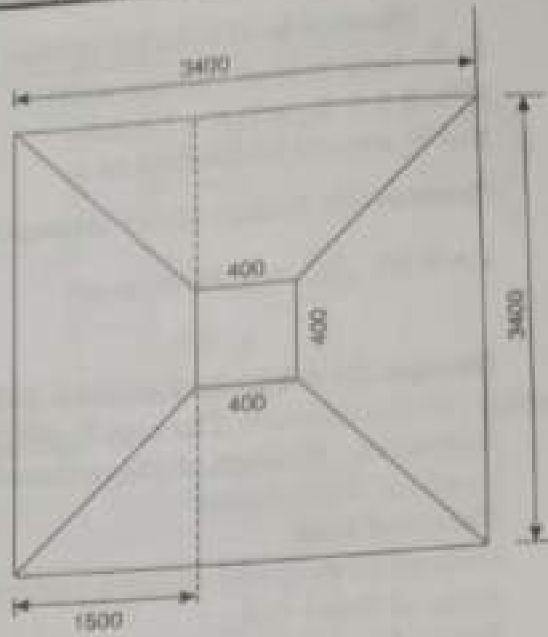

Fig. 28.12

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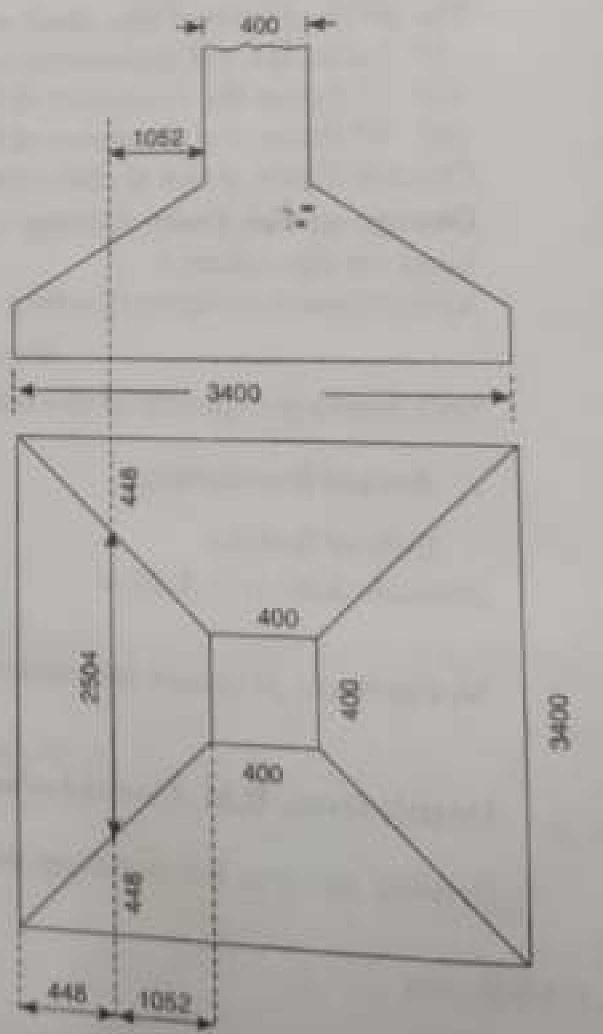

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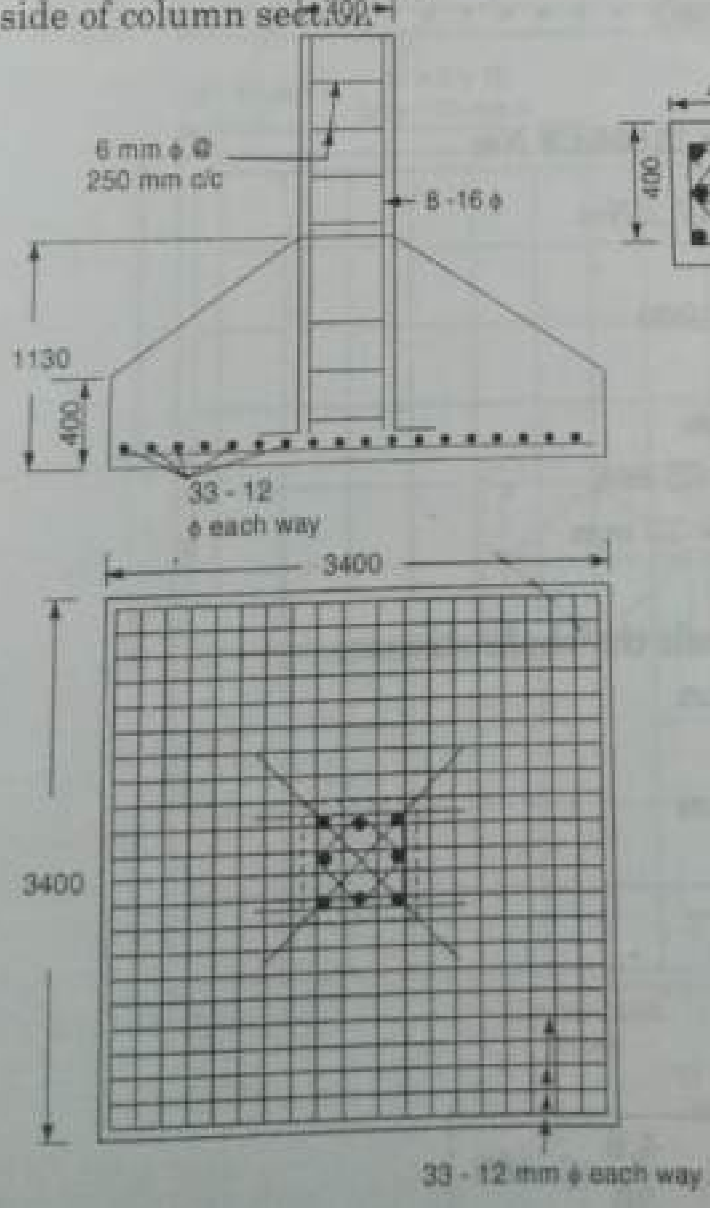

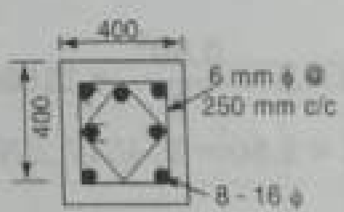

$K_s = 0.5 + \beta_c = 0.5 + 1 = 1.5$

Permissible design shear stress $= K_s \times 0.25$

$= 1 \times 0.25\ \sqrt{20} = 1.12\ N/mm^2$

Fig. 28.34. Region of transverse bending.

Design. 28.9 *Design a reinforced concrete combined rectangular footing for two columns A and B located 3.60 metres apart. The sizes of the columns are 400 mm × 400 mm and 600 mm × 600 mm and the loads on them are 1000 kN and 1500 kN respectively. The projection of the footing parallel to the length of the footing beyond the axis of the column A is limited to 590 mm. The safe bearing capacity of the soil is 280 kN/m². Use M 20 concrete and Fe 415 steel.*

Solution.

Total load on the two columns = 1000 + 1500 = 2500 kN

Approximate weight of foundation (10% of column load) = 250 kN

Total load transmitted to the soil = 2750 kN

Safe bearing capacity of the soil = 280 kN/m^2

$\therefore$ Area of the foundation $= \frac{2750}{280} = 9.821\ m^2$

Distance of the resultant column load from the axis of column $A = \frac{1500 \times 3.60}{2500} = 2.16\ m$

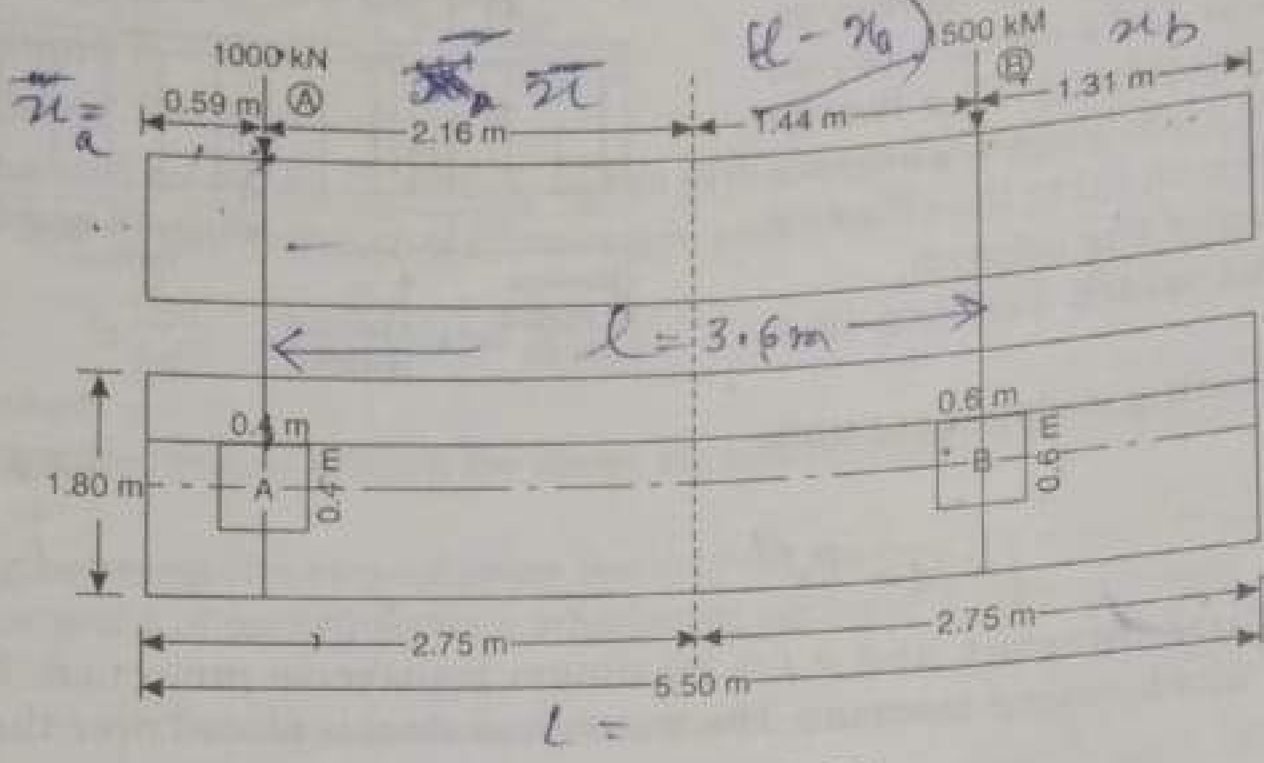

Fig. 28.25.

Distance of the resultant column load from the left edge of the footing = 0.59 + 2.16 = 2.75 m

For the condition that the resultant of the column loads must pass through the centroid of the foundation plan, length of the footing = $L = 2 \times 2.75 = 5.50\ m$

$\therefore$ Width of the footing $= B = \frac{9.821}{5.50} = 1.79\ m$ say 1.80 m

Fig. 28.29 shows the position of the two columns on the foundation plan.

Net upward pressure intensity on the footing.

$$= p = \frac{1000 + 1500}{5.5 \times 1.8} = 252.52\ kN/m^2$$

Depth of the footing

(i) ***Punching shear consideration under the column A***

Punching load = Column load – Reaction of soil on column area

$= 1000 - 252.52 \times 0.40^2 = 959.60\ kN = 959600\ N$

Factored punching load $= 1.5 \times 959600 = 1439400\ N$

Design punching shear stress $= 1.80\ N/mm^2$

Equating punching shear resistance to the factored punching load

$4 \times 400 \times D \times 1.80 = 1439400 \quad \therefore \quad D = 499.8\ mm$

(ii) ***Punching shear consideration under the column B***

Punching load $= 1500 - 252.52 \times 0.60^2 = 1409\ kN$

Factored punching load $= 1.5 \times 1409 = 2113.5\ kN = 2113500\ N$

Equating punching shear resistance to the factored punching load

$4 \times 600 \times D \times 1.80 = 2113500 \quad \therefore \quad D = 489.2\ mm$

(iii) ***B.M. Consideration***

Factored load on column $A = 1.5 \times 1000 = 1500\ kN$

Factored load on column $B = 1.5 \times 1500 = 2250\ kN$

Reaction of soil on the footing per *metre* run $= \frac{1500 + 2250}{5.50} = 681.818\ kN/m$

B.M. under the coloumn $A = M_u = +\ 681.818 \times \frac{0.59^2}{2} = +\ 118.67\ kNm$

B.M. under the column $B = M_u = +\ 681.818 \times \frac{1.31^2}{2} = +\ 585.03\ kNm$

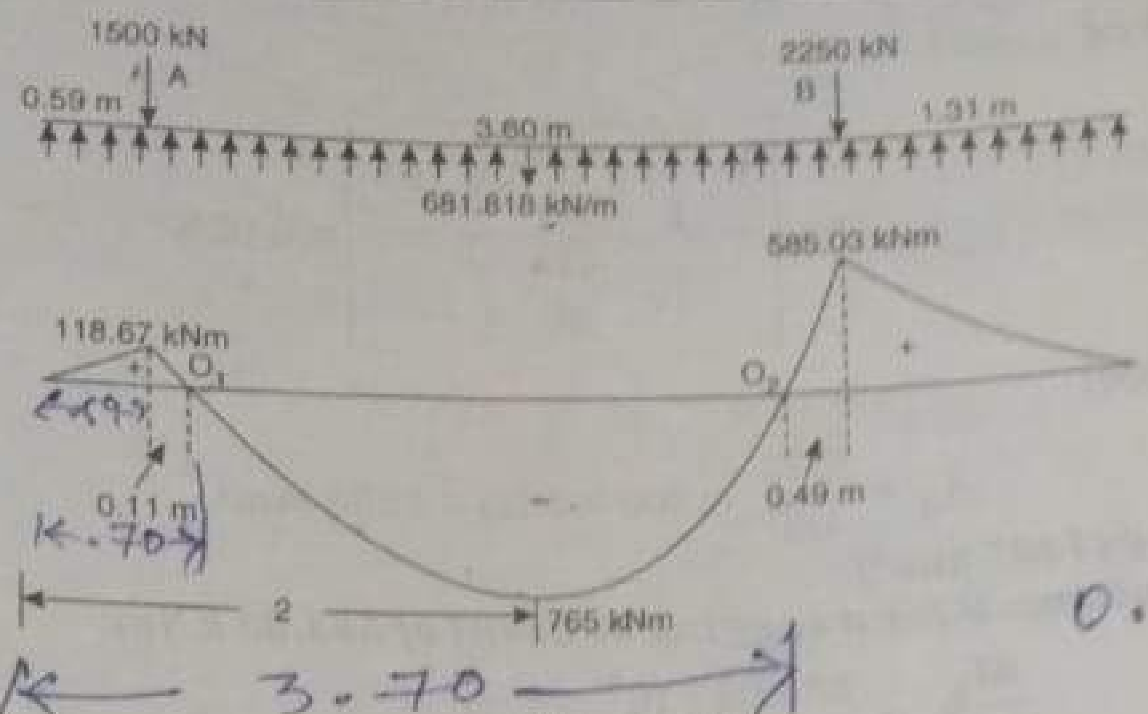

Fig. 28.36

The maximum hogging moment occurs at a section where the shear force is zero. Let this section be x metres from the left end.

Equating the shear force to zero,

$681.818\,x - 1500 = 0$

$\therefore \quad x = 2.20\ m$

$$\therefore \quad M_{max} = 681.818 \times \frac{2.20^2}{2} - 1500\,(2.20 - 0.59) = -765\ kNm$$

Points of Contraflexure

Equating the general expression for bending moment to zero,

$$681.818\,\frac{x^2}{2} - 1500\,(x - 0.59) = 0$$

or $x^2 - 4.4\,x + 2.596 = 0$

Solving we get, $x = 0.70\ m$ and $3.70\ m$ from the left end.

Equating $M_{u,\,lim}$ to M_{max}

$0.138\,f_{ck}\,bd^2 = 0.138 \times 20 \times 1800\,d^2 = 765 \times 10^6 \qquad \therefore \quad d = 392.4\ mm$

Providing 16 mm ϕ bars at a clear cover of 60 mm

Effective cover to the centre of steel = 60 + 8 = 68 mm

Overall depth = 392.4 + 68 = 452.4 mm

For economic use of steel increase the depth by 40%

$\therefore$ Overall depth = 1.40 × 452-4 = 634 mm

Provide an overall depth of 650 mm

Actual effective depth = d = 650 − 68 = 582 mm

Reinforcement from B.M. Consideration

Top steel for maximum hogging moment of 765 kNm

$$\frac{M_{max}}{bd^2} = \frac{765 \times 10^6}{1800 \times 582^2} = 1.255$$

Percentage of steel required

$$p_t = 50\left[\frac{1 - \sqrt{1 - \frac{4.6}{20} \times 1.255}}{\frac{415}{20}}\right] = 0.377$$

$$\therefore \quad A_{st} = \frac{0.377}{100}\,(1800 \times 582) = 3950\ mm^2$$

Provide 20 bars of 16 mm ϕ (4020 mm²)

Bottom steel under the column A for a sagging moment of 118.67 kNm

$$\frac{M_a}{bd^2} = \frac{118.67 \times 10^6}{1800 \times 582^2} = 0.195$$

Design. 28.18. A building rests on six columns 450 mm × 450 mm arranged as shown in Fig. 28.85. Each central column carries a load of 800 kN and the end columns carry 500 kN each. Design a raft foundation for the column. The design shall also allow for a wind load moment of 1200 kNm about the base of the raft. Use M 20 concrete and Fe. 415 steel.

Solution. Total load on the columns $= 2 \times 800 + 4 \times 500 = 3600\ kN$

Approximate weight of foundation

@ 10% column loads $= 360\ kN$

Total $= 3960\ kN$

Total moment about the base $= 1200\ kNm$

$\therefore$ Eccentricity of the load $= \dfrac{1200}{3960} = 0.303\ m$

Extreme pressure intensity at the base

$$= \frac{3960}{12 \times 7}\left\{1 \pm \frac{6 \times 0.303}{7}\right\}\ kN/m^2$$

$\therefore$ $P_{max} = 59.4\ kN/m^2$

and $P_{min} = 34.9\ kN/m^2$

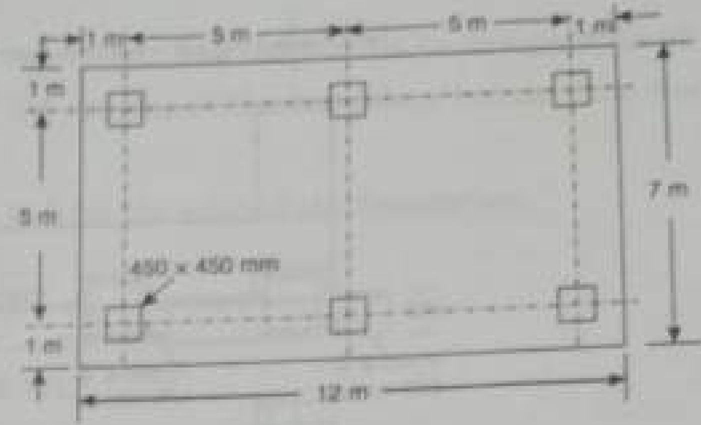

Fig. 28.85.

Pressure due to weight of foundation $= \dfrac{360}{12 \times 7} = 4.3\ kN/m^2$

$\therefore$ Net pressure,

$$p'_{max} = 59.4 - 4.3 = 55.1\ kN/m^2$$
$$p'_{min} = 34.9 - 4.3 = 30.6\ kN/m^2$$

Since the columns are 450 *mm* × 450 *mm*, let the width of the main beams be 500 *mm*.

Cantilevering projection $= 1.00 - 0.25 = 0.75\ m$

Upward pressure at 0.75 *m* from the edge

$$= 55.1 - \frac{55.1 - 30.6}{7} \times 0.75 = 52.5\ kN/m^2$$

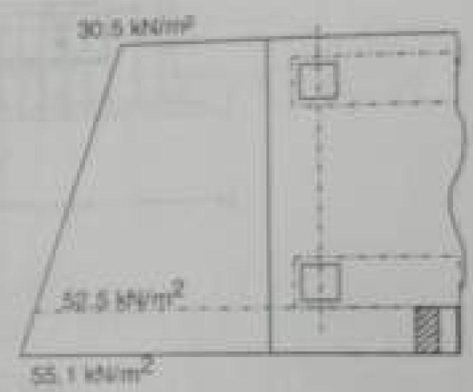

Fig. 28.86.

Consider a one *metre* wide strip of the cantilever slab.

Upward load on the cantilever $= \dfrac{55.1 + 52.5}{2} \times 0.75\ kN = 40.4\ kN$

This acts at $\dfrac{52.5 + 2 \times 55.1}{52.5 + 55.1} \times \dfrac{0.75}{3}$ *metre* from the edge of beam

$= 0.39\ m$ from the edge of beam

$\therefore$ Maximum bending moment for the cantilever slab per *metre* width

$= 40.4 \times 0.39\ kNm = 15.7\ kNm$

Factored moment $= 1.5 \times 15.7 = 23.55\ kNm$

Equating $M_{u,lim}$ to M_u

$$0.138 f_{ck} bd^2 = 0.138 \times 20 \times 1000\ d^2 = 23.55 \times 10^6$$
$$\therefore\ d = 93\ mm$$

Provide an overall depth of 180 mm

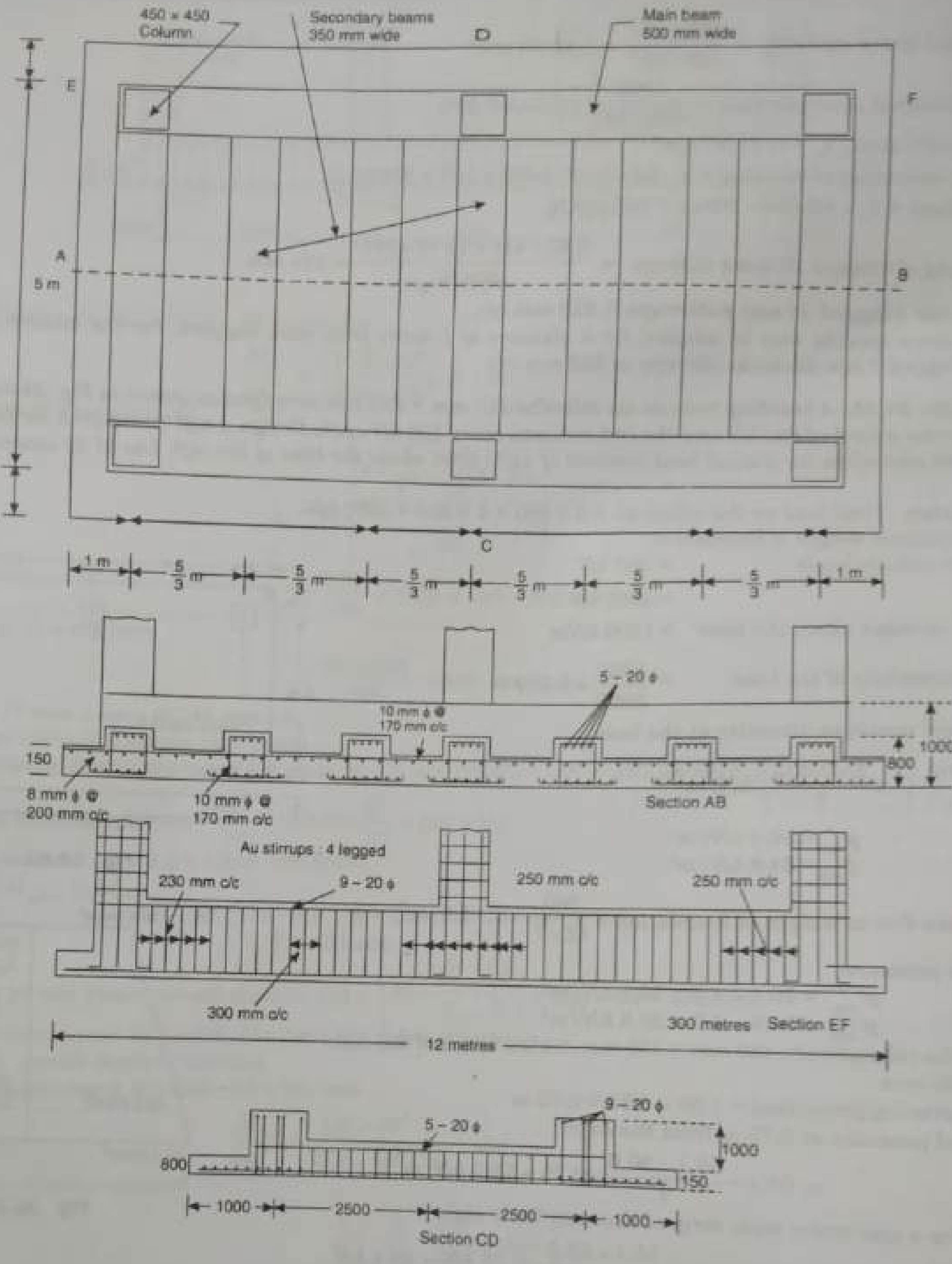

Fig. 28.87.

Providing 10 *mm* ϕ bars at a clear cover of 60 *mm*,
Effective depth $d = 180 - (60 + 5) = 115\ mm$

$$\frac{M_u}{bd^2} = \frac{23.55 \times 10^6}{1000 \times 115^2} = 1.78$$

Percentage of steel required

$$p_t = 50\left[\frac{1-\sqrt{1-\frac{4.6}{20}\times 1.78}}{\frac{415}{20}}\right] = 0.56\%$$

$$A_{st} = \frac{0.56}{100}\times 1000\times 115 = 644\ mm^2$$

Spacing of 10 *mm* ϕ bars $= \frac{79\times 1000}{644} = 122\ mm$

Provide 10 mm ϕ bars @ 120 mm c/c

Distribution steel $= \frac{0.12}{100}\times 1000\times 180 = 216\ mm^2$

Spacing of 8 *mm* ϕ bars $= \frac{50\times 1000}{216} = 231\ mm$

Provide 8 mm ∠ bars @ 200 mm c/c

Continuous Slab It is proposed to provide secondary beams at $\frac{5}{3}$ *m* centres. The slab should be designed for an upward pressure corresponding to a section at a distance equal to half the spacing of the secondary beams from the centre of the main beams.

In our case, the pressure is calculated at a distance of $1+\frac{5}{6} = 1.83\ m$ from the edge of the foundation.

The pressure at this section $= 55.1 - \frac{55.1-30.6}{7}\times 1.83 = 48.7\ kN/m2$

Maximum bending moment per metre width $= \frac{48.7}{12}\times\left(\frac{5}{3}\right)^2 = 11.27\ kNm$

Factored moment, $M_u = 1.5\times 11.27 = 16.905\ kNm.$

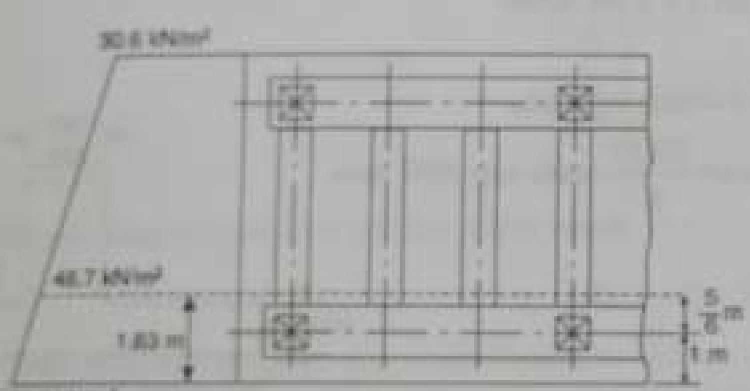

Fig. 28.88.

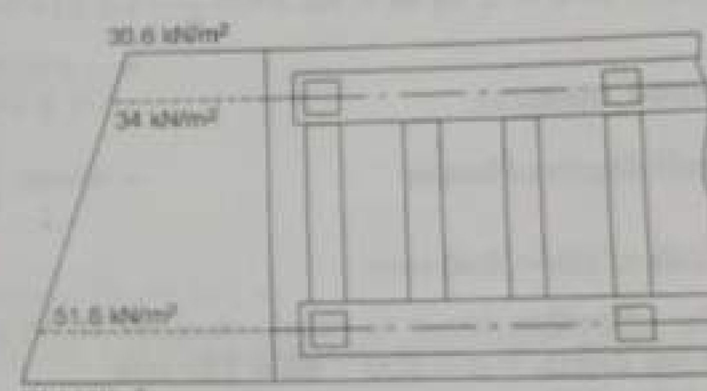

Fig. 28.89.

$$\frac{M_u}{bd^2} = \frac{16.905\times 10^6}{1000\times 115^2} = 1.278$$

Percentage of steel required

$$p_t = 50\left[\frac{1-\sqrt{1-\frac{4.6}{20}\times 1.278}}{\frac{415}{50}}\right] = 0.385\%$$

$$A_{st} = \frac{0.385}{100}\times 1000\times 115 = 443\ mm^2$$

Spacing of 10 *mm* ϕ bars $= \frac{79\times 1000}{443} = 178\ mm$

Provide 10 mm ϕ bars @ 170 mm c/c.

Secondary beams. These are designed as T - beams.

The loading on an intermediate secondary beam is less than that on the end secondary beam. The loading on the end secondary beam varies

From $51.6 \times 1\frac{5}{6} = 94.6\ kN/m$

to $34.1 \times 1\frac{5}{6} = 62.5\ kN/m$

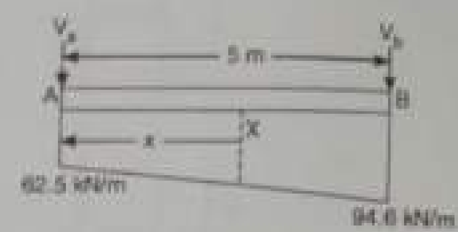

Fig. 28.90.

Total load $= \frac{62.5+94.6}{2} \times 5 = 392.8\ kN$

This acts at $= \left(\frac{62.5+2\times 94.6}{62.5\times 94.6}\right)\times\frac{5}{3} m$ from $A = 2.67\ m$ from A

Taking moments about A, we have

$$\therefore \quad V_b \times 5 = 392.8 \times 2.67$$
$$\therefore \quad V_b = 209.7\ kN$$
$$V_a = 392.8 - 209.7 = 183.1\ kN$$

Shear force at any section X distant x m from A

$$S = 62.5\,x + \frac{1}{2}\,x \times \frac{32.1\,x}{5} - 183.1\ kN$$
$$\therefore \quad S = 62.5\,x + 3.21\,x^2 - 183.1\ kN$$

$$\text{B.M. at section } X = M = 62.5\,\frac{x^2}{2} + 3.21\,\frac{x^3}{3} - 183.1\,x\ kNm$$

[Expression for M is obtained easily by integrating the expression for S]

Maximum Bending Moment. This occurs at a section where the shear force is zero. Equating the general expression for shear force to zero, we have,

$$62.5\,x + 3.21\,x^2 - 183.1 = 0$$
$$x^2 + 19.47\,x - 57.05 = 0$$

Solving, we get, $x = 2.58\ m$

$$\therefore \quad \text{B.M. at } x = 2.58\ m = 62.5\times\frac{2.58^2}{2} + 3.21\times\frac{2.58^3}{3} - 183.1\times 2.58\ kNm$$
$$= -246.02\ kNm$$

Factored moment $= -1.5 \times 246.02 = -369.03\ kNm$

Overall depth of beam $=$ about $\frac{1}{6}$ of span$=\frac{5000}{6}$ *mm say* 800 *mm*

Breadth of rib = 350 *mm*

Let the effective cover to reinforcement be 80 *mm*

Effective depth = d = 800 − 80 = 720 *mm*

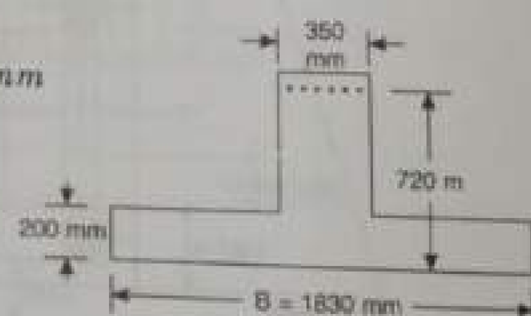

Fig. 28.91.

Breadth of Flange

This shall be taken as the lesser of the following

(*i*) $1 + \frac{5}{6} = 1.83\ m = 1830\ mm$

(*ii*) $\frac{l}{6} + b_r + 6d_s = \frac{5000}{6} + 350 + 6 \times 180 = 2263\ mm$

Hence width of flange B = 2263 *mm*. Assuming the neutral axis to lie within the flange and equating the ultimate moment of resistance to the factored moment,

$$0.36 \times 20 \times 1830\,x_u\,(720 - 0.42\,x_u) = 369.03 \times 10^6$$
$$x_u\,(720 - 0.42\,x_u) = 28007.741$$
$$x_u^2 - 1714.29\,x_u + 66685.10 = 0$$
$$x_u = 39.82\ mm \text{ (Less than 180 } mm)$$

Total compression = Total tension

$$0.36 \times 20 \times 1830 \times 39.82 = 0.87 \times 415\,A_{st}$$
$$\therefore \quad A_{st} = 1453.2\ mm^2$$

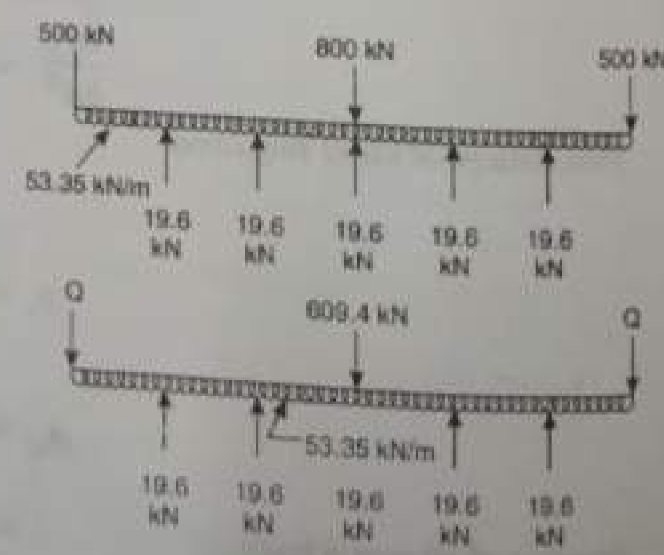

Fig. 28.92

Provide 5 bars of 20 mm ϕ (1570 mm^2)

Shear Analysis

Factored shear at the end $B = 1.5 \times 209.7 = 314.55\ kN$

Factored shear at the end $A = 1.5 \times 183.1 = 274.65\ kN$

Nominal shear stress at the end $B = \dfrac{314.55 \times 10^3}{350 \times 720} = 1.25\ N/mm^2$

Nominal shear stress at the end $A = \dfrac{274.65 \times 10^3}{350 \times 720} = 1.09\ N/mm^2$

Percentage of steel provided $= \dfrac{A_{st}}{bd} \times 100 = \dfrac{1570}{350 \times 720} \times 100 = 0.62\%$

For 0.62% steel $\tau_c = 0.52\ N/mm^2$

Shear resistance of concrete $S_c = 0.52 \times 350 \times 720 = 131040\ N = 131.04\ kN$

Position of this section in the zone of negative shear

$1.5\,(62.5\,x + 3.21\,x^2 - 183.1) = -131.04$

$x^2 + 19.47\,x - 29.83 = 0$

$x = 1.43\ m$ from the end A.

Position of this section in the zone of positive shear

$1.5\,(62.5\,x + 3.21\,x^2 - 183.1) = +131.04$

$x^2 + 19.47\,x - 84.25 = 0$

$\therefore \quad x = 3.65\ m$ from the end A

Main Beams

Each main beam is subjected to the following loads :

(*i*) Upward concentrated loads transmitted by secondary beams.

(*ii*) Upward uniformly distributed load transferred by the slab cantilevering from the main beams.

(*iii*) Downward column loads.

Maximum load transmitted by an end secondary beam.

$= 209.7\ kN$

$\therefore$ Maximum load transmitted by an intermediate secondary beam

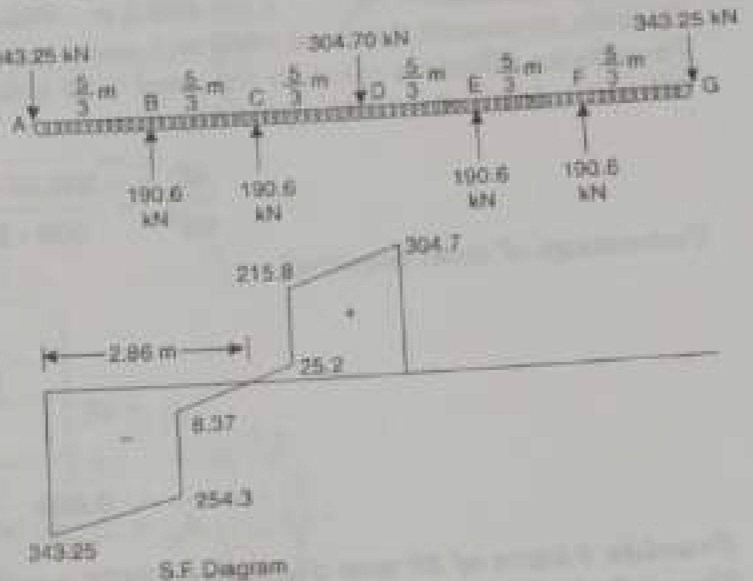

Fig. 28.93

$$= \left[\frac{\frac{5}{3}}{1\frac{5}{6}}\right] \times 209.7\ kN = 190.6\ kN$$

Uniformly distributed load on the main beam

$$= \frac{1}{2}\left[55.1 + 51.6\right] \times 1\ kN/m = 53.35\ kN/m$$

The load system acting on the main beam is shown in Fig. 28.92. Let the resultant point load at each end be Q.

Resolving the forces on the main beam, vertically, we have,

$$2Q + 609.4 = 190.6 \times 4 + 53.35 \times 10$$

$$\therefore \quad Q = 343.25\ kN$$

S.F. Calculations

$$S_a = -343.25\ kN$$

$$S_{ba} = -343.25 + 53.35 \times \frac{5}{3} = -254.3\ kN$$

$$S_{bc} = -254.4 + 190.6 = -63.7\ kN$$

$$S_{cb} = -63.7 + 53.35 \times \frac{5}{3} = +25.2\ kN$$

$$S_{cd} = +25.2 + 190.6 = +215.8\ kN$$

$$S_{dc} = +215.8 + 53.35 \times \frac{5}{3} = +304.7\ kN$$

B.M. Calculations

$$M_a = 0$$

$$M_b = -343.25 \times \frac{5}{3} + \frac{53.35}{2}\left(\frac{5}{2}\right)^2 = -498\ kNm$$

$$M_c = -343.25 \times \frac{10}{3} + 190.6 \times \frac{5}{3} + \frac{53.35}{2}\left(\frac{10}{3}\right)^2 = -530.1\ kNm$$

$$M_d = -343.25 \times 5 + 190.6 \times \frac{10}{3} + \frac{53.25}{2} \times 5^2 = -96.5\ kNm$$

Point of zero shear between B and C

Let at a distance of x *metres* the S.F. be zero

$\therefore$

$\therefore$ $53.35\,x + 190.6 = 343.25$

$x = 2.86\ m$

$\therefore$ B.M. at $x = 2.86\ m$ $= M_{max} = -343.25 \times 2.86 + \frac{53.35}{2}(2.86)^2 + 190.6\left(2.86 - \frac{5}{3}\right) kNm$

$= -536.7\ kNm$

Factored moment $= M_u = -1.5 \times 536.7 = -805.05\ kNm$

Provide an overall depth of 1000 *mm* and a width of 500 *mm*

Provide an effective cover to top steel = 80 *mm*

Effective depth $d = 1000 - 80 = 920\ mm$

$$\frac{M_u}{bd^2} = \frac{805.05 \times 10^6}{500 \times 920^2} = 1.902$$

Percentage of steel required

$$p_t = 50\left[\frac{1 - \sqrt{1 - \frac{4.6}{20} \times 1.902}}{\frac{415}{20}}\right] = 0.602\%$$

$$A_{st} = \frac{0.602}{100} \times 500 \times 920 = 2769.2\ mm^2$$

Provide 9 bars of 20 mm diameter (2826 mm²)

Shear Reinforcement

Nominal shear stress due to factored shear force of $1.5 \times 343.25\ kN$

$$= \frac{1.5 \times 343.25 \times 10^3}{500 \times 920} = 1.12\ N/mm^2$$

Nominal shear stress due to factored shear force of $1.5 \times 254.3\ kN$

$$= \frac{1.5 \times 254.3 \times 10^3}{500 \times 920} = 0.83\ N/mm^2$$

Nominal shear stress due to factored shear force of $1.5 \times 304.7\ kN$

$$= \frac{1.5 \times 304.7 \times 10^3}{500 \times 920} = 0.99\ N/mm^2$$

Beam between A and B, and F and G

$$V_u = 1.5 \times 343.25 = 514.875\ kN$$

Percentage of steel $= \frac{2826}{500 \times 920} \times 100 = 0.614\%$

Corresponding to 0.614% steel, $\tau_c = 0.51\ N/mm^2$.

Shear resistance of concrete $= \tau_c\ bd = 0.51 \times 500 \times 920 = 234600\ N$

Net shear $V_s = 514875 - 234600 = 280275\ N$

Spacing of 4 legged 8 *mm* ϕ stirrups $= \frac{0.87 \times 415 \times 4 \times 50 \times 920}{280275} = 237\ mm$

Provide 4 legged 8 mm ϕ stirrups @ 230 mm c/c

Beam between *C* and *D*, and *D* and *E*

$$V_u = 1.5 \times 304.7 = 457.05\ kN$$

Shear resistance of concrete = 234600 N

Net shear $V_s = 457050 - 234600 = 222450\ N$

Spacing of 4 legged 8 mm ϕ stirrups $= \frac{0.87 \times 415 \times 4 \times 50 \times 920}{222450} = 298\ mm$

Provide 4 legged 8 mm ϕ stirrups @ 250 mm c/c

Module 10

Compression Members

Lesson 21

Definitions, Classifications, Guidelines and Assumptions

Instructional Objectives:

At the end of this lesson, the student should be able to:

- define effective length, pedestal, column and wall,
- classify the columns based on types of reinforcement, loadings and slenderness ratios,
- identify and explain the functions of bracing in a braced column,
- determine the minimum and maximum percentage of longitudinal reinforcement,
- determine the minimum numbers and diameter of bars in rectangular and circular columns,
- determine the longitudinal reinforcement in a pedestal,
- determine the type, pitch and diameter of lateral ties of columns after determining the longitudinal steel,
- state the assumptions in the design of compression member by limit state of collapse,
- determine the strain distribution lines of a compression member subjected to axial load with or without the moments about one or both the axes,
- explain the need of the minimum eccentricity to be considered in the design of compression members.

10.21.1 Introduction

Compression members are structural elements primarily subjected to axial compressive forces and hence, their design is guided by considerations of strength and buckling. Figures 10.21.1a to c show their examples: pedestal, column, wall and strut. While pedestal, column and wall carry the loads along its length *l* in vertical direction, the strut in truss carries loads in any direction. The letters *l*, *b* and *D* represent the unsupported vertical length, horizontal lest lateral dimension, width and the horizontal longer lateral dimension, depth. These compression members may be made of bricks or reinforced concrete. Herein, reinforced concrete compression members are only discussed.

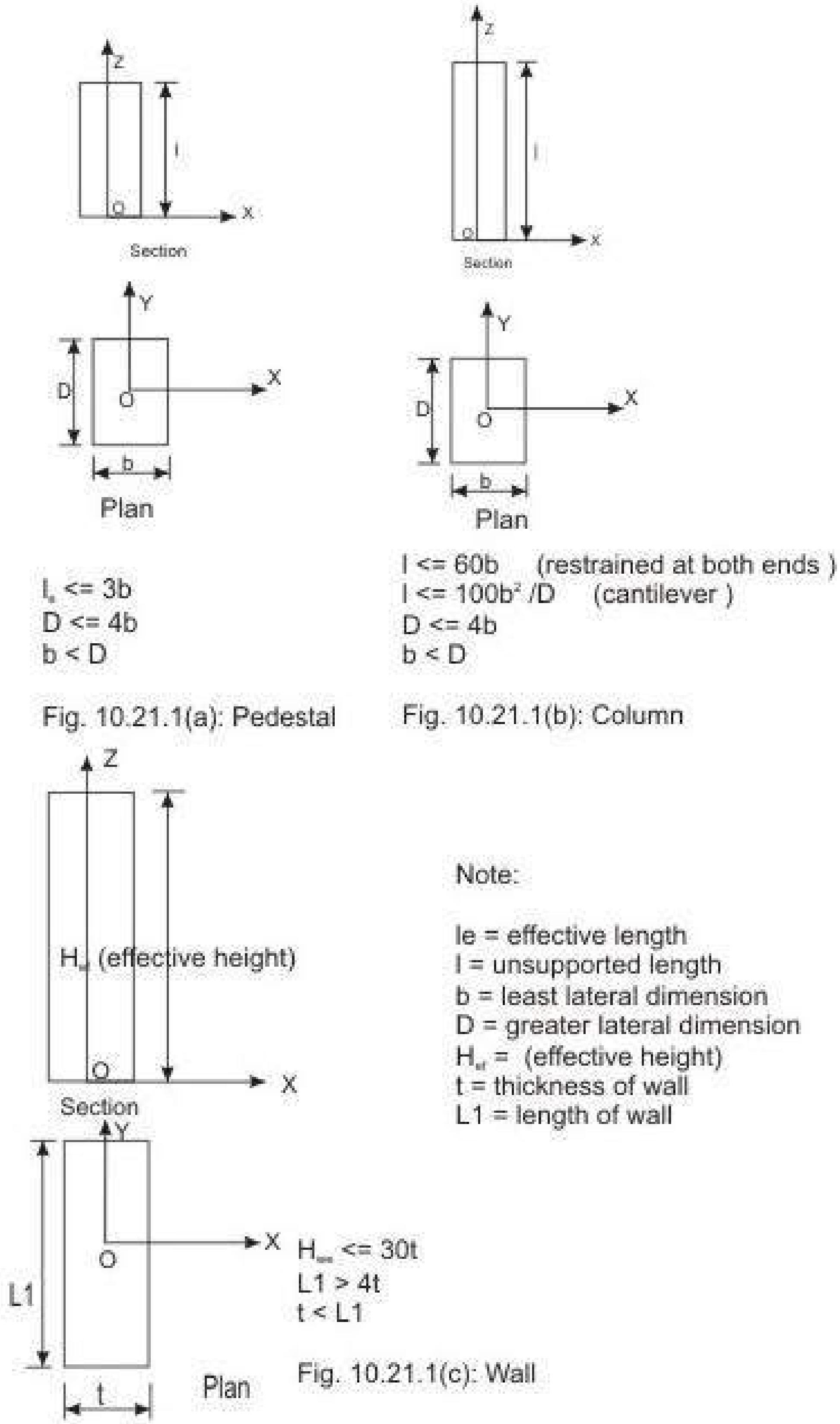

Fig. 10.21.1: Pedestal, column and wall

This module is intended to explain the definition of some common terminologies and to illustrate the design of compression members and other related issues. This lesson, however, explain the definitions and classifications of columns depending on different aspects. Further, the recommendations of IS 456 to be followed in the design are discussed regarding the longitudinal and lateral reinforcing bars. The assumptions made in the design of compression member by limit sate of collapse are illustrated.

10.21.2 Definitions

(a) Effective length: The vertical distance between the points of inflection of the compression member in the buckled configuration in a plane is termed as effective length l_e of that compression member in that plane. The effective length is different from the unsupported length l of the member, though it depends on the unsupported length and the type of end restraints. The relation between the effective and unsupported lengths of any compression member is

$$l_e = k\,l \tag{10.1}$$

where k is the ratio of effective to the unsupported lengths. Clause 25.2 of IS 456 stipulates the effective lengths of compression members (vide Annex E of IS 456). This parameter is needed in classifying and designing the compression members.

(b) Pedestal: Pedestal is a vertical compression member whose effective length l_e does not exceed three times of its least horizontal dimension b (cl. 26.5.3.1h, Note). The other horizontal dimension D shall not exceed four times of b (Fig.10.21.1a).

(c) Column: Column is a vertical compression member whose unsupported length l shall not exceed sixty times of b (least lateral dimension), if restrained at the two ends. Further, its unsupported length of a cantilever column shall not exceed $100b^2/D$, where D is the larger lateral dimension which is also restricted up to four times of b (vide cl. 25.3 of IS 456 and Fig.10.21.1b).

(d) Wall: Wall is a vertical compression member whose effective height H_{we} to thickness t (least lateral dimension) shall not exceed 30 (cl. 32.2.3 of IS 456). The larger horizontal dimension i.e., the length of the wall L is more than 4t (Fig.10.21.1c).

10.21.3 Classification of Columns Based on Types of Reinforcement

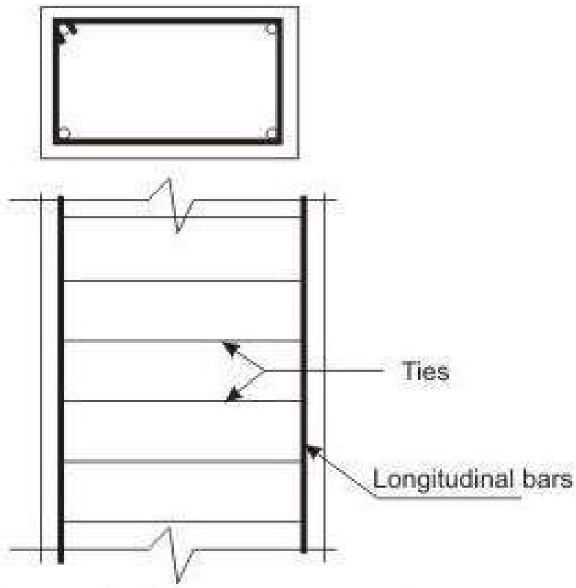

Fig. 10.21.2(a): Tied column

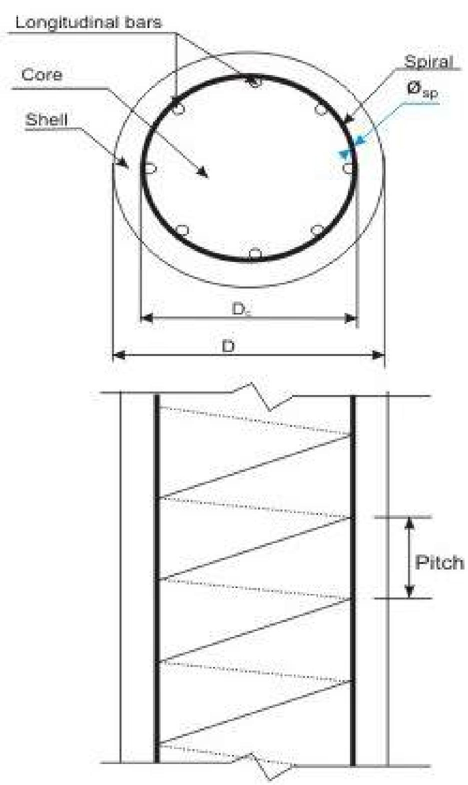

Fig. 10.21.2b: Column with helical reinforcement

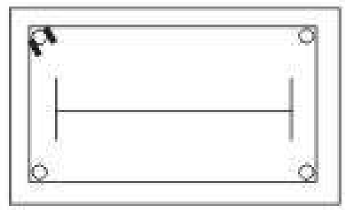

Fig. 10.21.2(c): Composite column (steel section)

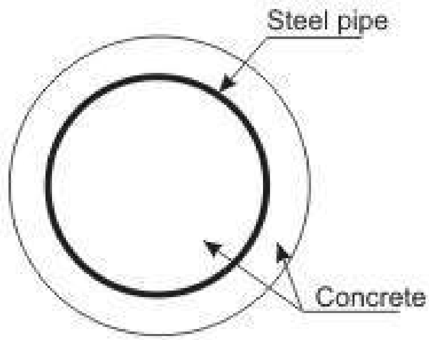

Fig. 10.21.2(d): Composite column (steel pipe)

Fig. 10.21.2: Tied, helically bound & composite columns

Based on the types of reinforcement, the reinforced concrete columns are classified into three groups:

(i) Tied columns: The main longitudinal reinforcement bars are enclosed within closely spaced lateral ties (Fig.10.21.2a).

(ii) Columns with helical reinforcement: The main longitudinal reinforcement bars are enclosed within closely spaced and continuously wound spiral reinforcement. Circular and octagonal columns are mostly of this type (Fig.10.21.2b).

(iii) Composite columns: The main longitudinal reinforcement of the composite columns consists of structural steel sections or pipes with or without longitudinal bars (Fig.20.21.2c and d).

Out of the three types of columns, the tied columns are mostly common with different shapes of the cross-sections viz. square, rectangular, T-, L-, cross etc. Helically bound columns are also used for circular or octagonal shapes of cross-sections. Architects prefer circular columns in some specific situations for the functional requirement. This module, accordingly takes up these two types (tied and helically bound) of reinforced concrete columns.

10.21.4 Classification of Columns Based on Loadings

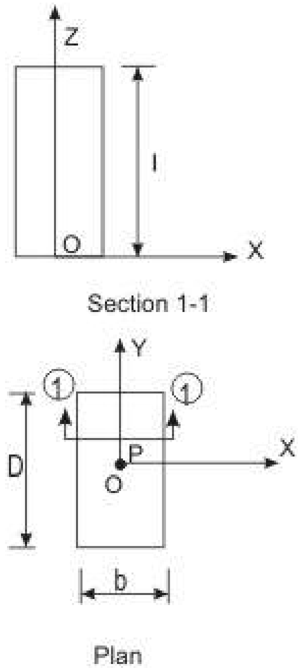

Fig. 10.21.3(a): Axial loading (concentric)

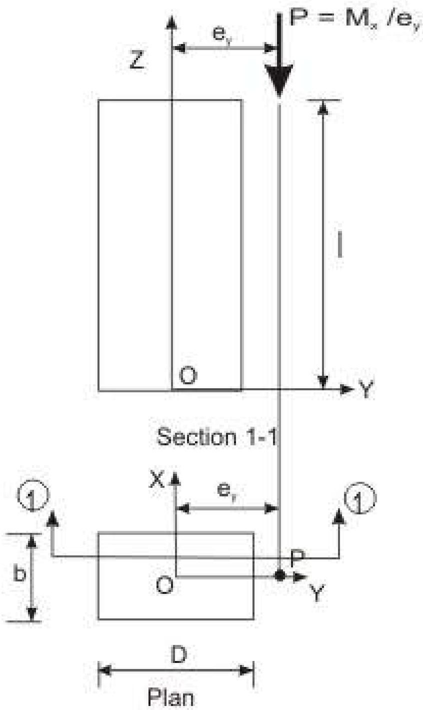

Fig. 10.21.3(b): Axial loading with uniaxial bending

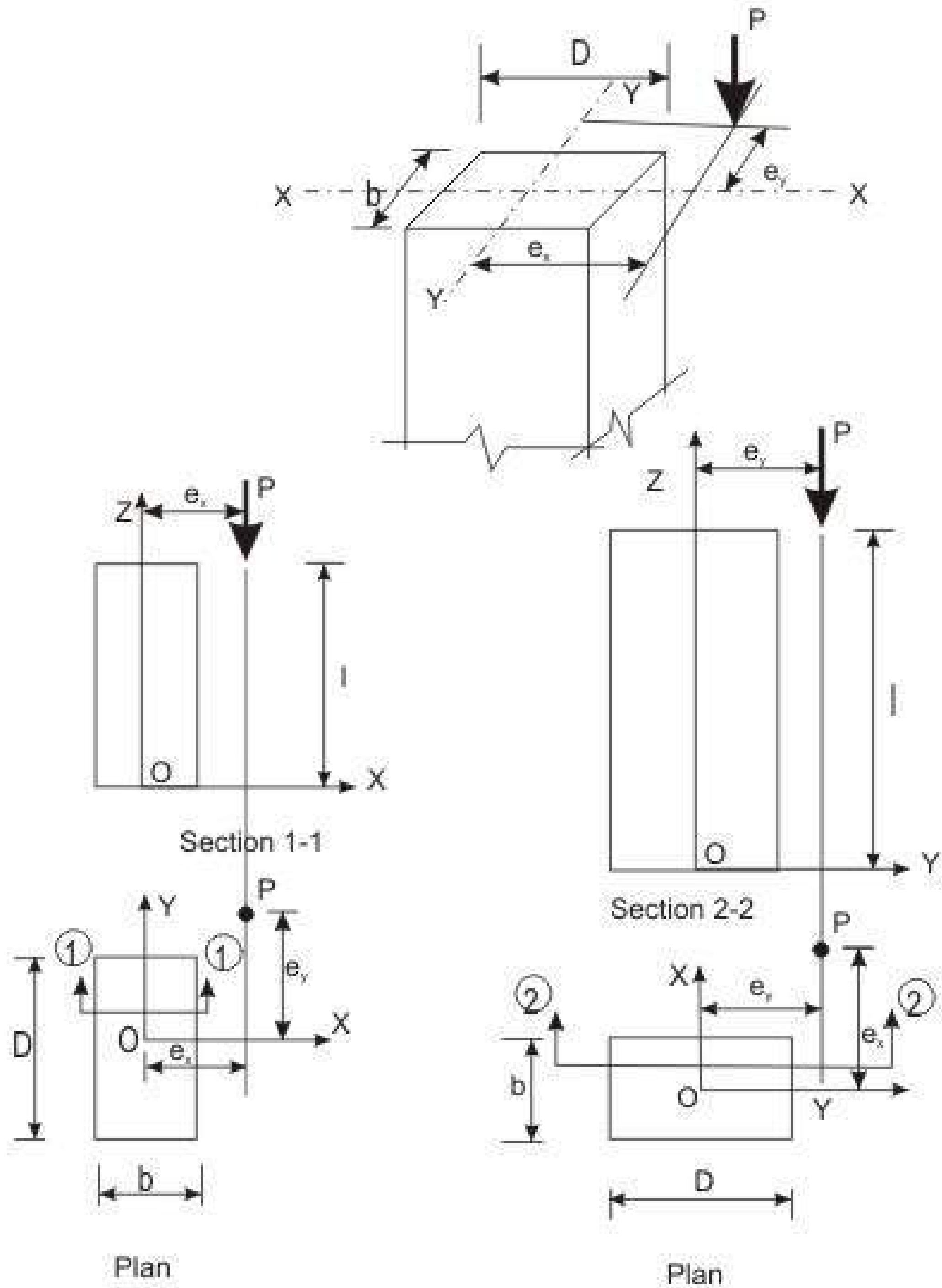

Fig. 10.21.3(c): Axial loading with biaxial bending

Fig. 10.21.3: Concentric and eccentric loadings on columns

Columns are classified into the three following types based on the loadings:

(i) Columns subjected to axial loads only (concentric), as shown in Fig.20.21.3a.

(ii) Columns subjected to combined axial load and uniaxial bending, as shown in Fig.10.21.3b.

(iii) Columns subjected to combined axial load and bi-axial bending, as shown in Fig.10.21.3c.

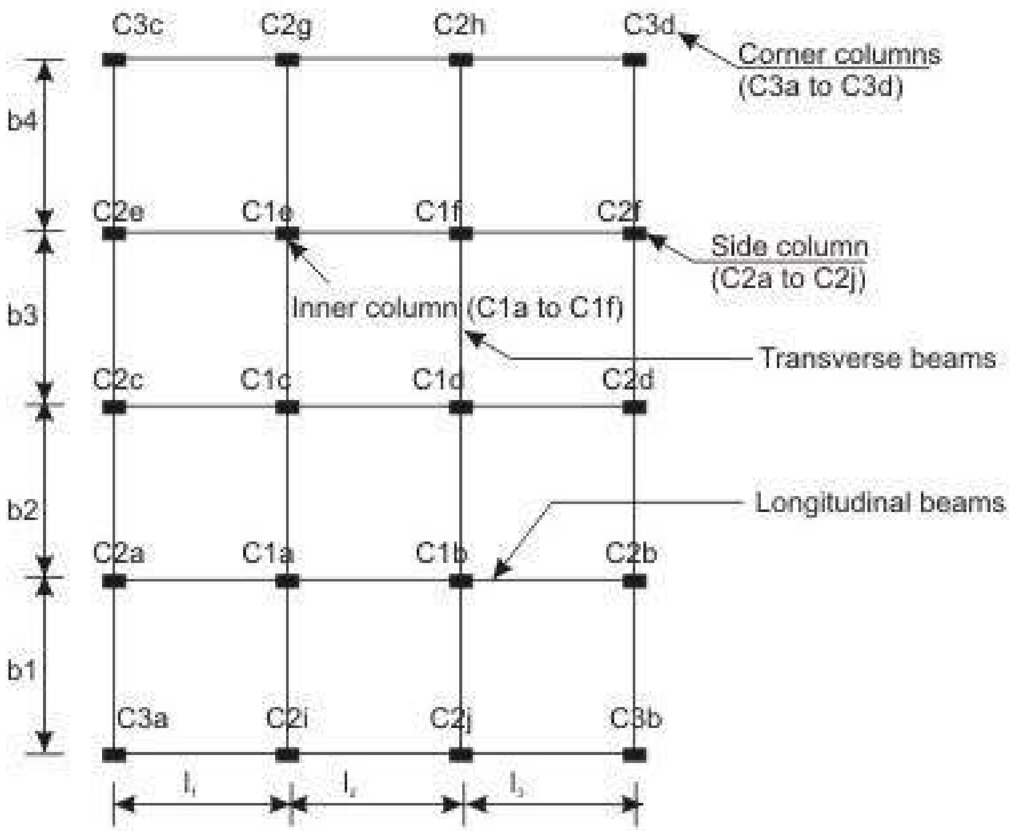

Fig. 10.21.4: Grid of beams and columns

Figure 10.21.4 shows the plan view of a reinforced concrete rigid frame having columns and inter-connecting beams in longitudinal and transverse directions. From the knowledge of structural analysis it is well known that the bending moments on the left and right of columns for every longitudinal beam will be comparable as the beam is continuous. Similarly, the bending moments at the two sides of columns for every continuous transverse beam are also comparable (neglecting small amounts due to differences of l_1, l_2, l_3 and b_1, b_2, b_3, b_4). Therefore, all internal columns (C1a to C1f) will be designed for axial force only. The side columns (C2a to C2j) will have axial forces with uniaxial bending moment, while the four corner columns (C3a to C3d) shall have axial forces with bi-axial bending moments. Thus, all internal columns (C1a to C1f), side columns

(C2a to C2j) and corner columns (C3a to C3d) are the columns of type (i), (ii) and (iii), respectively.

It is worth mentioning that pure axial forces in the inside columns is a rare case. Due to rigid frame action, lateral loadings and practical aspects of construction, there will be bending moments and horizontal shear in all the inside columns also. Similarly, side columns and corner columns will have the column shear along with the axial force and bending moments in one or both directions, respectively. The effects of shear are usually neglected as the magnitude is very small. Moreover, the presence of longitudinal and transverse reinforcement is sufficient to resist the effect of column shear of comparatively low magnitude. The effect of some minimum bending moment, however, should be taken into account in the design even if the column is axially loaded. Accordingly, cls. 39.2 and 25.4 of IS 456 prescribes the minimum eccentricity for the design of all columns. In case the actual eccentricity is more than the minimum, that should be considered in the design.

10.21.5 Classification of Columns Based on Slenderness Ratios

Columns are classified into the following two types based on the slenderness ratios:

(i) Short columns

(ii) Slender or long columns

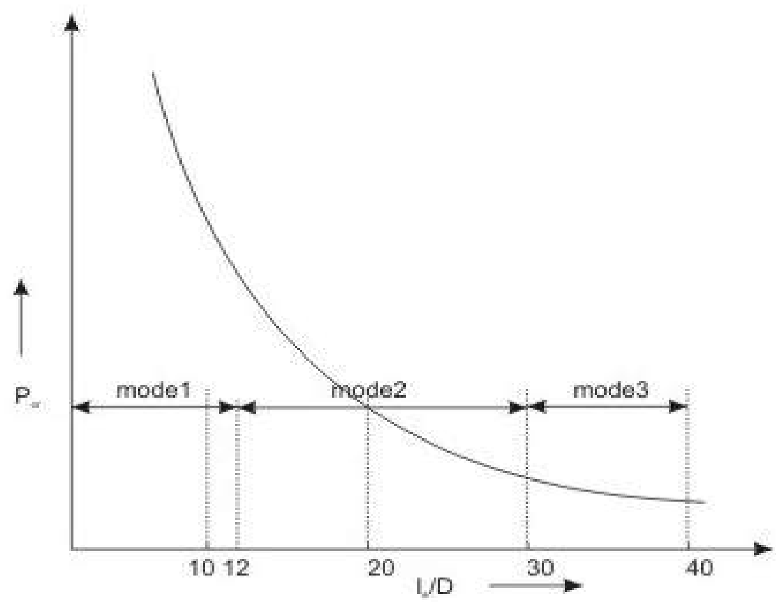

Fig. 10.21.5: Modes of failure of columns

Figure 10.21.5 presents the three modes of failure of columns with different slenderness ratios when loaded axially. In the mode 1, column does not undergo any lateral deformation and collapses due to material failure. This is known as compression failure. Due to the combined effects of axial load and moment a short column may have material failure of mode 2. On the other hand, a slender column subjected to axial load only undergoes deflection due to beam-column effect and may have material failure under the combined action of direct load and bending moment. Such failure is called combined compression and bending failure of mode 2. Mode 3 failure is by elastic instability of very long column even under small load much before the material reaches the yield stresses. This type of failure is known as elastic buckling.

The slenderness ratio of steel column is the ratio of its effective length l_e to its least radius of gyration r. In case of reinforced concrete column, however, IS 456 stipulates the slenderness ratio as the ratio of its effective length l_e to its least lateral dimension. As mentioned earlier in sec. 10.21.2(a), the effective length l_e is different from the unsupported length, the rectangular reinforced concrete column of cross-sectional dimensions b and D shall have two effective lengths in the two directions of b and D. Accordingly, the column may have the possibility of buckling depending on the two values of slenderness ratios as given below:

Slenderness ratio about the major axis $= l_{ex}/D$

Slenderness ratio about the minor axis $= l_{ey}/b$

Based on the discussion above, cl. 25.1.2 of IS 456 stipulates the following:

A compression member may be considered as short when both the slenderness ratios l_{ex}/D and l_{ey}/b are less than 12 where l_{ex} = effective length in respect of the major axis, D = depth in respect of the major axis, l_{ey} = effective length in respect of the minor axis, and b = width of the member. It shall otherwise be considered as a slender compression member.

Further, it is essential to avoid the mode 3 type of failure of columns so that all columns should have material failure (modes 1 and 2) only. Accordingly, cl. 25.3.1 of IS 456 stipulates the maximum unsupported length between two restraints of a column to sixty times its least lateral dimension. For cantilever columns, when one end of the column is unrestrained, the unsupported length is restricted to $100b^2/D$ where b and D are as defined earlier.

10.21.6 Braced and unbraced columns

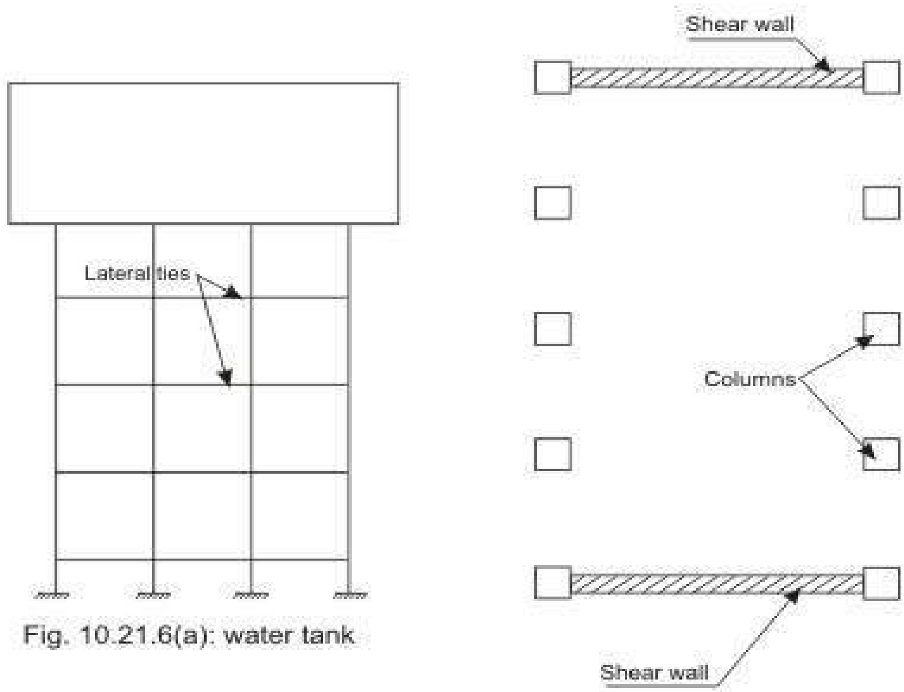

Fig. 10.21.6(a): water tank

Fig. 10.21.6(b): Tall building frame

Fig. 10.21.6: Bracing of columns

It is desirable that the columns do not have to resist any horizontal loads due to wind or earthquake. This can be achieved by bracing the columns as in the case of columns of a water tank or tall buildings (Figs.10.21.6a and b). Lateral tie members for the columns of water tank or shear walls for the columns of tall buildings resist the horizontal forces and these columns are called braced columns. Unbraced columns are supposed to resist the horizontal loads also. The bracings can be in one or more directions depending on the directions of the lateral loads. It is worth mentioning that the effect of bracing has been taken into account by the IS code in determining the effective lengths of columns (vide Annex E of IS 456).

10.21.7 Longitudinal Reinforcement

The longitudinal reinforcing bars carry the compressive loads along with the concrete. Clause 26.5.3.1 stipulates the guidelines regarding the minimum and maximum amount, number of bars, minimum diameter of bars, spacing of bars etc. The following are the salient points:

(a) The minimum amount of steel should be at least 0.8 per cent of the gross cross-sectional area of the column required if for any reason the provided area is more than the required area.

(b) The maximum amount of steel should be 4 per cent of the gross cross-sectional area of the column so that it does not exceed 6 per cent when bars from column below have to be lapped with those in the column under consideration.

(c) Four and six are the minimum number of longitudinal bars in rectangular and circular columns, respectively.

(d) The diameter of the longitudinal bars should be at least 12 mm.

(e) Columns having helical reinforcement shall have at least six longitudinal bars within and in contact with the helical reinforcement. The bars shall be placed equidistant around its inner circumference.

(f) The bars shall be spaced not exceeding 300 mm along the periphery of the column.

(g) The amount of reinforcement for pedestal shall be at least 0.15 per cent of the cross-sectional area provided.

10.21.8 Transverse Reinforcement

Transverse reinforcing bars are provided in forms of circular rings, polygonal links (lateral ties) with internal angles not exceeding 135^o or helical reinforcement. The transverse reinforcing bars are provided to ensure that every longitudinal bar nearest to the compression face has effective lateral support against buckling. Clause 26.5.3.2 stipulates the guidelines of the arrangement of transverse reinforcement. The salient points are:

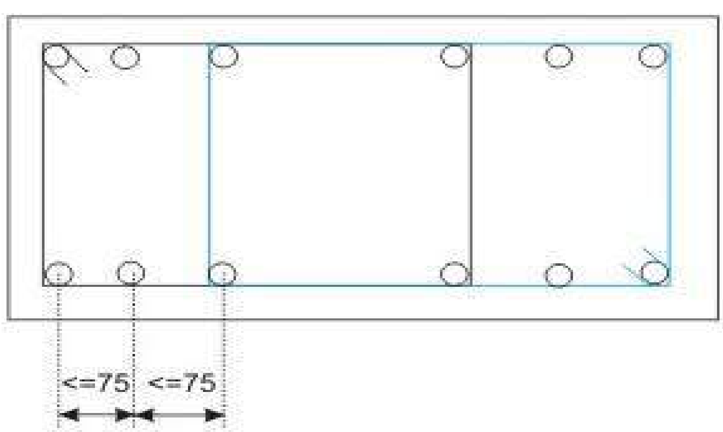

Fig. 10.21.7: Lateral tie (Scheme 1)

(a) Transverse reinforcement shall only go round corner and alternate bars if the longitudinal bars are not spaced more than 75 mm on either side (Fig.10.21.7).

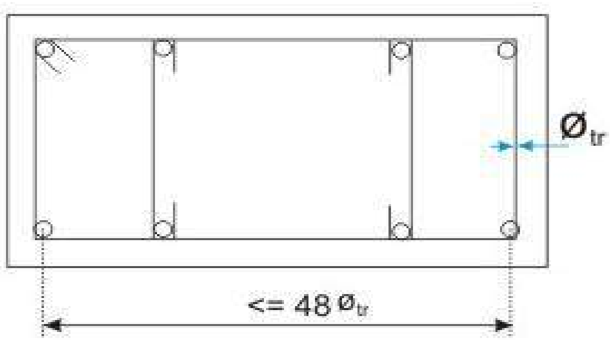

Fig. 10.21.8: Lateral tie (Scheme 2)

(b) Longitudinal bars spaced at a maximum distance of 48 times the diameter of the tie shall be tied by single tie and additional open ties for in between longitudinal bars (Fig.10.21.8).

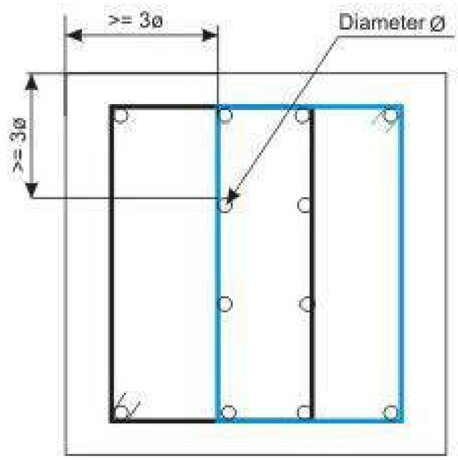

Fig. 10.21.9: Lateral tie (Scheme 3)

(c) For longitudinal bars placed in more than one row (Fig.10.21.9): (i) transverse reinforcement is provided for the outer-most row in accordance with (a) above, and (ii) no bar of the inner row is closer to the nearest compression face than three times the diameter of the largest bar in the inner row.

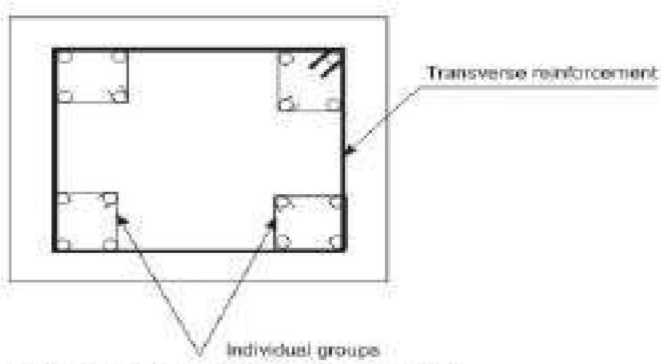

Fig. 10.21.10: Lateral tie (Scheme 4)

(d) For longitudinal bars arranged in a group such that they are not in contact and each group is adequately tied as per (a), (b) or (c) above, as appropriate, the transverse reinforcement for the compression member as a whole may be provided assuming that each group is a single longitudinal bar for determining the pitch and diameter of the transverse reinforcement as given in sec.10.21.9. The diameter of such transverse reinforcement should not, however, exceed 20 mm (Fig.10.21.10).

10.21.9 Pitch and Diameter of Lateral Ties

(a) Pitch: The maximum pitch of transverse reinforcement shall be the least of the following:

(i) the least lateral dimension of the compression members;

(ii) sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied; and

(iii) 300 mm.

(b) Diameter: The diameter of the polygonal links or lateral ties shall be not less than one-fourth of the diameter of the largest longitudinal bar, and in no case less than 6 mm.

10.21.10 Helical Reinforcement

(a) Pitch: Helical reinforcement shall be of regular formation with the turns of the helix spaced evenly and its ends shall be anchored properly by providing one and a half extra turns of the spiral bar. The pitch of helical reinforcement shall be determined as given in sec.10.21.9 for all cases except where an increased load on the column is allowed for on the strength of the helical reinforcement. In such cases only, the maximum pitch shall be the lesser of 75 mm and one-sixth of the core diameter of the column, and the minimum pitch shall be the lesser of 25 mm and three times the diameter of the steel bar forming the helix.

(b) Diameter: The diameter of the helical reinforcement shall be as mentioned in sec.10.21.9b.

10.21.11 Assumptions in the Design of Compression Members by Limit State of Collapse

It is thus seen that reinforced concrete columns have different classifications depending on the types of reinforcement, loadings and slenderness ratios. Detailed designs of all the different classes are beyond the scope here. Tied and helically reinforced short and slender columns subjected to axial loadings with or without the combined effects of uniaxial or biaxial bending will be taken up. However, the basic assumptions of the design of any of the columns under different classifications are the same. The assumptions (i) to (v) given in sec.3.4.2 of Lesson 4 for the design of flexural members are also applicable here. Furthermore, the following are the additional assumptions for the design of compression members (cl. 39.1 of IS 456).

(i) The maximum compressive strain in concrete in axial compression is taken as 0.002.

(ii) The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending and when there is no tension on the section shall be 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.

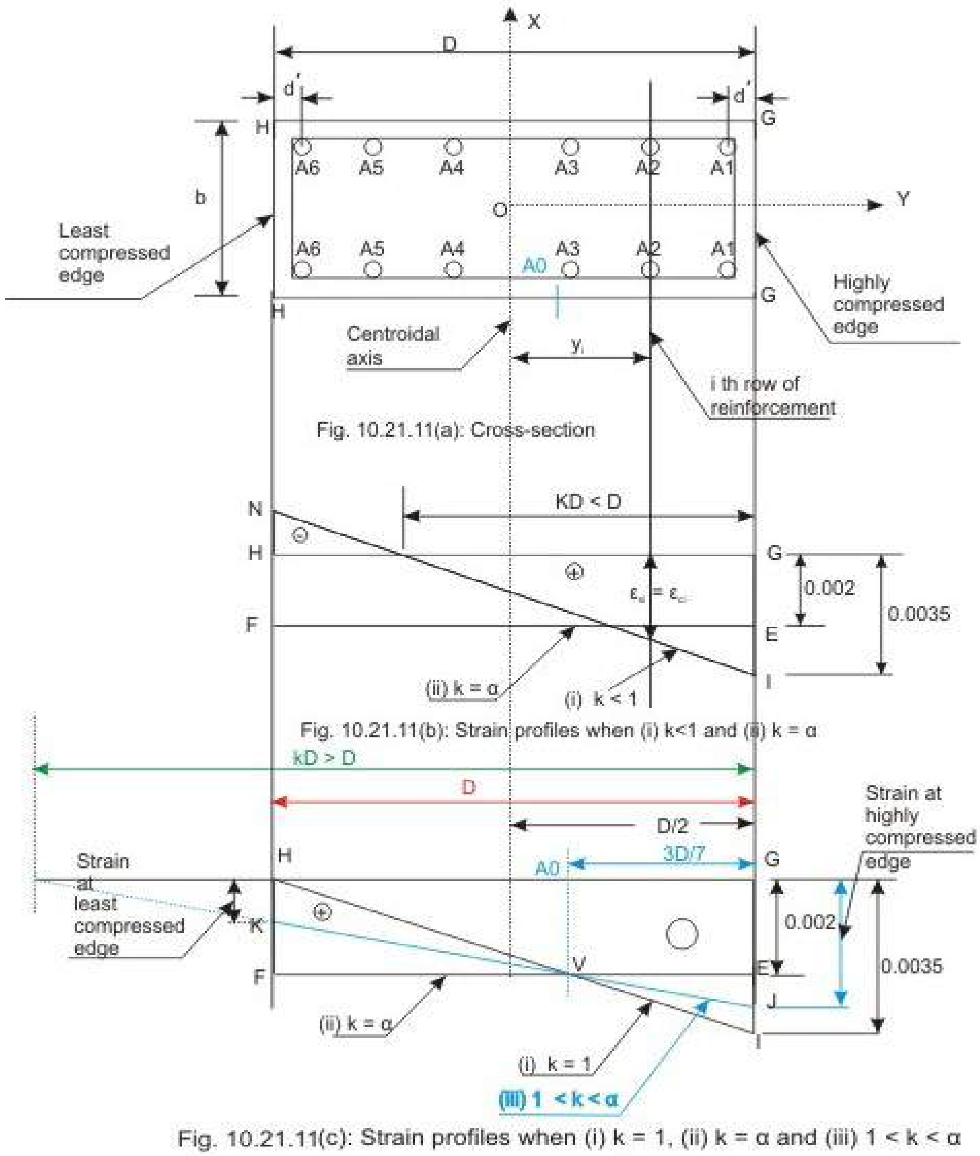

Fig. 10.21.11: Strain profiles for different positions of neutral axis

The assumptions (i) to (v) of section 3.4.2 of Lesson 4 and (i) and (ii) mentioned above are discussed below with reference to Fig.10.21.11a to c presenting the cross-section and strain diagrams for different location of the neutral axis.

The discussion made in sec. 3.4.2 of Lesson 4 regarding the assumptions (i), (iii), (iv) and (v) are applicable here also. Assumption (ii) of sec.3.4.2 is also applicable here when kD, the depth of neutral axis from the highly compressed right edge is within the section i.e., $k < 1$. The corresponding strain profile IN in Fig.10.21.11b is for particular value of P and M such that the maximum compressive strain is 0.0035 at the highly compressed right edge and tensile strain develops at the opposite edge. This strain profile is very much similar to that of a beam in flexure of Lesson 4.

The additional assumption (i) of this section refers to column subjected axial load P only resulting compressive strain of maximum (constant) value of 0.002 and for which the strain profile is EF in Fig.10.21.11b. The neutral axis is at infinity (outside the section).

Extending the assumption of the strain profile IN (Fig.10.21.11b), we can draw another strain profile IH (Fig.10.21.11c) having maximum compressive strain of 0.0035 at the right edge and zero strain at the left edge. This strain profile 1H along with EF are drawn in Fig.10.21.11c to intersect at V. From the two similar triangles EVI and GHI, we have

EV/GH = 0.0015/0.0035 = 3/7, which gives

EV = $3D/7$
(10.2)

The point V, where the two profiles intersect is assumed to act as a fulcrum for the strain profiles when the neutral axis lies outside the section. Another strain profile JK drawn on this figure passing through the fulcrum V and whose neutral axis is outside the section. The maximum compressive strain GJ of this profile is related to the minimum compressive strain HK as explained below.

GJ = GI – IJ = GI – 0.75 HK, as we can write IJ in term of HK from two similar triangles JVI and HVK:

IJ/HK = VE/VF = 0.75.

The value of the maximum compressive strain GJ for the profile JK is, therefore, 0.0035 minus 0.75 times the strain HK on the least compressed edge. This is the assumption (ii) of this section (cl. 39.1b of IS 456).

10.21.12 Minimum Eccentricity

Section 10.21.4 illustrates that in practical construction, columns are rarely truly concentric. Even a theoretical column loaded axially will have accidental eccentricity due to inaccuracy in construction or variation of materials etc. Accordingly, all axially loaded columns should be designed considering the minimum eccentricity as stipulated in cl. 25.4 of IS 456 and given below (Fig.10.21.3c)

$e_{x\,min}\ \geq$ greater of)l/500 + D/30) or 20 mm

(10.3)

$e_{y\,min}\ \geq$ greater of)l/500 + b/30) or 20 mm

where l, D and b are the unsupported length, larger lateral dimension and least lateral dimension, respectively.

10.21.13 Practice Questions and Problems with Answers

Q.1: Define effective length, pedestal, column and wall.

A.1: See sec. 10.21.2.

Q.2: Classify the columns based on types of reinforcement.

A.2: See sec. 10.21.3

Q.3: Classify the columns based on loadings.

A.3: See sec. 10.21.4.

Q.4: Classify the columns based on slenderness ratios.

A.4: See Sec. 10.21.5

Q.5: Explain braced and unbraced columns.

A.5: See sec. 10.21.6.

Q.6: Answer the following:

(a) What are the minimum and maximum amounts of longitudinal reinforcement in a column?

(b) What are the minimum numbers of longitudinal bars in rectangular and circular columns?

(c) What is the amount of longitudinal reinforcement in a pedestal?

(d) What is the maximum pitch of transverse reinforcement in a column?

(e) What is the diameter of lateral ties in a column?

A.6: (a) 0.8% and 4%

(b) 4 and 6

(c) 0.15% of cross-sectional area of the pedestal

(d) See sec. 10.21.9(a)

(e) See sec. 10.21.9(b).

Q.7: Explain the assumptions of determining the strain distribution lines in a column subjected to axial force and biaxial bending.

A.7: See sec. 10.21.11(i) and (ii).

Q.8: State the minimum eccentricity of a rectangular column for designing.

A.8: See sec. 10.21.12.

10.21.14 References

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10.21.15 Test 21 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions carrying equal marks.

TQ.1: Define effective length, pedestal, column and wall.

A.TQ.1: See sec. 10.21.2.

TQ.2: Classify the columns separately based on loadings and slenderness ratios.

A.TQ.2: See secs. 10.21.4 and 5.

TQ.3: Explain braced and unbraced columns.

A.TQ.3: See sec. 10.21.6.

TQ.4: Answer the following:

(a) What are the minimum and maximum amounts of longitudinal reinforcement in a column?

(b) What are the minimum numbers of longitudinal bars in rectangular and circular columns?

(c) What is the amount of longitudinal reinforcement in a pedestal?

(d) What is the maximum pitch of transverse reinforcement in a column?

(e) What is the diameter of lateral ties in a column?

A.TQ.4: (a) 0.8% and 4%

(b) 4 and 6

(c) 0.15% of cross-sectional area of the pedestal

(d) See sec. 10.21.9(a)

(e) See sec. 10.21.9(b).

TQ.5: Explain the assumptions of determining the strain distribution lines in a column subjected to axial force and biaxial bending.

A.TQ.5: See sec. 10.21.11(i) and (ii).

10.21.16 Summary of this Lesson

This lesson defines the effective length, pedestal, column and wall. Three different classifications of columns based on types of reinforcement, loadings slenderness ratio are explained. The need and functions of bracings are illustrated. The guidelines of IS 456 are discussed regarding the types, arrangement, minimum numbers and diameter of bars, pitch and other aspects of longitudinal and transverse reinforcement of columns. The assumptions needed for the design of compression members are illustrated. The determination of strain distribution lines are explained depending on the location of the neutral axis. The need for considering the minimum eccentricity and its amount are explained.

Module
10
Compression Members

Lesson
22
Short Axially Loaded Compression Members

Instructional Objectives:

At the end of this lesson, the student should be able to:

- state additional assumptions regarding the strengths of concrete and steel for the design of short axially loaded columns,
- specify the values of design strengths of concrete and steel,
- derive the governing equation for the design of short and axially loaded tied columns,
- derive the governing equation for the design of short and axially loaded spiral columns,
- derive the equation to determine the pitch of helix in spiral columns,
- apply the respective equations to design the two types of columns by direct computation,
- use the charts of SP-16 to design these two types of columns subjected to axial loads as per IS code.

10.22.1 Introduction

Tied and helically bound are the two types of columns mentioned in sec.10.21.3 of Lesson 21. These two types of columns are taken up in this lesson when they are short and subjected to axially loads. Out of several types of plan forms, only rectangular and square cross-sections are covered in this lesson for the tied columns and circular cross-section for the helically bound columns. Axially loaded columns also need to be designed keeping the provision of resisting some moments which normally is the situation in most of the practical columns. This is ensured by checking the minimum eccentricity of loads applied on these columns as stipulated in IS 456. Moreover, the design strengths of concrete and steel are further reduced in the design of such columns. The governing equations of the two types of columns and the equation for determining the pitch of the helix in continuously tied column are derived and explained. The design can be done by employing the derived equation i.e., by direct computation or by using the charts of SP-16. Several numerical examples are solved to explain the design of the two types of columns by direct computation and using the charts of SP-16.

10.22.2 Further Assumptions Regarding the Strengths of Concrete and Steel

All the assumptions required for the derivation of the governing equations are given in sec.10.21.11 of Lesson 21. The stress-strain diagrams of mild steel (Fe 250) and cold worked deformed bars (Fe 415 and Fe 500) are given in Figs.1.2.3 and 4, respectively of Lesson 2. The stress block of compressive part of concrete is given in Fig.3.4.1.9 of Lesson 4, which is used in the design of beam by limit state of collapse. The maximum design strength of concrete is shown as constant at 0.446 f_{ck} when the strain ranges from 0.002 to 0.0035. The maximum design stress of steel is 0.87 f_y.

Sections 10.21.4 and 12 of Lesson 21 explain that all columns including the short axially loaded columns shall be designed with a minimum eccentricity (cls. 25.4 and 39.2 of IS 456). Moreover, the design strengths of concrete and steel are further reduced to 0.4 f_{ck} and 0.67 f_y, respectively, to take care of the minimum eccentricity of 0.05 times the lateral dimension, as stipulated in cl.39.3 of IS 456. It is noticed that there is not attempt at strain compatibility. Also the phenomenon of creep has not been directly considered.

$e_{x\,min} \geq$ greater of ($l/500 + D/30$) or 20 mm

(10.3)

$e_{y\,min} \geq$ greater of ($l/500 + b/30$) or 20 mm

The maximum values of l_{ex}/D and l_{ey}/b should not exceed 12 in a short column as per cl.25.1.2 of IS 456. For a short column, when the unsupported length $l = l_{ex}$ (for the purpose of illustration), we can assume $l = 12\,D$ (or $12b$ when b is considered). Thus, we can write the minimum eccentricity = $12D/500 + D/30 = 0.057D$, which has been taken as $0.05D$ or $0.05b$ as the maximum amount of eccentricity of a short column.

It is, therefore, necessary to keep provision so that the short columns can resist the accidental moments due to the allowable minimum eccentricity by lowering the design strength of concrete by ten per cent from the value of $0.446f_{ck}$, used for the design of flexural members. Thus, we have the design strength of concrete in the design of short column as $(0.9)(0.446f_{ck}) = 0.4014f_{ck}$, say 0.40 f_{ck}. The reduction of the design strength of steel is explained below.

For mild steel (Fe 250), the design strength at which the strain is 0.002 is $f_y/1.15 = 0.87f_y$. However, the design strengths of cold worked deformed bars (Fe 415 and Fe 500) are obtained from Fig.1.2.4 of Lesson 2 or Fig.23A of IS 456. Table A of SP-16 presents the stresses and corresponding strains of Fe 415 and Fe 500. Use of Table A of SP-16 is desirable as it avoids error while reading from figures (Fig.1.2.4 or Fig.23A, as mentioned above). From Table A of SP-16, the corresponding design strengths are obtained by making linear interpolation. These values of design strengths for which the strain is 0.002 are as follows:

(i) Fe 415: $\{0.9f_{yd} + 0.05f_{yd}(0.002 - 0.00192)/(0.00241 - 0.00192)\} = 0.908f_{yd} = 0.789f_y$

(ii) Fe 500: $\{0.85f_{yd} + 0.05f_{yd}(0.002 - 0.00195)/(0.00226 - 0.00195)\} = 0.859f_{yd} = 0.746f_y$

A further reduction in each of three values is made to take care of the minimum eccentricity as explained for the design strength of concrete. Thus, the acceptable design strength of steel for the three grades after reducing 10 per cent from the above mentioned values are $0.783f_y$, $0.710f_y$ and $0.671f_y$ for Fe 250, Fe 415 and Fe 500, respectively. Accordingly, cl. 39.3 of IS 456 stipulates $0.67f_y$ as the design strength for all grades of steel while designing the short columns. Therefore, the assumed design strengths of concrete and steel are $0.4f_{ck}$ and $0.67f_y$, respectively, for the design of short axially loaded columns.

10.22.3 Governing Equation for Short Axially Loaded Tied Columns

Factored concentric load applied on short tied columns is resisted by concrete of area A_c and longitudinal steel of areas A_{sc} effectively held by lateral ties at intervals (Fig.10.21.2a of Lesson 21). Assuming the design strengths of concrete and steel are $0.4f_{ck}$ and $0.67f_y$, respectively, as explained in sec. 10.22.2, we can write

$$P_u = 0.4f_{ck} A_c + 0.67f_y A_{sc} \tag{10.4}$$

where P_u = factored axial load on the member,

f_{ck} = characteristic compressive strength of the concrete,

A_c = area of concrete,

f_y = characteristic strength of the compression reinforcement, and

A_{sc} = area of longitudinal reinforcement for columns.

The above equation, given in cl. 39.3 of IS 456, has two unknowns Ac and A_{sc} to be determined from one equation. The equation is recast in terms of A_g, the gross area of concrete and p, the percentage of compression reinforcement employing

$$A_{sc} = pA_g/100 \tag{10.5}$$

$$A_c = A_g(1 - p/100) \tag{10.6}$$

Accordingly, we can write

$$P_u/A_g = 0.4f_{ck} + (p/100)\,(0.67f_y - 0.4f_{ck}) \tag{10.7}$$

Equation 10.7 can be used for direct computation of A_g when P_u, f_{ck} and f_y are known by assuming p ranging from 0.8 to 4 as the minimum and maximum percentages of longitudinal reinforcement. Equation 10.4 also can be employed to determine A_g and p in a similar manner by assuming p. This method has been illustrated with numerical examples and is designated as Direct Computation Method.

On the other hand, SP-16 presents design charts based on Eq.10.7. Each chart of charts 24 to 26 of SP-16 has lower and upper sections. In the lower section, P_u/A_g is plotted against the reinforcement percentage $p(= 100A_s/A_g)$ for different grades of concrete and for a particular grade of steel. Thus, charts 24 to 26 cover the three grades of steel with a wide range of grades of concrete. When the areas of cross-section of the columns are known from the computed value of P_u/A_g, the percentage of reinforcement can be obtained directly from the lower section of the chart. The upper section of the chart is a plot of P_u/A_g versus P_u for different values of A_g. For a known value of P_u, a horizontal line can be drawn in the upper section to have several possible A_g values and the corresponding P_u/A_g values. Proceeding vertically down for any of the selected P_u/A_g value, the corresponding percentage of reinforcement can be obtained. Thus, the combined use of upper and lower sections of the chart would give several possible sizes of the member and the corresponding A_{sc} without performing any calculation. It is worth mentioning that there may be some parallax error while using the charts. However, use of chart is very helpful while deciding the sizes of columns at the preliminary design stage with several possible alternatives.

Another advantage of the chart is that, the amount of compression reinforcement obtained from the chart are always within the minimum and maximum percentages i.e., from 0.8 to 4 per cent. Hence, it is not needed to examine if the computed area of steel reinforcement is within the allowable range as is needed while using Direct Computation Method. This method is termed as SP-16 method while illustrating numerical examples.

10.22.4 Governing Equation of Short Axially Loaded Columns with Helical Ties

Columns with helical reinforcement take more load than that of tied columns due to additional strength of spirals in contributing to the strength of columns. Accordingly, cl. 39.4 recommends a multiplying factor of 1.05 regarding the strength of such columns. The code further recommends that the ratio of volume of helical reinforcement to the volume of core shall not be less than 0.36 (A_g/A_c – 1) (f_{ck}/f_y), in order to apply the additional strength factor of 1.05 (cl. 39.4.1). Accordingly, the governing equation of the spiral columns may be written as

$$P_u = 1.05(0.4\, f_{ck}\, A_c + 0.67\, f_y\, A_{sc}) \quad (10.8)$$

All the terms have been explained in sec.10.22.3.

Earlier observations of several investigators reveal that the effect of containing holds good in the elastic stage only and it gets lost when spirals reach the yield point. Again, spirals become fully effective after spalling off the concrete cover over the spirals due to excessive deformation. Accordingly, the above two points should be considered in the design of such columns. The first point is regarding the enhanced load carrying capacity taken into account by the multiplying factor of 1.05. The second point is maintaining specified ratio of volume of helical reinforcement to the volume of core, as specified in cl.39.4.1 and mentioned earlier.

The second point, in fact, determines the pitch p of the helical reinforcement, as explained below with reference to Fig.10.21.2b of Lesson 21.

Volume of helical reinforcement in one loop = $\pi(D_c - \phi_{sp})\, a_{sp}$ (10.9)

Volume of core = $(\pi/4)\, D_c^2\, p$ (10.10)

where D_c = diameter of the core (Fig.10.21.2b)

ϕ_{sp} = diameter of the spiral reinforcement (Fig.10.21.2b)

a_{sp} = area of cross-section of spiral reinforcement

p = pitch of spiral reinforcement (Fig.10.21.2b)

To satisfy the condition of cl.39.4.1 of IS 456, we have

$$\{\pi(D_c - \phi_{sp})\, a_{sp}\}/\{(\pi/4)\, D_c^2\ p\} \geq 0.36(A_g / A_c - 1)\,(f_{ck} / f_y)$$

which finally gives

$$p \leq 11.1(D_c - \phi_{sp})\, a_{sp}\ f_y /(D^2 - D_c^2)\ f_{ck}$$

(10.11)

Thus, Eqs.10.8 and 11 are the governing equations to determine the diameter of column, pitch of spiral and area of longitudinal reinforcement. It is worth mentioning that the pitch *p* of the spiral reinforcement, if determined from Eq.10.11, automatically satisfies the stipulation of cl.39.4.1 of IS 456. However, the pitch and diameter of the spiral reinforcement should also satisfy cl. 26.5.3.2 of IS 456:2000.

10.22.5 Illustrative Examples

Problem 1:

Design the reinforcement in a column of size 400 mm x 600 mm subjected to an axial load of 2000 kN under service dead load and live load. The column has an unsupported length of 4.0 m and effectively held in position and restrained against rotation in both ends. Use M 25 concrete and Fe 415 steel.

Solution 1:

Step 1: To check if the column is short or slender

Given l = 4000 mm, b = 400 mm and D = 600 mm. Table 28 of IS 456 = $l_{ex} = l_{ey}$ = 0.65(l) = 2600 mm. So, we have

l_{ex}/D = 2600/600 = 4.33 < 12

l_{ey}/b = 2600/400 = 6.5 < 12

Hence, it is a short column.

Step 2: Minimum eccentricity

$e_{x\,min}$ = Greater of (l_{ex}/500 + D/30) and 20 mm = 25.2 mm

$e_{y\,min}$ = Greater of (l_{ey}/500 + b/30) and 20 mm = 20 mm

0.05 D = 0.05(600) = 30 mm > 25.2 mm (= $e_{x\,min}$)

$0.05\ b = 0.05(400) = 20 \text{ mm} = 20 \text{ mm} (= e_{y\ min})$

Hence, the equation given in cl.39.3 of IS 456 (Eq.10.4) is applicable for the design here.

Step 3: Area of steel

Fro Eq.10.4, we have

$$P_u = 0.4\ f_{ck}\ A_c + 0.67\ f_y\ A_{sc} \qquad \ldots. \quad (10.4)$$

$$3000(10^3) = 0.4(25)\{(400)(600) – A_{sc}\} + 0.67(415)\ A_{sc}$$

which gives,

$$A_{sc} = 2238.39 \text{ mm}^2$$

Provide 6-20 mm diameter and 2-16 mm diameter rods giving 2287 mm^2 (> 2238.39 mm^2) and p = 0.953 per cent, which is more than minimum percentage of 0.8 and less than maximum percentage of 4.0. Hence, o.k.

Step 4: Lateral ties

The diameter of transverse reinforcement (lateral ties) is determined from cl.26.5.3.2 C-2 of IS 456 as not less than (i) $\phi/4$ and (ii) 6 mm. Here, ϕ = largest bar diameter used as longitudinal reinforcement = 20 mm. So, the diameter of bars used as lateral ties = 6 mm.

The pitch of lateral ties, as per cl.26.5.3.2 C-1 of IS 456, should be not more than the least of

(i) the least lateral dimension of the column = 400 mm

(ii) sixteen times the smallest diameter of longitudinal reinforcement bar to be tied = 16(16) = 256 mm

(iii) 300 mm

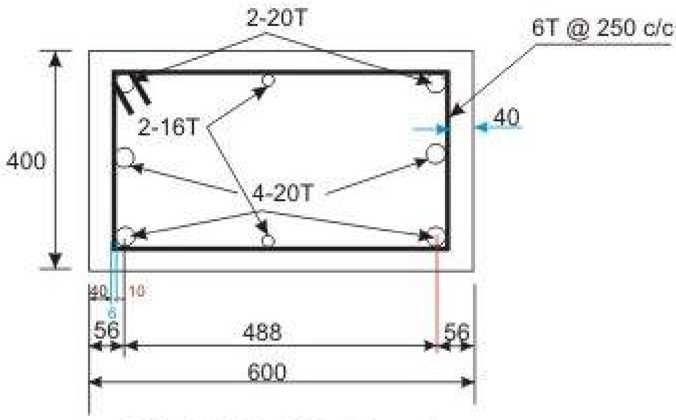

Fig. 10.22.1: Problem 1

Let us use p = pitch of lateral ties = 250 mm. The arrangement of longitudinal and transverse reinforcement of the column is shown in Fig. 10.22.1.

Problem 2:

Design the column of Problem 1 employing the chart of SP-16.

Solution 2:

Steps 1 and 2 are the same as those of Problem 1.

Step 3: Area of steel

$P_u/A_g = 3000(10^3)/(600)(400) = 12.5$ N/mm^2

From the lower section of Chart 25 of SP-16, we get p = 0.95% when P_u/A_g = 12.5 N/mm^2 and concrete grade is M 25. This gives A_{sc} = 0.95(400)(600)/100 = 2288 mm^2. The results of both the problems are in good agreement. Marginally higher value of A_{sc} while using the chart is due to parallax error while reading the value from the chart. Here also, 6-20 mm diameter bars + 2-16 mm diameter bars (A_{sc} provided = 2287 mm^2) is o.k., though it is 1 mm^2 less.

Step 4 is the same as that of Problem 1. Figure 10.22.1, thus, is also the figure showing the reinforcing bars (longitudinal and transverse reinforcement) of this problem (same column as that of Problem 1).

Problem 3:

Design a circular column of 400 mm diameter with helical reinforcement subjected to an axial load of 1500 kN under service load and live load. The column has an unsupported length of 3 m effectively held in position at both ends but not restrained against rotation. Use M 25 concrete and Fe 415 steel.

Solution 3:

Step 1: To check the slenderness ratio

Given data are: unsupported length l = 3000 mm, D = 400 mm. Table 28 of Annex E of IS 456 gives effective length $l_e = l$ = 3000 mm. Therefore, l_e/D = 7.5 < 12 confirms that it is a short column.

Step 2: Minimum eccentricity

e_{min} = Greater of (l/500 + D/30) or 20 mm = 20 mm

0.05 D = 0.05(400) = 20 mm

As per cl.39.3 of IS 456, e_{min} should not exceed 0.05D to employ the equation given in that clause for the design. Here, both the eccentricities are the same. So, we can use the equation given in that clause of IS 456 i.e., Eq.10.8 for the design.

Step 3: Area of steel

From Eq.10.8, we have

$$P_u = 1.05(0.4\, f_{ck}\, A_c + 0.67\, f_y\, A_{sc}) \qquad \ldots\ (10.8)$$

$$A_c = A_g - A_{sc} = 125714.29 - A_{sc}$$

Substituting the values of P_u, f_{ck}, A_g and f_y in Eq.10.8,

$$1.5(1500)(10^3) = 1.05\{0.4(25)(125714.29 - A_{sc}) + 0.67(415)\, A_{sc}\}$$

we get the value of A_{sc} = 3304.29 mm^2. Provide 11 nos. of 20 mm diameter bars (= 3455 mm^2) as longitudinal reinforcement giving p = 2.75%. This p is between 0.8 (minimum) and 4 (maximum) per cents. Hence o.k.

Step 4: Lateral ties

It has been mentioned in sec.10.22.4 that the pitch p of the helix determined from Eq.10.11 automatically takes care of the cl.39.4.1 of IS 456. Therefore, the pitch is calculated from Eq.10.11 selecting the diameter of helical reinforcement from cl.26.5.3.2 d-2 of IS 456. However, automatic satisfaction of cl.39.4.1 of IS 456 is also checked here for confirmation.

Diameter of helical reinforcement (cl.26.5.3.2 d-2) shall be not less than greater of (i) one-fourth of the diameter of largest longitudinal bar, and (ii) 6 mm.

Therefore, with 20 mm diameter bars as longitudinal reinforcement, the diameter of helical reinforcement = 6 mm.

From Eq.10.11, we have

Pitch of helix $p \leq 11.1(D_c - \phi_{sp})\, a_{sp}\, f_y/(D^2 - D_c^2)\, f_{ck}$ … (10.11)

where $D_c = 400 - 40 - 40 = 320$ mm, $\phi_{sp} = 6$ mm, $a_{sp} = 28$ mm^2, $f_y = 415$ N/mm^2, $D = 400$ mm and $f_{ck} = 25$ N/mm^2.

So, $p \leq 11.1(320 - 6)\,(28)\,(415)/(400^2 - 320^2)\,(25) \leq 28.125$ mm

As per cl.26.5.3.2 d-1, the maximum pitch is the lesser of 75 mm and 320/6 = 53.34 mm and the minimum pitch is lesser of 25 mm and 3(6) = 18 mm. We adopt pitch = 25 mm which is within the range of 18 mm and 53.34 mm. So, provide 6 mm bars @ 25 mm pitch forming the helix.

Checking of cl. 39.4.1 of IS 456

The values of helical reinforcement and core in one loop are obtained from Eqs.10.8 and 9, respectively. Substituting the values of D_c, ϕ_{sp}, a_{sp} and pitch p in the above two equations, we have

Volume of helical reinforcement in one loop = 27632 mm^3 and

Volume of core in one loop = 2011428.571 mm^3.

Their ratio = 27632/2011428.571 = 0.0137375

$0.36(A_g/A_c - 1)\,(f_{ck}/f_y) = 0.012198795$

It is, thus, seen that the above ratio (0.0137375) is not less than $0.36(A_g/A_c - 1)\,(f_{ck}/f_y)$.

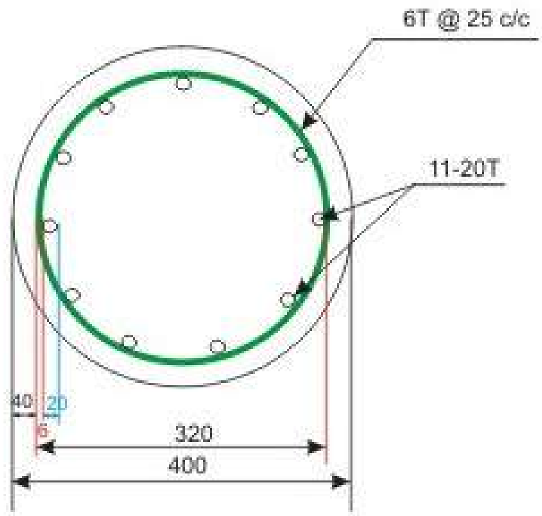

Fig. 10.22.2: Problem 3

Hence, the circular column of diameter 400 mm has eleven longitudinal bars of 20 mm diameter and 6 mm diameter helix with pitch p = 25 mm. The reinforcing bars are shown in Fig.10.22.2.

10.22.6 Practice Questions and Problems with Answers

Q.1: State and explain the values of design strengths of concrete and steel to be considered in the design of axially loaded short columns.

A.1: See sec. 10.22.2.

Q.2: Derive the governing equation for determining the dimensions of the column and areas of longitudinal bars of an axially loaded short tied column.

A.2: See sec. 10.22.2.

Q.3: Derive the governing equation for determining the diameter and areas of longitudinal bars of an axially loaded circular spiral short column.

A.3: First and second paragraph of sec. 10.22.4.

Q.4: Derive the expression of determining the pitch of helix in a short axially loaded spiral column which satisfies the requirement of IS 456.

A.4: See third paragraph onwards up to the end of sec. 10.22.4.

Q.5: Design a short rectangular tied column of b = 300 mm having the maximum amount of longitudinal reinforcement employing the equation given in cl.39.3 of IS 456, to carry an axial load of 1200 kN under service dead load and live load using M 25 and Fe 415. The column is effectively held in position at both ends and restrained against rotation at one end. Determine the unsupported length of the column.

A.5:

Step 1: Dimension *D* and area of steel A_{sc}

Substituting the values of P_u = 1.5(1200) = 1800 kN and A_{sc} = 0.04(300)D in Eq.10.4, we have

$$1800(10^3) = 0.4(25)(300D)(1 - 0.04) + 0.67(415)(0.04)(300D)$$

we get$D$ = 496.60 mm. Use 300 mm x 500 mm column.

A_{sc} = 0.04(300)(500) = 6000 mm^2, provide 4-36 mm diameter + 4-25 mm diameter bars to give 4071 + 1963 = 6034 mm^2 > 6000 mm^2.

Step 2: Lateral ties

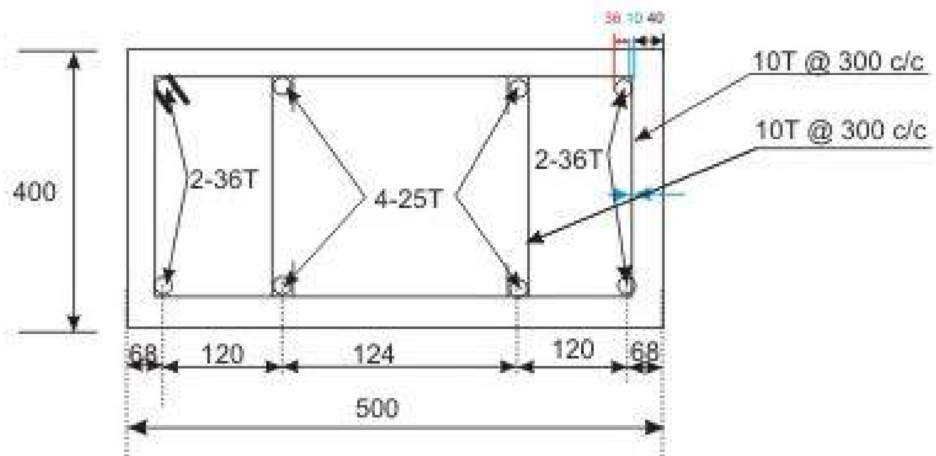

Fig. 10.22.3: Q.5

Diameter of lateral ties shall not be less than the larger of (i) 36/4 = 9 mm and (ii) 6 mm. Use 10 mm diameter bars as lateral ties.

Pitch of the lateral ties p shall not be more than the least of (i) 300 mm, (ii) 16(25) = 400 mm and (iii) 300 mm.

So, provide 10 mm diameter bars @ 300 mm c/c. The reinforcement bars are shown in Fig.10.22.3.

The centre to centre distance between two corner longitudinal bas along 500 mm direction is 500 – 2(4) + 10 + 18) = 364 mm which is less than 48 (diameter of lateral tie). Hence, the arrangement is satisfying Fig.9 of cl. 26.5.3.2 b-2 of IS 456.

Step 3: Unsupported length

As per the stipulation in cl. 25.1.2 of IS 456, the column shall be considered as short if l_{ex} = 12(D) = 6000 mm and l_{ey} = 12(300) = 3600 mm. For the type of support conditions given in the problem, Table 28 of IS 456 gives unsupported length is the least of (i) $l = l_{ex}/0.8$ = 6000/0.8 = 7500 mm and (ii) $l_{ey}/0.8$ = 3600/0.8 = 4500 mm. Hence, the unsupported length of the column is 4.5 m if the minimum eccentricity clause (cl. 39.3) is satisfied, which is checked in the next step.

Step 4: Check for minimum eccentricity

According to cl. 39.4 of IS 456, the minimum eccentricity of 0.05b or 0.05D shall not exceed as given in cl. 25.4 of IS 456. Thus, we have

(i) 0.05(500) = l/500 + 500/30 giving l = 4165 mm

(ii) 0.05(300) = l/500 + 300/10 giving l = 2500 mm

Therefore, the column shall have the unsupported length of 2.5 m.

Q.6: (a) Suggest five alternative dimensions of square short column with the minimum longitudinal reinforcement to carry a total factored axial load of 3000 kN using concrete of grades 20, 25, 30, 35 and 40 and Fe 415. Determine the respective maximum unsupported length of the column if it is effectively held in position at both ends but not restrained against rotation. Compare the given factored load of the column with that obtained by direct computation for all five alternative columns.

(b) For each of the five alternative sets of dimensions obtained in (a), determine the maximum factored axial load if the column is having maximum longitudinal reinforcement (i) employing SP-16 and (ii) by direct computation.

A.6:

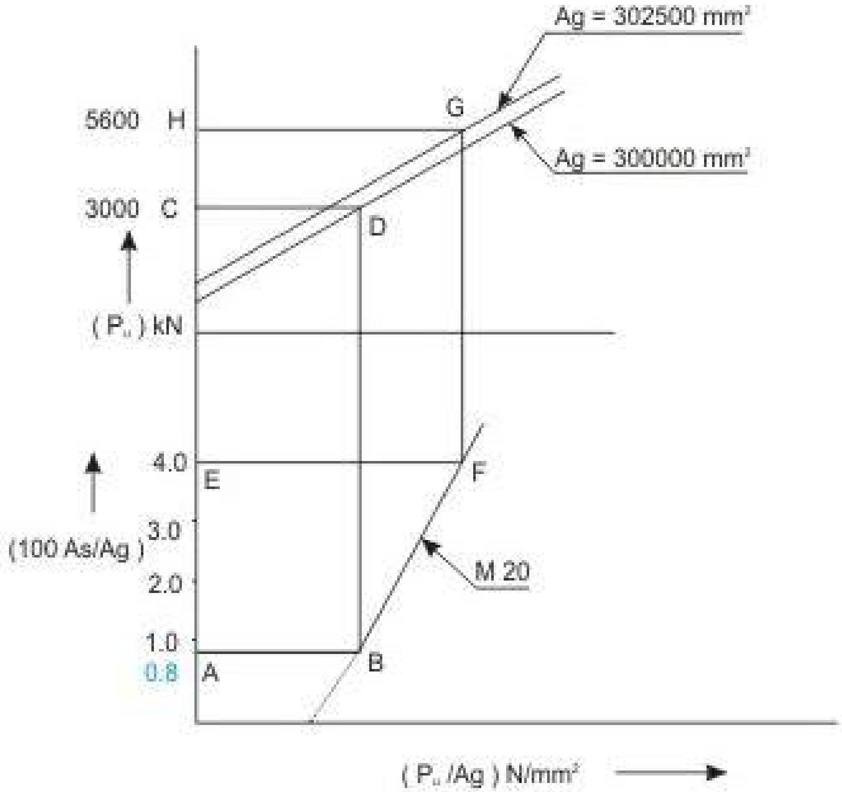

Fig. 10.22.4: Q.6, Chart 25 of SP-16
(not to scale)

Solution of Part (a):

Step 1: Determination of A_g and column dimensions b (= D)

Chart 25 of SP-16 gives all the dimensions of five cases. The two input data are P_u = 3000 kN and 100 A_s/A_g = 0.8. In the lower section of Chart 25, one horizontal line AB is drawn starting from A where p = 0.8 (Fig.10.22.4) to meet the lines for M 20, 25, 30, 35 and 40 respectively. In Fig.10.22.4, B is the meeting point for M 20 concrete. Separate vertical lines are drawn from these points of intersection to meet another horizontal line CD from the point C where P_u = 3000 kN in the upper section of the figure. The point D is the intersecting point. D happens to be on line when A_g = 3000 cm^2. Otherwise, it may be in between two liens with different values of A_g. For M 20, A_g = 3000 cm^2. However, in case the point is in between two lines with different values of A_g, the particular A_g has to be computed by linear interpolation. Thus, all five values of A_g are obtained.

The dimension b = D = $\sqrt{300000}$ = 550 mm. Other four values are obtained similarly. Table 10.1 presents the values of A_g and D along with other results as explained below.

Step 2: Unsupported length of each column

The unsupported length l is determined from two considerations:

(i) Clause 25.1.2 of IS 456 mentions that the maximum effective length l_{ex} is 12 times b or D (as $b = D$ here for a square column). The unsupported length is related to the effective length depending on the type of support. In this problem Table 28 of IS 456 stipulates $l = l_{ex}$. Therefore, maximum value of $l = 12\,D$.

(ii) The minimum eccentricity of cl. 39.3 should be more than the same as given in cl. 25.4. Assuming them to be equal, we get $l/500 + D/30 = D/20$, which gives $l = 8.33D$. For the column using M 20 and Fe 415, the unsupported length = 8.33(550) = 4581 mm. All unsupported lengths are presented in Table 10.1 using the equation

$$l = 8.33\,D \tag{1}$$

Step 3: Area of longitudinal steel

Step 1 shows that the area provided for the first case is 550 mm x 550 mm = 302500 mm^2, slightly higher than the required area of 300000 mm^2 for the practical aspects of construction. However, the minimum percentage of the longitudinal steel is to the calculated as 0.8 per cent of area required and not area provided (vide cl. 26.5.3.1 b of IS 456). Hence, for this case A_{sc} = 0.8(300000)/100 = 2400 mm^2. Provide 4-25 mm diameter + 4-12 mm diameter bars (area = 1963 + 452 = 2415 mm^2). Table 10.1 presents this and other areas of longitudinal steel obtained in a similar manner.

Step 4: Factored load by direct computation

Equation 10.4 is employed to calculate the factored load by determining A_c from A_g and A_{st}. With a view to comparing the factored loads, we will use the values of A_g as obtained from the chart and not the A_g actually provided. From Eq.10.4, we have

$$P_u \text{ from direct computation} = 0.4(f_{ck})(0.992\,A_g) + 0.67(f_y)(0.008)A_g$$

or $$P_u = A_g(0.3968\,f_{ck} + 0.00536\,f_y) \tag{2}$$

For the first case when A_g = 300000 mm^2, f_{ck} = 20 N/mm^2, and f_y = 415 N/mm^2, Eq.(2) gives P_u = 3048.12 kN. This value and other values of factored loads obtained from the direct computation are presented in Table 10.1.

Table 10.1 Results of Q.6a (Minimum Longitudinal Steel), given factored P_u = 3000 kN

Concrete grade	Gross area of concrete (A_g)		$b = D$ (cm)	Area of steel (A_{sc})			P_u by direct computation	l (m)
	Required (cm^2)	Provided (cm^2)		Required (cm^2)	Provided (cm^2)	Bars		
M 20	3000	3025	55	24	24.15	4-25 + 4-12	3048.12	4.581
M 25	2500	2500	50	20	20.60	4-20 + 4-16	3036.10	4.165
M 30	2200	2209	47	17.60	17.85	2.25 + 4-16	3108.25	3.915
M 35	1800	1806	42.5	14.40	14.57	2-28 + 2-12	2900.23	3.540
M 40	1600	1600	40	12.80	13.06	2-20 + 6-12	2895.42	3.332

Solution of Part (b):

Step 1: Determination of P_u

Due to the known dimensions of the column section, the A_g is now known. With known A_g and reinforcement percentage $100A_s/A_g$ as 4 per cent, the factored P_u shall be determined. For the first case, when $b = D =$ 550 mm, A_g = 302500 mm^2. In Chart 25, we draw a horizontal line EF from E, where $100A_s/A_g$ = 4 in the lower section of the chart (see Fig.10.22.4) to meet the M 20 line at F. Proceeding vertically upward, the line FG intersects the line A_g = 302500 at G. A horizontal line towards left from G, say GH, meets the load axis at H where P_u = 5600 kN. Similarly, P_u for other sets are determined and these are presented in Table 10.2, except for the last case when M 40 is used, as this chart has ended at p = 3.8 per cent.

Step 2: Area of longitudinal steel

The maximum area of steel, 4 per cent of gross area of column = 0.04(550)(550) = 12100 mm^2. Provide 12-36 mm diameter bars to have the actual area of steel = 12214 mm^2 > 12100 mm^2, as presented in Table 10.2.

Step 3: Factored P_u from direct computation

From Eq,10.4, as in Step 4 of the solution of Part (a) of this question, we have

$$P_u = 0.4 f_{ck} (A_g - A_{sc}) + 0.67 f_y A_{sc} \quad (3)$$

Substituting the values of A_g and A_{sc} actually provided, we get the maximum P_u of the same column when the longitudinal steel is the maximum. For the first case when A_g = 302500 mm^2, A_{sc} = 12214 mm^2, f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2, we get P_u = 5718.4 kN. This value along with other four values are presented in Table 10.2.

Remarks:

Tables 10.1 and 10.2 reveal that two sets of results obtained from charts of SP-16 and by direct computation methods are in good agreement. However, values obtained from the chart are marginally different from those obtained by direct computation both on the higher and lower sides. These differences are mainly due to personal error (parallax error) while reading the values with eye estimation from the chart.

Table 10.2 Results of Q.6(b) (Maximum Longitudinal Steel) given the respective A_g

Concrete grade	b = D (cm)	Gross concrete area (A_s) (cm^2)	Area of steel (A_{sc})			P_u = Factored load	
			Required (cm^2)	Provided (cm^2)	Bars (No.)	SP-chart (kN)	Direct Computation (kN)
M 20	55	3025	121	122.14	12-36	5600	5718.4
M 25	50	2500	100	101.06	8-36 + 4-25	5200	5208.9
M 30	47	2209	88.36	88.97	8-32 + 4-28	5000	5017.8
M 35	42.5	1806.25	72.25	73.69	12-28	4500	4474.5
M 40	40	1600	64	64.46	8-28 + 4-32	Not available	4249.2

Q.7: Design a short, helically reinforced circular column with minimum amount of longitudinal steel to carry a total factored axial load of 3000 kN with the same support condition as that of Q.6, using M 25 and Fe 415. Determine its unsupported length. Compare the results of the dimension and area of longitudinal steel with those of Q.6(a) when M 25 and Fe 415 are used.

A.7:

Step 1: Diameter of helically reinforced circular column

As per cl. 39.4 of IS 456, applicable for short spiral column, we get from Eq.10.8

$$P_u = 1.05(0.4 f_{ck} A_c + 0.67 f_y A_{sc}) \quad \text{.... (10.8)}$$

Given data are: P_u = 3000 kN, $A_c = \pi/4 (D^2)(0.992)$, $A_{sc} = 0.008(\pi/4) D^2$, f_{ck} = 25 N/mm^2 and f_y = 415 N/mm^2. So, we have

$$3000(10^3) = 1.05(12.1444)(\pi/4)D^2$$

giving D = 547.2 mm and A_g = 235264.2252 mm^2. Provide diameter of 550 mm.

Step 2: Area of longitudinal steel

Providing 550 mm diameter, the required A_g has been exceeded. Clause 26.5.3.1b stipulates that the minimum amount of longitudinal bar shall be determined on the basis of area required and not area provided for any column. Accordingly, the area of longitudinal steel = 0.008 A_g = 0.008(235264.2252) = 1882.12 mm^2. Provide 6-20 mm diameter bars (area = 1885 mm^2) as longitudinal steel, satisfying the minimum number of six bar (cl. 26.5.3.1c of IS 456).

Step 3: Helical reinforcement

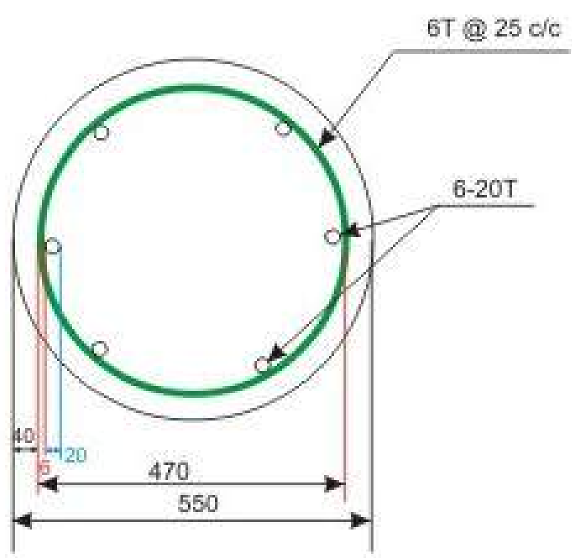

Fig. 10.22.5: Q.7

Minimum diameter of helical reinforcement is greater of (i) 20/4 or (ii) 6 mm. So, provide 6 mm diameter bars for the helical reinforcement (cl. 26.5.3.2d-2 of IS 456). The pitch of the helix p is determined from Eq.10.11 as follows:

$$p \leq 11.1(D_c - \phi_{sp})\, a_{sp} f_y/(D^2 - D_c^2)\, f_{ck} \quad \text{.... (10.11)}$$

Using D_c = 550 – 40 – 40 = 470 mm, ϕ_{sp} = 6 mm, a_{sp} = 28 mm^2, D = 550 mm, f_{ck} = 25 N/mm^2 and f_y = 415 N/mm^2, we get

$$p \leq 11.1(470 - 6)(28)(415)/(550^2 - 470^2)(25) \leq 29.34 \text{ mm}$$

Provide 6 mm diameter bar @ 25 mm c/c as helix. The reinforcement bars are shown in Fig. 10.22.5. Though use of Eq.10.11 automatically checks the stipulation of cl. 39.4.1 of IS 456, the same is checked as a ready reference in Step 4 below.

Step 4: Checking of cl. 39.4.1 of IS 456

Volume of helix in one loop = $\pi (D_c - \phi_{sp})\, a_{sp}$ …. (10.9)

Volume of core = $(\pi/4)\, D_c^2\, (p)$ …. (10.10)

The ratio of Eq.10.9 and Eq.10.10 = $4(D_c - \phi_{sp})\, a_{sp}/D_c^2\, p$

= 4(470 – 6)(28)/(470)(470)(25) = 0.009410230874

This ratio should not be less than $0.36(A_g/A_c - 1)(f_{ck}/f_y)$

= $0.36\{(D^2/D_c^2) - 1)\}\,(f_{ck}/f_y)$ = 0.008011039177

Hence, the stipulation of cl. 39.4.1 is satisfied.

Step 5: Unsupported length

The unsupported length shall be the minimum of the two obtained from (i) short column requirement given in cl. 25.1.2 of IS 456 and (ii) minimum eccentricity requirement given in cls. 25.4 and 39.3 of IS 456. The two values are calculated separately:

(i) $l = l_e = 12D = 12(550) = 6600$ mm

(ii) $l/500 + D/30 = 0.05\, D$ gives l = 4583.3 mm

So, the unsupported length of this column = 4.58 m.

Step 6: Comparison of results

Table 10.3 presents the results of required and actual gross areas of concrete and area of steel bars, dimensions of column and number and diameter of longitudinal reinforcement of the helically reinforced circular and the square columns of Q.6(a) when M 20 and Fe 415 are used for the purpose of comparison.

Table 10.3 Comparison of results of circular and square columns with minimum longitudinal steel (P_u = 3000 kN, M 25, Fe 415)

Column shape and Q.No.	Gross concrete area			Area of steel		
	Required (cm^2)	Provided (cm^2)	Dimension *D* (cm)	Required (cm^2)	Provided (cm^2)	Bar dia. and No. (mm,No.)
Circular (Q.7)	2352.64	2376.78	55	18.82	18.85	6-20
Square (Q.6(a)	2500	2500	50	20	20.6	4-20 + 4-16

10.22.7 References

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10.22.8 Test 22 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions carrying equal marks.

TQ.1: Derive the expression of determining the pitch of helix in a short axially loaded spiral column which satisfies the requirement of IS 456. (20 marks)

A.TQ.1: See third paragraph onwards up to the end of sec. 10.22.4.

TQ.2: Design a square, short tied column of $b = D = 500$ mm to carry a total factored load of 4000 kN using M 20 and Fe 415. Draw the reinforcement diagram.

(30 marks)

A.TQ.2:

Step 1: Area of longitudinal steel

Assuming p as the percentage of longitudinal steel, we have A_c = (500)(500)(1 – 0.01p), A_{sc} = (500)(500)(0.01p), f_{ck} = 20 N/mm^2 and f_y = 415 N/mm^2. Using these values in Eq.10.4

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{st} \quad \text{.... (10.4)}$$

or $$4000000 = 0.4(20)(250000)(1 - 0.01p) + 0.67(415)(250000)(0.01p)$$

we get p = 2.9624, which gives A_{sc} = (2.9624)(500)(500)/100 = 7406 mm^2. Use 4-36 + 4-25 + 4-22 mm diameter bars (4071 + 1963 + 1520) = 7554 mm^2 > 7406 mm^2 as longitudinal reinforcement.

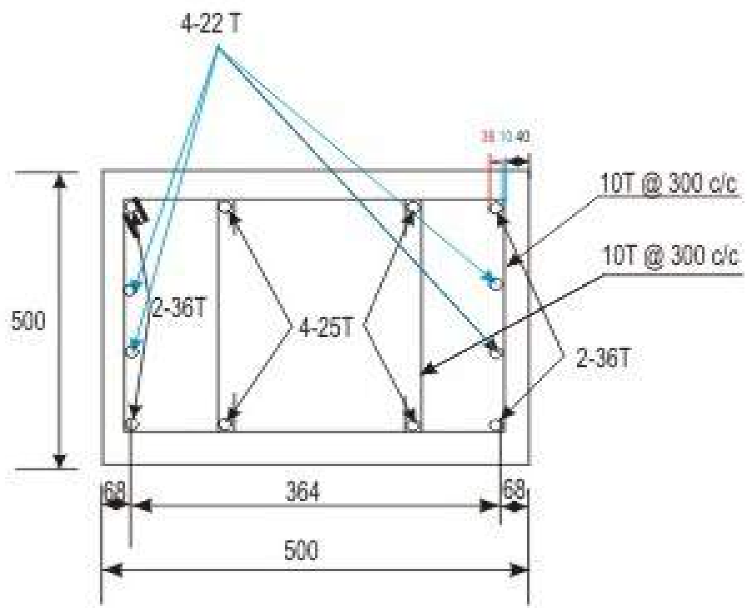

Fig. 10.22.6: TQ.2

Step 2: Lateral ties

Diameter of tie is the greater of (i) 36/4 and (ii) 6 mm. Provide 10 mm diameter lateral ties.

The pitch of the lateral ties is the least of (i) 500 mm, (ii) 16(22) = 352 mm, and (iii) 300 mm. Provide 10 mm diameter @ 300 mm c/c. The reinforcement bars are shown in Fig.10.22.6. It is evident that the centre to centre distance between two corner bars = 364 mm is less than 48 times the diameter of lateral ties = 480 mm (Fig.9 of cl. 26.5.3.2b-2 of IS 456).

10.22.9 Summary of this Lesson

This lesson illustrates the additional assumptions made regarding the strengths of concrete and steel for the design of short tied and helically reinforced columns subjected to axial loads as per IS 456. The governing equations for determining the areas of cross sections of concrete and longitudinal steel are derived and explained. The equation for determining the pitch of the helix for circular columns is derived. Several numerical examples are solved to illustrate the applications of the derived equations and use of charts of SP-16 for the design of both tied and helically reinforced columns. The results of the same problem by direct computation are compared with those obtained by employing the charts of SP-16 are compared. The differences of results, if any, are discussed. Understanding the illustrative examples and solving the practice and test problems will explain the applications of the equation and use of charts of SP-16 for designing these two types of columns.

Module
10
Compression Members

Lesson 25

Design of Short Columns under Axial Load with Uniaxial Bending

Instructional Objectives:

At the end of this lesson, the student should be able to:

- state the two types of problems that can be solved using the design charts of SP-16,
- mention the three sets of design charts specifying their parameters,
- state the approximations, limitations and usefulness of the design charts of SP-16,
- mention the different steps of solving the analysis type of problems using the design charts of SP-16,
- mention the different steps of solving the design type of problems using the design charts of SP-16,
- apply the methods in solving both types of problems using the design charts of SP-16.

10.25.1 Introduction

Lesson 24 explains the procedure of preparing the design charts of short rectangular reinforced concrete columns under axial load with uniaxial bending. It is also mentioned that similar design charts can be prepared for circular and other types of cross-sections of columns by dividing the cross-section into several strips. This lesson explains the design of rectangular and circular short columns with the help of design charts.

It is known that the design of columns by direct computations involves several trials and hence, time taking. On the other hand, design charts are very useful in getting several alternative solutions quickly. Further, design charts are also used for the analysis of columns for safety etc. However, there are limitations of using the design charts, which are mentioned in this lesson. Several numerical problems are solved in this lesson for the purpose of illustration covering both analysis and design types of problems using the design charts of SP-16.

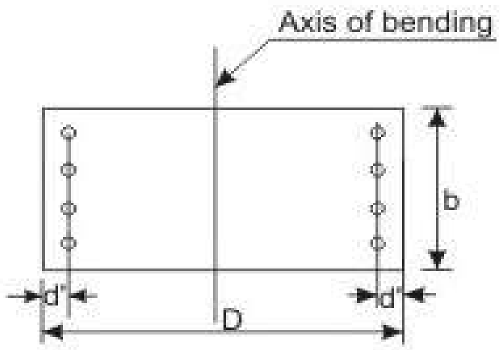

Fig.10.25.1: Rectangular column section - reinforcement distributed equally on two sides

10.25.2 Design Charts of SP-16

SP-16 has three sets of design charts prepared by following the procedure explained in Lesson 24 for rectangular and circular types of cross-sections of columns. The three sets are as follows:

(i) Charts 27 to 38 are the first set of twelve charts for rectangular columns having symmetrical longitudinal steel bars in two rows (Fig.10.25.1) for three grades of steel (Fe 250, Fe 415 and Fe 500) and each of them has four values of *d'/D* ratios (0.05, 0.10, 0.15 and 0.20).

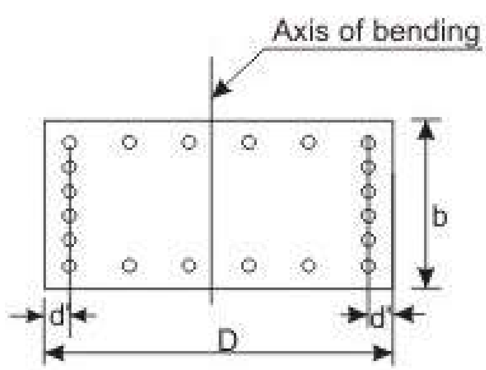

Fig.10.25.2: Rectangular column section - reinforcement distributed equally on four sides

(ii) Charts 39 to 50 are the second set of twelve charts for rectangular columns having symmetrical longitudinal steel bars (twenty numbers) distributed equally on four sides (in six rows, Fig.10.25.2) for three grades of steel (Fe 250, Fe 415 and Fe 500) and each of them has four values of *d'/D* ratios (0.05, 0.10, 0.15 and 0.20).

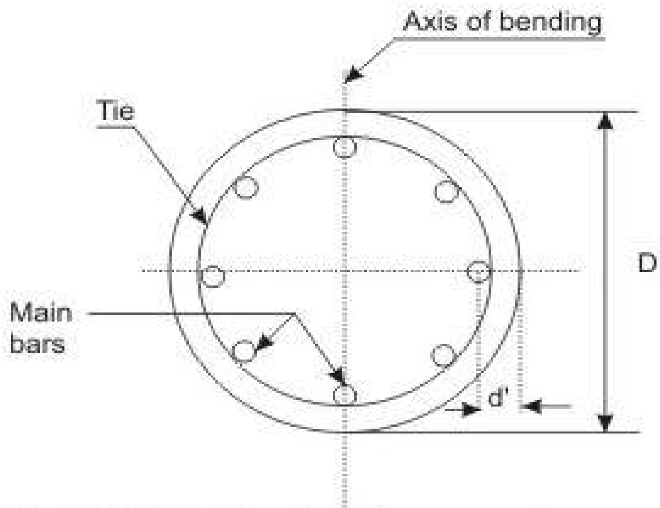

Fig.10.25.3: Circular column section - reinforcement uniformly distributed circumferencially

(iii) The third set of twelve charts, numbering from 51 to 62, are for circular columns having eight longitudinal steel bars of equal diameter and uniformly spaced circumferentially (Fig.10.25.3) for three grades of steel (Fe 250, Fe 415 and Fe 500) and each of them has four values of *d'/D* ratios (0.05, 0.10, 0.15 and 0.20).

All the thirty-six charts are prepared for M 20 grade of concrete only. This is a justified approximation as it is not worthwhile to have separate design charts for each grade of concrete.

10.25.3 Approximations and Limitations of Design Charts of SP-16

(i) Approximations

The following are the approximations of the design charts of SP-16:

(a) Grade of concrete

As mentioned in the earlier section, all the design chars of SP-16 assume the constant grade M 20 of concrete. However, each chart has fourteen plots having different values of the parameter p/f_{ck} ranging from zero to 0.26 at an interval of 0.02. The designer, thus, can make use of the actual grade of concrete by multiplying the p/f_{ck} obtained from the plot with the actual f_{ck} for the particular grade of concrete to partially compensate the approximation.

(b) The *d'/D* ratio

The three sets of charts have four fixed values of *d'/D* ratios (0.05, 0.10, 0.15 and 0.20). However, in the practical design, the *d'/D* ratio may be different from those values. In such situations intermediate values are determined by making linear interpolations.

(c) Equal distribution of twenty longitudinal steel bars on four sides of rectangular columns

In spite of the above consideration, the design charts may be used without significant error for any number of bars greater than eight provided the bars are distributed equally on four sides.

(d) Longitudinal bars in circular columns

Though the design charts are prepared considering eight bars uniformly placed circumferentially, they may generally be used for any number of bars greater than six, uniformly placed circumferentially.

(ii) Limitations

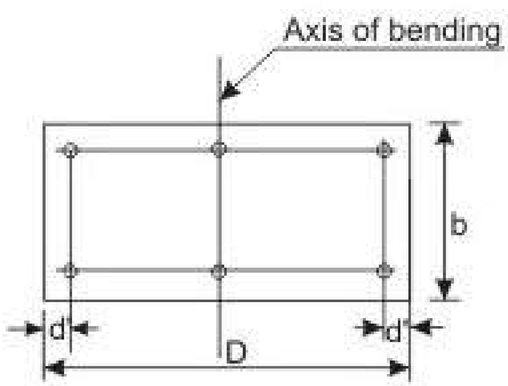

Fig.10.25.4: Six-bars arrangement

The following are the limitations of the design chars of SP-16:

(a) Longitudinal bars equally distributed on four sides of rectangular columns

Twenty bars, when equally placed on four sides, are placed in six rows avoiding any bar on the two axes. However, there will be bars on the axes for odd number of rows. A very common type is the 6-bar arrangement (Fig.10.25.4). Such arrangements, though symmetrical, are not covered in the design charts of SP-16. In such cases, the designer has to make his own assumptions judiciously in order to use the available charts of SP-16. Alternatively, he has to prepare the actual design chart depending on the bar arrangement to get accurate results.

(b) Unsymmetrical arrangement of longitudinal bars in rectangular cross-sections

It is not covered in the charts.

(c) Non-uniform placing of longitudinal bars in circular cross-sections

It is not covered in the charts.

(d) Cross-sections other than rectangular or circular like, I, T, H, X etc.

These are not covered in the charts.

The items under b, c and d, though rare, should be taken care of by preparing the respective design chars as and when needed.

(e) Concluding remarks

In spite of the above approximations and limitations, use of SP-16 has several advantages even by making some more approximations if the charts are not directly applicable. In the note of cl.39.5 of IS 456, the following is recommended, which is worth reproducing:

"The design of members subject to combined axial load and uniaxial bending will involve lengthy calculation by trial and error. In order to overcome these difficulties interaction diagrams may be used. These have been prepared and published by BIS in "SP-16 Design aids for reinforced concrete to IS 456'."

Accordingly, the use of SP-16 is explained in the following sections for the solutions of both analysis and design types of problems.

10.25.4 Use of Design Charts in the Analysis Type of Problems

In many situations, it becomes necessary to assess the safety of a column with known cross-section dimensions, and longitudinal and transverse steel reinforcing bars. The objective is to examine if the column can resist some critical values of P_u or M_u or pairs of P_u and M_u, as may be expected to be applied on the column. This is done by comparing if the given values of pair of P_u and M_u are less than the respective strength capacities pair of P_u and M_u. The word "given" shall be used in the suffix of pairs of P_u and M_u to indicate that they are the given values for which the column has to be examined. The strength capacities of the column, either P_u or M_u alone or pair of P_u and M_u, will not have any suffix. Thus, the designer shall assess

(pair of P_u and M_u)$_{given}$ < pair of P_u and M_u, as strength capacities
(10.53)

This type of problem is known as analysis type of problem. The three steps are given below while using design charts of SP-16 for solving such problems.

Step 1: Selection of the design chart

The designer has to select a particular design chart, specified by the chart number, from the known value of *d'/D* and the grade of steel for circular columns; and considering also the distribution of longitudinal steel bars equally on two or four sides for the rectangular columns.

Step 2: Selection of the particular curve

The designer shall select the particular curve out of the family of fourteen curves in the chart selected in Step 1. The selection of the curve shall be made from the value of p/f_{ck} parameter which is known.

Step 3: Assessment of the column

This can be done in any of the three methods selecting two of the three parameters as known and comparing the third parameter to satisfy Eq.10.53. The parameters are $P_u/f_{ck}\,bD$, $M_u/f_{ck}\,bD^2$ and p/f_{ck} for rectangular columns. For circular columns the breadth *b* shall be replaced by *D* (the diameter of the column).

10.25.5 Use of Design Charts in the Design Type of Problems

It is explained in sec.10.24.2 of Lesson 24 that the design of columns mainly involves with the determination of percentage of longitudinal steel *p*, either assuming or knowing the dimensions *b* and *D*, grades of concrete and steel, distribution of longitudinal bars in two or multiple rows and *d'/D* from the analysis or elsewhere. However, the designer has to confirm the assumed data or revise them, if needed. The use of design charts of SP-16 is explained below in four steps while designing columns:

Step 1: Selection of the design chart

As in step 1 of sec.10.25.4, the design chart is selected from the assumed values of the parameter as explained in step 1 of sec.10.25.4. The only difference is that, here the assumed parameter may be revised, if required.

Step 2: Determination of the percentage of longitudinal steel

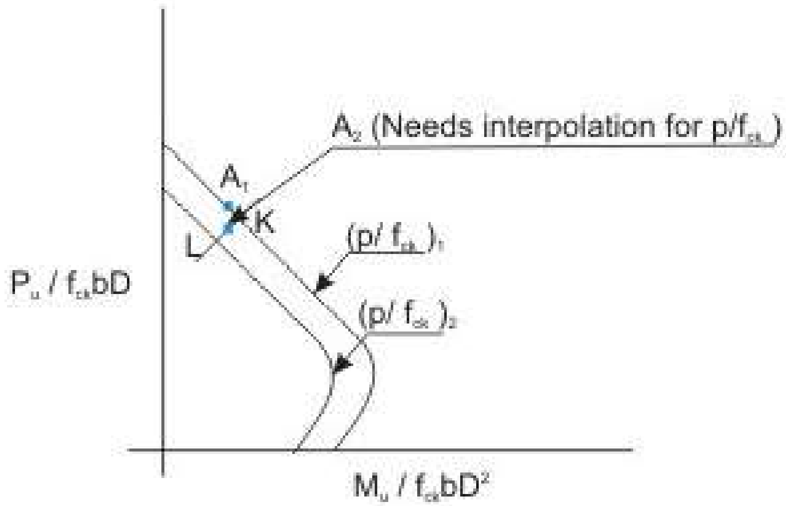

Fig.10.25.5: Determination of p/ f_{ck} by linear interpolation (not to scale)

The two parameters ($P_u/f_{ck}\, bD$) and ($M_u/f_{ck}\, bD^2$) are known and the point A is located on the design chart with these two coordinates (Fig.10.25.5). The point may be like A1, on a particular curve of specified p/f_{ck}, or like A2, in between two such curves having two values of p/f_{ck}, the difference between the two values of p/f_{ck} is 0.02. In the first case, the corresponding p/f_{ck} is obtained directly as specified on the curve. While, in the second case, liner interpolation is to be done by drawing a line KL perpendicular to the two curves and passing through the point A2.

The percentage of longitudinal steel is obtained by multiplying the p/f_{ck}, so obtained, by the actual grade of concrete (which may be different from M 20 though the chart is prepared assuming M 20 only). Thus, percentage of longitudinal steel,

$$p = (p/f_{ck}) \text{ (Actual } f_{ck}) \tag{10.54}$$

This percentage of longitudinal steel (obtained from Eq.10.54) is a tentative value and shall be confirmed after finalizing the assumed data, i.e., *d'/D, b, D* etc.

Step 3: Design of transverse reinforcement

This should be done before confirming *d'/D* as the diameter of the lateral tie has a role in finalizing *d'*. The design of transverse reinforcement shall be done following the procedure explained in secs.10.21.8 and 10.21.9 of Lesson 21.

Step 4: Revision of the design, if required

If the value of d'/D changes in step 3 requiring any change of other dimension etc., the repetition of steps 1 to 3 are needed. Otherwise, the design is complete.

10.25.6 Illustrative Examples

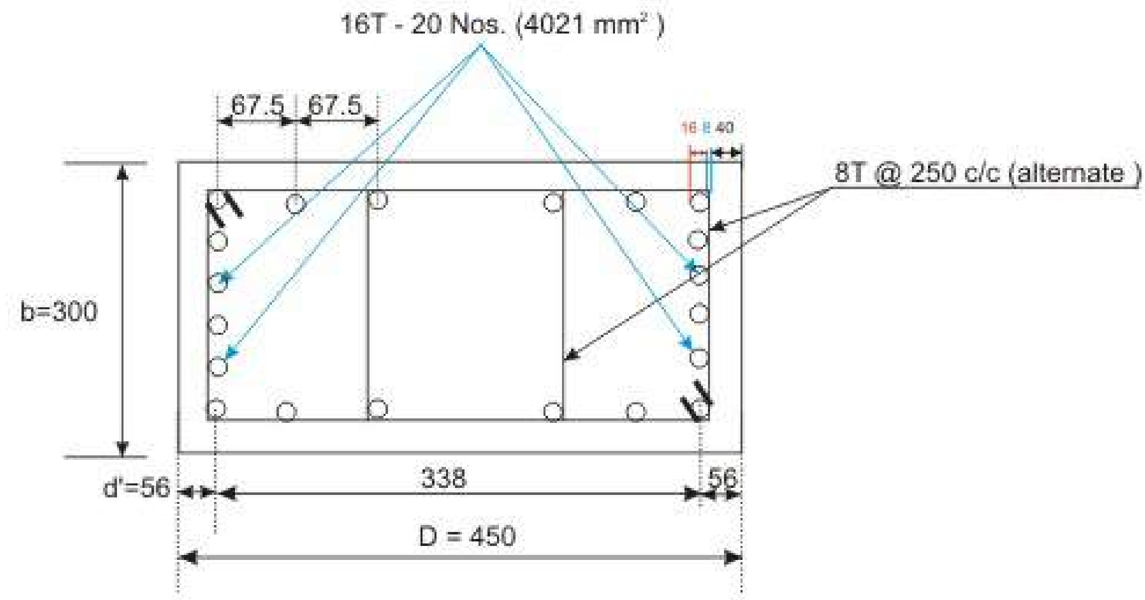

Fig. 10.25.6: Tied column of Problem 1

Problem 1:

Figure 10.25.6 shows a rectangular short reinforced concrete column using M 25 and Fe 415. Analyse the safety of the column when subjected to P_u = 1620 kN and M_u = 170 kNm.

Solution 1:

This is an analysis type of problems. The data given are: b = 300 mm, D = 450 mm, d' = 56 mm, A_{sc} = 4021 mm^2 (20 bars of 16 mm diameter), f_{ck} = 25 N/mm^2, f_y = 415 N/mm^2, P_u = 1620 kN and M_u = 170 kNm. So, we have d'/D = 56/450 = 0.1244, $P_u/f_{ck}bD$ = 0.48, $M_u/f_{ck}bD^2$ = 0.111934 and p/f_{ck} = 0.11914.

Step 1: Selection of design chart

From the given data: d'/D = 0.1244, f_y = 415 N/mm^2 and longitudinal steel bars are equally distributed on four sides, the charts selected are 44 (for d'/D =

0.1) and 45 (for d'/D = 0.15). Linear interpolation has to done with the values obtained from these two charts.

Step 2: Selection of the particular curve

From the given value of p/f_{ck} = 0.11914, the two curves having p/f_{ck} = 0.1 and 0.12 are selected from both the charts (No. 44 and 45). Here also, linear interpolation has to be done.

Step 3: Assessment of the column

In order to assess the column, we select the two given parameters p/f_{ck} and $P_u/f_{ck}bd^2$ to determine the third parameter $Mu/f_{ck}bD^2$ to compare its value with the given parameter $M_u/f_{ck}bD^2$. However, the value of $M_u/f_{ck}bD^2$ is obtained by doing linear interpolation two times: once with respect to p/f_{ck} and the second time with respect to d'/D. The results are furnished in Table 10.9 below:

Table 10.9: Values of $M_u/f_{ck}bD^2$ when $(P_u/f_{ck}bD^2)_{\text{given}}$ = 0.48 and $(p/f_{ck})_{\text{given}}$ = 0.11914; and d'/D = 0.1244

Sl. No.	p/f_{ck}	d'/D		
		0.1	0.15	0.1244
1	0.1	0.1*	0.09**	0.09512***
2	0.12	0.12*	0.107**	0.113656***
3	0.11914	0.1194***	0.10649***	0.1130941***

Note: * Values obtained from chart 44
** Values obtained from chart 45
*** Linearly interpolated values

Thus, the moment capacity of the column is obtained from the final value of $M_u/f_{ck}bD^2$ = 0.1130941 as

$$M_u = (0.1130941)(25)(300)(450)(450) \text{ Nmm} = 171.762 \text{ kNm},$$

which is higher than the given M_u = 170 kNm. Hence, the column can be subjected to the pair of given P_u and M_u as 1620 kN and 170 kNm, respectively.

Problem 2:

Design a short spiral column subjected to P_u = 2100 kN and M_u = 187.5 kNm using M 25 and Fe 415. The preliminary diameter of the column may be taken as 500 mm.

Solution 2:

Step 1: Selection of design chart

With the given f_y = 415 N/mm^2 and assuming d'/D = 0.1, the chart selected for this problem is Chart 56.

Step 2: Determination of the percentage of longitudinal steel

With the given f_{ck} = 25 N/mm^2 and assuming the given D = 500 mm, we have:

$P_u/f_{ck}D^2$ = 2100000/25(500)(500) = 0.336, and

$M_u/f_{ck}D^3$ = 187.5(10^6)/25(500)(500)(500) = 0.06

The particular point A (Fig.10.25.5) having coordinates of $P_u/f_{ck}D^2$ = 0.336 and $M_u/f_{ck}D^3$ = 0.06 in Chart 56 gives: p/f_{ck} = 0.08. Hence, p = 0.08(25) = 2 per cent (see Eq.10.54).

A_{sc} = 0.02(π)(500)(500)/4 = 3928.57 mm^2

Provide 8-25 mm diameter bars to have A_{sc} actually provided = 3927 mm^2. Marginally less amount of steel than required will be checked considering the enhancement of strength for spiral columns as stipulated in cl.39.4 of IS 456.

Step 3: Design of transverse reinforcement

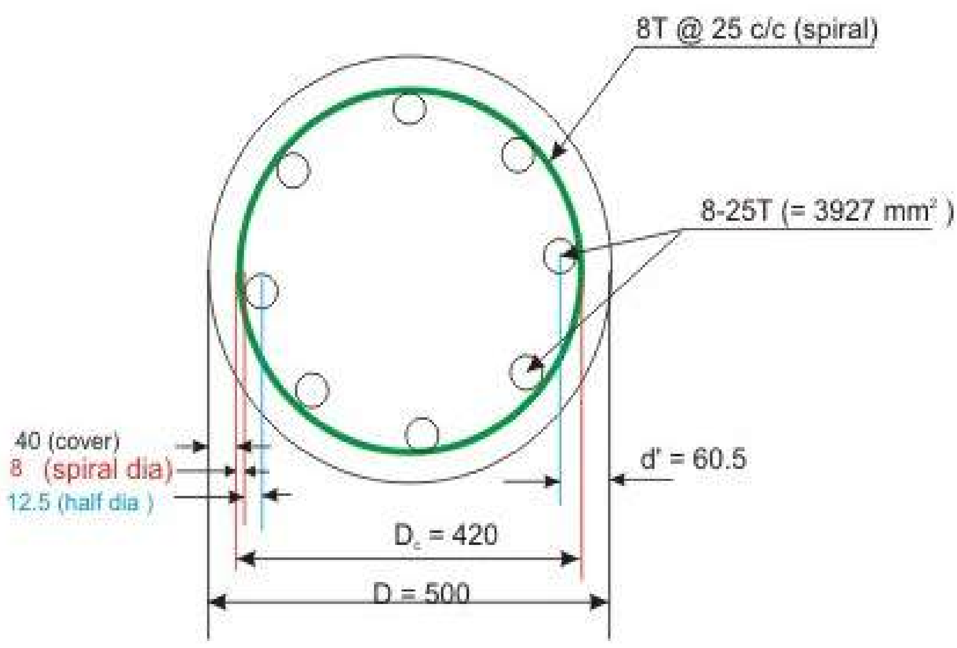

Fig. 10.25.7: Spiral column of Problem 2

The diameter of the helical reinforcement is taken as 8 mm (> 25 mm/4). The pitch *p* of the spiral is determined from Eq.10.11 of Lesson 22, which satisfies the stipulation in cl.39.4.1 of IS 456. From Eq.10.11, we have the pitch of the spiral *p* as:

$$p \leq 11.1(D_c - \phi_{sp})\, a_{sp}\, f_y/(D^2 - D_c^2)\, f_{ck} \quad \ldots. \quad (10.11)$$

where, D_c = 500 – 40 – 40 = 420 mm, D = 500 mm, f_{ck} = 25 N/mm^2, f_y = 415 N/mm^2, ϕ_{sp} = 8 mm and a_{sp} = 50 mm^2.

Using the above values in Eq.10.11, we have $p \leq 25.716$ mm. As per cl.26.5.3.2d1, regarding the pitch of spiral: $p \ngtr 420/6$ (= 70 mm), $p \nless 25$ mm and $p \nless 24$ mm. So, pitch of the spiral = 25 mm is o.k. Figure 10.25.7 presents the cross-section with reinforcing bars of the column.

Step 4: Revision of the design, if required

Providing 25 mm diameter longitudinal steel bars and 8 mm diameter spirals, we have d' = 40 + 8 + 12.5 = 60.5 mm. This gives d'/D = 60.5/500 = 0.121. In step 1, d'/D is assumed as 0.1. So, the revision of the design is needed.

However, as mentioned in step 2, the area of steel required is not provided and this may be offset considering the enhanced strength of the spiral column, as stipulated in cl.39.4 of IS 456.

We, therefore, assess the strength of the designed column, when d'/D = 0.121 and A_{sc} = 3927 mm^2, if it can be subjected to P_u = 2100 kN and M_u = 187.5 kNm.

For the purpose of assessment, we determine the capacity P_u of the column when M_u = 187.5 kNm. Further, the revised d'/D = 0.121 needs to interpolate the values from Charts 56 (for d'/D = 0.1) and 57 (for d'/D = 0.15). The value of p/f_{ck} = 0.08 and $M_u/f_{ck}bD^3$ = 0.06. Table 10.10 presents the results.

Table 10.10: Value of $P_u/f_{ck}bD^2$ when $M_u/f_{ck}D^3$ = 0.06 and p/f_{ck} = 0.08

Sl.No.	d'/D	$P_u/f_{ck}D^2$
1	0.1	0.336 (from Chart 56)
2	0.15	0.30 (from Chart 57)
3	0.121	0.32088 (Interpolated value)

From Table 10.10, thus, we get,

$P_u/f_{ck}D^2$ = 0.32088, which gives P_u = (0.32088)(25)(500)(500) = 2005.5 kN.

Considering the enhanced strength as 1.05 times as per cl.39.4 of IS 456, the actual capacity of this column is (1.05)(2005.5) = 2105 kN > 2100 kN.

Thus, the design is safe to carry P_u = 2100 kN and M_u = 187.5 kNm.

10.25.7 Practice Questions and Problems with Answers

Q.1: Name the two types of problems that can be solved using the design charts of SP-16.

A.1: See sec. 10.25.1.

Q.2: Mention the three different sets of design charts available in SP-16 mentioning the number of charts and the parameters for their identification.

A.2: See sec. 10.25.2.

Q.3: State the approximations, limitations and usefulness of the design charts of SP-16 in solving the analysis and design type of problems of short columns.

A.3: See sec. 10.25.3.

Q.4:

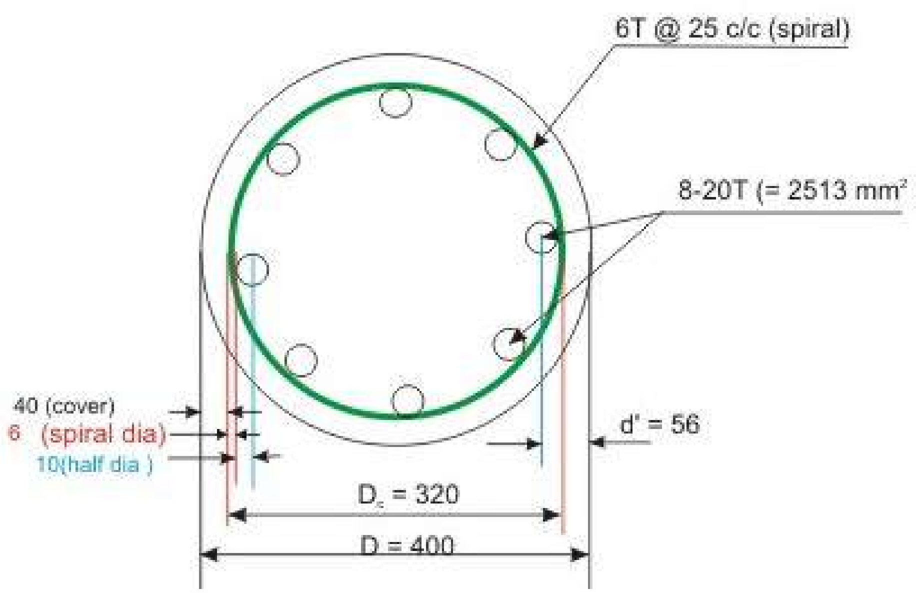

Fig. 10.25.8: Spiral column of Q. 4

Assess the safety of the spiral column shown in Fig.10.25.8 using M 20 and Fe 415 when subjected to P_u = 1200 kN and M_u = 64 kNm, considering the enhanced strength of the spiral column.

A.4: In this problem, the given data are: D = 400 mm, d' = 40 + 6 + 10 = 56 mm, A_{sc} = 2513 mm^2 (8-20 mm diameter bars), f_{ck} = 20 N/mm^2, f_y = 415 N/mm^2, P_u = 1200 kN and M_u = 64 kNm.

Step 1: Selection of the design charts

With f_y = 415 N/mm^2 and d'/D = 56/400 = 0.14, we select two charts nos. 56 (for d'/D = 0.1) and 57 (for d'/D = 0.15). We have to interpolate the values obtained from these two charts.

Step 2: Selection of the particular curve

From the given data we have p/f_{ck} = 0.0999488 $\cong$ 0.1. So, we select the curve for p/f_{ck} = 0.1 in the two charts (Nos. 56 and 57).

Step 3: Assessment of the column

For the purpose of assessment, we select the two parameters p/f_{ck} and $M_u/f_{ck}D^3$ and determine the values of $P_u/f_{ck}D^2$ from the two charts for interpolation. The results are presented in Table 10.11 below.

Table 10.11: Values of $p/f_{ck}D^2$ and $M_u/f_{ck}D^3$ and p/f_{ck} = 0.1

Sl.No.	d'/D	$P_u/f_{ck}D^2$
1	0.1	0.444 (from Chart 56)
2	0.15	0.422 (from Chart 57)
3	0.14	0.4264 (Interpolated value)

From Table 10.11, thus, we get $P_u/f_{ck}D^2$ = 0.4264, which gives P_u = 1364.48 kN.

It may be noted that for more accuracy another set of values of d'/D = 0.08 is required. The interpolated value, thus obtained, shall be strictly applicable when p/f_{ck} = 0.0999488. However, for all practical designs, such accuracy is not required.

Further, as per cl.39.4 of IS 456, the enhanced capacity of the spiral column = 1.05(1364.48) = 1432.704 kN, which is more than 1200 kN. It is also seen that the column is safe even without considering the enhanced capacity as the P_u = 1364.48 kN > 1200 kN.

Q.5:

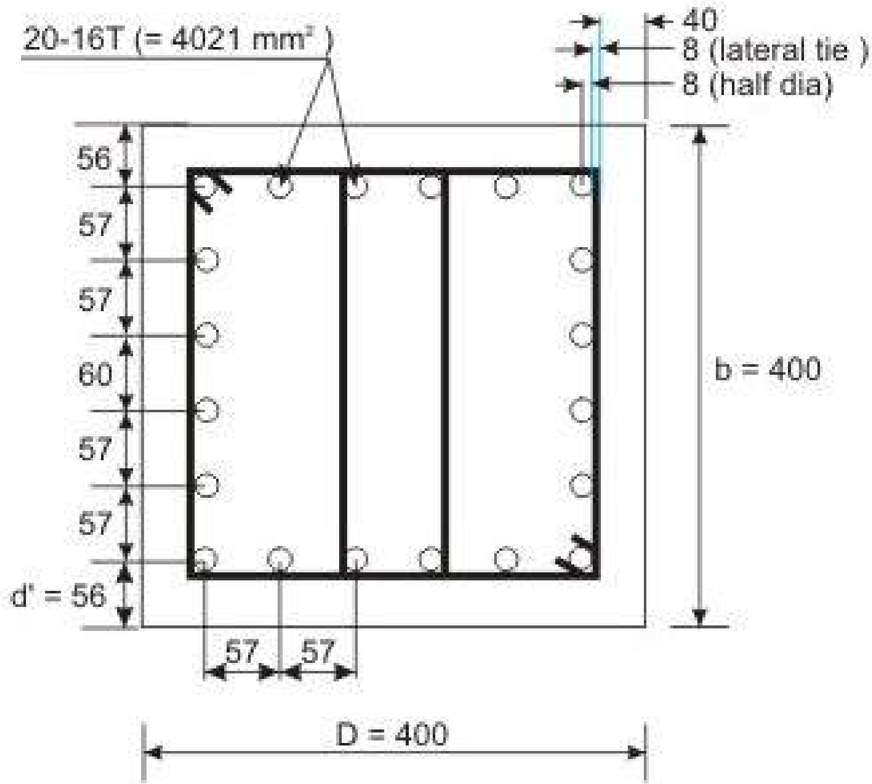

Fig .10.25.9: Square column of Q. 5

Design a short square tied column to carry P_u = 2240 kN and M_u = 112 kNm using M 25 and Fe 415, and assuming the dimension b = D = 400 mm, as shown in Fig.10.25.9.

A.5: The data given are: b = D = 400 mm, P_u = 2240 kN, M_u = 112 kNm, f_{ck} = 25 N/mm^2 and f_y = 415 N/mm^2.

Step 1: Selection of the design chart

With the given data of f_y = 415 N/mm^2 and assuming d'/D = 0.15, we have to refer to Chart 45.

Step 2: Determination of percentage of longitudinal steel

Using the values of f_{ck} = 25 N/mm^2 and assuming b = D = 400 mm as given, we have $P_u/f_{ck}D^2$ = 0.56 and $M_u/f_{ck}D^3$ = 0.07.

From Chart 45, we get p/f_{ck} = 0.1, giving p = 2.5 per cent. Accordingly,

A_{sc} = 2.5(400)(400)/100 = 4000 mm^2. Provide 20 bars of 16 mm diameter ($A_{sc(\text{provide})}$ = 4021 mm^2).

Step 3: Design of lateral tie

The arrangement of lateral tie shall be like Fig.18 of IS 456 as the longitudinal bars are not spaced more than 75 mm on either side (cl.26.5.3.2b1 of IS 456). The pitch of the lateral tie of diameter 8 mm is kept at 250 mm c/c satisfying the stipulation in cl.26.5.3.2c1 of IS 456. Figure 10.25.9 presents the cross-section with reinforcing bars of the column.

Step 4: Revision of the design, if required

The value of *d'* is now 56 mm which gives d'/D = 0.14. Accordingly, the assumed value of d'/D in step 1 as 0.15 is not valid. So, we have to check the capacity of the column interpolating the values when d'/D = 0.1 and 0.15 from Charts 44 and 45, respectively. Further, the longitudinal steel provided gives p/f_{ck} = 0.100525, which also is different from 0.1 as obtained in step 2 of this problem. Though the difference is marginal, both the interpolations are done for the academic interest and results are presented in Table 10.12 below. In assessing the capacity of this column, we keep p/f_{ck} = 0.100125 and $P_u/f_{ck}D^2$ = 0.56 as constants and determine the value of $M_u/f_{ck}D^3$ by two linear interpolations.

Table 10.12: Values of $M_u/f_{ck}D^3$ when $P_u/f_{ck}bD^2$ = 0.56 and p/f_{ck} = 0.10025

Sl. No.	p/f_{ck}	d'/D		
		0.1	0.15	0.14
1	0.1	0.1*	0.07**	0.072***
2	0.12	0.08*	0.09**	0.092***
3	0.100525	0.080525***	0.070525***	0.072525***

Note: * Values obtained from Chart 44
** Values obtained from Chart 45
*** Linearly interpolated values

So, the capacity of the column M_u = (0.072525)(25)(400)(400)(400) Nmm = 116 kNm > 112 kNm.

Hence, the design of the column is o.k.

10.25.8 References

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10.25.9 Test 25 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

TQ.1: Mention the three different sets of design charts available in SP-16 mentioning the number of charts and the parameters for their identification. (10 marks)

A.TQ.1: See sec. 10.25.2.

TQ.2: State the approximations, limitations and usefulness of the design charts of SP-16 in solving the analysis and design type of problems of short columns. (10 marks)

A.TQ.2: See sec. 10.25.3.

TQ.3:

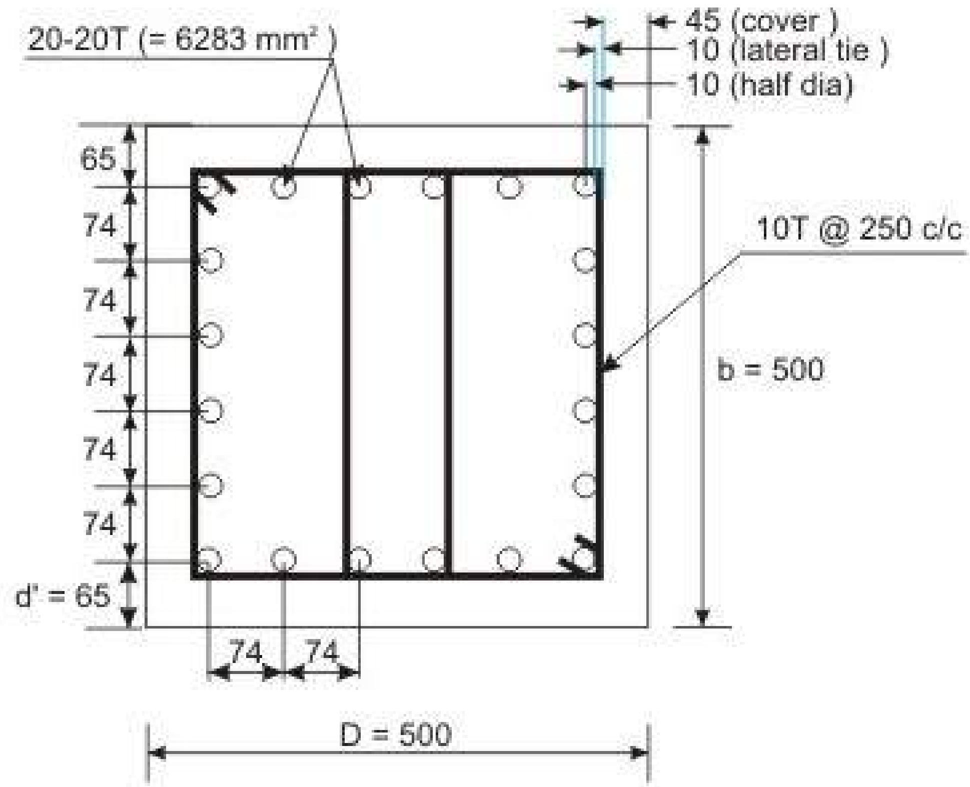


Fig .10.25.10: Square column of TQ. 3

Check the short square column of Fig.10.25.10 to carry P_u = 3250 kN and M_u = 250 kNm using M 25 and Fe 415. (30 marks)

A.TQ.3: Given data are: b = D = = 500 mm, A_{sc} = 6283 mm^2 (20 bars of 20 mm diameter), f_{ck} = 25 N/mm^2, f_y = 415 N/mm^2, P_u = 3250 kN and M_u = 250 kNm.

Step 1: Selection of design chart

From Fig.10.25.10, we get d' = 65 mm giving d'/D = 0.13, and given f_y = 415 N/mm^2, we select Charts 44 (for d'/D = 0.1) and 45 (for d'/D = 0.15). We have to interpolate the values to get the result when d'/D = 0.13.

Step 2: Selection of the particular curve

With p = 628300/(500)(500) = 2.5132 per cent, we get p/f_{ck} = 0.100528 $\cong$ 0.1. Accordingly, the curve for p/f_{ck} = 0.1 is to be used in Charts 44 and 45.

Step 3: Assessment of the column

For the assessment, we keep $P_u/f_{ck}D^2$ = 3250/25(500)(500) = 0.52 and p/f_{ck} = 0.1 as constants to determine $M_u/f_{ck}D^3$ from two charts. The results are given in Table 10.13 below.

Table 10.13: Values of $M_u/f_{ck}D^3$ when $P_u/f_{ck}D^2$ = 0.52 and p/f_{ck} = 0.1

Sl.No.	d'/D	$M_u/f_{ck}D^3$
1	0.1	0.09 (from Chart 44)
2	0.15	0.08 (from Chart 45)
3	0.13	0.084 (Interpolated value)

So, we get $M_u/f_{ck}D^3$ = 0.084, giving M_u = (0.084)(25)(500)(500)(500) = 262.5 kNm > 250 kNm.

Hence, the column is safe to carry P_u = 3250 kN and M_u = 250 kNm.

10.25.10 Summary of this Lesson

This lesson explains the approximations, limitations and usefulness of the three sets of design charts available in SP-16 for the purpose of solving analysis and design types of reinforced concrete columns. The use of design charts has been illustrated in several steps for the solution of both analysis and design types of problems.

Several numerical problems in illustrative examples, practice problem and test will help in understanding the use of design charts to solve the two types of problems.

Module 10

Compression Members

Lesson 26

Short Compression Members under Axial Load with Biaxial Bending

Instructional Objectives:

At the end of this lesson, the student should be able to:

- understand the behaviour of short columns under axial load and biaxial bending,
- understand the concept of interaction surface,
- identify the load contour and interaction curves of P_u-M_u in a interaction surface,
- mention the limitation of direct application of the interaction surface in solving the problems,
- explain the simplified method of design and analysis of short columns under axial load and biaxial bending,
- apply the IS code method in designing and analysing the reinforced concrete short columns under axial load and biaxial bending.

10.26.1 Introduction

Beams and girders transfer their end moments into the corner columns of a building frame in two perpendicular planes. Interior columns may also have biaxial moments if the layout of the columns is irregular. Accordingly, such columns are designed considering axial load with biaxial bending. This lesson presents a brief theoretical analysis of these columns and explains the difficulties to apply the theory for the design. Thereafter, simplified method, as recommended by IS 456, has been explained with the help of illustrative examples in this lesson.

10.26.2 Biaxial Bending

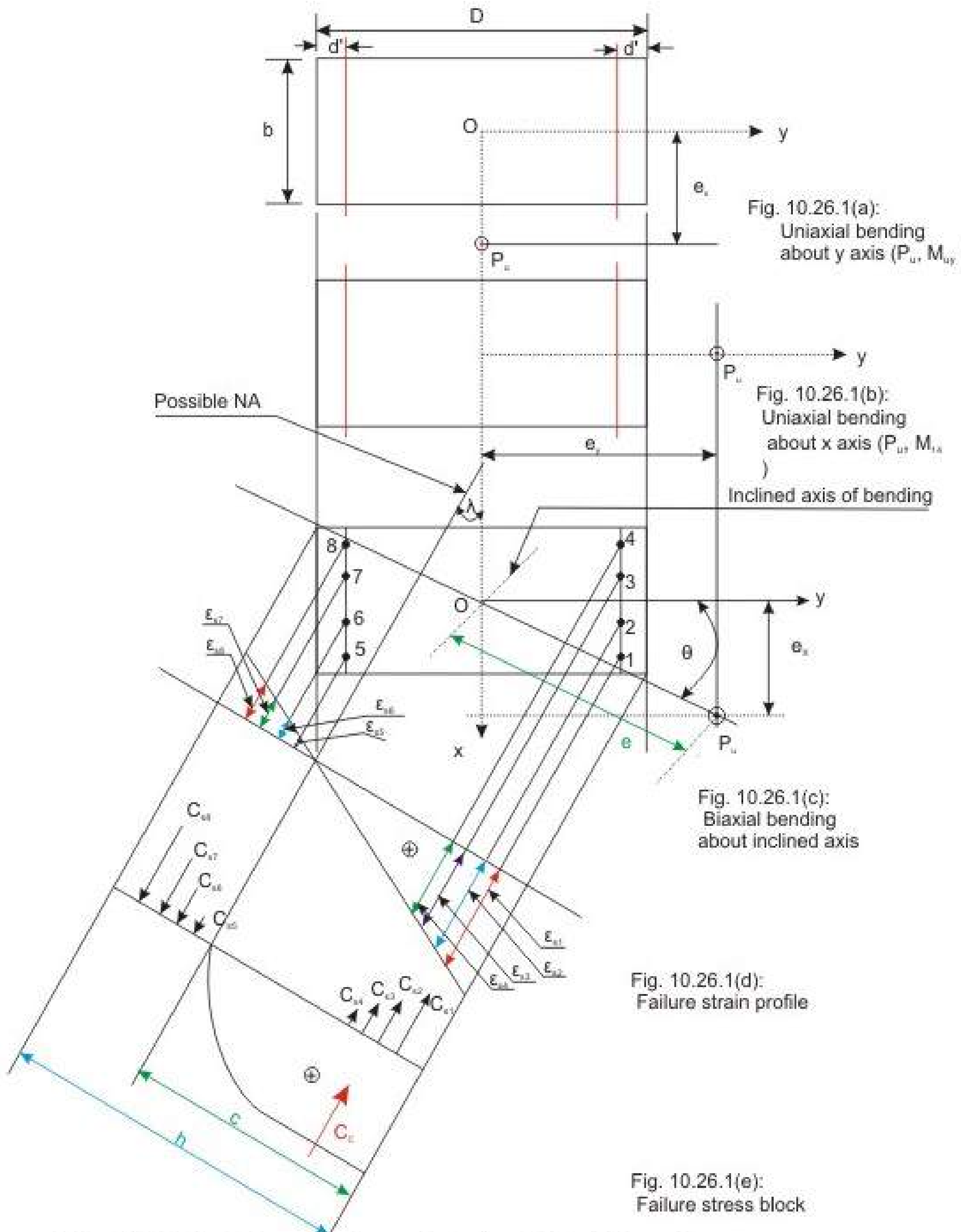


Fig. 10.26.1: Column under uniaxial and biaxial bending

Figures 10.26.1a and b present column section under axial load and uniaxial bending about the principal axes *x* and *y*, respectively. Figure 10.26.1c presents the column section under axial load and biaxial bending. The eccentricities e_x and e_y of Fig.10.26.1c are the same as those of Fig.10.26.1a (for e_x) and Fig.10.26.1b (for e_y), respectively. Thus, the biaxial bending case (case c) is the resultant of two uniaxial bending cases a and b. The resultant eccentricity e, therefore, can be written as (see Fig.10.26.1c):

$$e = (e_x^2 + e_y^2)^{1/2}$$
(10.55)

Designating the moments of cases a, b and c by M_{ux}, M_{uy} and M_u, respectively, we can write:

$$M_u = (M_{ux}^2 + M_{uy}^2)^{1/2}$$
(10.56)

and the resultant M_u is acting about an inclined axis, so that

$$\tan\theta = e_x/e_y = M_{uy}/M_{ux}$$
(10.57)

the angle of inclination θ is measured from y axis.

This inclined resultant axis shall also be the principal axis if the column section including the reinforcing bars is axisymmetric. In such a situation, the biaxial bending can be simplified to a uniaxial bending with the neutral axis parallel to the resultant axis of bending.

The reinforced concrete column cross-sections are, in general, non-axisymmetric with reference to the longitudinal axis and, therefore, the neutral axis is not parallel to the resultant axis of bending (θ is not equal to λ in Fig.10.26.1c). Moreover, it is extremely laborious to find the location of the neutral axis with successive trials. However, failure strain profile and stress block can be drawn for a given location of the neutral axis. Figs.10.25.1d and e present the strain profile and stress block, respectively, of the section shown in Fig.10.25.1c.

10.26.3 Interaction Surface

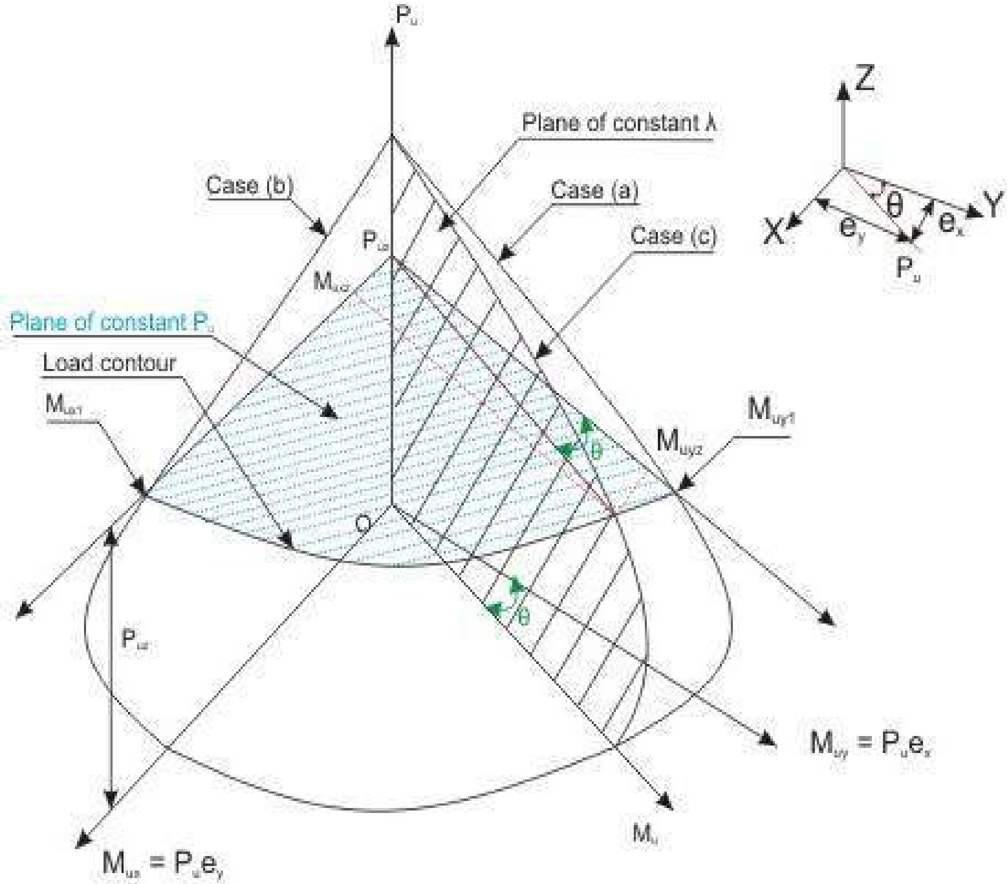


Fig. 10.26.2: Interaction diagram under axial load and biaxial bending

Figure 10.26.2 can be visualised as a three-dimensional plot of P_u-M_{ux}-M_{uy}, wherein two two-dimensional plots of P_u-M_{uy} and P_u-M_{us} are marked as case (a) and case (b), respectively. These two plots are the interaction curves for the columns of Figs.10.26.1a and b, respectively. The envelope of several interaction curves for different axes will generate the surface, known as interaction surface.

The interaction curve marked as case (c) in Fig.10.26.2, is for the column under biaxial bending shown in Fig.10.26.1c. The corresponding axis of bending is making an angle θ with the y axis and satisfies Eq.10.57. It has been explained in Lesson 24 that a column subjected to a pair of P and M will be safe if their respective values are less than P_u and M_u, given by its interaction curve. Extending the same in the three-dimensional figure of interaction surface, it is also acceptable that a column subjected to a set of P_u, M_{uy} and M_{ux} is safe if the set of values lies within the surface. Since P_u is changing in the direction of z, let us designate the moments and axial loads as mentioned below:

M_{uxz} = design flexural strength with respect to major axis xx under biaxial loading, when $P_u = P_{uz}$,

M_{uyz} = design flexural strength with respect to minor axis *yy* under biaxial loading, when $P_u = P_{uz}$,

M_{ux1} = design flexural strength with respect to major axis *xx* under uniaxial loading, when $P_u = P_{uz}$, and

M_{uy1} = design flexural strength with respect to minor axis *yy* under uniaxial loading, when $P_u = P_{uz}$.

The above notations are also shown in Fig.10.26.2.

All the interaction curves, mentioned above, are in planes perpendicular to *xy* plane. However, the interaction surface has several curves parallel to *xy* plane, which are planes of constant P_u. These curves are known as load contour, one such load contour is shown in Fig.10.26.2, when $P_u = P_{uz}$. Needless to mention that the load is constant at all points of a load contour. These load contour curves are also interaction curves depicting the interaction between the biaxial bending capacities.

10.26.4 Limitation of Interaction Surface

The main difficulty in preparing an exact interaction surface is that the neutral axis for the case (c) of Fig.10.26.1c will not, in general, be perpendicular to the line joining the loading point P_u and the centre of the column (Fig.10.26.1c). This will require several trials with c and λ, where c is the distance of the neutral axis and λ angle made by the neutral axis with the x axis, as shown in Fig.10.26.1c. Each trial will give a set of P_u, M_{ux} and M_{uy}. Only for a particular case, the neutral axis will be perpendicular to the line joining the load point P_u to the centre of the column. This search makes the process laborious. Moreover, several trials with c and λ, giving different values of h (see Fig.10.26.1c), may result in a failure surface with wide deviations, particularly as the value of P_u will be increasing.

Accordingly, the design of columns under axial load with biaxial bending is done by making approximations of the interaction surface. Different countries adopted different approximate methods. Clause 39.6 of IS 456 recommends one method based on Bresler's formulation, also known as "Load Contour Method", which is taken up in the following section. (For more information, please refer to: "Design Criteria for Reinforced Columns under Axial Load and Biaxial Bending", by B. Bresler, J. ACI, Vol.32, No.5, 1960, pp.481-490).

10.26.5 IS Code Method for Design of Columns under Axial Load and Biaxial Bending

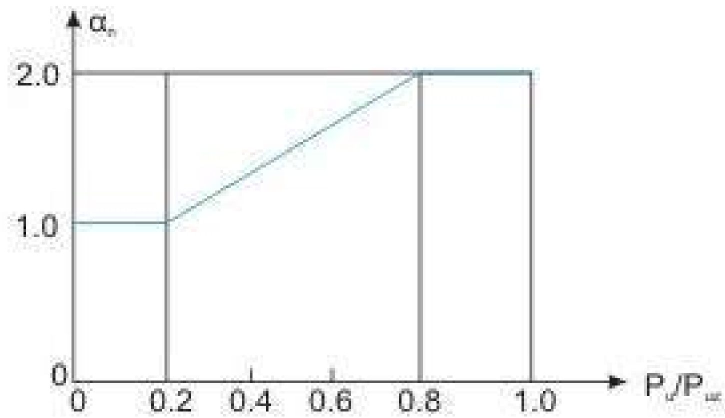


Fig. 10.26.3: Exponent α_n versus P_u / P_{uz}

IS 456 recommends the following simplified method, based on Bresler's formulation, for the design of biaxially loaded columns. The relationship between M_{uxz} and M_{uyz} for a particular value of P_u = P_{uz}, expressed in non-dimensional form is:

$$(M_{ux} / M_{ux1})^{\alpha n} + (M_{uy} / M_{uy1})^{\alpha n} \leq 1$$
(10.58)

where M_{ux} and M_{uy} = moments about x and y axes due to design loads, and

α_n is related to P_u/P_{uz}, (Fig.10.26.3), where

P_{uz} = $0.45\, f_{ck}\, A_c + 0.75\, f_y\, A_{sc}$

= $0.45\, A_g + (0.75\, f_y - 0.45\, f_{ck})\, A_{sc}$
(10.59)

where A_g = gross area of the section, and

A_{sc} = total area of steel in the section

M_{uxz}, M_{uyz}, M_{ux1} and M_{uy1} are explained in sec.10.26.3 earlier.

It is worth mentioning that the quantities M_{ux}, M_{uy} and P_u are due to external loadings applied on the structure and are available from the analysis, whereas M_{ux1}, M_{uy1} and P_{uz} are the capacities of the column section to be considered for the design.

Equation 10.58 defines the shape of the load contour, as explained earlier (Fig.10.26.2). That is why the method is also known as "Load Contour Method". The exponent α_n of Eq.10.58 is a constant which defines the shape of the load contour and depends on the value of P_u. For low value of the axial load, the load contour is approximated as a straight line and, in that case, α_n = 1. On the other hand, for high values of axial load, the load contour is approximated as a quadrant of a circle, when α_n = 2. For intermediate load values, the value of α_n lies between 1 and 2. Chart 64 of SP-16 presents the load contour and Fig.10.26.3 presents the relationship between α_n and P_u/P_{uz}. The mathematical relationship between α_n and P_u/P_{uz} is as follows:

α_n = 1.0, when $P_u/P_{uz} \le 0.2$

α_n = 0.67 + 1.67 P_u/P_{uz}, when $0.2 < (P_u/P_{uz}) < 0.8$

α_n = 2.0, when $(P_u/P_{uz}) \ge 0.8$
(10.60)

10.26.6 Solution of Problems using IS Code Method

The IS code method, as discussed in sec.10.26.5, can be employed to solve both the design and analysis types of problems. The only difference between the design and analysis type of problems is that a trial section has to be assumed including the percentage of longitudinal reinforcement in the design problems. However, these data are available in the analysis type of problems. Therefore, a guide line is given in this section for assuming the percentage of longitudinal reinforcement for the design problem. Further, for both types of problems, the eccentricities of loads are to be verified if they are more than the corresponding minimum eccentricities, as stipulated in cl.25.4 of IS 456. Thereafter, the relevant steps are given for the solution of the two types of problems.

(a) Selection of trial section for the design type of problems

As mentioned in sec.10.24.2(i) of Lesson 24, the preliminary dimensions are already assumed during the analysis of structure (mostly statically indeterminate). Thus, the percentage of longitudinal steel is the one parameter to be assumed from the given P_u, M_{ux}, M_{uy}, f_{ck} and f_y. Pillai and Menon (Ref. No. 4) suggested a simple way of considering a moment of approximately 15 per cent in excess (lower percentage up to 5 per cent if P_u/P_{uz} is relatively high) of the resultant moment

$$M_u = (1.15)(M_{ux}^2 + M_{uy}^2)^{1/2}$$
(10.61)

as the uniaxial moment for the trial section with respect to the major principal axis *xx*, if $M_{ux} \ge M_{uy}$; otherwise, it should be with respect to the minor principal axis.

The reinforcement should be assumed to be distributed equally on four sides of the section.

(b) Checking the eccentricities e_x and e_y for the minimum eccentricities

Clause 25.4 of IS 256 stipulates the amounts of the minimum eccentricities and are given in Eq.10.3 of sec.10.21.11 of Lesson 21. However, they are given below as a ready reference.

$$e_{xmin} \geq \text{greater of } (l/500 + b/30) \text{ or } 20 \text{ mm}$$

…. (10.3)

$$e_{ymin} \geq \text{greater of } (l/500 + D/30) \text{ or } 20 \text{ mm}$$

where l, b and D are the unsupported length, least lateral dimension and larger lateral dimension, respectively. The clause further stipulates that for the biaxial bending, it is sufficient to ensure that the eccentricity exceeding the minimum value about one axis at a time.

(c) Steps for the solution of problems

The following are the steps for the solution of both analysis and design types of problems while employing the method recommended by IS 456.

(i) Verification of eccentricities

It is to be done determining $e_x = M_{ux}/P_u$ and $e_y = M_{uy}/P_u$ from the given data of P_u, M_{ux} and M_{uy}; and e_{xmin} and e_{ymin} from Eq.10.3 from the assumed b and D and given l.

(ii) Assuming a trial section including longitudinal reinforcement

This step is needed only for the design type of problem, which is to be done as explained in (a) above.

(iii) Determination of M_{ux1} and M_{uy1}

Use of design charts should be made for this. M_{ux1} and M_{uy1}, corresponding to the given P_u, should be significantly greater than M_{ux} and M_{uy}, respectively. Redesign of the section should be done if the above are not satisfied for the design type of problem only.

(iv) Determination of P_{uz} and α_n

The values of P_{uz} and α_n can be determined from Eqs.10.59 and 10.60, respectively. Alternatively, P_{uz} can be obtained from Chart 63 of SP-16.

(v) Checking the adequacy of the section

This is done either using Eq.10.58 or using Chart 64 of SP-16.

10.26.7 Illustrative Example

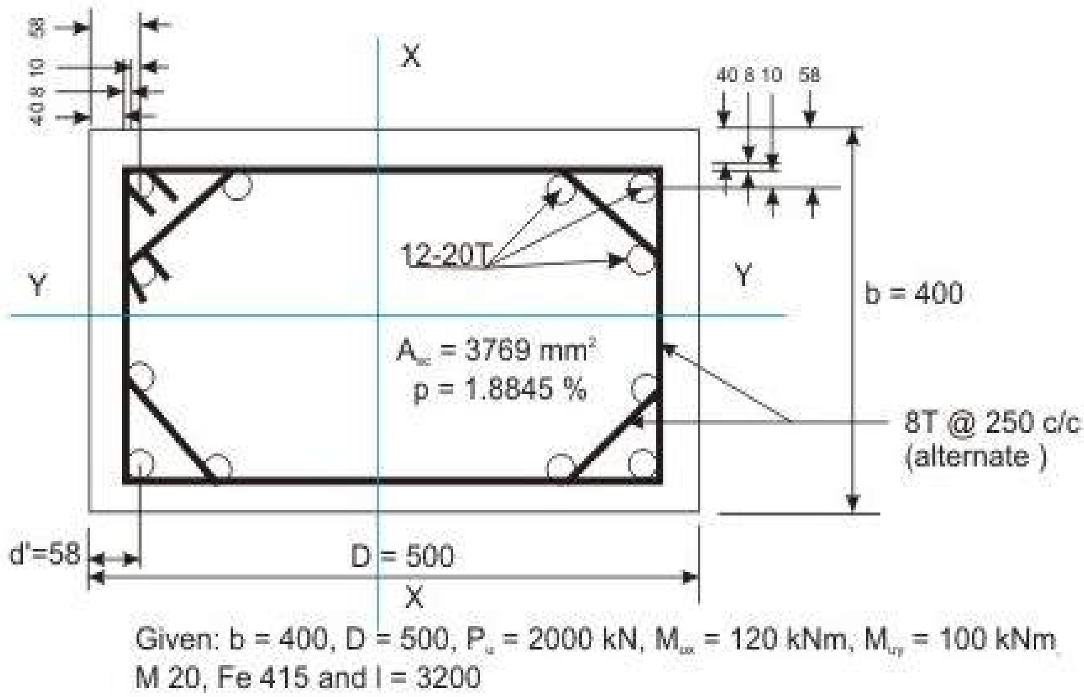


Fig. 10.26.4: Problem 1

Problem 1:

Design the reinforcement to be provided in the short column of Fig.10.26.4 is subjected to P_u = 2000 kN, M_{ux} = 130 kNm (about the major principal axis) and M_{uy} = 120 kNm (about the minor principal axis). The unsupported length of the column is 3.2 m, width b = 400 mm and depth D = 500 mm. Use M 25 and Fe 415 for the design.

Solution 1:

Step 1: Verification of the eccentricities

Given: l = 3200 mm, b = 400 mm and D = 500 mm, we have from Eq.10.3 of sec.10.26.6b, the minimum eccentricities are:

e_{xmin} = greater of (3200/500 + 400/30) and 20 mm = 19.73 mm or 20 mm = 20 mm

e_{ymin} = greater of (3200/500 + 500/30) and 20 mm = 23.07 mm or 20 mm = 23.07 mm

Again from P_u = 2000 kN, M_{ux} = 130 kNm and M_{uy} = 120 kNm, we have e_x = M_{ux}/P_u = 130(10^6)/2000(10^3) = 65 mm and e_y = M_{uy}/P_u = 120(10^6)/2000(10^3) = 60 mm. Both e_x and e_y are greater than e_{xmin} and e_{ymin}, respectively.

Step 2: Assuming a trial section including the reinforcement

We have b = 400 mm and D = 500 mm. For the reinforcement, $M_u = 1.15(M_{ux}^2 + M_{uy}^2)^{1/2}$, from Eq.10.61 becomes 203.456 kNm. Accordingly, we get

$P_u/f_{ck}bD$ = 2000(10^3)/(25)(400)(500) = 0.4

$M_u/f_{ck}bD^2$ = 203.456(10^6)/(25)(400)(500)(500) = 0.0814

Assuming d' = 60 mm, we have d'/D = 0.12. From Charts 44 and 45, the value of p/f_{ck} is interpolated as 0.06. Thus, p = 0.06(25) = 1.5 per cent, giving A_{sc} = 3000 mm^2. Provide 12-20 mm diameter bars of area 3769 mm^2, actual p provided = 1.8845 per cent. So, p/f_{ck} = 0.07538.

Step 3: Determination of M_{ux1} and M_{uy1}

We have $P_u/f_{ck}bD$ = 0.4 and p/f_{ck} = 0.07538 in step 2. Now, we get $M_{ux1}/f_{ck}bD^2$ from chart corresponding to d' = 58 mm (Fig.10.26.4) i.e., d'/D = 0.116. We interpolate the values of Charts 44 and 45, and get $M_{ux1}/f_{ck}bD^2$ = 0.09044. So, M_{ux1} = 0.0944(25)(400)(500(500)(10^{-6}) = 226.1 kNm.

For M_{ux1}, d'/b = 58/400 = 0.145. In a similar manner, we get M_{uy1} = 0.0858(25)(400)(400)(500)(10^{-6}) = 171.6 kNm.

As M_{ux1} and M_{uy1} are significantly greater than M_{ux} and M_{uy}, respectively, redesign of the section is not needed.

Step 4: Determination of P_{uz} and α_n

From Eq.10.59, we have P_{uz} = 0.45(25)(400)(500) + {0.75(415) - 0.45(25)}(3769) = 3380.7 kN.

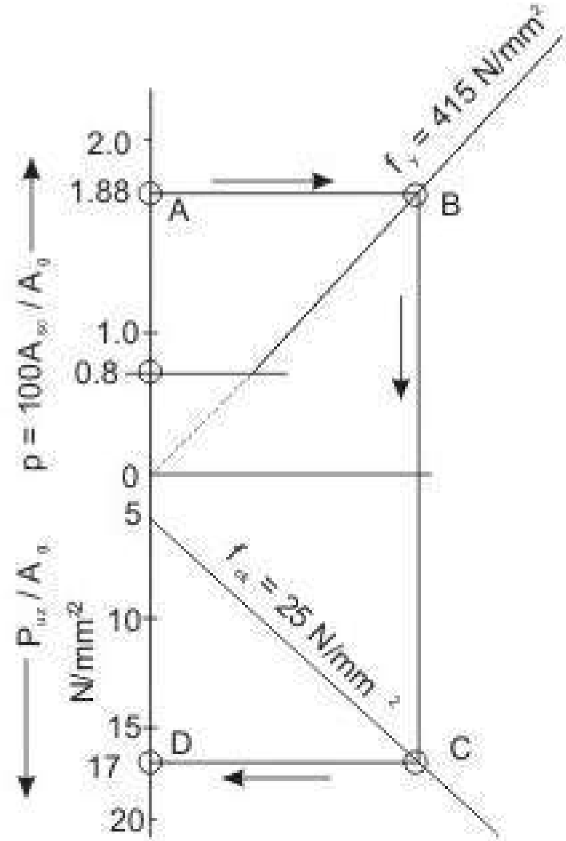


Fig. 10.26.5: Chart 63 of SP-16 in Problem 1
(not to scale)

Alternatively, Chart 63 may be used to find P_{uz} as explained. From the upper section of Chart 63, a horizontal line AB is drawn at p = 1.8845, to meet the Fe 415 line B (Fig.10.26.5). A vertical line BC is drawn from B to meet M 25 line at C. Finally, a horizontal line CD is drawn from C to meet P_{uz}/A_g at 17. This gives P_{uz} = 17(400)(500) = 3400 kN. The difference between the two values, 19.3 kN is hardly 0.57 per cent, which is due to the error in reading the value from the chart. However, any one of the two may be employed.

Now, the value of α_n is obtained from Eq.10.60 for P_u/P_{uz} = 2000/3380.7 = 0.5916, i.e., 0.2 < P_u/P_{uz} < 0.8, which gives, α_n = 0.67 + 1.67 (P_u/P_{uz}) = 1.658. Alternatively, α_n may be obtained from Fig.10.26.3, drawn to scale.

Step 5: Checking the adequacy of the section

Using the values of M_{ux}, M_{ux1}, M_{uy}, M_{uy1} and α_n in Eq.10.58, we have $(130/226.1)^{1.658} + (120/171.6)^{1.658} = 0.9521 < 1.0$. Hence, the design is safe.

Alternatively, Chart 64 may be used to determine the point (M_{ux}/M_{ux1}), (M_{uy}/M_{uy1}) is within the curve of P_u/P_{uz} = 0.5916 or not.

Here, M_{ux}/M_{ux1} = 0.5749 and M_{uy}/M_{uy1} = 0.6993. It may be seen that the point is within the curve of P_u/P_{uz} = 0.5916 of Chart 64 of SP-16.

Step 6: Design of transverse reinforcement

As per cl.26.5.3.2c of IS 456, the diameter of lateral tie should be > (20/4) mm diameter. Provide 8 mm diameter bars following the arrangement shown in Fig.10.26.4. The spacing of lateral tie is the least of :

(a) 400 mm = least lateral dimension of column,

(b) 320 mm = sixteen times the diameter of longitudinal reinforcement (20 mm),

(c) 300 mm

Accordingly, provide 8 mm lateral tie alternately @ 250 c/c (Fig.10.26.4).

10.26.8 Practice Questions and Problems with Answers

Q.1: Explain the behaviour of a short column under biaxial bending as the resultant of two uniaxial bending.

A.1: See sec. 10.26.2

Q.2: Draw one interaction surface for a short column under biaxial bending and show typical interaction curves and load contour curve. Explain the safety of a column with reference to the interaction surface when the column is under biaxial bending.

A.2: See sec.10.26.3 and Fig.10.26.2.

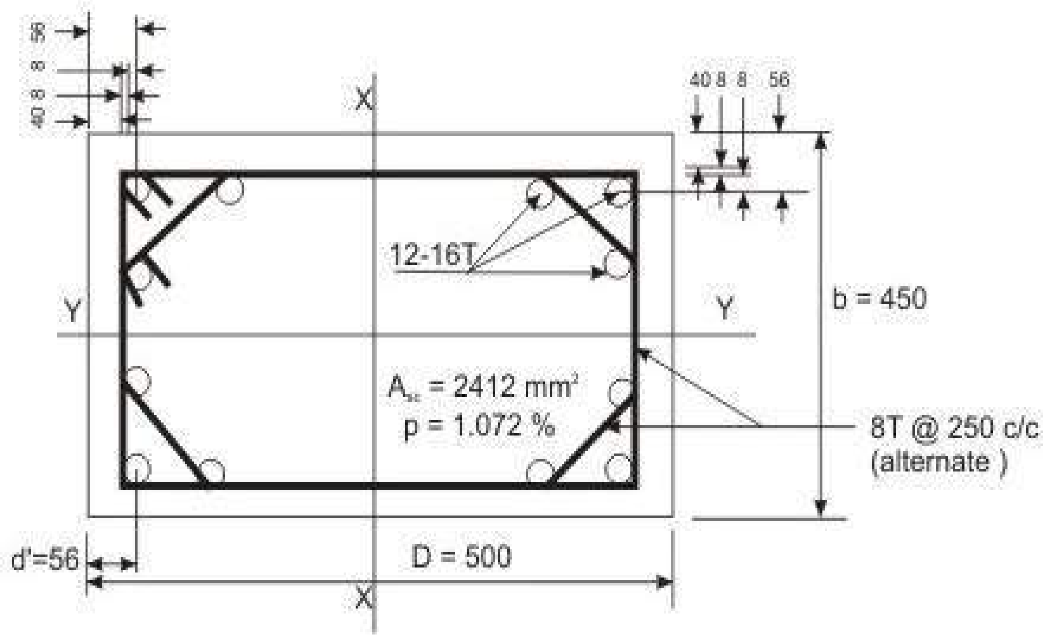
Q.3: Discuss the limitation of the interaction curve.

A.3: See sec.10.26.4.

Q.4: Illustrate the IS code method of design of columns under biaxial bending.

A.4: See sec.10.26.5.

Q.5:



Given: b = 450, D = 500, P_u = 1600 kN, M_{ux} = 120 kNm, M_{uy} = 100 kNm, M 20, Fe 415, l = 3200, A_{sc} = 2412 mm², tie 8T @ 250 c/c (alternate)

Fig. 10.26.6: Q.5

Analyse the safety of the short column of unsupported length 3.2 m, b = 450 mm, D = 500 mm, as shown in Fig.10.26.6, having 12-16 mm diameter bars as longitudinal reinforcement and 8 mm diameter bars as lateral tie @ 250 mm c/c, when subjected to P_u = 1600 kN, M_{ux} = 120 kNm and M_{uy} = 100 kNm. Use M 25 and Fe 415.

A.5:

Step 1: Verification of the eccentricities

From the given data: l = 3200 mm, b = 450 mm and D = 500 mm,

e_{xmin} = 3200/500 + 450/30 = 21.4 > 20 mm, so, 21.4 mm

e_{ymin} = 3200/500 + 5000/30 = 23.06 > 20 mm, so, 23.06 mm

$e_x = M_{ux}/P_u = 120(10^3)/1600 = 75$ mm

$e_y = M_{uy}/P_u = 100(10^3)/1600 = 62.5$ mm

So, the eccentricities e_x and e_y are >> e_{xmin} and e_{ymin}.

Step 2: Determination of M_{ux1} and M_{uy1}

Given data are: b = 450 mm, D = 500 mm, f_{ck} = 25 N/mm^2, f_y = 415 N/mm^2, P_u = 1600 kN, M_{ux} = 120 kNm, M_{uy} = 100 kNm and A_{sc} = 2412 mm^2 (12-16 mm diameter bars).

We have p = (100)(2412)/(450)(500) = 1.072 per cent, and d'/D = 56/500 = 0.112, d'/b = 56/450 = 0.124, $P_u/f_{ck}bD$ = 1600/(25)(450)(500) = 0.2844 and p/f_{ck} = 1.072/25 = 0.043. We get $M_{ux1}/f_{ck}bD^2$ from Charts 44 and 45 as 0.09 and 0.08, respectively. Linear interpolation gives $M_{ux1}/f_{ck}bD^2$ for d'/D = 0.112 as 0.0876. Thus,

$$M_{ux1} = (0.0876)(25)(450)(500)(500) = 246.376 \text{ kNm}$$

Similarly, interpolation of values (0.09 and 0.08) from Charts 44 and 45, we get $M_{uy1}/f_{ck}db^2$ = 0.085 for d'/b = 0.124. Thus

$$M_{uy1} = (0.085)(25)(500)(450)(450) = 215.156 \text{ kNm}$$

Step 3: Determination of P_{uz} and α_n

From Eq.10.59, P_{uz} = 0.45(25)(450)(500) + {0.75(415) - 0.45(25)}(2412) = 3254.85 kN. This gives P_u/P_{uz} = 1600/3254.85 = 0.491574.

From Eq.10.60, α_n = 0.67 + 1.67(P_u/P_{uz}) = 0.67 + 1.67(0.491574) = 1.4909.

Step 4: Checking the adequacy of the section

From Eq.10.58, we have: $(120/246.376)^{1.4909} + (100/215.156)^{1.4909}$ = 0.6612 < 1.

Hence, the section is safe to carry P_u = 1600 kN, M_{ux} = 120 kNm and M_{uy} = 100 kNm.

10.26.9 References

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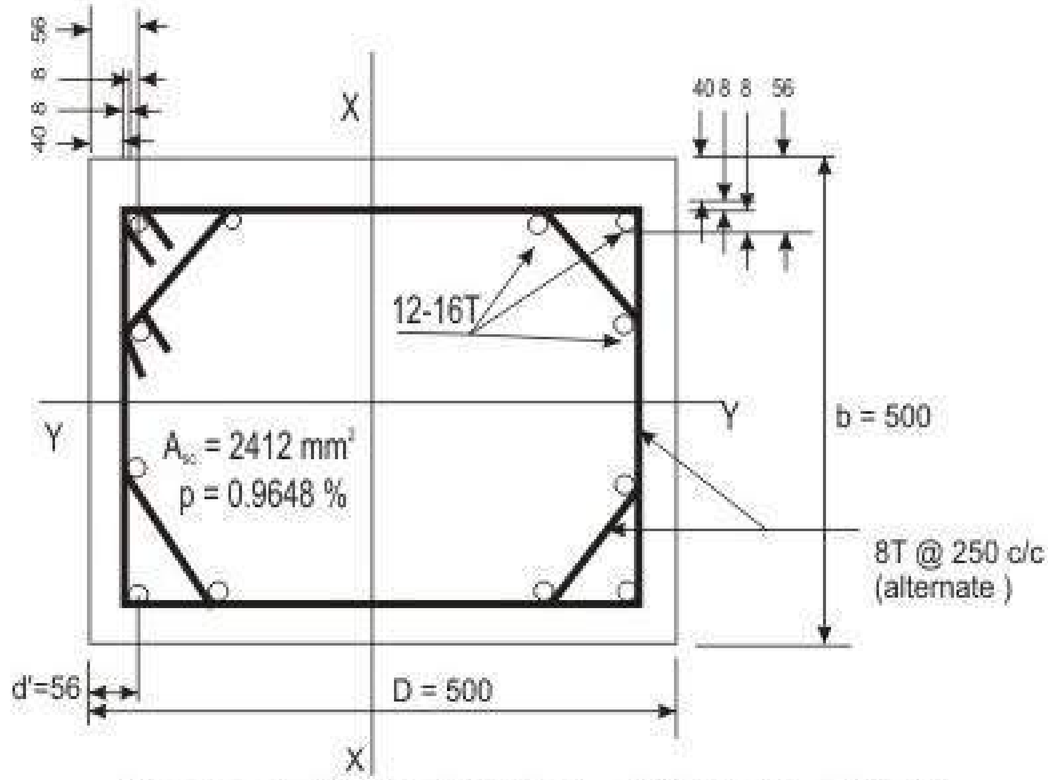
10. Textbook of R.C.C, by G.S.Birdie and J.S.Birdie, Wiley Eastern Limited, New Delhi, 1993.
11. Design of Concrete Structures, 13th Edition, by Arthur H. Nilson, David Darwin and Charles W. Dolan, Tata McGraw-Hill Publishing Company Limited, New Delhi, 2004.
12. Concrete Technology, by A.M.Neville and J.J.Brooks, ELBS with Longman, 1994.
13. Properties of Concrete, 4th Edition, 1st Indian reprint, by A.M.Neville, Longman, 2000.
14. Reinforced Concrete Designer's Handbook, 10th Edition, by C.E.Reynolds and J.C.Steedman, E & FN SPON, London, 1997.
15. Indian Standard Plain and Reinforced Concrete – Code of Practice (4th Revision), IS 456: 2000, BIS, New Delhi.
16. Design Aids for Reinforced Concrete to IS: 456 – 1978, BIS, New Delhi.

10.26.10 Test 26 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

TQ.1:



Given: b = D = 500, P_u = 1800 kN, M_{ux} = 160 kNm, M_{uy} = 150 kNm, M 20, Fe 415, l = 3500, A_{sc} = 2412 mm², tie 8T @ 250 c/c (alternate)

Fig. 10.26.7: TQ.1

Analyse the safety of the short square column of unsupported length = 3.5 m, b = D = 500 mm, as shown in Fig.10.26.7, with 12-16 mm diameter bars as longitudinal reinforcement and 8 mm diameter bars as lateral tie @ 250 mm c/c, when subjected to P_u = 1800 kN, M_{ux} = 160 kNm and M_{uy} = 150 kNm.

A.TQ.1:

Step 1: Verification of the eccentricities

From the given data: l = 3500 mm, b = D = 500 mm, we have

e_{min} in both directions (square column) = (3500/500) + (500/30) = 23.67 mm

e_x = 160(10^3)/1800 = 88.88 mm and e_y = 150(10^3)/1800 = 83.34 mm

Therefore, e_x and e_y >> e_{min}.

Step 2: Determination of M_{ux1} and M_{uy1}

We have the given data: $b = D = 500$ mm, $f_{ck} = 25$ N/mm^2, $f_y = 415$ N/mm^2, $P_u = 1800$ kN, $M_{ux} = 160$ kNm, $M_{uy} = 150$ kNm and $A_{sc} = 2412$ mm^2 (12-16 mm diameter bars).

The percentage of longitudinal reinforcement $p = 241200/(500)(500) = 0.9648$ per cent, and $d'/D = 56/500 = 0.112$ and $p/f_{ck} = 0.9648/25 = 0.03859$. Linear interpolation of values of $M_{ux1}/f_{ck}bD^2$ from Charts 44 and 45 for $d'/D = 0.112$ is obtained as 0.08. Thus,

$$M_{ux1} = (0.08)(25)(500)(500)(500) = 250 \text{ kNm}$$

$$M_{uy1} = M_{ux1} = 250 \text{ kNm (square column)}$$

Step 3: Determination of P_{uz} and α_n

From Eq.10.59,

$$P_{uz} = 0.45(25)(500)(500) + \{0.75(415) - 0.45(25)\}(2415) = 3536.1 \text{ kN.}$$

$$P_u/P_{uz} = 1800/3536.1 = 0.509.$$

From Eq.10.60, $\alpha_n = 0.67 + 1.67(0.509) = 1.52$.

Step 4: Checking the adequacy of the section

From Eq.10.58, we have: $(160/250)^{1.52} + (150/250)^{1.52} = 0.967 < 1$.

Hence, the section can carry $P_u = 1800$ kN, $M_{ux} = 160$ kNm and $M_{uy} = 150$ kNm.

10.26.11 Summary of this Lesson

This lesson explains the behaviour of short columns under axial load and biaxial bending with the help of interaction surface, visualised as a three-dimensional plot of P_u-M_{ux}-M_{uy}. The interaction surface has a set of interaction curves of P_u-M_u and another set of interaction curves of M_{uxz}-M_{uyz} at constant P_{uz}, also known as load contour. The design and analysis of short columns are also explained with the help of derived equations and design charts of SP-16. Numerical examples in the illustrative example, practice problems and test will help in understanding the application of the theory in solving the analysis and design types of problems of short columns under axial load and biaxial bending.

Module
10
Compression Members

Lesson
24
Preparation of Design Charts

Instructional Objectives:

At the end of this lesson, the student should be able to:

- identify a design chart and understand the differences between a design chart and interaction diagram of *P* and *M*,
- name the major design parameters of short columns subjected to axial loads and uniaxial bending,
- state the design parameters assumed before the design,
- state the design parameter actually designed for the column,
- explain the roles of each of the design parameters in increasing the strength capacities of column,
- name the two non-dimensional design parameters to prepare the design charts,
- derive the governing equations in four separate cases while preparing the design charts,
- mention the various points at which the values of the two non-dimensional parameters are determined to prepare the design charts,
- prepare the design chart of any short and rectangular column subjected to axial loads and uniaxial moment.

10.24.1 Introduction

Lesson 23 illustrates the different steps of determining the capacities of a short, rectangular, reinforced with steel bars, concrete column. Several pairs of collapse strengths P_u and M_u are to be determined for a column with specific percentage of longitudinal steel bars assuming different positions of the neutral axis. A designer has to satisfy that each of the several pairs of P_u and M_u, obtained from the structural analysis, is less than or equal to the respective strengths in form of pairs of P_u and M_u obtained from determining the capacities for several locations of the neutral axis. Thus, the design shall involve several trials of a particular cross-section of a column for its selection.

On the other hand, it is also possible to prepare non-dimensional interaction diagram selecting appropriate non-dimensional parameters. This would help to get several possible cross-sections with the respective longitudinal steel bars. This lesson explains the preparation of such non-dimensional interaction diagrams which are also known as design charts.

Similar design charts of circular and other types of cross-sections can be prepared following the same procedure as that of rectangular cross-section. However, the stress block parameters, explained in Lesson 23, are to be established separately by summing up the forces and moment of several strips by dividing the cross-section of columns into the strips. This lesson is restricted to columns of rectangular cross-section which are symmetrically reinforced.

10.24.2 Design Parameters

The following are the four major design parameters to be determined for any column so that it has sufficient pairs of strengths (P_u and M_u) to resist all critical pairs obtained from the analysis:

(i) dimensions *b* and *D* of the rectangular cross-section,

(ii) longitudinal steel reinforcing bars - percentage *p*, nature of distribution (equally on two or four sides) and *d'/D*,

(iii) grades of concrete and steel, and

(iv) transverse reinforcement.

The roles and importance of each of the above four parameters are elaborated below:

(i) Dimensions b and D of the rectangular cross-section

The strength of column depends on the two dimensions *b* and *D*. However, preliminary dimensions of *b* and *D* are already assumed for the analysis of structure, which are usually indeterminate statically. In the subsequent redesign, these dimensions may be revised, if needed, inviting re-analysis with the revised dimensions.

(ii) Longitudinal steel reinforcing bars

It is a very important consideration to utilise the total area of steel bars effectively. The total area of steel, expressed in percentage *p* ranges from the minimum 0.8 to the maximum 4 per cent of the gross area of the cross-section. The bars may be distributed either equally on two sides or on all four sides judiciously having two or multiple rows of steel bars. The strain profiles of Fig.10.23.2 reveals that the rows of bars may be all in compression or both compression and tension depending on the location of the neutral axis. Accordingly, the total strength of the longitudinal bars is determined by adding all the individual strengths of bars of different rows. The effective cover *d'*, though depends on the nominal cover, has to be determined from practical considerations of housing all the steel bars.

(iii) Grades of concrete and steel

The dimensions *b* and *D* of the cross-section and the amount of longitudinal steel bars depend on the grades of concrete and steel.

(iv) Transverse reinforcement

The transverse reinforcement, provided in form of lateral ties or spirals, are important for the following advantages in

(a) preventing premature / local buckling of the longitudinal bars,

(b) improving ductility and strength by the effect of confinement of the core concrete,

(c) holding the longitudinal bars in position during construction, and

(d) providing resistance against shear and torsion, if present.

However, the transverse reinforcement does not have a major contribution in influencing the capacities of the column. Moreover, the design of transverse reinforcement involves selection of bar diameter and spacing following the stipulations in the design code. The bar diameter of the transverse reinforcement also depends on the bar diameter of longitudinal steel. Accordingly, the transverse reinforcement is designed after finalizing other parameters mentioned above.

It is, therefore, clear that the design of columns mainly involves the determination of percentage of longitudinal reinforcement *p*, either assuming or knowing the dimensions *b* and *D*, grades of concrete and steel, distribution of longitudinal bars in two or multiple rows and *d'/D* ratio from the analysis or elsewhere. Needless to mention that any designed column should be able to resist several critical pairs of P_u and M_u obtained from the analysis of the structure. It is also a fact that several trials may be needed to arrive at the final selection revising any or all the assumed parameters. Accordingly, the design charts are prepared to give the results for the unknown parameter quickly avoiding lengthy calculations after selecting appropriate non-dimensional parameters.

Based on the above considerations and making the design simple, quick and fairly accurate, the following are the two non-dimensional parameters:

For axial load: $P_u/f_{ck}bD$

For moment: $M_u/f_{ck}bD^2$

The characteristic strength of concrete f_{ck} has been associated with the non-dimensional parameters as the grade of concrete does not improve the strength of the column significantly. The design charts prepared by SP-16 are assuming the constant value of f_{ck} for M 20 to avoid different sets of design charts for different grades of concrete. However, separate design charts are presented in SP-16 for three grades of steel (Fe 250, Fe 415 and Fe 500), four values of d'/D (0.05, 0.1, 0.15 and 0.2) and two types of distribution of longitudinal steel (distributed equally on two and four sides). Accordingly there are twenty-four design charts for the design of rectangular columns. Twelve separate design charts are also presented in SP-16 for circular sections covering the above mentioned three grades of steel and for values of d'/D ratio.

However, the unknown parameter p, the percentage of longitudinal reinforcement has been modified to p/f_{ck} in all the design charts of SP-16, so that for grades other than M 20, the more accurate value of p can be obtained by multiplying the p/f_{ck} with the actual grade of concrete used in the design of that column.

However, this lesson explains that it is also possible to prepare design chart taking into consideration the actual grade of concrete. As mentioned earlier, the design charts are prepared getting the pairs of values of P_u and M_u in non-dimensional form from the equations of equilibrium for different locations of the neutral axis. We now take up the respective non-dimensional equations for four different cases as follows:

(a) When the neutral axis is at infinity, i.e., $kD = \infty$, pure axial load is applied on the column.

(b) When the neutral axis is outside the cross-section of the column, i.e., $\infty > kD \geq D$.

(c) When the neutral axis is within the cross-section of the column, i.e., $kD < D$.

(d) When the column behaves like a steel beam.

10.24.3 Non-dimensional Equation of Equilibrium when $k = \infty$, (Pure Axial Load)

Figures 10.23.2b and c of Lesson 23 present the strain profile EF and the corresponding stress block for this case. As the load is purely axial, we need to express the terms C_c and C_s of Eq.10.35 of sec.10.23.10 of Lesson 23. The total compressive force due to concrete of constant stress of 0.446 f_{ck} is:

$$C_c = 0.446\ f_{ck}\ b\ D \qquad (10.37)$$

However, proper deduction shall be made for the compressive force of concrete not available due to the replacement by steel bars while computing C_s.

The force of longitudinal steel bars in compression is now calculated. The steel bars of area $pbD/100$ are subjected to the constant stress of f_{sc} when the strain is 0.002. Subtracting the compressive force of concrete of the same area $pbD/100$, we have,

$$C_s = (pbD/100)\ (f_{sc} - 0.446\ f_{ck}) \qquad (10.38)$$

Thus, we have from Eq.10.35 of sec.10.23.10 of Lesson 23 after substituting the expressions of C_c and C_s from Eqs.10.37 and 10.38,

$$P_u = 0.446\ f_{ck}\ b\ D + (pbD/100)\ (f_{sc} - 0.446\ f_{ck}) \qquad (10.39)$$

Dividing both sides of Eq.10.39 by $f_{ck}\ bD$, we have

$$(P_u/f_{ck}\ bD) = 0.446 + (p/100\ f_{ck})\ (f_{sc} - 0.446\ f_{ck}) \qquad (10.40)$$

Thus, Eq.10.40 is the only governing equation for this case to be considered.

10.24.4 Non-dimensional Equations of Equilibrium when Neutral Axis is Outside the Section ($\infty > kD \geq D$)

Figures 10.23.3b and c of Lesson 23 present the strain profile JK and the corresponding stress block for this case. The expressions of C_c, C_s and appropriate lever arms are determined to write the two equations of equilibrium (Eqs.10.35 and 36) of Lesson 23. While computing C_c, the area of parabolic stress block is determined employing the coefficient C_1 from Table 10.4 of Lesson 23. Similarly, the coefficient C_2, needed to write the moment equation, is obtained from Table 10.4 of Lesson 23. The forces and the corresponding lever arms of longitudinal steel bars are to be considered separately and added for each of the n rows of the longitudinal bars. Thus, we have the first equation as,

$$P_u = C_1 f_{ck} bD + \sum_{i=1}^{n} (p_i \, bD/100)(f_{si} - f_{ci})$$

(10.41)

where C_1 = coefficient for the area of stress block to be taken from Table 10.4 of Lesson 23,

p_i = A_{si}/bD where A_{si} is the area of reinforcement in the i^{th} row,

f_{si} = stress in the i^{th} row of reinforcement, taken positive for compression and negative for tension,

f_{ci} = stress in concrete at the level of the i^{th} row of reinforcement, and

n = number of rows of reinforcement.

Here also, the deduction of the compressive force of concrete has been made for the concrete replaced by the longitudinal steel bars.

Dividing both sides of Eq.10.41 by $f_{ck}bD$, we have

$$(P_u/f_{ck}bD) = C_1 + \sum_{i=1}^{n} (p_i /100 \, f_{ck})(f_{si} - f_{ci})$$

(10.42)

Similarly, the moment equation (Eq.10.36) becomes,

$$M_u = C_1 f_{ck}bD\,(D/2 - C_2D) + \sum_{i=1}^{n} (p_i \, bD/100)(f_{si} - f_{ci})\, y_i$$

(10.43)

where C_2 = coefficient for the distance of the centroid of the compressive stress block of concrete measured from the highly compressed right edge and is taken from Table 10.4 of Lesson 23, and

y_i = the distance from the centroid of the section to the i^{th} row of reinforcement, positive towards the highly compressed right edge and negative towards the least compressed left edge.

Dividing both sides of Eq.10.43 by $f_{ck}bD^2$, we have

$$(M_u/f_{ck}bD^2) = C_1(0.5 - C_2) + \sum_{i=1}^{n} (p_i /100 \, f_{ck})(f_{si} - f_{ci})(y_i/D)$$

(10.44)

Equations 10.42 and 10.44 are the two non-dimensional equations of equilibrium in this case when $\infty < kD \leq D$.

10.24.5 Non-dimensional Equations of Equilibrium when the Neutral Axis is within the Section (kD < D)

The strain profile IN and the corresponding stress block of concrete are presented in Figs.10.23.4b and c for this case. Following the same procedure of computing C_c, C_s and the respective lever arms, we have the first equation as

$$P_u = 0.36 f_{ck} kbD + \sum_{i=1}^{n} (p_i bD/100)(f_{si} - f_{ci}) \tag{10.45}$$

Dividing both sides of Eq.10.45 by $f_{ck}bD$, we have

$$P_u/f_{ck}bD = 0.36 k + \sum_{i=1}^{n} (p_i/100 f_{ck})(f_{si} - f_{ci}) \tag{10.46}$$

and the moment equation (Eq.10.36) as

$$M_u = 0.36 f_{ck} kbD(0.5 - 0.42 k) D + \sum_{i=1}^{n} (p_i bD/100)(f_{si} - f_{ci})(y_i/D) \tag{10.47}$$

Dividing both sides of Eq.10.47 by $f_{ck}bD^2$, we have

$$(M_u/f_{ck}bD^2) = 0.36 k(0.5 - 0.42 k) + \sum_{i=1}^{n} (p_i/100 f_{ck})(f_{si} - f_{ci})(y_i/D) \tag{10.48}$$

where k = Depth of the neutral axis/Depth of column, mentioned earlier in sec.10.21.10 and Fig.10.21.11 of Lesson 21.

Equations 10.46 and 10.48 are the two non-dimensional equations of equilibrium in this case.

10.24.6 Non-dimensional Equation of Equilibrium when the Column Behaves as a Steel Beam

This is a specific situation when the column is subjected to pure moment $M_u = M_o$ only (Point 6 of the interaction diagram in Fig.10.23.1 of Lesson 23).

Since the column has symmetrical longitudinal steel on both sides of the centroidal axis of the column, the column will resist the pure moment by yielding of both tensile and compressive steel bars (i.e., $f_{si} = 0.87\ f_y = f_{yd}$). Thus, we have only one equation (Eq.10.36 of Lesson 23), which becomes

$$M_u = \sum_{i=1}^{n} (p_i\, bD/100)\,(0.87\, f_y)\,(y_i/D)$$

(10.49)

Dividing both sides of Eq.10.49 by $f_{ck}\ bD^2$, we have

$$(M_u/f_{ck}bD^2) = \sum_{i=1}^{n} (p_i/100\ f_{ck})\,(0.87\, f_y)\,(y_i/D)$$

(10.50)

Equation 10.50 is the equation of equilibrium in this case.

10.24.7 Preparation of Design Charts

Design charts are prepared employing the equations of four different cases as given in secs.10.24.3 to 6. The advantage of employing the equations is that the actual grade of concrete can be taken into account, though it may not be worthwhile to follow this accurately. However, preparation of interaction diagram will help in understanding the behaviour of column with the change of neutral axis depth for the four cases mentioned in sec.10.24.2. The step by step procedure of preparing the design charts is explained below. It is worth mentioning that the values of $(P_u/f_{ck}bD)$ and $(M_u/f_{ck}bD^2)$ are determined considering different locations of the neutral axis for the four cases mentioned in sec.10.24.2.

Step 1: When the neutral axis is at infinity

The governing equation is Eq.10.40. The strain profile EF and the corresponding stress block are in Fig.10.23.2b and c of Lesson 23, respectively.

Step 2: When the column is subjected to axial load considering minimum eccentricity

Lesson 22 presents the design of short columns subjected to axial load only considering minimum eccentricity as stipulated in cl.29.3 of IS 456, employing Eq.10.4, which is as follows:

$P_u = 0.4\ f_{ck}\ b\ D + (pbD/100)\ (0.67\ f_y - 0.4\ f_{ck})$ …. (10.4)

Dividing both sides of Eq.10.4 by $f_{ck}\ bD$, we have

$(P_u/f_{ck}\, bD) = 0.4 + (p/100\, f_{ck})\, (0.67\, f_y - 0.4\, f_{ck})$
(10.51)

The P_u obtained from Eq.10.51 can also resist M_u as per cl.39.3 of IS 456. From the stipulation of cl. 39.3 of IS 456 and considering the maximum value of the minimum eccentricity as 0.05D, we have

$M_u = (P_u)\,(0.05)D = 0.02\, f_{ck}\, bD^2 + (0.05\, pbD^2/100)\, (0.67\, f_y - 0.4\, f_{ck})$

Dividing both sides of the above equation by $f_{ck}bD^2$, we have

$(M_u/f_{ck}\, bD^2) = 0.02 + (0.05p/100\, f_{ck})\, (0.67\, f_y - 0.4\, f_{ck})$
(10.52)

Equations 10.51 and 10.52 are the two equations to be considered in this case.

Step 3: When the neutral axis is outside the section

Figures 10.23.3b and c of Lesson 23 present one strain profile JK and the corresponding stress block, respectively, out of a large number of values of k from 1 to infinity, only values up to about 1.2 are good enough to consider, as explained in sec.10.23.5 of Lesson 23. Accordingly, we shall consider only one point, where k = 1.1, in this case. With the help of Eqs.10.42 and 10.44, Table 10.4 for the values of C_1 and C_2, Table 10.5 for the values of f_{si} and Eq.10.23 or Eq.10.27 for the values of f_{ci}, the non-dimensional parameters $P_u/f_{ck}\, bD$ and $M_u/f_{ck}\, bD^2$ are determined.

Step 4: When the neutral axis is within the section

One representative strain profile IU and the corresponding stress block are presented in Fig.10.23.4b and c, respectively, of Lesson 23. The following six points of the interaction diagram are considered satisfactory for preparing the design charts:

(a) Where the tensile stress of longitudinal steel is zero i.e., $kD = D - d'$,

(b) Where the tensile stress of longitudinal steel is $0.4 f_y d = 0.4(0.87\, f_y)$,

(c) Where the tensile stress of longitudinal steel is) $0.8 f_y d = 0.8(0.87\, f_y)$,

(d) Where the tensile stress of longitudinal steel is $f_y d = 0.87 f_y$ and strain $= 0.87 f_y/E_s$, i.e., the initial yield point,

(e) Where the tensile stress of longitudinal steel is $f_y d = 0.87 f_y$ and strain $= 0.87 f_y/E_s + 0.002$, i.e., the final yield point,

(f) When the depth of the neutral axis is $0.25D$.

For all six points, the respective strain profile and the corresponding stress blocks can be drawn. Therefore, values of ($P_u/f_{ck}\ bD$) and ($M_u/f_{ck}\ bD^2$) are determined from Eqs.10.46 and 10.48, using Table 10.5 for f_{sc} and Eq.10.34 for f_{ci}.

Step 5: When the column behaves like a steel beam

As explained in sec.10.24.6, Eq.10.50 is used to compute $M_u/f_{ck}\ bD^2$ in this case.

Step 6: Preparation of design chart

The ten pairs of ($P_u/f_{ck}\ bD$) and ($M_u/f_{ck}\ bD^2$) (one set each in steps 1, 2, 3 and 5 and six sets in step 4) can be plotted to prepare the desired design chart.

One illustrative example is taken up in the next section.

10.24.8 Illustrative Example

Problem 1:

Prepare a design chart for a rectangular column with 3 per cent longitudinal steel distributed equally on two faces using M 25 and Fe 415, and considering $d'/D = 0.15$.

Solution 1:

The solution of this problem is explained in six steps of the earlier section.

Step 1: When the neutral axis is at infinity

Figures 10.23.2b and c present the strain profile EF and the corresponding stress block, respectively. Using the values of p = 3 per cent, f_{ck} = 25 N/mm^2 and determining the value of f_{sc} = 327.7388 N/mm^2 (using linear interpolation from the values of Table 10.5 of Lesson 23), we get the value of ($P_u/f_{ck}\ bD$) from Eq.10.40 as

$$(P_u/f_{ck}\ bD) = 0.8259.$$

Step 2: When the column is subjected to axial load considering minimum eccentricity

Using the value of p = 3 per cent, f_{ck} = 25 N/mm^2 and f_y = 415 N/mm^2 in Eqs.10.51 and 10.52 of sec.10.24.6, we have

$(P_u/f_{ck}\, bD)$ = 0.7217

$(M_u/f_{ck}\, bD^2)$ = 0.0361

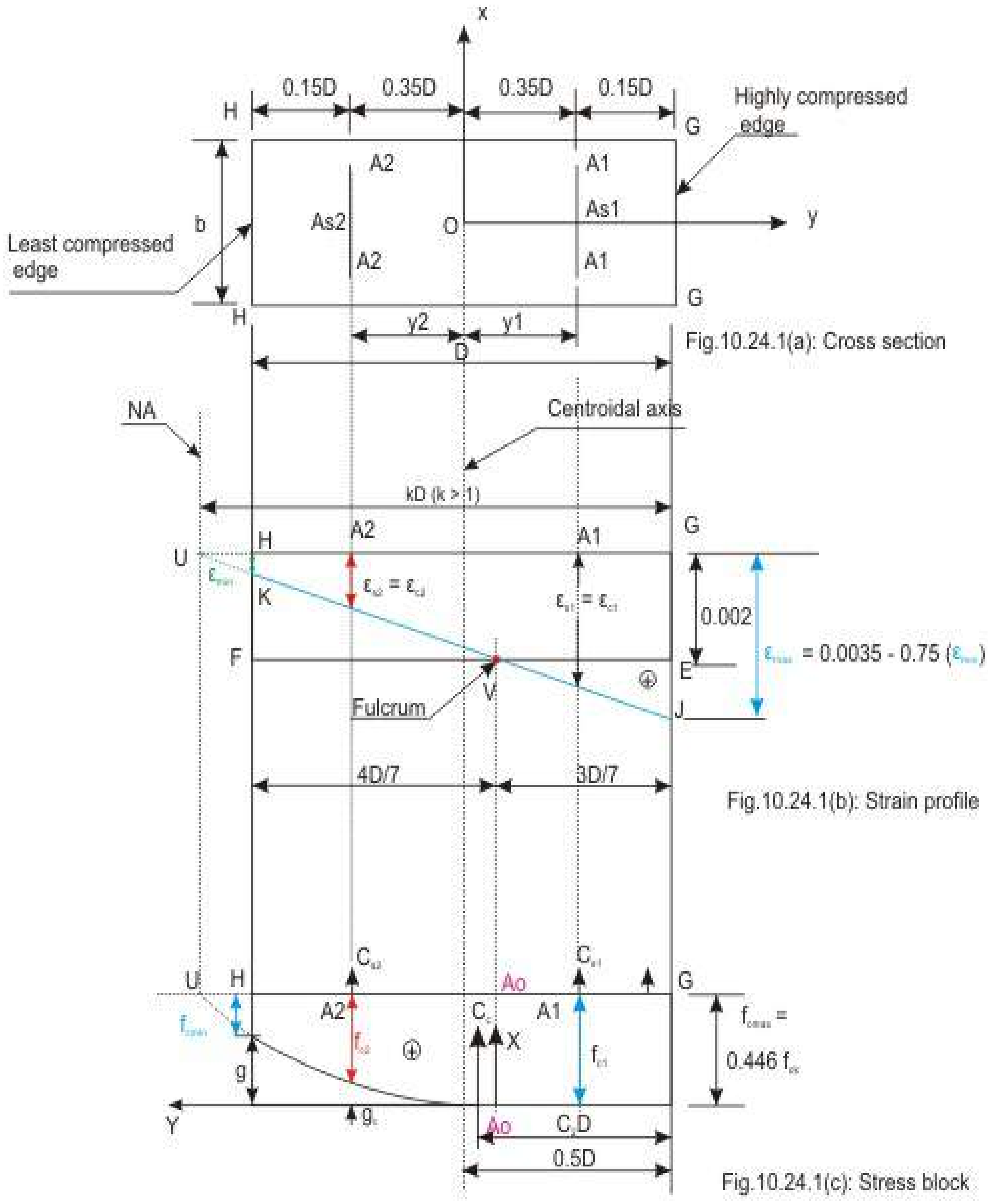


Fig.10.24.1: Problem 1 and Q. 3 (step 3, k=1.1)

Step 3: When the neutral axis depth = 1.1 *D*

Figures 10.24.1a, b and c show the section of the column, strain profile JK and the corresponding stress block, respectively, for this case. We use Eqs.10.42 and 10.44 for determining the value of $(P_u/f_{ck}\, bD)$ and $(M_u/f_{ck}\, bD^2)$ for this case using k = 1.1, f_{ck} = 25 N/mm^2, p_1 = p_2 = 1.5 , y_1/D = 0.35 and y_2/D = -0.35. Values of C_1, C_2, f_{s1} and f_{s2}, f_{c1} and f_{c2} are obtained from equations mentioned in Step 3 of sec.10.24.6. The values of all the quantities are presented in Table 10.6A, mentioning the source equation no., table no. etc. to get the two non-dimensional parameters as given below:

$(P_u/f_{ck}\,bD)$ = 0.67405
$(M_u/f_{ck}\,bD^2)$ = 0.06370

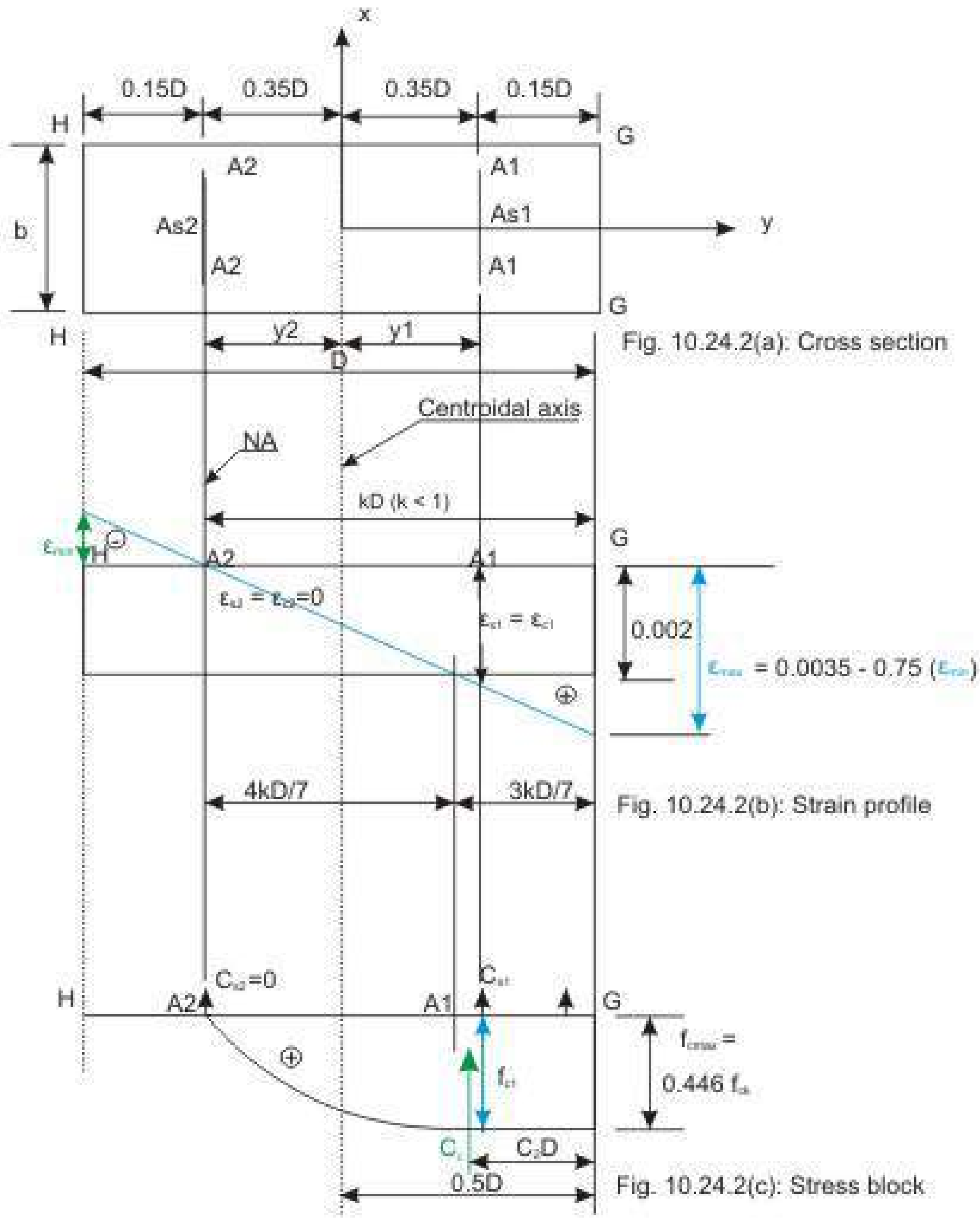


Fig. 10.24.2: Problem 1 and Q. 3 (step 4, fs2 = 0)

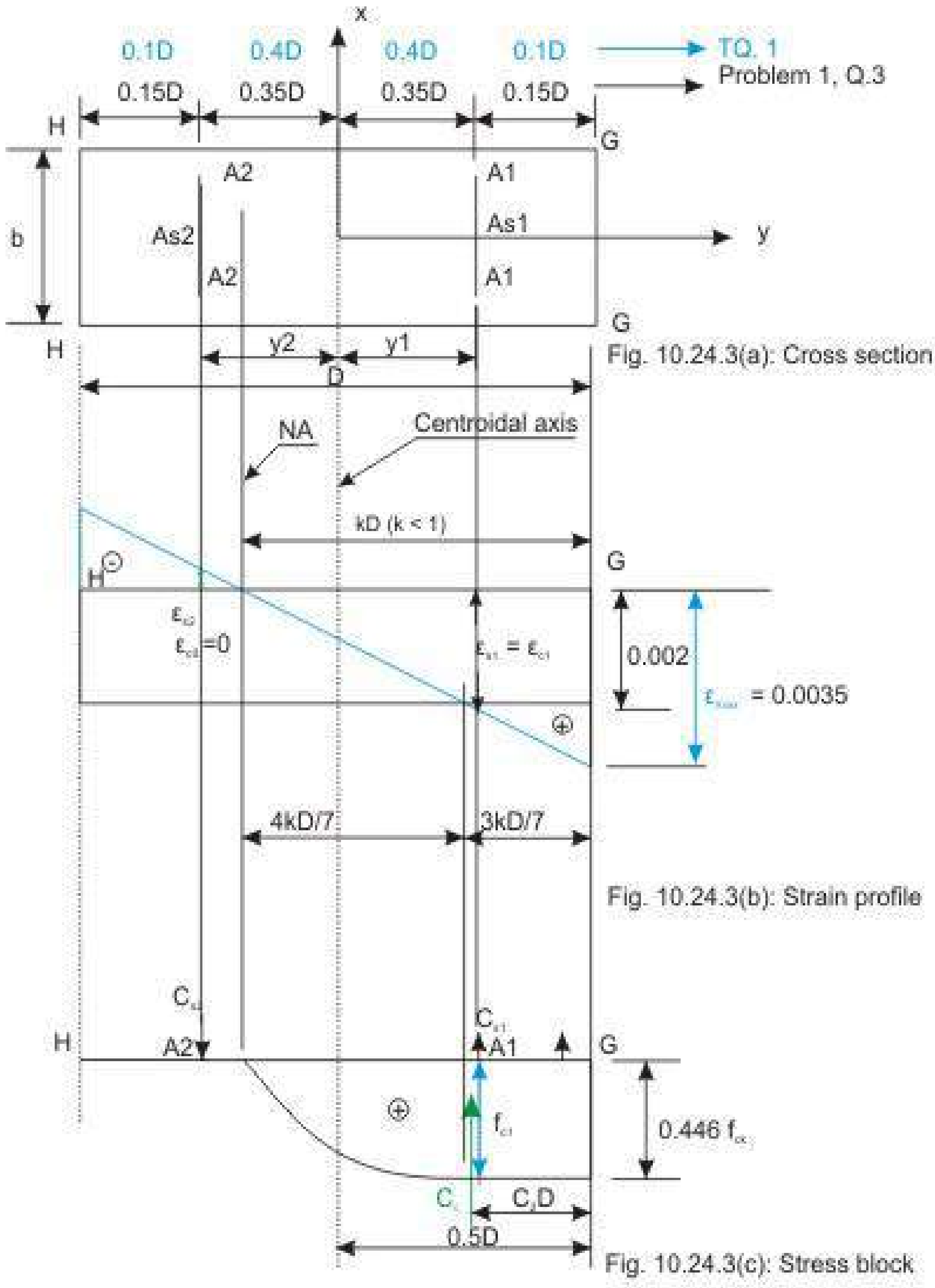


Fig. 10.24.3: Problem 1, Q.3 and TQ. 1 (step 4, fs2 = $-0.4f_{yd}$)

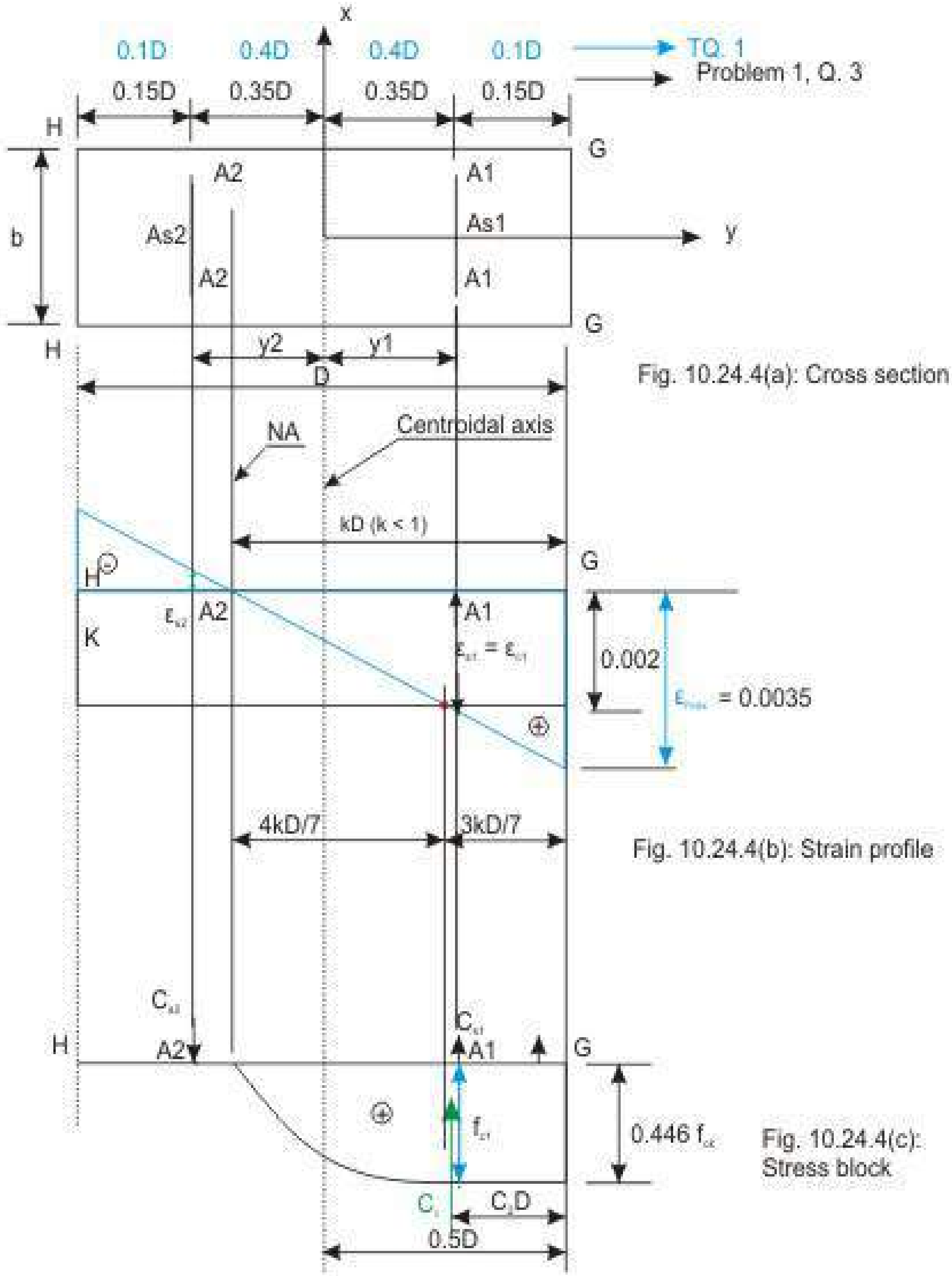


Fig. 10.24.4.: Problem 1, Q.3 and TQ. 1 (step 4, fs2 = - 0.8f_{yd})

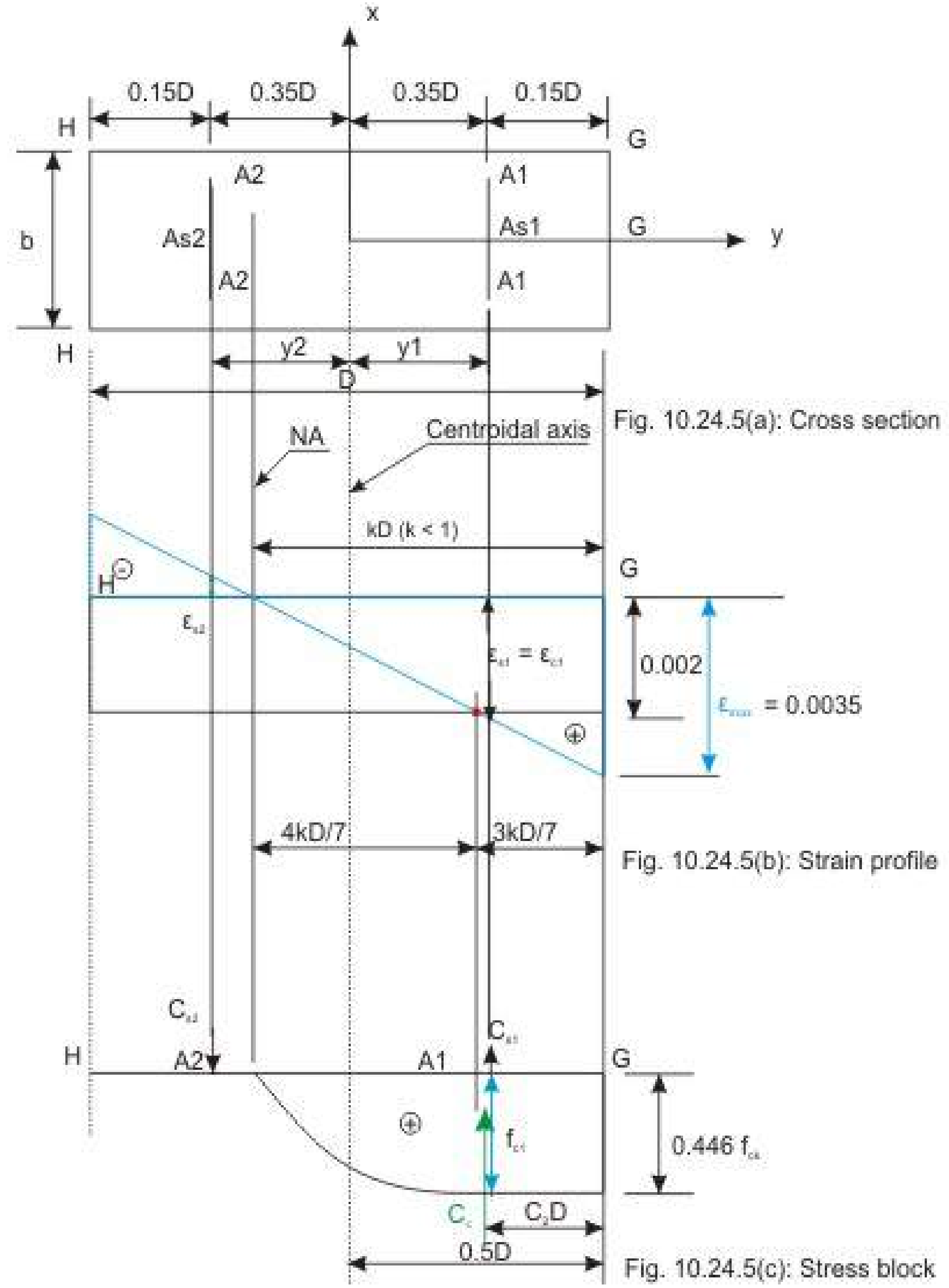


Fig. 10.24.5: Problem 1 and Q.3 (step 4, f_{s2} = - f_{yd} , initial yield)

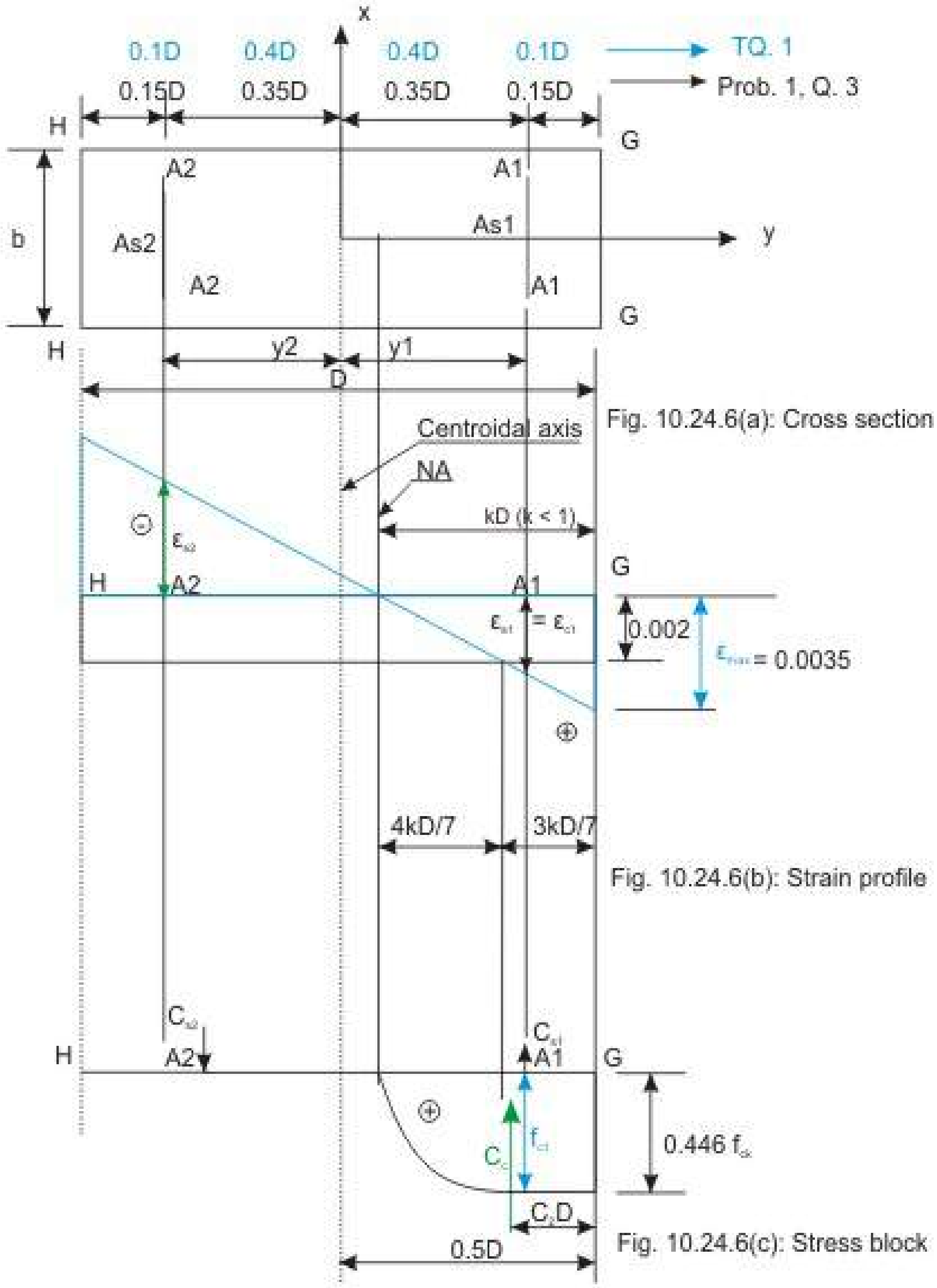


Fig. 10.24.6: Problem 1, Q.3 and TQ. 1 (step 4, $f_{s2} = f_{yd}$),final yield

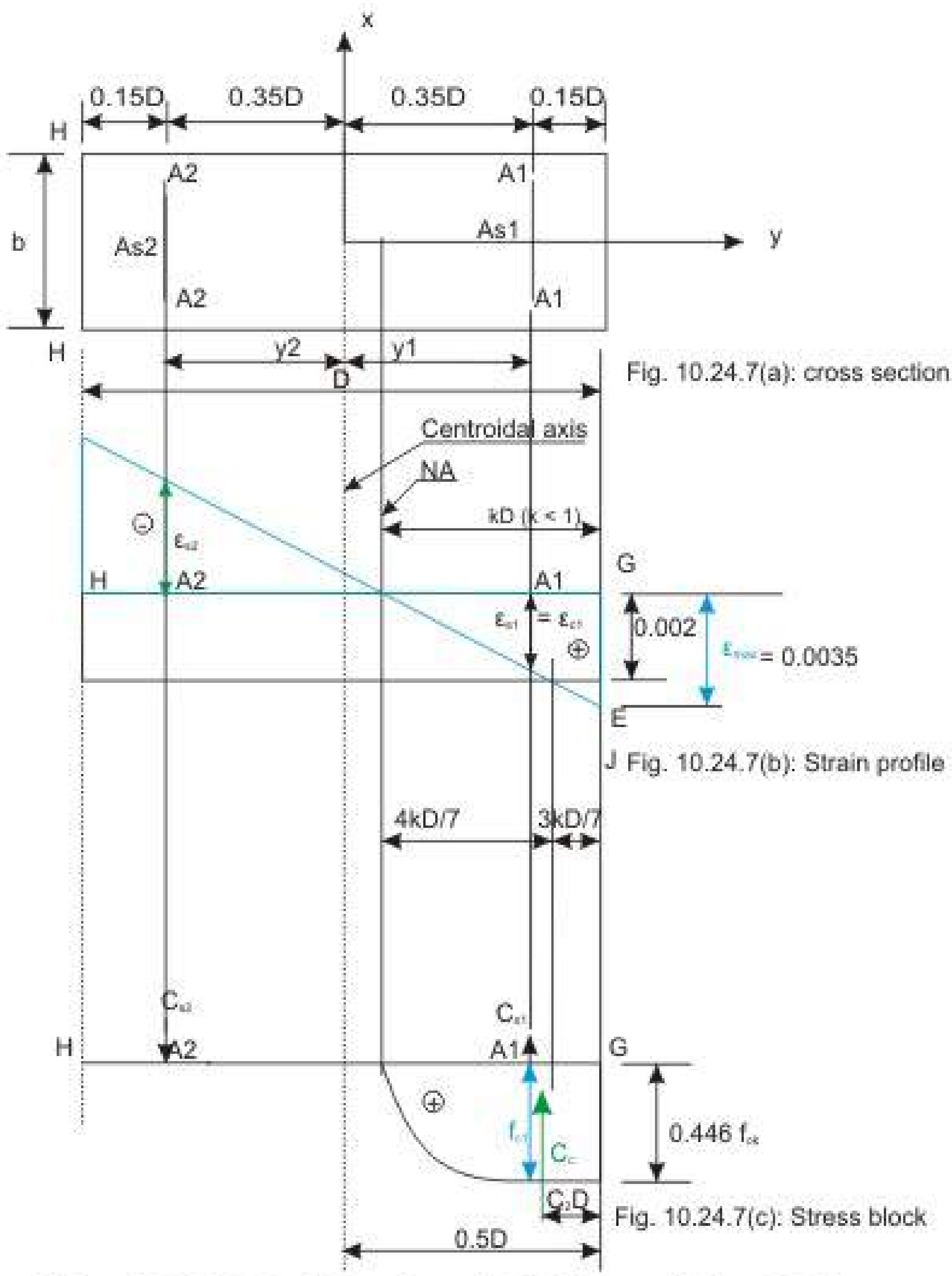


Fig. 10.24.7: Problem 1 and Q.3 (step 4, k = 0.25)

Step 4: When the neutral axis is within the section

In Step 4 of section 10.24.6, six different locations of neutral axis are mentioned; five of them (a to e) are specified by the magnitude of f_{s2} (tensile) of longitudinal steel and one of them is specified by the value of k = 0.25. The values of all the quantities are presented in Tables 10.6A and B, mentioning the source equation no., table no. etc.

Figures 10.24.2 to 10.24.7 present the respective strain profiles and the corresponding stress block separately for all six different locations of the neutral axis.

Table 10.6A Parameters and results of Problem 1 of Section 10.24.8

Given data: f_{ck} = 25 N/mm^2, f_y = 415 N/mm^2, p = 3 per cent, $p_1 = p_2$ = 1.5 per cent,

d'/D = 0.15

Note: Units of f_{si}, f_{sc} and f_{ci} are in N/mm^2, (-) minus sign indicates tensile strain or stress.

Sl.No.	Given Description	k = 1.1	f_{s2} = 0	$f_{s2} = -0.4 f_{yd}$	$f_{s2} = 0.8 f_{yd}$
1	Sec. No.	10.24.7	10.24.7	10.24.7	10.24.7
2	Step No.	3	4	4	4
3	Fig. No.	10.24.1	10.24.2	10.24.3	10.24.4
4	$\varepsilon_{s1} = \varepsilon_{c1}$	0.002829	0.00288	0.00275	0.00263
5	$\varepsilon_{s2} = \varepsilon_{c2}$	0.000744	0.0	-0.00072	-0.00144
6	Table No. of f_{si} and f_{sc}	10.5	10.5	10.5	10.5
7	f_{s1}	352.407	352.871	351.669	348.392
8	f_{s2}	148.914	0.0	-144.42	-288.84
9	f_{sc}	NA	NA	NA	NA
10	Eq.Nos. of f_{ci}	10.23 and 10.27	10.34	10.34	10.34
11	f_{c1}	11.15	11.15	11.15	11.15
12	f_{c2}	6.757	0.0	0.0	0.0
13	Table No. of C_1 and C_2	10.4	NA	NA	NA
14	C_1	0.384	NA	NA	NA
15	C_2	0.443	NA	NA	NA
16	y_1/D	+0.35	+0.35	+0.35	+0.35
17	y_2/D	-0.35	-0.35	-0.35	-0.35
18	k	1.1	0.85	0.7046	0.6017
19	Eq.No. of $P_u/f_{ck}\,bD$	10.42	10.46	10.46	10.46
20	$P_u/f_{ck}\,bD$	0.6740	0.5110	0.3713	0.2457
21	Eq.No. of $M_u/f_{ck}\,bD^2$	10.44	10.48	10.48	10.48
22	$M_u/f_{ck}\,bD^2$	0.0643	0.1155	0.1536	0.1850

Table 10.6B Parameters and results of Problem 1 of Section 10.24.8

Given data: f_{ck} = 25 N/mm^2, f_y = 415 N/mm^2, p = 3 per cent, p_1 = p_2 = 1.5 per cent,

d'/D = 0.15

Note: Units of f_{si}, f_{sc} and f_{ci} are in N/mm^2, (-) minus sign indicates tensile strain or stress.

Sl.No.	Given Description	f_{s2} = - f_{yd} (Initial yield)	f_{s2} = - f_{yd} (Final yield)	k = 0.25
1	Sec. No.	10.24.7	10.24.7	10.24.7
2	Step No.	4	4	4
3	Fig. No.	10.24.5	10.24.6	10.24.7
4	$\varepsilon_{s1} = \varepsilon_{c1}$	0.00256	0.00221	0.0014
5	$\varepsilon_{s2} = \varepsilon_{c2}$	-0.00180	-0.00380	-0.0084
6	Table No. of f_{si} and f_{sc}	10.5	10.5	10.5
7	f_{s1}	346.754	335.484	281.0
8	f_{s2}	-361.05	-361.05	-361.05
9	f_{sc}	NA	NA	NA
10	Eq.Nos. of f_{ci}	10.34	10.34	10.34
11	f_{c1}	11.15	11.15	10.146
12	f_{c2}	0.0	0.0	0.0
13	Table No. of C_1 and C_2	NA	NA	NA
14	C_1	NA	NA	NA
15	C_2	NA	NA	NA
16	y_1/D	+0.35	+0.35	+0.35
17	y_2/D	-0.35	-0.35	-0.35
18	k	0.5607	0.4072	0.25
19	Eq.No. of $P_u/f_{ck}\,bD$	10.46	10.46	10.46
20	$P_u/f_{ck}\,bD$	0.1866	0.1246	0.0353
21	Eq.No. of $M_u/f_{ck}\,bD^2$	10.48	10.48	10.48
22	$M_u/f_{ck}\,bD^2$	0.1997	0.1921	0.1680

Step 5: When the column behaves like a steel beam

For this case, the parameter ($M_u/f_{ck}\,bD^2$) is determined from Eq.10.50 using p_1 = p_2 = 1.5 per cent, f_{ck} = 25 N/mm^2, f_y = 415 N/mm^2, y_1/D = 0.35 and y_2/D = -0.35. Thus, we get

$(M_u/f_{ck}\,bD^2) = 0.15164$

Step 6: Final results of design chart

The values of ten pairs of $(P_u/f_{ck}\,bD)$ and $(M_u/f_{ck}\,bD^2)$ as obtained in steps 1 to 5 are presented in Sl. Nos. 1 to 10 of Table 10.6C. The design chart can be prepared by plotting these values.

Table 10.6C Final values of $P_u/f_{ck}\,bD$ and $M_u/f_{ck}\,bD^2$ of Problem 1 of Section 10.24.8

Sl. No.	Particulars about the point	$P_u/f_{ck}\,bD$	$M_u/f_{ck}\,bD^2$
1	$k = \alpha$	0.8259	0.0
2	Minimum eccentricity	0.7217	0.0361
3	$k = 1.1$	0.6740	0.0643
4	$f_{s2} = 0$	0.5110	0.1155
5	$f_{s2} = (-)0.4\,f_{yd}$	0.3713	0.1536
6	$f_{s2} = (-)0.8\,f_{yd}$	0.2457	0.1850
7	$f_{s2} = (-)\,f_{yd}$ (Initial yield)	0.1866	0.1997
8	$f_{s2} = (-)\,f_{yd}$ (Final yield)	0.1246	0.1921
9	$k = 0.25$	0.0353	0.1680
10	Steel Beam	0.0	0.1516

10.24.9 Practice Questions and Problems with Answers

Q.1: Why do we need to have non-dimensional design chart?

A.1: See sec. 10.24.1

Q.2: Name the different design parameters while designing a column. Mention which one is the most important parameter.

A.2: See sec. 10.24.2.

Q.3: Prepare a design chart for a rectangular column within three per cent longitudinal steel, equally distributed on two faces, using M 25 and Fe 250 and considering $d'/D = 0.15$.

.A.3: The solution of this problem is obtained following the same six steps of Problem 1 of sec.10.24.8, except that the grade of steel here is Fe 250. Therefore, the final results and all the parameters are presented in Table 10.7 avoiding explaining step by step again.

Table 10.7 Final values of $P_u/f_{ck}\ bD$ and $M_u/f_{ck}\ bD^2$ of Q.3 of Section 10.24.9

Sl. No.	Particulars about the point	$P_u/f_{ck}\ bD$	$M_u/f_{ck}\ bD^2$
1	$k = \alpha$	0.6936	0.0
2	Minimum eccentricity	0.5890	0.0295
3	k = 1.1	0.5931	0.0354
4	$f_{s2} = 0$	0.4298	0.0871
5	$f_{s2} = (-)0.4\ f_{yd}$	0.3438	0.1113
6	$f_{s2} = (-)0.8\ f_{yd}$	0.2645	0.1323
7	$f_{s2} = (-)\ f_{yd}$ (Initial yield)	0.2268	0.1421
8	$f_{s2} = (-)\ f_{yd}$ (Final yield)	0.1559	0.1395
9	k = 0.25	0.0839	0.1248
10	Steel Beam	0.0	0.0914

10.24.10 References

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10. Textbook of R.C.C, by G.S.Birdie and J.S.Birdie, Wiley Eastern Limited, New Delhi, 1993.
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14. Reinforced Concrete Designer's Handbook, 10th Edition, by C.E.Reynolds and J.C.Steedman, E & FN SPON, London, 1997.
15. Indian Standard Plain and Reinforced Concrete – Code of Practice (4th Revision), IS 456: 2000, BIS, New Delhi.
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10.24.11 Test 24 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

TQ.1: Determine the parameters including the two non-dimensional parameters, *Pu* and *Mu* of a rectangular reinforced concrete short column of b = 370 mm, D = 530 mm, d'/D = 0.1 and having 8-25 mm diameter bars as longitudinal steel distributed equally on two sides using M 20 and Fe 415 for each of the following three cases:

(a) when $f_{s2} = -0.4 f_{yd}$

(b) when $f_{s2} = -0.8 f_{yd}$

(c) when $f_{s2} = - f_{yd}$ (at final yield)

(16 + 17 + 17 = 50)

A.TQ.1: This problem can be solved following the same procedure of explained in Step 4b, c and d of sec.10.24.7. The step by step calculations are not shown here and the final results are presented in Table 10.8.

Table 10.8 Parameters and results of TQ.1 of Section 10.24.11

Given data: f_{ck} = 20 N/mm^2, f_y = 415 N/mm^2, b = 370 mm, D = 530 mm, Longitudinal steel = 8-25 mm diameter equally distributed on two sides, d'/D = 0.15

Sl.No.	Given Description	$f_{s2} = -0.4 f_{yd}$	$f_{s2} = 0.8 f_{yd}$	$f_{s2} = - f_{yd}$ (Final yield)
1	Sec. No.	10.24.7	10.24.7	10.24.7
2	Step No.	4	4	4
3	Fig. No.	10.24.3	10.24.4	10.24.6
4	$\varepsilon_{s1} = \varepsilon_{c1}$	0.0030	0.0029	0.0027
5	$\varepsilon_{s2} = \varepsilon_{c2}$	-0.00072	-0.00144	-0.0038
6	Table No. of f_{si} and f_{sc}	10.5	10.5	10.5
7	f_{s1}	354.1702	353.468	349.956

8	f_{s2}	-144.42	-288.84	-361.05
9	f_{sc}	NA	NA	NA
10	Eq.Nos. of f_{ci}	10.34	10.34	10.34
11	f_{c1}	11.15	11.15	11.15
12	f_{c2}	0.0	0.0	0.0
13	Table No. of C_1 and C_2	NA	NA	NA
14	C_1	NA	NA	NA
15	C_2	NA	NA	NA
16	y_1/D	+0.4	+0.4	+0.4
17	y_2/D	-0.4	-0.4	-0.4
18	k	0.7461	0.6371	0.4311
19	Eq.No. of $P_u/f_{ck}\,bD$	10.46	10.46	10.46
20	$P_u/f_{ck}\,bD$	0.3690	0.2572	0.1452
21	P_u (kN)	1447.225	1008.792	569.568
22	Eq.No. of $M_u/f_{ck}\,bD^2$	10.48	10.48	10.48
23	$M_u/f_{ck}\,bD^2$	0.1481	0.1799	0.1899
24	M_u (kNm)	307.777	374.125	394.779

10.24.12 Summary of this Lesson

This lesson explains the procedure of the preparation of design charts of rectangular reinforced concrete short columns subjected to axial load and uniaxial moment. Different positions of the neutral axis due to different pairs of P_u and M_u give rise to different strain profiles and stress blocks. Accordingly, the column may collapse when subjected to any pair of axial load and moment exceeding the capacities of the column. Design charts are very much useful to design the column avoiding lengthy numerical computations. Illustrative example, practice and test problems will help in understanding each step of the procedure to prepare the design chart.

Module
10
Compression Members

Lesson 27

Slender Columns

Instructional Objectives:

At the end of this lesson, the student should be able to:

- define a slender column,
- give three reasons for its increasing importance and popularity,
- explain the behaviour of slender columns loaded concentrically,
- explain the behaviour of braced and unbraced single column or a part of rigid frame, bent in single or double curvatures,
- roles and importance of additional moments due to P- Δ effect and moments due to minimum eccentricities in slender columns,
- identify a column if sway or nonsway type,
- understand the additional moment method for the design of slender columns,
- apply the equations or use the appropriate tables or charts of SP-16 for the complete design of slender columns as recommended by IS 456.

11.27.1 Introduction

Slender and short are the two types of columns classified on the basis of slenderness ratios as mentioned in sec.10.21.5 of Lesson 21. Columns having both l_{ex}/D and l_{ey}/b less than twelve are designated as short and otherwise, they are slender, where l_{ex} and l_{ey} are the effective lengths with respect to major and minor axes, respectively; and D and b are the depth and width of rectangular columns, respectively. Short columns are frequently used in concrete structures, the design of such columns has been explained in Lessons 22 to 26, loaded concentrically or eccentrically about one or both axes. However, slender columns are also becoming increasingly important and popular because of the following reasons:

(i) the development of high strength materials (concrete and steel),

(ii) improved methods of dimensioning and designing with rational and reliable design procedures,

(iii) innovative structural concepts – specially, the architect's expectations for creative structures.

Accordingly, this lesson explains first, the behaviour of slender elastic columns loaded concentrically. Thereafter, reinforced concrete slender columns loaded concentrically or eccentrically about one or both axes are taken up. The design of slender columns has been explained and illustrated with numerical examples for easy understanding.

10.27.2 Concentrically Loaded Columns

It has been explained in Lessons 22 to 26 that short columns fail by reaching the respective stresses indicating their maximum carrying capacities. On the other hand, the slender or long columns may fail at a much lower value of the load when sudden lateral displacement of the member takes place between the ends. Thus, short columns undergo material failure, while long columns may fail by buckling (geometric failure) at a critical load or Euler's load, which is much less in comparison to that of short columns having equal area of cross-section. The buckling load is termed as Euler's load as Euler in 1744 first obtained the value of critical load for various support conditions. For more information, please refer to Additamentum, "De Curvis elasticis", in the "Methodus inveiendi Lineas Curvas maximi minimive proprietate gaudentes" Lausanne and Geneva, 1744. An English translation of this work is given in Isis No.58, Vol.20, p.1, November 1933.

The general expression of the critical load P_{cr} at which a member will fail by buckling is as follows:

$$P_{cr} = \pi^2 EI/(kl)^2$$

where E is the Young's modulus I is the moment of inertia about the axis of bending, l is the unsupported length of the column and k is the coefficient whose value depends on the degree of restraints at the supports. Expressing moment of inertia $I = Ar^2$, where A is the area of cross-section of the column and r is the radius of gyration, the above equations can be written as,

$$P_{cr} = \pi^2 EA/(kl/r)^2 \quad (10.62)$$

Thus, P_{cr} of a particular column depends upon kl/r or slenderness ratio. It is worth mentioning that kl is termed as effective length l_e of the column.

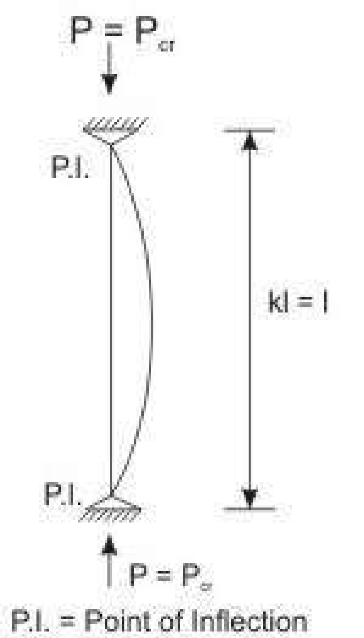

Fig. 10.27.1: Column - hinged at both ends, (k = 1)

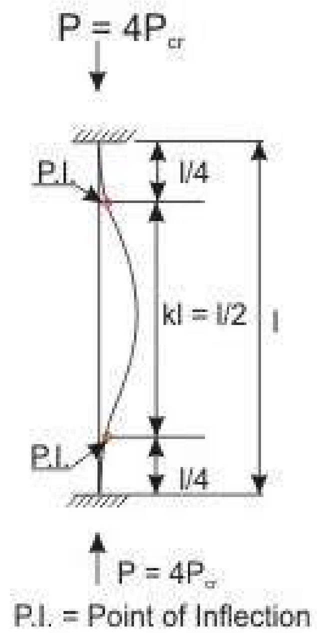

Fig. 10.27.2: Column - fixed at both ends, (k = 0.5)

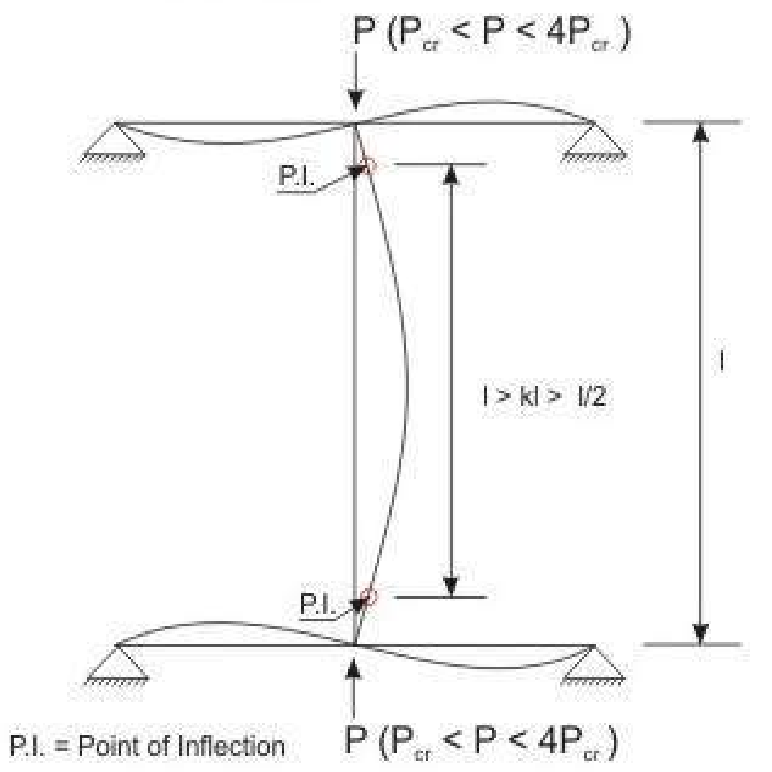

Fig.10.27.3: Column supported on cross-beams (0.5 < k < 1)

Figures 10.27.1 and 2 show two elastic slender columns having hinge supports at both ends and fixed supports against rotation at both ends, respectively. Figure 10.27.3 presents a column of real structure whose end supports are not either hinged or fixed. It has supports partially restrained against rotation by the top and bottom beams. Each of the three figures shows the respective buckled shape, points of inflection *PI*s (points of zero moment), the distance between the *PI*s and the value of *k*. All the three columns, having supports at both ends, have the *k* values less than one or at most one. By providing supports at both ends, one end of the column is prevented from undergoing lateral movement or sidesway with respect to the other end.

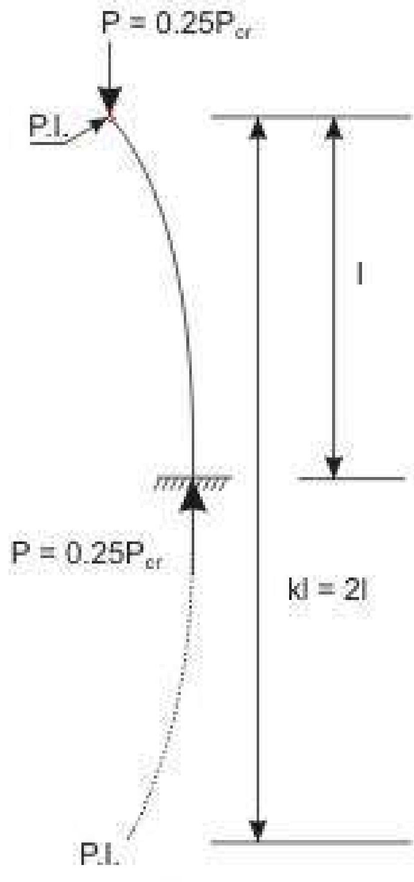

Fig. 10.27.4: Column - fixed at one end and free at other end, (k = 2)

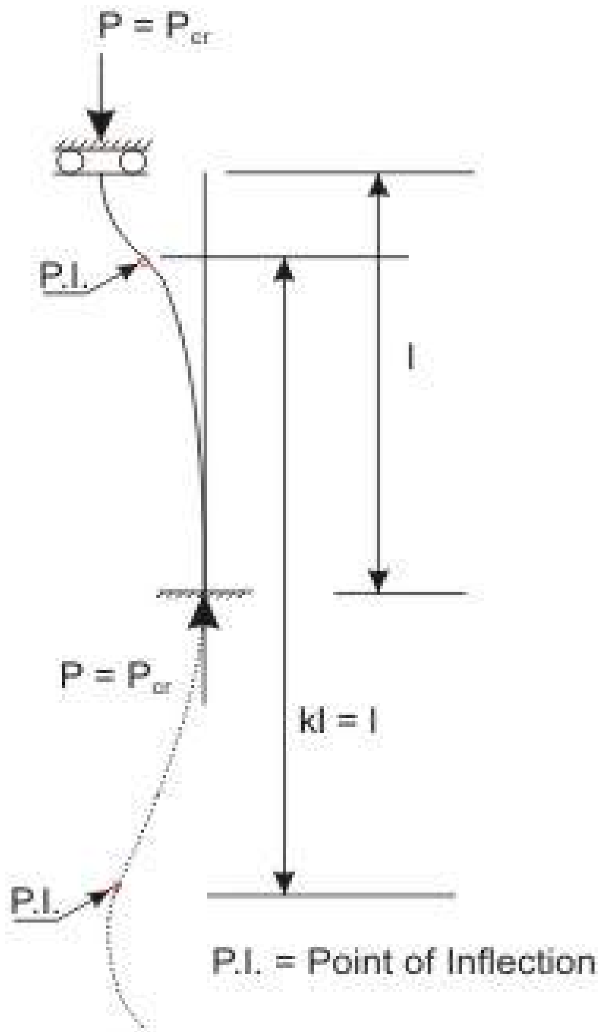

Fig. 10.27.5: Column rotationally fixed at both end, (k = 1)

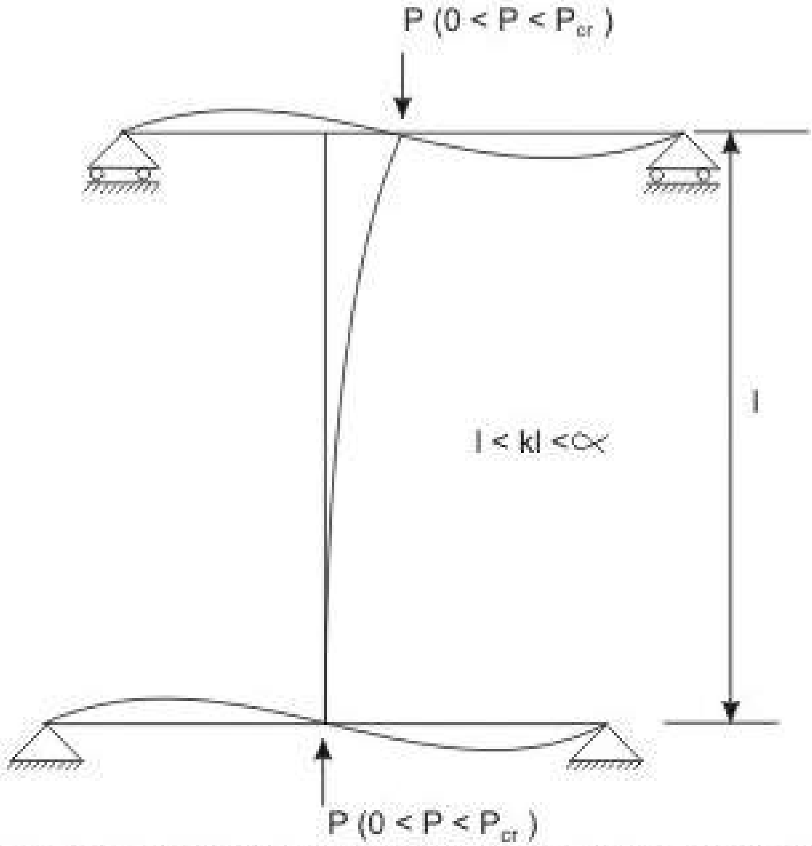

Fig.10.27.6: Column partially restrained at both ends
($1 < k < \infty$)

However, cantilever columns are entirely free at one end, as shown in Fig.10.27.4. Figure 10.27.5 shows another type of column, rotationally fixed at both ends but one end can move laterally with respect to the other. Like that of Fig.10.27.3, a real column, not hinged, fixed or entirely free but restrained by top and bottom beams, where sideway can also take place. Each of these three figures, like those of Figs.10.27.1 to 3, presents the respective buckled shape, points of inflection (*PI*s), if any, the distance between the *PI*s and the value of *k*. All these columns have the respective *k* values greater than one or at least one.

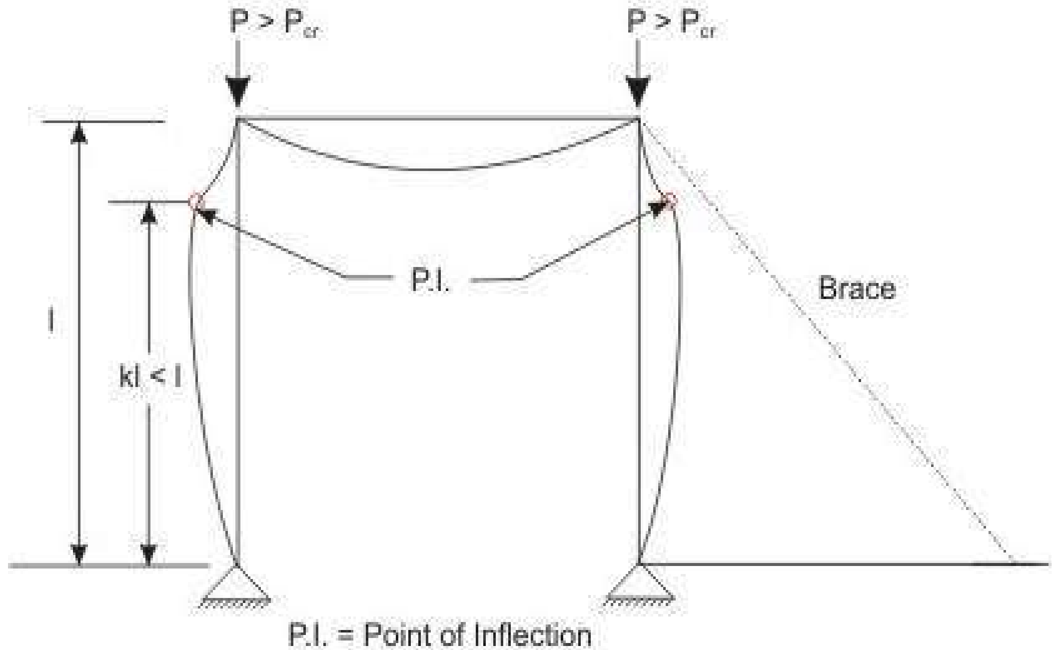

Fig. 10.27.7: Braced portal frame (k < 1)

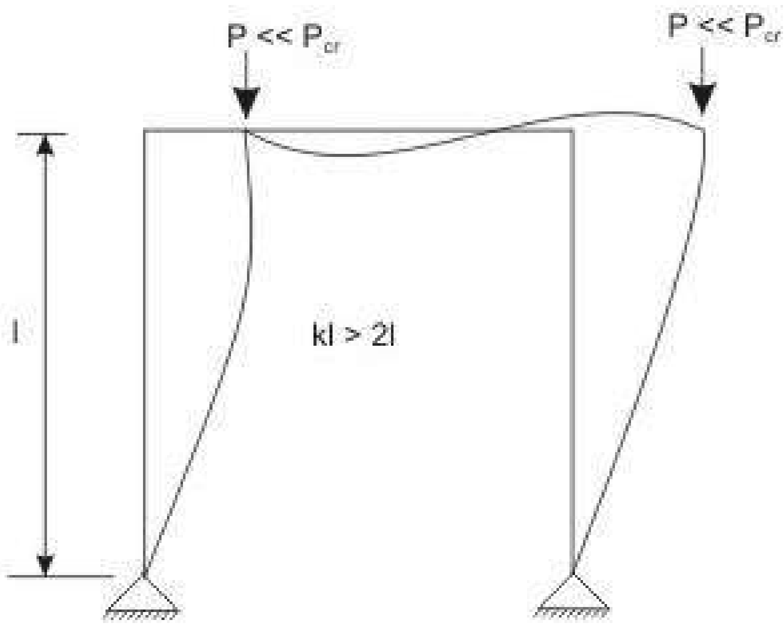

Fig. 10.27.8: Unbraced portal frame (k > 2)

Figures 10.27.7 and 8 present two reinforced concrete portal frames, a typical reinforced concrete rigid frame. Columns of Fig.10.27.7 are prevented from sidesway and those of Fig.10.27.8 are not prevented from sidesway, respectively, when subjected to concentric loadings. The buckled configuration of the frame, prevented from sidesway (Fig.10.27.7) is similar to that of Fig.10.27.3, except that the lower ends of the portal frame are hinged. One of the two points of inflection (*PI*s) is at the lower end of the column, while the other *PI* is slightly below the upper end of the column, depending on the degree of restraint. The value of k for such a frame is thus less than 1. The critical load is, therefore, slightly more than P_{cr} of the hinge-hinge column of Fig.10.27.1. The buckled configuration of the other portal frame of Fig.10.27.8, where sidesway is not prevented, is similar to the column of Fig.10.27.4 when it is made upside down, except that the upper end is not fixed but partially restrained by the supporting beam. In this case, the value of k exceeds 2, depending on the degree of restraint. One of the two *PI*s is at the bottom of the column. The critical load of the column of Fig.10.27.8 is much less than that of the column of Fig.10.27.1.

Table 10.14: Critical loads in terms of P_{cr} of hinge-hinge column and effective lengths $l_e = kl$ of elastic and reinforced concrete columns with different boundary conditions and for a constant unsupported length l

Sl. No.	Support conditions	Critical load P_{cr}	Effective length $l_e = kl$	Fig. No.
(A) Elastic single columns				
1.	Hinged at both ends, no sidesway	P_{cr}	l	10.27.1
2.	Fixed against rotation at both ends – no sidesway	$4P_{cr}$	0.5 l	10.27.2
3.	Partially restrained against rotation by top and bottom cross-beams, no sidesway	Between P_{cr} and $4P_{cr}$	$l > kl > l/2$	10.27.3
4.	Fixed at one end and entirely free at other end – sidesway not prevented	0.25 P_{cr}	2 l, one *PI* is on imaginary extension	10.27.4
5.	Rotationally fixed at both ends – sidesway not prevented	P_{cr}	l, one *PI* is on imaginary extension	10.27.5
6.	Partially restrained against rotation at both ends – sidesway not prevented	Between zero and slightly less than P_{cr} *	$l < kl < \alpha$	10.27.6
(B) Reinforced concrete columns				
7.	Hinged portal frame – no sidesway	$> P_{cr}$	$kl < l$	10.27.7
8.	Hinged portal frame – sidesway not prevented	$<< P_{cr}$	$kl > 2\, l$	10.27.8

Notes: 1. Buckled shapes are half sine wave between two points of inflection (*PI*s).

2. * The critical load is slightly less than P_{cr} of hinge-hinge column (Sl.No.1), when cross-beams are very rigid compared to columns, i.e., the case under Sl.No.6 approaches the case under Sl.No.1.

   The critical load is zero when cross-beams are very much flexible compared to columns, i.e., the case under Sl.No.6 approaches to hinge-hinge column of Sl.No.1, allowing sidesway. In that case, it becomes unstable and hence, carries zero load.

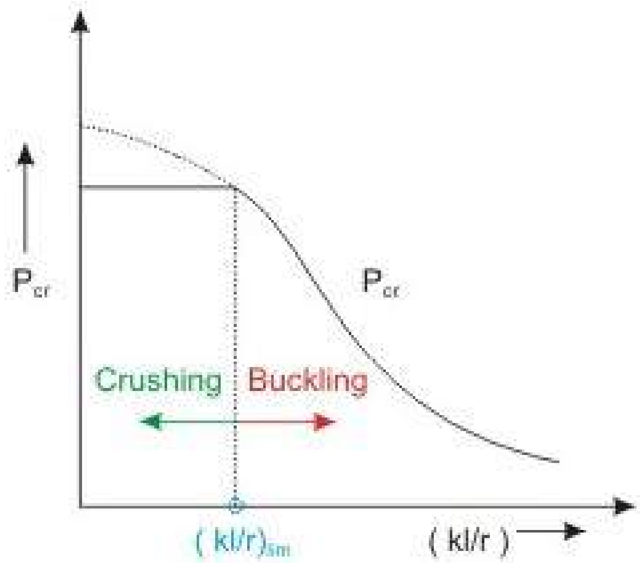

Fig. 10.27.9: Effect of slenderness on strength

Table 10.14 presents the critical load in terms of that of hinge-hinge column P_{cr} and effective lengths l_e (equal to the distance between two points of inflection *PI*s = *kl*) of elastic and reinforced concrete columns for a constant value of the unsupported length *l*.

The stress-strain curve of concrete, as shown in Fig.1.2.1 of Lesson 2, reveals that the initial tangent modulus of concrete E_c is much higher than E_t (tangent modulus at higher stress level). Taking this into account in Eq.10.62, Fig.10.27.9 presents a plot of buckling load P_{cr} versus *kl/r*. It is evident from the plot that the critical load is reducing with increasing slenderness ratio. For very short columns, the limiting factored concentric load estimated from Eq.10.39 of Lesson 24 will be found to be less than the critical load, determined from Eq.10.62. The column, therefore, will fail by direct crushing and not by buckling. We can also find out the limiting value of *kl/r* when the crushing load and the buckling load are the same. The $(kl/r)_{lim}$ is shown in Fig.10.27.9. The limiting value of *kl/r* also indicates that a column having *kl/r* more than $(kl/r)_{lim}$ will fail by buckling, while columns having any value of *kl/r* less than $(kl/r)_{lim}$ will fail by crushing of concrete.

The following are the observations of the discussions about the concentrically loaded columns:

1. As the slenderness ratio *kl/r* increases, the strength of concentrically loaded column decreases.

2. The effective length of columns either in single members or parts of rigid frames is between 0.5*l* and *l*, if the columns are prevented from sidesway by bracing or otherwise. The actual value depends on the degree of end restraints.

3. The effective length of columns either in single members or parts of rigid frames is always greater than one, if the columns are not prevented from sidesway. The actual value depends on the degree of end restraints.

4. The critical load of braced frame against sidesway is always significantly larger than that of the unbraced frame.

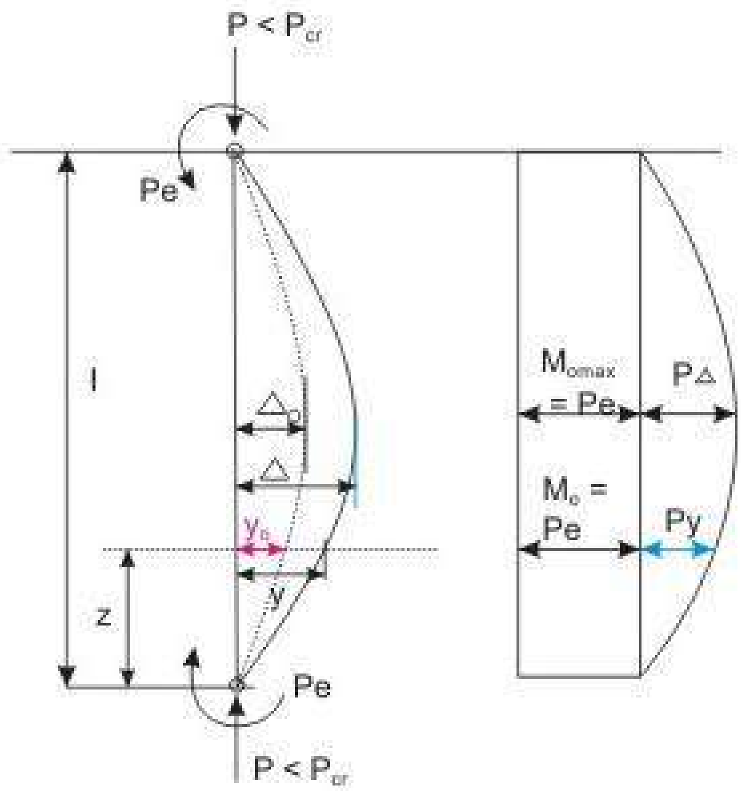

Fig. 10.27.10(a): Deflections Fig. 10.27.10(b): Moments

Fig. 10.27.10: Column bent in single curvature, (H = 0)

10.27.3 Slender Columns under Axial Load and Uniaxial Moment

(A) Columns bent in single curvature

Figure 10.27.10a shows a column bent in single curvature under axial load P less than its critical load P_{cr} with constant moment Pe. The deflection profile marked by dotted line is due to the constant moment. However, there will be additional moment of Py at a distance z from the origin (at the bottom of column) which will deflect the column further, as shown by the solid line. The constant moment Pe and additional moment Py are shown in Fig.10.27.10b. Thus, the total moment becomes

$$M = M_o + Py = P(e + y) \quad (10.63)$$

The maximum moment is $P(e + \Delta)$ at the mid-height of the column. This, we can write

$$M_{max} = M_o + P\Delta = P(e + \Delta) \quad (10.64)$$

This is known as P - Δ effect.

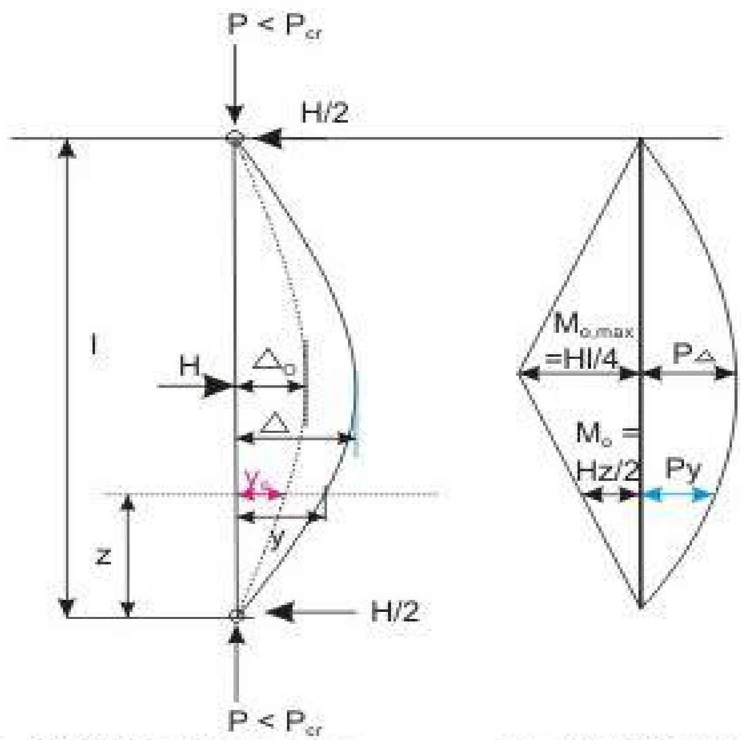

Fig. 10.27.11(a): Deflections Fig. 10.27.11(b): Moments

Fig. 10.27.11: Column bent in single curvature, (H = H)

Figure 10.27.11a shows another column whose bending is caused by a transverse load *H*. The bending moment at a distance *z* from the origin (bottom of the column) is $Hz/2$ causing deflection of the column marked by dotted line in the figure. The axial load *P*, less than its critical load P_{cr}, causes additional moment resulting in further deflection, marked by solid line in the figure. This additional deflection produces additional moment of *Py* at a section *z* from the origin. The two bending moment diagrams are shown in Fig.10.27.11b. Here again, the total moment is

$$M = M_o + Py = Hz/2 + Py$$
(10.65)

The maximum moment at the mid-height of the column is

$$M_{max} = M_{o,max} + P\Delta = Hl/4 + P\Delta$$
(10.66)

The total moment in Eqs.10.63 and 10.65 consists of the moment M_o that acts in the presence of *P* and the additional moment caused by *P* (= *Py*). The deflections *y* can be computed from y_o, the deflections without the axial load from the expression

$$y = y_o[1/\{1 - (P/P_{cr})\}]$$
(10.67)

From Eq.10.64, we have

$$M_{max} = M_o + P\Delta = M_o + P\Delta_o[1/\{1 - (P/P_{cr})\}]$$
(10.68)

Equation 10.68 can be written as

$$M_{max} = M_o \frac{1 + \psi(P/P_{cr})}{1 - (P/P_{cr})}$$
(10.69)

where ψ depends on the type of loading and generally varies between ± 0.20. Since P/P_{cr} is always less than one, we can ignore $\psi(P/P_{cr})$ term of Eq.10.69, to have

$$M_{max} = M_o/\{1 - (P/P_{cr})\}$$
(10.70)

where $1/\{1 - (P/P_{cr})\}$ is the moment magnification factor. In both the cases above (Figs.10.27.10 and 11), a direct addition of the maximum moment caused by transverse load or otherwise, to the maximum moment caused by *P* gives the total maximum moment as that is the most unfavourable situation. However, this is not the case for situation taken up in the following.

(B) Columns bent in double curvature

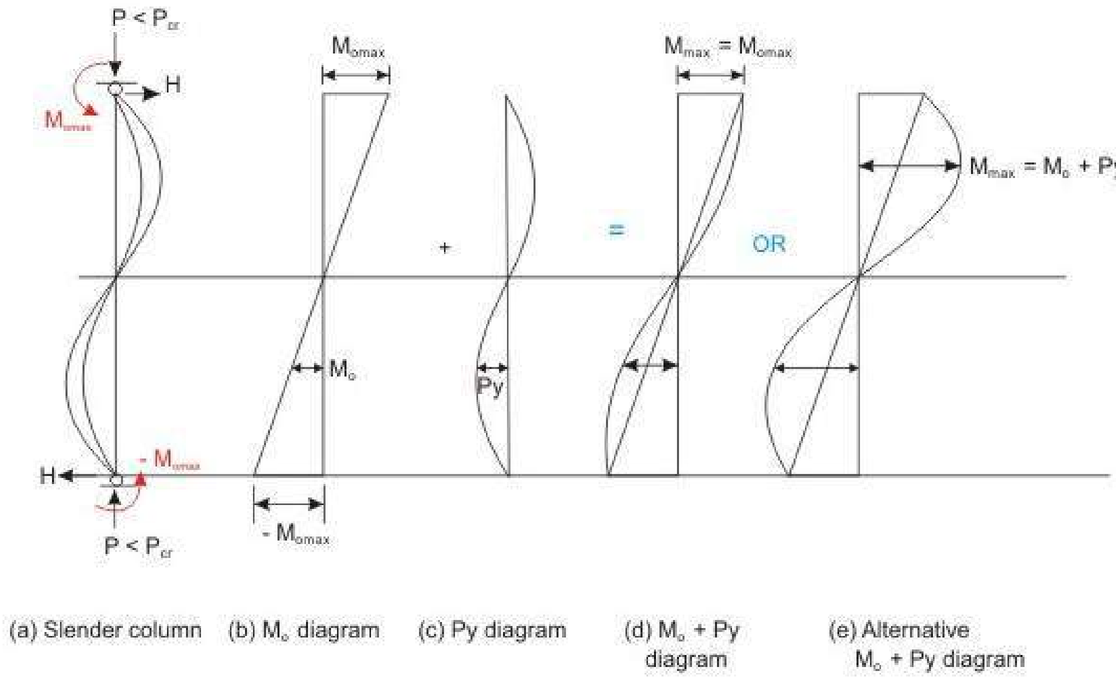

Fig. 10.27.12: Slender column under axial load and bending, bent in double curvature

Figure 10.27.12a shows a column subjected to equal end moment of opposite signs. From the moment diagrams M_o and Py (Figs.10.27.12b and c), it is clear that though M_o moments are maximum at the ends, the Py moments are maximum at some distance from the ends. The total moment can be either as shown in d or in e of Fig.10.27.12. In case of Fig.10.27.12d, the maximum moment remains at the ends and in Fig.10.27.12e, the maximum moment is at some distance from the ends, where M_o is comparatively smaller than $M_{o\,max}$ at the ends. Accordingly, the total maximum moment is moderately higher than $M_{o\,max}$.

From the above, it is evident that the moment M_o will be magnified most strongly if the section of $M_{o\,max}$ coincides with the section of maximum value of y, as in the case of column bent in single curvature of Figs.10.27.10 and 11. Similarly, if the two moments are unequal but of same sign as in Fig.10.27.10, the moment M_o will be magnified but not so much as in Fig.10.27.10. On the other hand, if the unequal end moments are of opposite signs and cause bending in double curvature, there will be little or no magnification of M_o moment.

This dependence of moment magnification on the relative magnitudes of the two moments can be expressed by modifying the earlier Eq.10.70 as

$M_{max} = M_o C_m/\{1 - (P/P_{cr})\}$
(10.71)

where $C_m = 0.6 + 0.4(M_1/M_2) \geq 0.4$
(10.72)

The moment M_1 is smaller than M_2 and M_1/M_2 is positive if the moments produce single curvature and negative if they produce double curvature. It is further seen from Eq.10.72 that $C_m = 1$, when $M_1 = M_2$ and in that case, Eq.10.71 becomes the same as Eq.10.70.

For the column of Fig.10.27.12a, the deflections caused by M_o are magnified when axial load P is applied. The deflection can be obtained from

$y = y_o [1/\{1 - (P/4P_{cr})\}]$
(10.73)

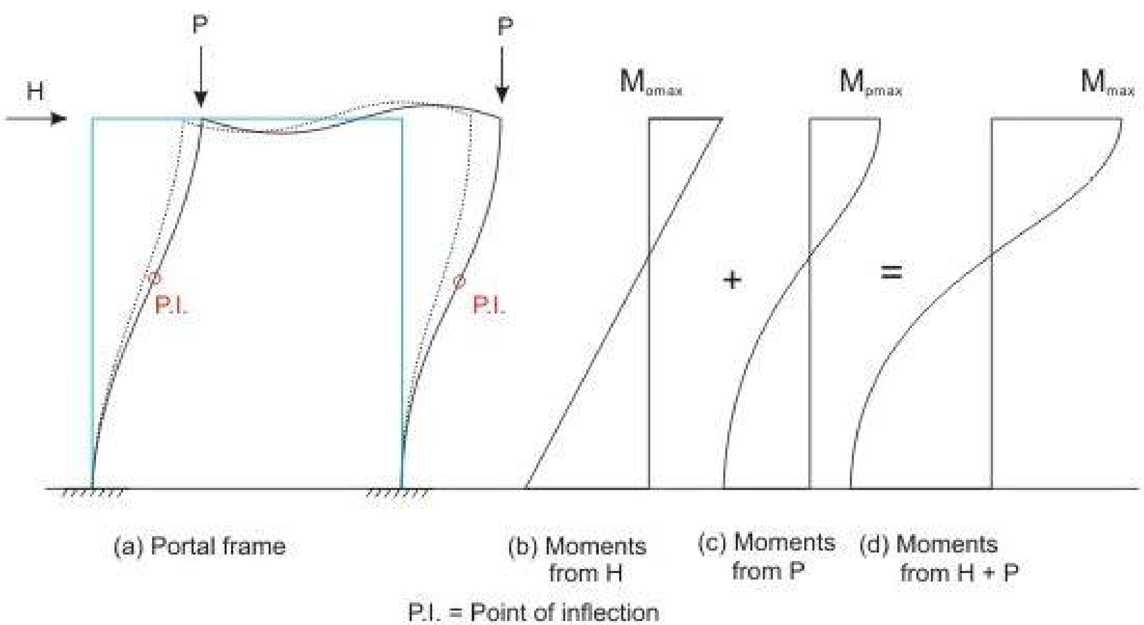

Fig. 10.27.13: Fixed portal frame - laterally unbraced

(C) Portal frame laterally unbraced and braced

Here, the sidesway can occur only for the entire frame simultaneously. A fixed portal frame, shown in Fig.10.27.13a, is under horizontal load H and compression force P. The moments due to H and P and the total moment diagrams are shown in Fig.10.27.13b, c and d, respectively. The deformations of the frame due to H are shown in Fig.10.27.13a by dotted curves, while the solid curves are the magnified deformations. It is observed that the maximum values of positive and negative M_o are at the ends of the column where the maximum values of positive and negative moments due to *P* also occur. Thus, the total moment shall be at the ends as the two effects are fully additive.

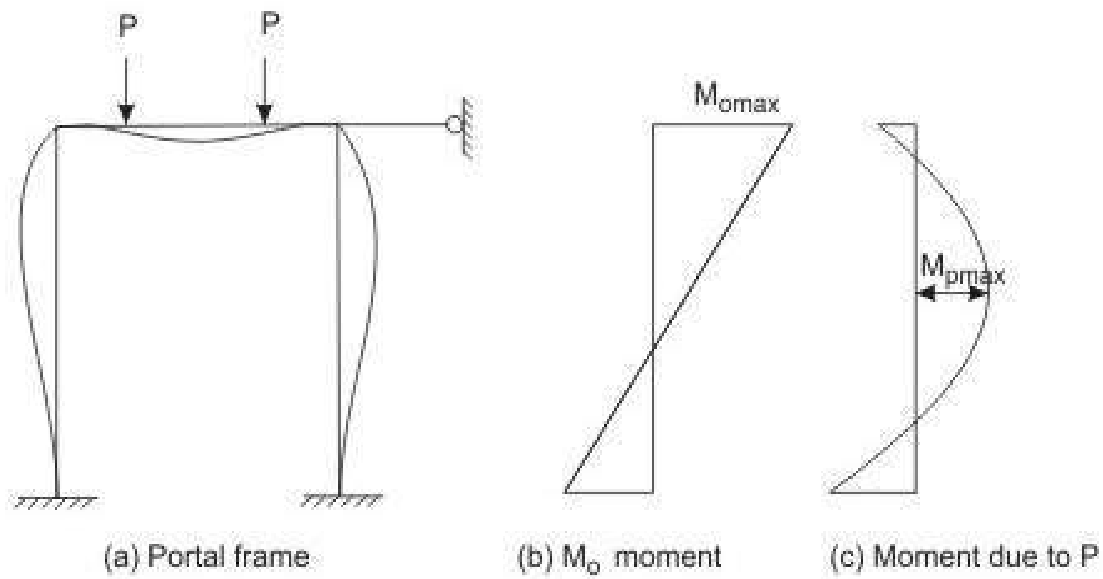

Fig. 10.27.14: Fixed portal frame - laterally braced

Figure 10.27.14a shows a fixed portal frame, laterally braced so that no sidesway can occur. Figures 10.27.14b and c show the moments M_o and due to *P*. It is seen that the maximum values of the two different moments do not occur at the same location. As a result, the magnification of the moment either may not be true or shall be small.

(D) Columns with different slenderness ratios

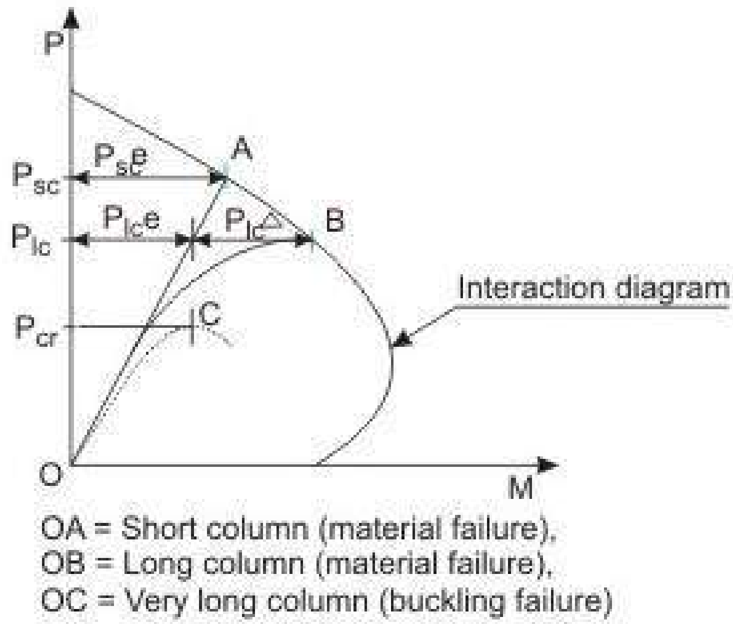

Fig. 10.27.15: Behaviour of slender column

Figure 10.27.15 shows the interaction diagram of *P* and *M* at the mid-height section of the column shown in Fig.10.27.10. Three loading paths OA, OB and OC are also shown in the figure for three columns having the same cross-sectional area and the eccentricity of loads but with different slenderness ratios. The three columns are loaded with increasing *P* and *M* (at constant eccentricity) up to failure. The loading path OA is linear indicating $\Delta = 0$, i.e., for a very short column. It should be noted that Δ should be theoretically zero only when either the effective length or the eccentricity is zero. In a practical short column, however, some lateral deflection shall be there, which, in turn will cause additional moment not more than five per cent of the primary moment and may be neglected. The loading path OA terminates at point A of the interaction diagram, which shows the failure load P_{sc} of the short column with moment $M_{sc} = P_{sc}\,e$. The short column fails by crushing of concrete at the mid-height section. This type of failure is designated as material failure, either a tension failure or a compression failure depending on the location of the point A on the interaction curve.

The load path OB is for a long column, where the deflection Δ caused by increasing value of *P* is significant. Finally, the long column fails at load P_{lc} and moment $M_{lc} = P_{lc}(e + \Delta)$. The loading path OB further reveals that the secondary moment $P_{lc}\Delta$ is comparable to the primary moment $P_{lc}\,e$. Moreover, the failure load and the primary moment of the long column P_{lc} and $P_{lc}\,e$, respectively, are less than those of the short column (P_{sc} and $P_{sc}\,e$, respectively), though both the columns have the same cross-sectional areas and eccentricities but different slenderness ratios. Here also, the mid-height section of the column undergoes material failure, either a compression failure or a tension failure, depending on the location of the point B on the interaction diagram.

The loading path OC, on the other hand, is for a very long column when the lateral deflection Δ is so high that the slope of the path *dP/dM* at C is zero. The column is so slender that the failure is due to buckling (instability) at a comparatively much low value of the load P_{cr}, though this column has the same cross-sectional area and the eccentricity of load as of the other two columns. Such instability failure occurs for very slender columns, specially when they are not braced.

The following points are summarised from the discussion made in sec.10.27.3.

1. Additional deflections and moments are caused by the axial compression force *P* in columns. The additional moments increase with the increase of *kl/r*, when other parameters are equal.

2. Laterally braced compression members and bent in single curvature have the same or nearby locations of the maxima of both M_o and *Py*. Thus, being fully additive, they have large moment magnification.

3. Laterally braced compression members and bent in double curvature have different locations of the maxima of both M_o and *Py*. As a result, the moment magnification is either less or zero.

4. Members of frames not braced laterally, the maxima of M_o and *Py* mostly occur at the ends of column and cause the maximum total moment at the ends of columns only. Additional moments and additional deflections increase with the increase of *kl/r*.

10.27.4 Effective Length of Columns

Annex E of IS 456 presents two figures (Figs.26 and 27) and a table (Table 26) to estimate the effective length of columns in frame structures based on a research paper, "Effective length of column in multistoreyed building" by R.H. Wood in The Structural Engineer Journal, No.7, Vol.52, July 1974. Figure 26 is for columns in a frame with no sway, while Fig.27 is for columns in a frame with sway. These two figures give the values of *k* (i.e., l_e/l) from two parameters β_1 and β_2 which are obtained from the following expression:

$$\beta = \sum K_c / \sum K_c + \sum K_b$$

(10.74)

where K_c and K_b are flexural stiffnesses of columns and beams, respectively. The quantities β_1 and β_2 at the top and bottom joints A and B, respectively, are determined by summing up the *K* values of members framing into a joint at top and bottom, respectively. Thus β_1 and β_2 for the frame shown in Fig.10.27.16 are as follows:

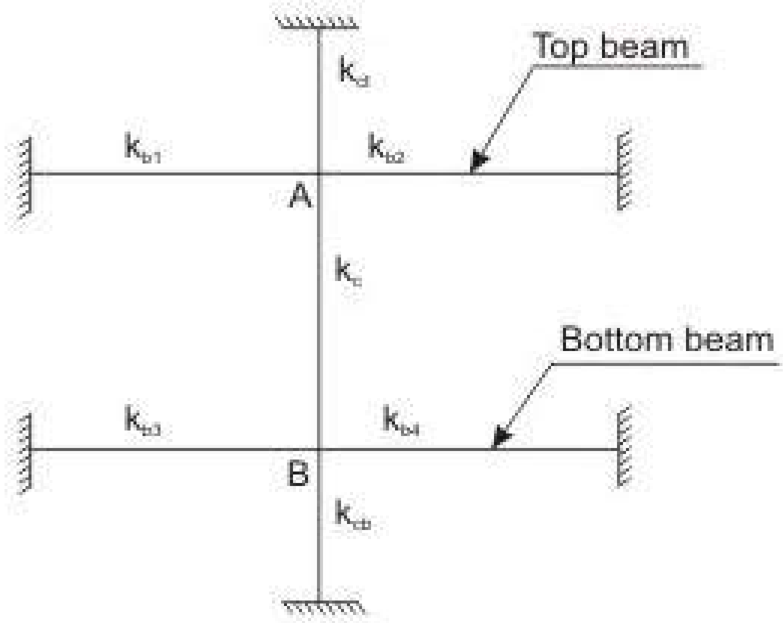

Fig. 10.27.16: Stiffness of columns in Wood's chart

$$\beta_1 = (K_c + K_{ct})/(K_c + K_{ct} + K_{b1} + K_{b2}) \quad (10.75)$$

$$\beta_2 = (K_c + K_{cb})/(K_c + K_{cb} + K_{b3} + K_{b4}) \quad (10.76)$$

However, assuming idealised conditions, the effective length in a given plane may be assessed from Table 28 in Annex E of IS 456, for normal use.

10.27.5 Determination of Sway or No Sway Column

Clause E-2 of IS 456 recommends the stability index Q to determine if a column is a no sway or sway type. The stability index Q is expressed as:

$$Q = \sum P_u \Delta_u / H_u h_z \quad (10.77)$$

where $\sum P_u$ = sum of axial loads on all columns in the storey,

Δ_u = elastically computed first-order lateral deflection,

H_u = total lateral force acting within the storey, and

h_z = height of the storey.

The column may be taken as no sway type if the value of Q is $\leq$ 0.4, otherwise, the column is considered as sway type.

10.27.6 Design of Slender Columns

The design of slender columns, in principle, is to be done following the same procedure as those of short columns. However, it is essential to estimate the total moment i.e., primary and secondary moments considering P-Δ effects. These secondary moments and axial forces can be determined by second-order rigorous structural analysis – particularly for unbraced frames. Further, the problem becomes more involved and laborious as the principle of superposition is not applicable in second-order analysis.

However, cl.39.7 of IS 456 recommends an alternative simplified method of determining additional moments to avoid the laborious and involved second-order analysis. The basic principle of additional moment method for estimating the secondary moments is explained in the next section.

10.27.7 Additional Moment Method

In this method, slender columns should be designed for biaxial eccentricities which include secondary moments (Py of Eq.10.63 and 10.65) about major and minor axes. We first consider braced columns which are bent symmetrically in single curvature and cause balanced failure i.e., $P_u = P_{ub}$.

(A) Braced columns bent symmetrically in single curvature and undergoing balanced failure

For braced columns bent symmetrically in single curvature, we have from Eqs.10.63 and 10.65,

$$M = M_o + Py = M_o + P\,e_a = M_o + M_a \qquad (10.78)$$

where P is the factored design load P_u, M are the total factored design moments M_{ux} and M_{uy} about the major and minor axes, respectively; M_o are the primary factored moments M_{oux} and M_{ouy} about the major and minor axes, respectively; M_a are the additional moments M_{ax} and M_{ay} about the major and minor axes, respectively and e_a are the additional eccentricities e_{ax} and e_{ay} along the minor and major axes, respectively. The quantities M_o and P of Eq.10.78 are known and hence, it is required to determine the respective values of e_a, the additional eccentricities only.

Let us consider the columns of Figs.10.27.10 and 11 showing Δ as the maximum deflection at the mid-height section of the columns. The column of Fig.10.27.10, having a constant primary moment M_o, causes constant curvature ϕ, while the column of Fig.10.27.11, having a linearly varying primary moment with a maximum value of $M_{o\ max}$ at the mid-height section of the column, has a linearly varying curvature with the maximum curvature of ϕ_{max} at the mid-height section the column. The two maximum curvatures can be expressed in terms of their respective maximum deflection Δ as follows:

The constant curvature (Fig.10.27.10) $\phi_{\max} = 8\Delta/l_e^2$
(10.79)

The linearly varying curvature (Fig.10.27.11) $\phi_{\max} = 12\Delta/l_e^2$
(10.80)

where l_e are the respective effective lengths kl of the columns. We, therefore, consider the maximum ϕ as the average value lying in between the two values of Eqs.10.79 and 80 as

$\phi_{\max} = 10\Delta/l_e^2$
(10.81)

Accordingly, the maximum additional eccentricities e_a, which are equal to the maximum deflections Δ, can be written as

$e_a = \Delta = \phi l_e^2 /10$
(10.82)

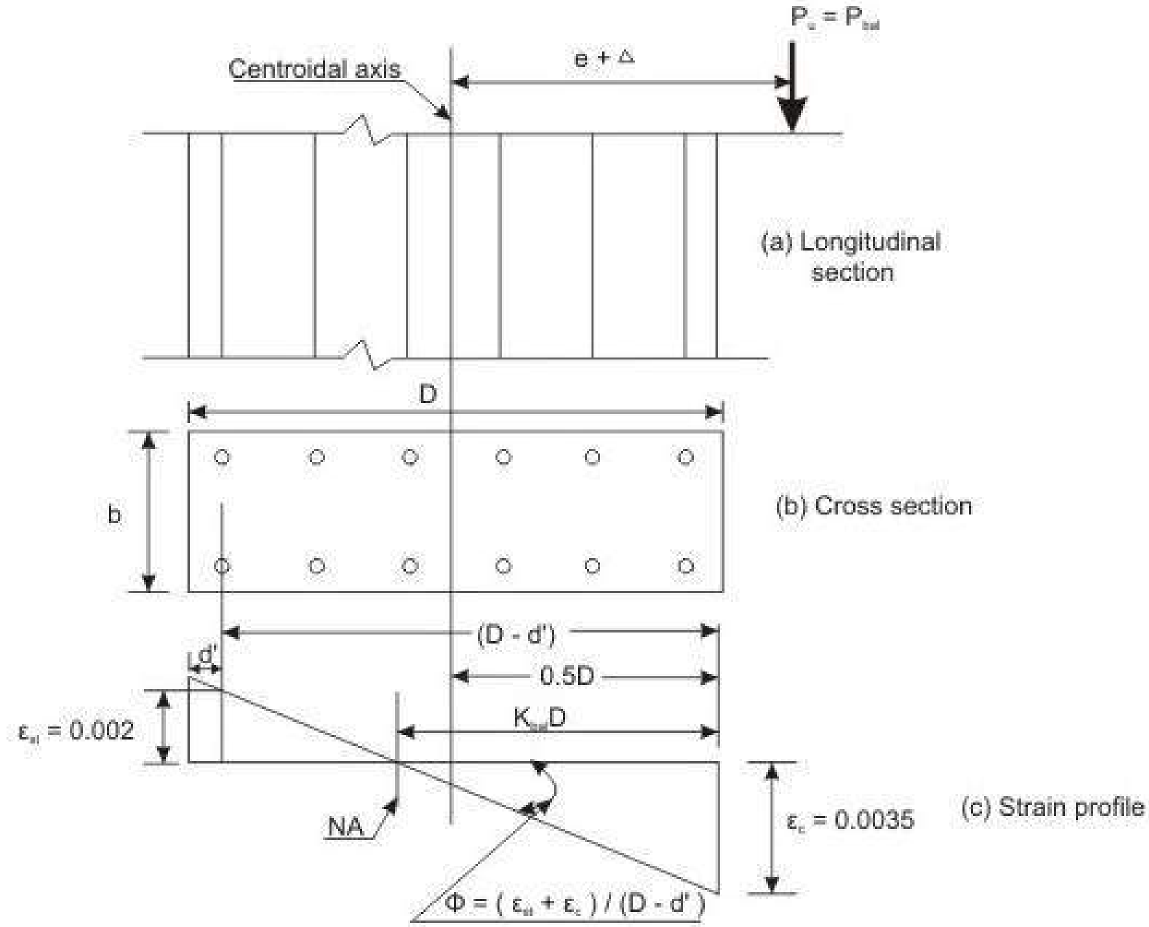

Fig. 10.27.17: Maximum curvature at mid-height section when $P_u = P_{bal}$

Assuming the column undergoes a balanced failure when $P_u = P_{ub}$, the maximum curvature at the mid-height section of the column, shown in Figs.10.27.17a and b, can be expressed as given below, assuming (i) the values of ε_c = 0.0035, ε_{st} = 0.002 and d'/D = 0.1, and (ii) the additional moment capacities are about eighty per cent of the total moment.

ϕ = eighty per cent of {(0.0035 + 0.002)/0.9D}(see Fig.10.27.17c)

or $\phi = 1/200D$ (10.83)

Substituting the value of ϕ in Eq.10.82,

$e_a = D(l_e/D)^2/2000$ (10.84)

Therefore, the additional moment M_a can be written as,

$M_a = Py = P\Delta = Pe_a = (PD/2000)(l_e/D)^2$ (10.85)

Thus, the additional moments M_{ax} and M_{ay} about the major and minor axes, respectively, are:

$M_{ax} = (P_u D/2000)(l_{ex}/D)^2$ (10.86)

$M_{ay} = (P_u b/2000)(l_{ey}/b)^2$ (10.87)

where P_u = axial load on the member,

l_{ex} = effective length in respect of the major axis,

l_{ey} = effective length in respect of the minor axis,

D = depth of the cross-section at right angles to the major axis, and

b = width of the member.

Clause 39.7.1 of IS 456 recommends the expressions of Eqs.10.86 and 87 for estimating the additional moments M_{ax} and M_{ay} for the design. These two expressions of the additional moments are derived considering the columns to be braced and bent symmetrically undergoing balanced failure. Therefore, proper modifications are necessary for different situations like braced columns with unequal end moments with the same or different signs, unbraced columns and columns causing compression failure i.e., when $P_u > P_{ub}$.

(B) Braced columns subjected to unequal primary moments at the two ends

For braced columns without any transverse loads occurring in the height, the primary maximum moment ($M_{o\,max}$ of Eq.10.64), with which the additional moments of Eqs.10.86 and 87 are to be added, is to be taken as:

$M_{o\,max} = 0.4\,M_1 + 0.6\,M_2$ (10.88)

and further $M_{o\,max} \geq 0.4\,M_2$ (10.89)

where M_2 is the larger end moment and M_1 is the smaller end moment, assumed to be negative, if the column is bent in double curvature.

To eliminate the possibility of total moment $M_{u\,max}$ becoming less than M_2 for columns bent in double curvature (see Fig.10.27.12) with M_1 and M_2 having opposite signs, another condition has been imposed as

$$M_{u\,max} \geq M_2 \quad (10.90)$$

The above recommendations are given in notes of cl.39.7.1 of IS 456.

(C) Unbraced columns

Unbraced frames undergo considerable deflection due to P-Δ effect. The additional moments determined from Eqs.10.86 and 87 are to be added with the maximum primary moment $M_{o\,max}$ at the ends of the column. Accordingly, we have

$$M_{o\,max} = M_2 + M_a \quad (10.91)$$

The above recommendation is given in the notes of cl.39.7.1 of IS 456.

(D) Columns undergoing compression failure ($P_u > P_{ub}$)

It has been mentioned in part A of this section that the expressions of additional moments given by Eqs.10.86 and 10.87 are for columns undergoing balanced failure (Fig.10.27.17). However, when the column causes compression failure, the e/D ratio is much less than that of balanced failure at relatively high axial loads. The entire section may be under compression causing much less curvatures. Accordingly, additional moments of Eqs.10.86 and 10.87 are to be modified by multiplying with the reduction factor k as given below:

(i) For $P_u > P_{ubx}$: $$k_{ax} = (P_{uz} - P_u)/(P_{uz} - P_{ubx}) \quad (10.92)$$

(ii) For $P_u > P_{uby}$: $$k_{ay} = (P_{uz} - P_u)/(P_{uz} - P_{uby}) \quad (10.93)$$

with a condition that k_{ax} and k_{ay} should be ≤ 1 (10.94)

where P_u = axial load on compression member

P_{uz} is given in Eq.10.59 of Lesson 26 and is,

$$P_{uz} = 0.45 f_{ck} A_c + 0.75 f_y A_{st} \quad \ldots (10.59)$$

P_{ubx}, P_{uby} = axial loads with respect to major and minor axes, respectively, corresponding to the condition of maximum compressive strain of 0.0035 in concrete and tensile strain of 0.002 in outermost layer of tension steel.

It is seen from Eqs.10.92 and 10.93 that the values of k (k_{ax} and k_{ay}) vary linearly from zero (when $P_u = P_{uz}$) to one (when $P_u = P_{ub}$). Since Eqs.10.92 and 10.93 are not applicable for $P_u < P_{ub}$, another condition has been imposed as given in Eq.10.94.

The above recommendations are given in cl.39.7.1.1 of IS 456.

The following discussion is very important for the design of slender columns.

Additional moment method is one of the methods of designing slender columns as discussed in A to D of this section. This method is recommended in cl.39.7 of IS 456 also. The basic concept here is to enhance the primary moments by adding the respective additional moments estimated in a simple way avoiding laborious and involved calculations of second-order structural analysis. However, these primary moments under eccentric loadings should not be less than the moments corresponding to the respective minimum eccentricity, as stipulated in the code. Hence, the primary moments in such cases are to be replaced by the minimum eccentricity moments. Moreover, all slender columns, including those under axial concentric loadings, are also to be designed for biaxial bending, where the primary moments are zero. In such cases, the total moment consisting of the additional moment multiplied with the modification factor, if any, in each direction should be equal to or greater than the respective moments under minimum eccentricity conditions. As mentioned earlier, the minimum eccentricity consideration is given in cl.25.4 of IS 456.

10.27.8 Illustrative Example

The following illustrative example is taken up to explain the design of slender columns. The example has been solved in step by step using (i) the equations of Lessons 21 to 27 and (ii) employing design charts and tables of SP-16, to compare the results.

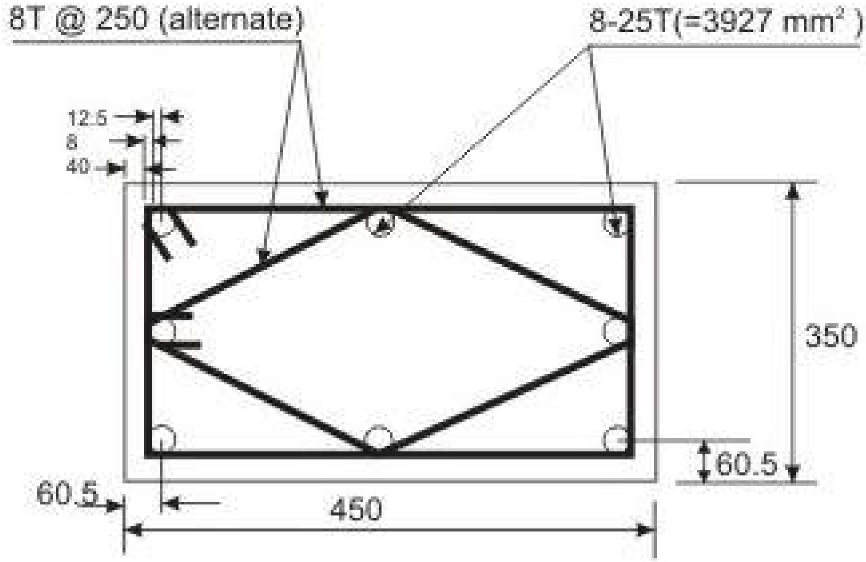

Fig. 10.27.18: Problem 1

Problem 1:

Determine the reinforcement required for a braced column against sidesway with the following data: size of the column = 350 x 450 mm (Fig.10.27.18); concrete and steel grades = M 30 and Fe 415, respectively; effective lengths l_{ex} and l_{ey} = 7.0 and 6.0 m, respectively; unsupported length l = 8 m; factored load P_u = 1700 kN; factored moments in the direction of larger dimension = 70 kNm at top and 30 kNm at bottom; factored moments in the direction of shorter dimension = 60 kNm at top and 30 kNm at bottom. The column is bent in double curvature. Reinforcement will be distributed equally on four sides.

Solution 1:

Step 1: Checking of slenderness ratios

l_{ex}/D = 7000/450 = 15.56 > 12,

l_{ey}/b = 6000/350 = 17.14 > 12.

Hence, the column is slender with respect to both the axes.

Step 2: Minimum eccentricities and moments due to minimum eccentricities (Eq.10.3 of Lesson21)

$e_{x\,min} = l/500 + D/30$ = 8000/500 + 450/30 = 31.0 > 20 mm

$e_{y\,min} = l/500 + b/30$ = 8000/500 + 350/30 = 27.67 > 20 mm

M_{ox} (Min. ecc.) = $P_u(e_{x\,min})$ = (1700) (31) (10^{-3}) = 52.7 kNm

M_{oy} (Min. ecc.) = $P_u(e_{y\,min})$ = (1700) (27.67) (10^{-3}) = 47.04 kNm

Step 3: Additional eccentricities and additional moments

Method 1: Using Eq. 10.84

$e_{ax} = D(l_{ex}/D)^2/2000$ = (450) $(7000/450)^2/2000$ = 54.44 mm

$e_{ay} = b(l_{ex}/b)^2/2000$ = (350) $(6000/350)^2/2000$ = 51.43 mm

$M_{ax} = P_u(e_{ax})$ = (1700) (54.44) (10^{-3}) = 92.548 kNm

$M_{ay} = P_u(e_{ay})$ = (1700) (51.43) (10^{-3}) = 87.43 kNm

Method 2: Table I of SP-16

For l_{ex}/D = 15.56, Table I of SP-16 gives:

e_{ax}/D = 0.1214, which gives e_{ax} = (0.1214) (450) = 54.63 mm

For l_{ey}/D = 17.14, Table I of SP-16 gives:

e_{ay}/b = 0.14738, which gives e_{ay} = (0.14738) (350) = 51.583 mm

It is seen that values obtained from Table I of SP-16 are comparable with those obtained by Eq. 10.84 in Method 1.

Step 4: Primary moments and primary eccentricities (Eqs.10.88 and 89)

$M_{ox} = 0.6M_2 - 0.4M_1$ = 0.6(70) – 0.4(30) = 30 kNm, which should be $\geq$ 0.4 M_2 (= 28 kNm). Hence, o.k.

$M_{oy} = 0.6M_2 - 0.4M_1$ = 0.6(60) – 0.4(30) = 24 kNm, which should be $\geq$ 0.4 M_2 (= 24 kNm). Hence, o.k.

Primary eccentricities:

$e_x = M_{ox}/P_u$ = (30/1700) (10^3) = 17.65 mm

$e_y = M_{oy}/P_u$ = (24/1700) (10^3) = 14.12 mm

Since, both primary eccentricities are less than the respective minimum eccentricities (see Step 2), the primary moments are revised to those of Step 2. So, M_{ox} = 52.7 kNm and M_{oy} = 47.04 kNm.

Step 5: Modification factors

To determine the actual modification factors, the percentage of longitudinal reinforcement should be known. So, either the percentage of longitudinal reinforcement may be assumed or the modification factor may be assumed which should be verified subsequently. So, we assume the modification factors of 0.55 in both directions.

Step 6: Total factored moments

M_{ux} = M_{ox} + (Modification factor) (M_{ax}) = 52.7 + (0.55) (92.548)

= 52.7 + 50.9 = 103.6 kNm

M_{uy} = M_{oy} + (Modification factor) (M_{ay}) = 47.04 + (0.55) (87.43)

= 47.04 + 48.09 = 95.13 kNm

Step 7: Trial section (Eq.10.61 of Lesson 26)

The trial section is determined from the design of uniaxial bending with P_u = 1700 kN and M_u = 1.15 $(M_{ux}^2 + M_{uy}^2)^{1/2}$. So, we have M_u = (1.15){$(103.6)^2$ + $(95.13)^2$}$^{1/2}$ = 161.75 kNm. With these values of P_u (= 1700 kN) and M_u (= 161.75 kNm), we use chart of SP-16 for the d'/D = 0.134. We assume the diameters of longitudinal bar as 25 mm, diameter of lateral tie = 8 mm and cover = 40 mm, to get d' = 40 + 8 + 12.5 = 60.5 mm. Accordingly, d'/D = 60.5/450 = 0.134 and d'/b = 60.5/350 = 0.173. We have:

$P_u/f_{ck}\, bD$ = $1700(10^3)/(30)(350)(450)$ = 0.3598

$M_u/f_{ck}\, bD^2$ = $161.75(10^6)/(30)(350)(450)(450)$ = 0.076

We have to interpolate the values of p/f_{ck} for d'/D = 0.134 obtained from Charts 44 (for d'/D = 0.1) and 45 (d'/D = 0.15). The values of p/f_{ck} are 0.05 and 0.06 from Charts 44 and 45, respectively. The corresponding values of p are 1.5 and 1.8 per cent, respectively. The interpolated value of p for d'/D = 0.134 is 1.704 per cent, which gives A_{sc} = (1.704)(350)(450)/100 = 2683.8 mm^2. We use 4-25 + 4-20 (1963 + 1256 = 3219 mm^2), to have p provided = 2.044 per cent giving p/f_{ck} = 0.068.

Step 8: Calculation of balanced loads P_b

The values of P_{bx} and P_{by} are determined using Table 60 of SP-16. For this purpose, two parameters k_1 and k_2 are to be determined first from the table. We have p/f_{ck} = 0.068, d'/D = 0.134 and d'/b = 0.173. From Table 60, k_1 = 0.19952 and k_2 = 0.243 (interpolated for d'/D = 0.134) for P_{bx}. So, we have: $P_{bx}/f_{ck}bD = k_1 + k_2\,(p/f_{ck})$ = 0.19952 + 0.243(0.068) = 0.216044, which gives P_{bx} = 0.216044(30)(350)(450)(10^{-3}) = 1020.81 kN.

Similarly, for P_{by}: d'/b = 0.173, p/f_{ck} = 0.068. From Table 60 of SP-16, k_1 = 0.19048 and k_2 = 0.1225 (interpolated for d'/b = 0.173). This gives $P_{by}/f_{ck}bD$ = 0.19048 + 0.1225(0.068) = 0.19881, which gives P_{by} = (0.19881)(30)(350)(450)(10^{-3}) = 939.38 kN.

Since, the values of P_{bx} and P_{by} are less than P_u, the modification factors are to be used.

Step 9: Determination of P_{uz}

Method 1: From Eq.10.59 of Lesson 26

$$P_{uz} = 0.45\, f_{ck}\, A_g + (0.75\, f_y - 0.45\, f_{ck})\, A_{sc}$$

$$= 0.45(30)(350)(450) + \{0.75(415) - 0.45(30)\}(3219) = 3084.71 \text{ kN}$$

Method 2: Using Chart 63 of SP-16

We get P_{uz}/A_g = 19.4 N/mm^2 from Chart 63 of SP-16 using p = 2.044 per cent. Therefore, P_{uz} = (19.4)(350)(450)(10^{-3}) = 3055.5 kN, which is in good agreement with that of Method 1.

Step 10: Determination of modification factors

Method 1: From Eqs.10.92 and 10.93

$$k_{ax} = (P_{uz} - P_u)/(P_{uz} - P_{ubx}) \quad \dots (10.92)$$

or k_{ax} = (3084.71 – 1700)/(3084.71 – 1020.81) = 0.671 and

$$k_{ay} = (P_{uz} - P_u)/(P_{uz} - P_{uby}) \quad \dots (10.93)$$

or k_{ay} = (3084.71 – 1700)/(3084.71 – 939.39) = 0.645

The values of the two modification factors are different from the assumed value of 0.55 in Step 5. However, the moments are changed and the section is checked for safety.

Method 2: From Chart 65 of SP-16

From Chart 65 of SP-16, for the two parameters, P_{bx}/P_{uz} = 1020.81/3084.71 = 0.331 and P_u/P_{uz} = 1700/3084.71 = 0.551, we get k_{ax} = 0.66. Similarly, for the two parameters, P_{by}/P_{uz} = 939.38/3084.71 = 0.3045 and P_u/P_{uz} = 0.551, we have k_{ay} = 0.65. Values of k_{ax} and k_{ay} are comparable with those of Method 1.

Step 11: Total moments incorporating modification factors

M_{ux} = M_{ox} (from Step 4) + (k_{ax}) M_{ax} (from Step 3)

= 52.7 + 0.671(92.548) = 114.8 kNm

M_{uy} = M_{oy} (from Step 4) + k_{ay} (M_{ay}) (from Step 3)

= 47.04 + (0.645)(87.43) = 103.43 kNm.

Step 12: Uniaxial moment capacities

The two uniaxial moment capacities M_{ux1} and M_{uy1} are determined as stated: (i) For M_{ux1}, by interpolating the values obtained from Charts 44 and 45, knowing the values of $P_u/f_{ck}bD$ = 0.3598 (see Step 7), p/f_{ck} = 0.068 (see Step 7), d'/D = 0.134 (see Step 7), (ii) for M_{uy1}, by interpolating the values obtained from Charts 45 and 46, knowing the same values of $P_u/f_{ck}bD$ and p/f_{ck} as those of (i) and d'/D = 0.173 (see Step 7). The results are given below:

(i) $M_{ux1}/f_{ck}bD^2$ = 0.0882 (interpolated between 0.095 and 0.085)

(ii) $M_{uy1}/f_{ck}bb^2$ = 0.0827 (interpolated between 0.085 and 0.08)

So, we have, M_{ux1} = 187.54 kNm and M_{uy1} = 136.76 kNm.

Step 13: Value of α_n

Method 1: From Eq.10.60 of Lesson 26

We have P_u/P_{uz} = 1700/3084.71 = 0.5511. From Eq.10.60 of Lesson 26, we have α_n = 0.67 + 1.67 (P_u/P_{uz}) = 1.59.

Method 2: Interpolating the values between (P_u/P_{uz}) = 0.2 and 0.6

The interpolated value of α_n = 1.0 + (0.5511 – 0.2)/0.6 = 1.5852. Both the values are comparable. We use α_n = 1.5852.

Step 14: Checking of column for safety

Method 1: From Eq.10.58 of Lesson 26

We have in Lesson 26:

$$(M_{ux} / M_{ux1})^{\alpha_n} + (M_{uy} / M_{uy1})^{\alpha_n} \le 1 \qquad \ldots \ (10.58)$$

Here, putting the values of M_{ux}, M_{ux1}, M_{uy}, M_{uy1} and α_n, we get: $(114.8/187.54)^{1.5452}$ + $(103.43/136.76)^{1.5852}$ = 0.4593 + 0.6422 = 1.1015. Hence, the section or the reinforcement has to be revised.

Method 2: Chart 64 of SP-16

The point having the values of (M_{ux}/M_{ux1}) = 114.8/187.54 = 0.612 and (M_{uy}/M_{uy1}) = 103.43/136.76 = 0.756 gives the value of P_u/P_z more than 0.7. The value of P_u/P_{uz} here is 0.5511 (see Step 13). So, the section needs revision.

We revise from Step 7 by providing 8-25 mm diameter bars (= 3927 mm^2, p = 2.493 per cent and p/f_{ck} = 0.0831) as the longitudinal reinforcement keeping the values of b and D unchanged. The revised section is checked furnishing the repeated calculations from Step 8 onwards. The letter R is used before the number of step to indicate this step as revised one.

Step R8: Calculation of balanced loads P_b

Table 60 of SP-16 gives k_1 = 0.19952, and k_2 = 0.243. We have p/f_{ck} = 0.0831 now. So, P_{bx} = {0.19952 + (0.243)(0.0831)} (30)(350)(450)(10^{-3}) = 1038.145 kN. Similarly, k_1 = 0.19048, k_2 = 0.1225 and p/f_{ck} = 0.0831 give P_{by} = {0.19048 + (0.1225)(0.0831)} (30)(350)(450)(10^{-3}) = 948.12 kN.

The values of P_{bx} and P_{by} are less than P_u (= 1700 kN). So, modification factors are to be incorporated.

Step R9: Determination of P_{uz} (Eq. 10.59 of Lesson 26)

P_{uz} = 0.45(30)(350)(450) + {0.75(415) – 0.45(30)}(3927) = 3295.514 kN.

Step R10: Determination of modification factors (Eqs.10.92 and 10.93)

k_{ax} = (3295.514 – 1700)/(3295.514 – 1038.145) = 0.707

k_{ay} = (3295.514 – 1700)/(3295.514 – 948.12) = 0.68

Step R11: Total moments incorporating modification factors

M_{ux} = 52.70 + 0.707(92.548) = 118.13 kNm

M_{uy} = 47.04 + 0.68(87.43) = 106.49 kNm

Step R12: Uniaxial moment capacities

Using Charts 44 and 45 for M_{ux1} and Charts 45 and 46 for M_{uy1}, we get (i) the coefficient 0.1032 (interpolating 0.11 and 0.10) and (ii) the coefficient 0.0954 (interpolating 0.1 and 0.09) for M_{ux1} and M_{uy1}, respectively.

M_{ux1} = (0.1032)(30)(350)(450)(450)(10^{-6}) = 219.429 kNm

M_{uy1} = (0.0954)(30)(450)(350)(350)(10^{-6}) = 157.77 kNm

Step R13: Value of α_n (Eq.10.60 of Lesson 26)

P_u/P_{uz} = 1700/3295.514 = 0.5158 which gives

α_n = 1 + (0.5158 – 0.2)/0.6 = 1.5263

Step R14: Checking of column for safety (Eq.10.58 of Lesson 26)

$(118.13/219.424)^{1.5263} + (106.49/157.77)^{1.5263}$ = 0.3886 + 0.5488 = 0.9374 < 1.0

Hence, the revised reinforcement is safe. The section is shown in Fig.10.27.18.

10.27.9 Practice Questions and Problems with Answers

Q.1: Define a slender column. Give three reasons for its increasing importance and popularity.

A.1: See sec. 10.27.1.

Q.2: Explain the behaviour of a slender column subjected to concentric loading. Explain Euler's load.

A.2: See sec.10.27.3.

Q.3: Choose the correct answer.

(A) As the slenderness ratio increases, the strength of concentrically loaded column:
(i) increases (ii) decreases

(B) For braced columns, the effective length is between

(i) *l* and 2*l* (ii) 0.5*l* and 2*l* (iii) 0.5*l* and *l*

(C) The critical load of a braced frame is

(i) always larger than that of an unbraced column

(ii) always smaller than that of an unbraced column

(iii) sometimes larger and sometimes smaller than that of an unbraced column

A.3: A. (ii), B. (iii), C. (i)

Q.4: Explain the behaviour of slender columns under axial load and uniaxial bending, bent in single curvature.

A.4: Part (A) of sec. 10.27.3.

Q.5: Explain the behaviour of slender columns under axial load and uniaxial bending, bent in double curvature.

A.5: Part (B) of sec. 10.27.3.

Q.6: Explain the behaviour of columns in portal frame both braced and unbraced.

A.6: Part (C) of sec. 10.27.3.

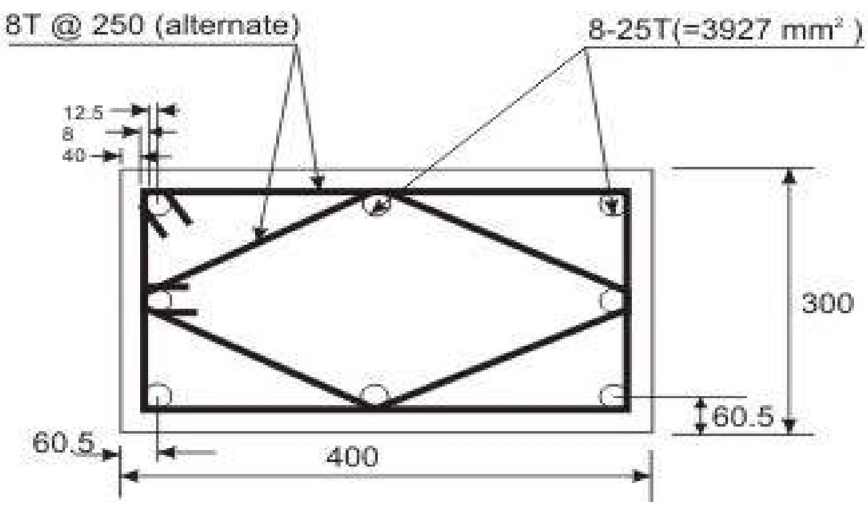

Fig. 10.27.19: Q.7

Q.7: Check the column of Fig.10.27.19, if subjected to an axial factored load of P_u = 1500 kN only when the unsupported length of the column = l = 8.0 m, l_{ex} = l_{ey} = 6.0 m, D = 400 mm, b = 300 mm, using concrete of M 20 and steel grade in Fe 415.

A.7: Solution:

Step 1: Slenderness ratios

L_{ex}/D = 6000/400 = 15 > 12

L_{ey}/b = 6000/300 = 20 > 12

The column is slender about both the axes.

Step 2: Minimum eccentricities and moments due to minimum eccentricities (Eq.10.3 of Lesson 21)

$e_{x\ min}$ = l/500 + D/30 = 8000/500 + 400/30 = 29.33 mm > 20 mm

$e_{y\ min}$ = 8000/500 + 300/30 = 26 mm > 20 mm

M_x due to min. ecc. = P_u ($e_{x\ min}$) = 1500(29.33) = 43.995 kNm

M_y due to min. ecc. = P_u ($e_{y\ min}$) = 1500(26.0) = 39.0 kNm

Step 3: Primary moments

Since the column is concentrically loaded, the primary moments are zero. Therefore, the additional moments must be greater than the respective moments due to minimum eccentricity.

Step 4: Additional eccentricities and moments (Eq.10.84)

e_{ax} = $D(l_{ex}/D)^2/2000$ = $400(6000/400)^2/2000$ = 45 mm > $e_{x\ min}$ (= 29.23 mm)

e_{ay} = $b(l_{ey}/b)^2/2000$ = $300(6000/300)^2/2000$ = 60 mm > $e_{y\ min}$ (= 26 mm)

Step 5: Calculation of balance loads P_{bx} and P_{by}

Given A_{sc} = 3927 mm^2 (8 bars of 25 mm diameter give p = 3.2725 per cent. So, p/f_{ck} = 0.1636. Using 8 mm diameter lateral tie, d' = 40 + 8 + 12.5 =

60.5 mm giving d'/D = 60.5/400 = 0.15125 $\cong$ 0.15 and d'/b = 60.5/300 = 0.2017 $\cong$ 0.20.

From Table 60 of SP-16, we get k_1 = 0.196 and k_2 = 0.061. Thus, we have:

$$P_{bx} = \{0.196 + (0.061)(0.1636)\}(20)(300)(400)(10^{-3}) = 494.35 \text{ kN}$$

Similarly, for P_{by}: k_1 = 0.184 and k_2 = -0.011, we get

$$P_{by} = \{0.184 - (0.011)(0.1636)\}(20)(300)(400)(10^{-3}) = 437.281 \text{ kN}$$

Since, P_{bx} and P_{by} are less than P_u (= 1500 kN), modification factors are to be incorporated.

Step 6: Determination of P_{uz} (Eq.10.59 of Lesson 26)

$$P_{uz} = 0.45(20)(300)(400) + \{0.75(415) – 0.45(20)\}(3927)(10^{-3}) = 2266.94 \text{ kN}$$

Step 7: Determination of modification factors

k_{ax} = (2266.94 – 1500)/(2266.94 – 494.35) = 0.433 and

k_{ay} = (2266.94 – 1500)/(2266.94 – 437.281) = 0.419

Step 8: Additional moments and total moments

M_{ax} = 1500(0.433)(45) = 29.2275 kNm

M_{ay} = 1500(0.419)(60) = 37.71 kNm

Since, primary moments are zero as the column is concentrically loaded, the total moment shall consist of the additional moments. But, as both the additional moments are less than the respective moment due to minimum eccentricity, the revised additional moments are: M_{ax} = 43.995 kNm and M_{ay} = 39.0 kNm, which are the total moments also.

Thus, we have:

M_{ux} = 43.995 kNm, M_{uy} = 39.0 kNm and P_u = 1500 kN.

Step 9: Uniaxial moment capacities

We have, $P_u/f_{ck}\, bD$ = {1500/(20)(300)(400)}(1000) = 0.625, p/f_{ck} = 0.1636 and d'/D = 0.15 for M_{ux1}; and d'/b = 0.2 for M_{uy1}. The coefficients are 0.11 (from Chart 45) and 0.1 (from Chart 46) for M_{ux1} and M_{uy1}, respectively. So, we get,

M_{ux1} = 0.11(20)(300)(400)(400)(10^{-6}) = 225.28 kNm, and

M_{uy1} = 0.1(20)(300)(300)(400)(10^{-6}) = 72.0 kNm

Step 10: Value of α_n (Eq.10.60 of Lesson 26)

Here, P_u/P_{uz} = 1500/2266.94 = 0.6617. So, we get

α_n = 1.0 + (0.4617/0.6) = 1.7695

Step 11: Checking the column for safety (Eq.10.58 of Lesson 26)

$$(M_{ux}/M_{ux1})^{\alpha_n} + (M_{uy}/M_{uy1})^{\alpha_n} \le 1$$

Here, $(43.995/225.28)^{1.7695} + (39.0/72.0)^{1.7695}$ = 0.0556 + 0.3379 = 0.3935 < 1

Hence, the column is safe to carry P_u = 1500 kN.

11.27.10 References

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11.27.11 Test 27 with Solutions

Maximum Marks = 50, Maximum Time = 30 minutes

Answer all questions.

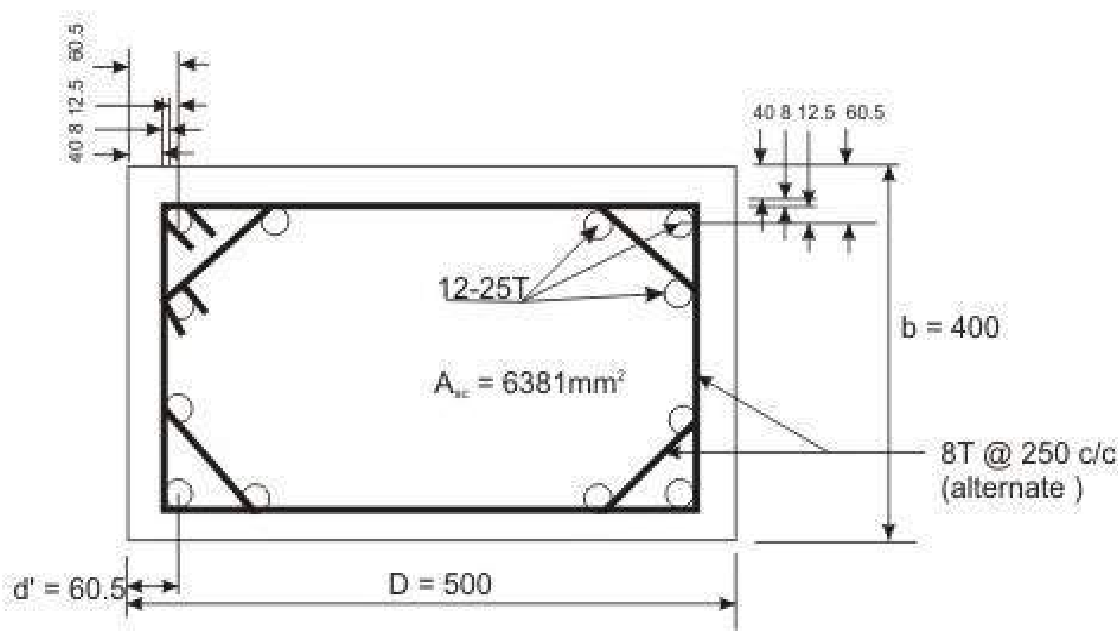


Fig. 10.27.20: TQ.1

TQ.1: Determine the primary, additional and total moments of the column shown in Fig.10.27.20 for the three different cases:

(i) When the column is braced against sidesway and is bent in single curvature.

(ii) When the column is braced against sidesway and is bent in double curvature.

(iii) When the column is unbraced.

Use the following data: P_u = 2000 kN, concrete grade = M 20, steel grade = Fe 415, unsupported length l = 8.0 m, l_{ex} = 7.0 m, l_{ey} = 6.0 m, A_{sc} = 6381 mm^2 (12-25 mm diameter bars), lateral tie = 8 mm diameter @ 250 mm c/c, d' = 60.5 mm, D = 500 mm and b = 400 mm. The factored moments are: 70 kNm at top and 40 kNm at bottom in the direction of larger dimension and 60 kNm at top and 30 kNm at bottom in the direction of shorter dimension.

A.TQ.1: Solution

The following are the common steps for all three cases.

Step 1: Slenderness ratios

l_{ex}/D = 7000/500 = 14 > 12 and l_{ey}/b = 6000/400 = 15 > 12

The column is slender about both axes.

Step 2: Minimum eccentricities and moments due to minimum eccentricities (Eq.10.3 of Lesson 21)

$e_{x\,min}$ = l/500 + D/30 = 8000/500 + 500/30 = 32.67 mm > 20 mm, and

$e_{y\,min}$ = l/500 + b/30 = 8000/500 + 400/30 = 29.34 mm > 20 mm

M_x (min. ecc.) = 2000(32.67)(10^{-3}) = 65.34 kNm, and

M_y (min. ecc.) = 2000(29.34)(10^{-3}) = 58.68 kNm

Step 3: Additional eccentricities and moments due to additional eccentricities (Eq.10.84)

e_{ax} = $D(l_{ex}/D)^2/2000$ = $500(7000/500)^2/2000$ = 49 mm > $e_{x\,min}$ (= 32.67 mm)

$e_{ay} = b(l_{ey}/b)^2/2000 = 400(6000/400)^2/2000 = 45$ mm $> e_{y\,min}$ (= 29.34 mm)

$M_{ax} = P_u(e_{ax}) = (2000)(49)(10^{-3}) = 98$ kNm, and

$M_{ay} = P_u(e_{ay}) = (2000)(45)(10^{-3}) = 90$ kNm

Step 4: Calculation of balanced loads

Using d'/D = 0.121 and p/f_{ck} = 3.1905/20 = 0.159525 in Table 60 of SP-16, we have k_1 = 0.20238 and k_2 = 0.2755 (by linear interpolation). This gives

$$P_{bx} = \{0.20238 + 0.2755(0.159525)\}(20)(400)(500)(10^{-3}) = 983.32 \text{ kN}$$

Similarly, d'/b = 0.15125 and p/f_{ck} = 0.159525 in Table 60 of SP-16 gives k_1 = 0.1957 and k_2 = 0.198625 (by linear interpolation). So, we get

$$P_{by} = \{0.1957 + 0.198625(0.159525)\}(20)(400)(500)(10^{-3}) = 909.54 \text{ kN}$$

Both P_{bx} and P_{by} are smaller than P_u (= 2000 kN). Hence, modification factors are to be incorporated.

Step 5: Calculation of P_{uz} (Eq.10.59 of Lesson 26)

$$P_{uz} = 0.45\, f_{ck}\, A_g + (0.75\, f_y - 0.45\, f_{ck})\, A_{sc}$$

$$= 0.45(20)(400)(500) + \{0.75(415) - 0.45(20)\}(6381) = 3728.66 \text{ kN}$$

Step 6: Modification factors and revised additional moments (Eqs.10.92 and 10.93)

k_{ax} = (3728.66 - 2000)/(3728.66 – 983.32) = 0.6297, and

k_{ay} = (3728.66 - 2000)/(3728.66 – 909.54) = 0.6132

The revised additional moments are:

$M_{ax} = 98(0.6297) = 61.71$ kNm, and

$M_{ay} = 90(0.6132) = 55.19$ kNm

Now, the different cases are explained.

Case (i): Braced column in single curvature

Primary moments = 0.4 M_1 + 0.6 M_2, but should be equal to or greater than 0.4 M_2 and moment due to minimum eccentricities. So, we get,

M_{ox} = largest of 58 kNm, 28 kNm and 65.34 kNm = 65.34 kNm

M_{oy} = largest of 48 kNm, 24 kNm and 58.68 kNm = 58.68 kNm

Additional moments are M_{ax} = 61.71 kNm and M_{ay} = 55.19 kNm (incorporating the respective modification factors).

Total moments = M_{ux} = M_{ox} + M_{ax} = 65.34 + 61.71 = 127.05 kNm > 65.34 kNm (moment due to minimum eccentricity), and

M_{uy} = M_{oy} + M_{ay} = 58.68 + 55.19 = 113.87 kNm > 58.68 kNm (moment due to minimum eccentricity).

Case (ii): Braced column in double curvature

Primary moments = - 0.4 M_1 + 0.6 M_2, but should be equal to or greater than 0.4M_2 and the moment due to minimum eccentricity. So, we get,

M_{ox} = largest of 26 kNm, 28 kNm and 65.34 kNm = 65.34 kNm

M_{oy} = largest of 24 kNm, 24 kNm and 58.68 kNm = 58.68 kNm

Additional moments are M_{ax} = 61.71 kNm and M_{ay} = 55.19 kNm

Final moments = M_{ux} = M_{ox} + M_{ax} = 65.34 + 61.71 = 127.05 kNm > 65.34 kNm (moment due to minimum eccentricity), and

M_{uy} = 58.68 + 55.19 = 113.87 kNm > 58.68 kNm (moment due to minimum eccentricity).

Case (iii): Unbraced column

Primary moments = M_2 and should be greater than or equal to moment due to minimum eccentricity.

M_{ox} = 70 kNm > 65.34 kNm (moment due to minimum eccentricity), and

M_{oy} = 60 kNm > 58.68 kNm (moment due to minimum eccentricity).

Additional moments are M_{ax} = 61.71 kNm and M_{ay} = 55.19 kNm

Final moments = M_{ux} = M_{ox} + M_{ax} = 70.0 + 61.71 = 131.71 kNm > 65.34 kNm (moment due to minimum eccentricity), and

M_{uy} = M_{oy} + M_{ax} = 60.0 + 55.19 = 115.19 kNm > 58.68 kNm (moment due to minimum eccentricity).

10.27.12 Summary of this Lesson

This lesson mentions the reasons of increasing importance and popularity of slender columns and explains the behaviour of slender columns loaded concentrically or eccentrically. The role of minimum eccentricity that cannot be avoided in any practical column is explained for slender columns. The moments due to minimum eccentricities in both directions should be taken into account for a slender column loaded concentrically as it should be designed under biaxial bending. On the other hand, the given primary moments are also to be checked so that they are equal to or greater than the respective moments due to minimum eccentricity for all slender columns.

Both braced and unbraced columns, bent in single or double curvatures, are explained. The importance of modification factors of the additional moments due to P-Δ effect is explained. Effective lengths and important parameter to determine the slenderness ratios are illustrated for different types of support conditions either in single column or when the column is a part of rigid frames. Additional moment method, a simple method for the design of slender columns, is explained, which is recommended in IS 456. Numerical problems in illustrative example, practice problem and test questions will help in understanding and applying the method for the design of slender columns, as stipulated in IS 456. Direct computations from the given equations as well as use of design charts and tables of SP-16 are illustrated for the design.

A beam is generally supported on a hinge at one end and a roller bearing at the other end. The reactions are determined by using static equilibrium equations. Such as beam is a statically determinate structure. If the ends of the beam are restrained/clamped/encastre/fixed then the moments are included at the ends by these restraints and this moments make the structural element to be a statically indeterminate structure or a redundant structure. These restraints make the slopes at the ends zero and hence in a fixed beam, the deflection and slopes are zero at the supports.

A continuous beam is one having more than one span and it is carried by several supports (minimum of three supports). Continuous beams are widely used in bridge construction. Consider a three bay of a building which carries the loads W_1, W_2 and W_3 in two ways.

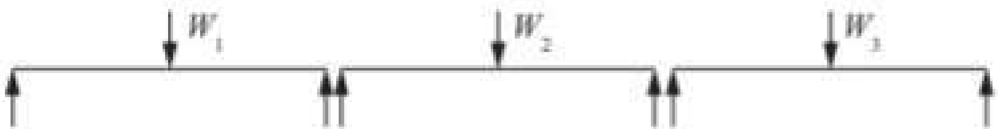


FIG. 11a Simply supported beam

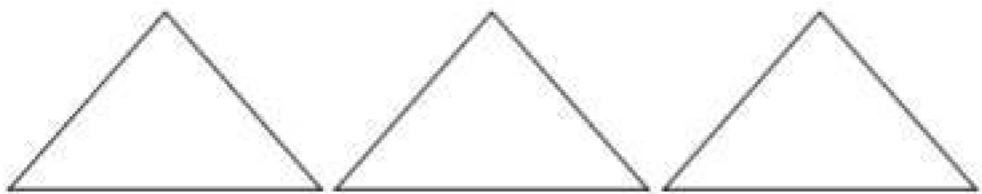


FIG. 11b Bending moment diagrams

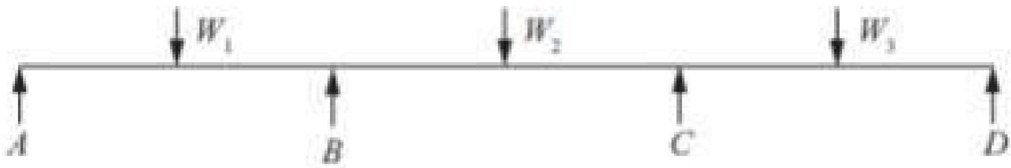


FIG. 11c Continuous beam

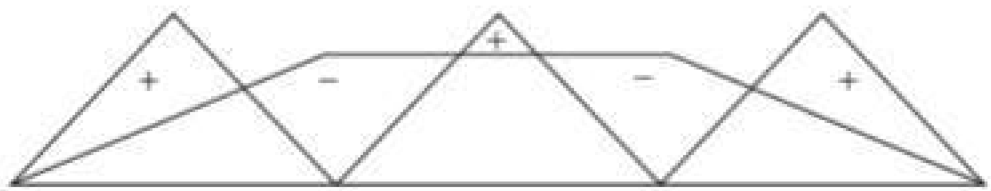


FIG. 11d Bending moment diagram

If the load is carried by the first case then the reactions of individual beams can be obtained by equilibrium equations alone. The beam deflects in the respective span and does not depend on the influence of adjacent spans.

In the second case, the equilibrium equations alone would not be sufficient to determine the end moments. The slope at an interior support *B* must be same on either side of the support. The magnitude of the slope can be influenced by not only the load on the spans either side of it but the entire loads on the span of the continuous beam. The redundants could be the reactions or the bending moments over the support. Clapeyron (1857) obtained the compatibility equation in term of the end slopes of the adjacent spans. This equation is called theorem of three moments which contain three of the unknowns. It gives the relationship between the loading and the moments over three adjacent supports at the same level.

11.1 DERIVATION OF CLAPEYRON'S THEOREM (THEOREM OF THREE MOMENTS)

Figure 11e shows two adjacent spans *AB* and *BC* of a continuous beam with two spans. The settlement of the supports are Δ_A, Δ_B and Δ_C and the deflected shape of the beam is shown in $A'B'C'$ (Fig. 11f).

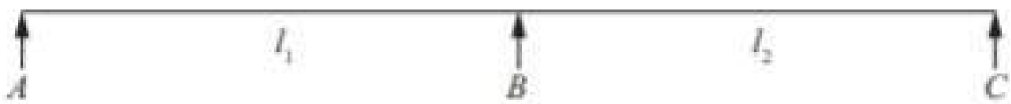


FIG. 11e

The primary structure is consisting of simply supported beams with imaginery hinges over each support (Fig 11g). Fig 11h shows the simply beam bending moment diagrams and Fig 11i shows the support moment diagram for the supports.

A compatibility equation is derived based on the fact that the end slopes of adjacent spans are equal in magnitude but opposite in sign. Using Fig 11f and the property similar triangles

$$\frac{GD}{DB'} = \frac{HF}{B'F}$$

$$\frac{\Delta_B - \Delta_A + \delta_A^B}{l_1} = \frac{\Delta_C - \Delta_B + \delta_C^B}{l_2}$$

$$\text{i.e.} \quad \frac{\delta_A^B}{l_1} + \frac{\delta_C^B}{l_2} = \frac{\Delta_A - \Delta_B}{l_1} + \frac{\Delta_C - \Delta_B}{l_2} \tag{i}$$

The displacements are obtained as follows.

$$\delta_A^B = \frac{1}{E_1 I_1}\left\{A_1\bar{x}_1 + \frac{1}{2}M_A l_1 \cdot \frac{l_1}{3} + \frac{1}{2}M_B \cdot l_1 \cdot 2l_1/3\right\} \tag{ii}$$

$$\delta_C^B = \frac{1}{E_2 I_2}\left\{A_2\bar{x}_2 + \frac{1}{2}M_C l_2 \cdot \frac{l_2}{3} + \frac{1}{2}M_B l_2 \cdot 2l_2/3\right\}$$

Combining the equations (i) and (ii)

$$\frac{M_A l_1}{E_1 I_1} + 2M_B\left(\frac{l_1}{E_1 I_1} + \frac{l_2}{E_2 I_2}\right) + M_C\frac{l_2}{E_2 I_2} + 6\left\{\frac{A_1\bar{x}_1}{E_1 I_1 l_1} + \frac{A_2\bar{x}_2}{E_2 I_2 l_1}\right\}$$

$$= 6\left\{\frac{\Delta_A - \Delta_B}{l_1} + \frac{\Delta_C - \Delta_B}{l_2}\right\} \tag{iii}$$

The above equation is called as Clapeyron's equation of three moments.

In a simplified form of an uniform beam section (EI = constant); when there are no settlement of supports

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 = -6\left(\frac{A_1\bar{x}_1}{l_1} + \frac{A_2\bar{x}_2}{l_2}\right) \tag{iv}$$

It is to be mentioned here that $\bar{x}_1$ and $\bar{x}_2$ are measured outwards in each span from the loads to the ends.

11.1.1 Procedure for Analysing the Continuous Beams using Theorem of Three Moments

(1) Draw simple beam moment diagram for each span of the beam. Compute the area of the above diagrams viz, $A_1, A_2 \ldots A_n$ and locate the centroid of such diagrams $\bar{x}_1, \bar{x}_2 \ldots \bar{x}_n$. It must be remembered that the distances $\bar{x}_1$, $\bar{x}_2 \ldots \bar{x}_n$ are the centroidal distances measured towards the ends of each span as shown in Fig. 11j.

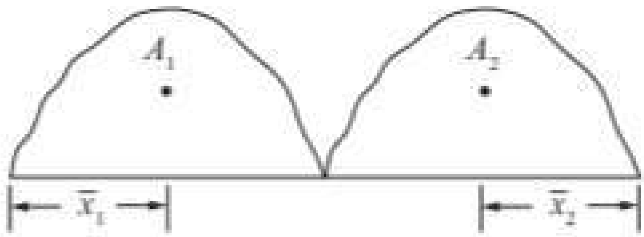


FIG. 11j Simple beam moment diagrams

(2) Identify the support moments which are to be determined viz, M_A, M_B and M_C

(3) Apply three moment equation for each pair of spans which results in an equation or equations which are to be solved simultaneously. If the beam is of uniform section (EI = constant) and no support settlements apply equation (iv) and in case the beam is non-uniform and the support settles/raises apply equation (iii).

(4) The solution of the equations gives the values of the support moments and the bending moment diagram can be drawn.

(5) The reactions at the supports and the shear force diagram can be obtained by using equilibrium equations.

11.2 APPLICATION OF THREE MOMENT EQUATION IN CASE OF BEAMS WHEN ONE OR BOTH OF THE ENDS ARE FIXED

11.2.1 Propped Cantilever Beam

Consider the propped cantilever beam of span AB, which is fixed at A and supported on a prop at B. It is subjected to uniformly distributed load over the entire span. The fixed end moment at the support A can be determined by using theorem of three moments.

FIG. 11k Propped cantilever beam

As the A is fixed support, extend the beam form A to A' of span 'zero length' and A' is simply supported.

(1) The simple beam moment diagram is a parabola with a central ordinate of $(wl^2/8)$. The centroid of this bending moment diagram (symmetrical parabola) is at a distance '$l/2$' from the supports A and B.

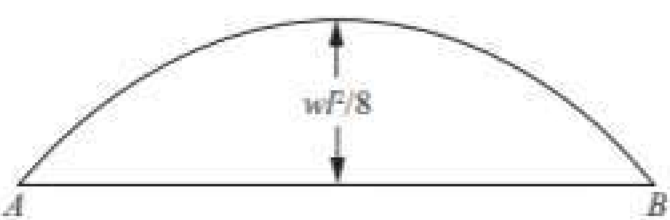


FIG. 11l Simple beam moment diagram

It's area is $A = \left(\frac{2}{3}\right)(l)\left(\frac{wl^2}{8}\right) = \frac{wl^3}{12}$.

(2) The support moment diagram is drawn as

FIG. 11m Pure moment diagram

(3) Apply three moment theorem for the span AB.

$$M'_A(0) + 2M_A(0+l) + 0 = -6\left(\frac{wl^3}{12}\right)\left(\frac{l}{2}\right)$$

$$\therefore \quad \boxed{M_A = -wl^2/8}$$

(4) The support reactions are computed by drawing the free body diagram as

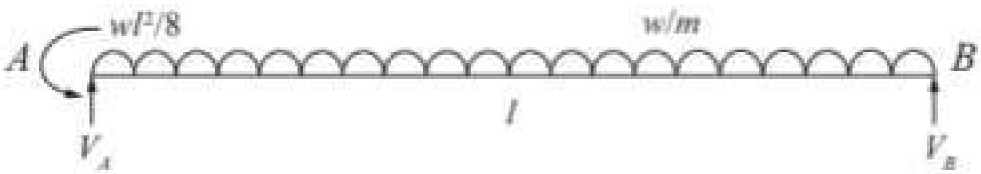


FIG. 11n Free body diagram

$$\sum V = 0; \qquad V_A + V_B = wl$$

$$\sum M_A = 0; \quad \frac{-wl^2}{8} + \frac{wl^2}{2} - V_B l = 0$$

and hence

$$\boxed{\begin{aligned} V_B &= \frac{3wl}{8} \\ V_A &= \frac{5wl}{8} \end{aligned}}$$

(5) Using the reactions, the shear force diagram and bending moment diagrams are obtained as

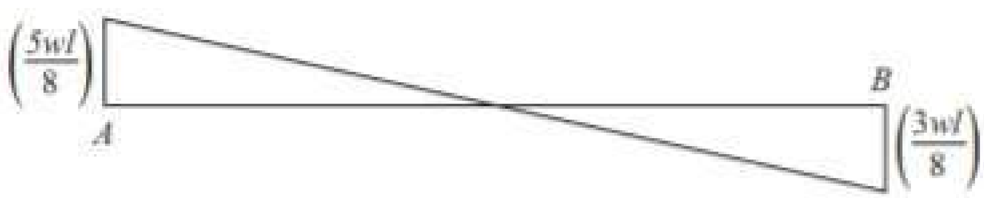


FIG. 11o Shear force diagram

The point of contraflexure is determined by equating the bending moment expression to zero and hence

$$\frac{5wl}{8}x - \frac{wx^2}{2} - \frac{wl^2}{8} = 0$$
$$l^2 + 4x^2 - 5lx = 0$$

Solving the above equation we get $x = l$ and

$$\boxed{x = 0.25l}$$

The location of maximum positive bending moment from support A is obtained by equating the shear force to zero.

$$\frac{5wl}{8} - wx = 0$$
$$\boxed{x = \frac{5l}{8}}$$

At this location, the maximum positive bending moment is obtained from

$$\text{Max} + \text{ve BM} = \frac{-wl^2}{8} + \left(\frac{5wl}{8}\right)\left(\frac{5l}{8}\right) - \frac{w(5l/8)^2}{2}$$
$$M_C = -\frac{wl^2}{8} + \frac{25wl^2}{64} - \frac{25wl^2}{128} = \frac{9wl^2}{128} = 0.07wl^2$$

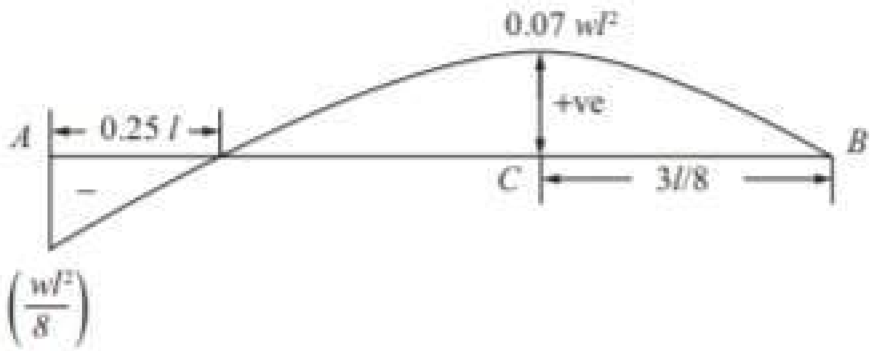


FIG. 11p Bending moment diagram

11.2.2 Beams with Both the Ends Fixed

Consider a beam AB of span l is fixed at both the ends. The beam is carrying a concentrated load of W at a distance of '$l/3$' from the fixed end A.

As the end A is a fixed support, extend this A to A' of span (l') of zero length and is also simply supported at A'. Likewise the end B is extended to B'.

The simply supported bending moment diagram is drawn with the maximum ordinate as $\frac{W \times (l/3) \times (2l/3)}{l} = 2Wl/9$.

The centroid of the unsymmetrical triangle is shown in Fig. 11.3j.

FIG. 11q Fixed beam

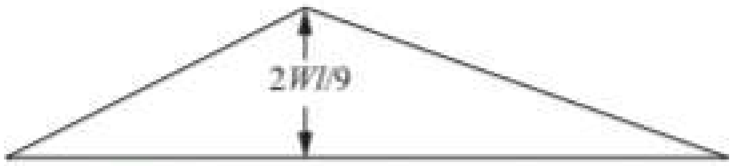


FIG. 11r Simple beam moment diagram

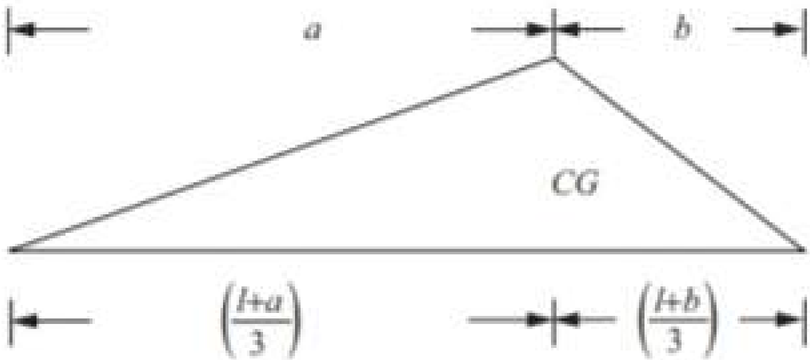


FIG. 11s Centroid of an unsymmetrical triangle

The centroid of the simply supported BMD is obtained using the above as $\left(\frac{4l}{9}\right)$ from A and $\left(\frac{5l}{9}\right)$ from B.

The area of the bending moment diagram is $\left(\frac{1}{2}\right)(l)\left(\frac{2Wl}{9}\right) = \frac{Wl^2}{9}$.

The support moment diagram can be drawn by identifying the support moments as M_A and M_B. Thus

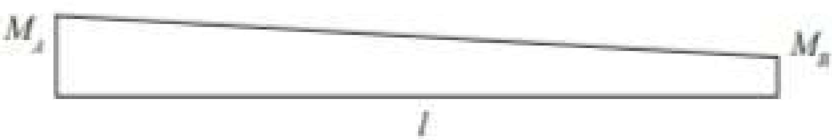


FIG. 11t Pure moment diagram

Applying three moment theorem for a pair of spans of $A'AB$ (Ref Eq (iv))

$$M'_A(0) + 2M_A(0+l) + M_B(l) = 0 - 6\left(\frac{Wl^2}{9}\right)\left(\frac{5l}{9}\right) \times 1/l$$

$$2M_A + M_B = -0.37\,Wl$$

Considering the next pair of spans ABB'

$$M_A l + 2M_B(l+0) + M_{B'}(0) = -6\left(\frac{Wl^2}{9}\right)\left(\frac{4l}{9}\right)$$

$$M_A + 2M_B = -0.296\ Wl$$

Thus the support moments are obtained by solving the above equations

$$\boxed{\begin{array}{l} M_A = -0.148\ Wl \\ M_B = -0.074\ Wl \end{array}}$$

Free body diagram to determine the reactions

FIG. 11u

Using the static equilibrium;

$$\sum V = O; \qquad V_A + V_B = W$$

$$\sum M_A = O; \qquad -0.148\ Wl + W\left(\frac{l}{3}\right) - V_B l + 0.074\ Wl = O$$

$$\boxed{\begin{array}{l} V_B = 0.26W \\ V_A = 0.74W \end{array}}$$

FIG. 11v Shearforce diagram

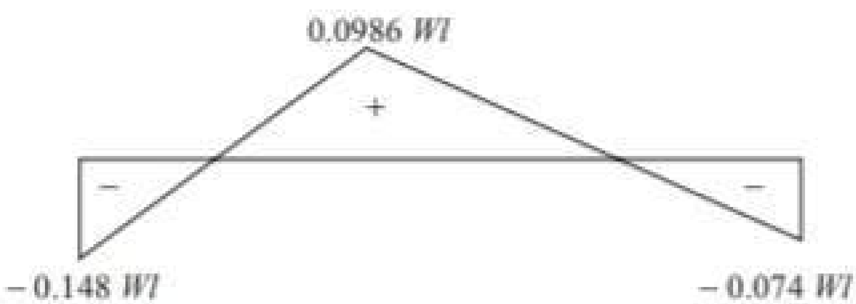
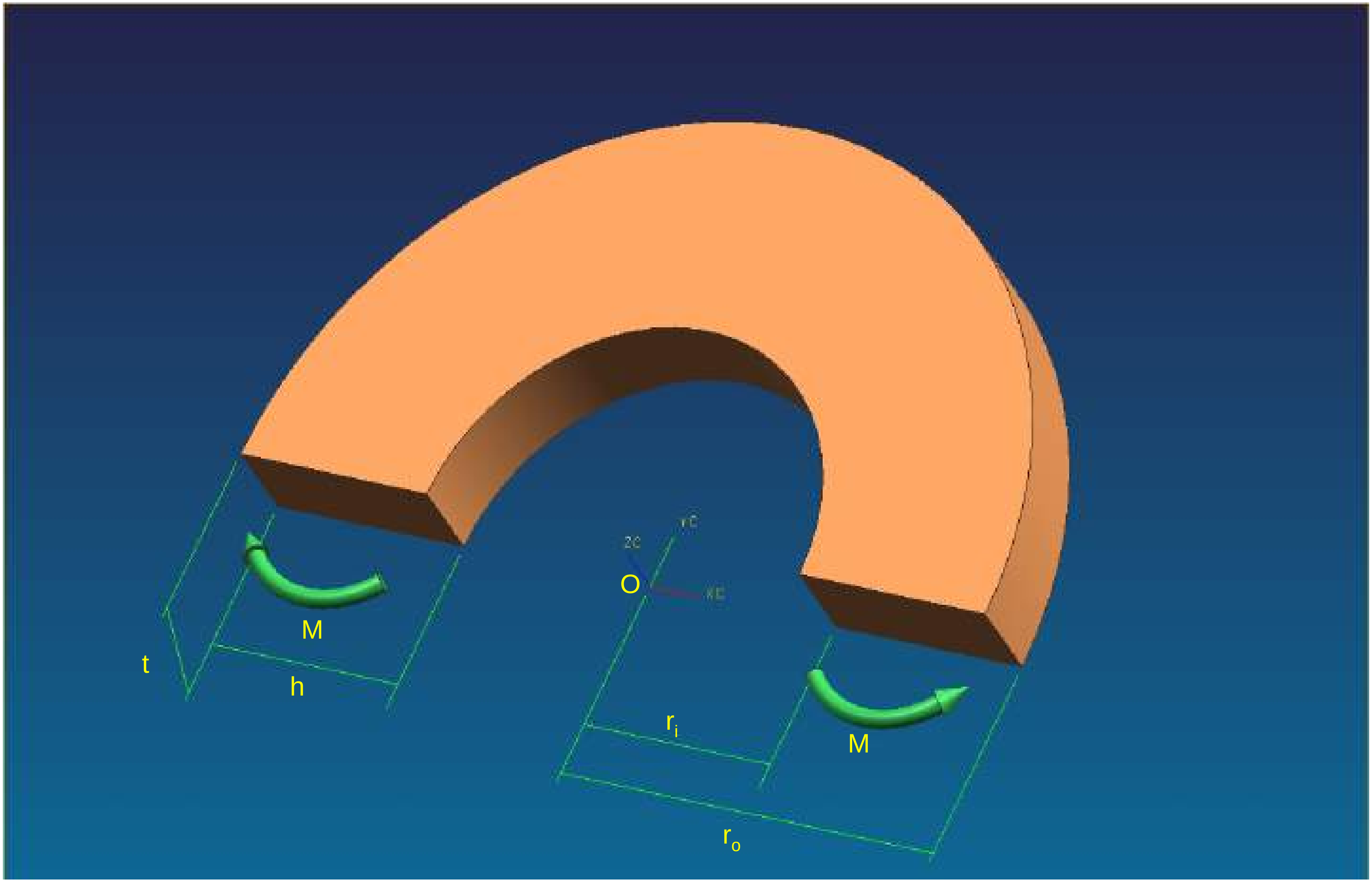


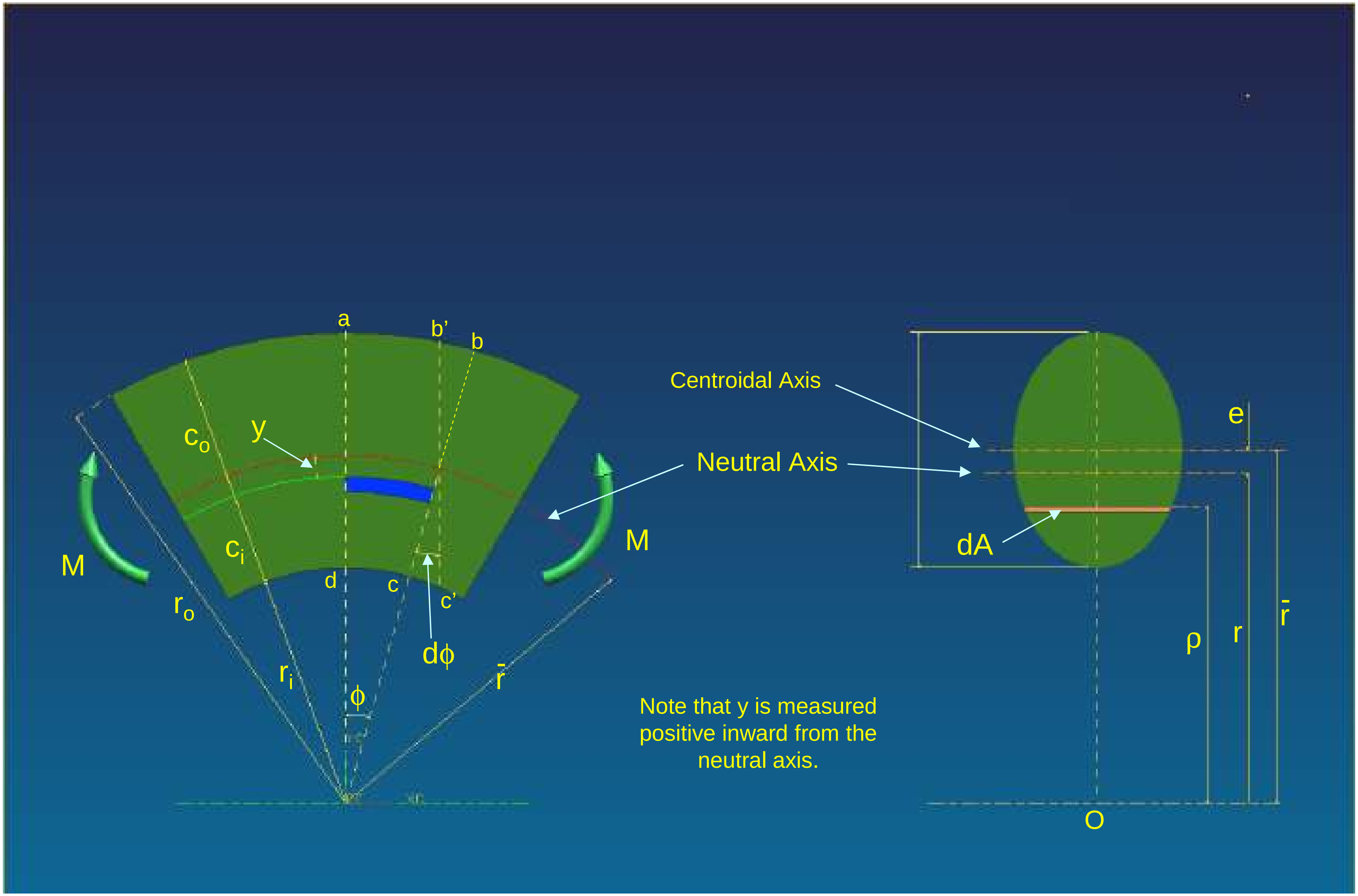
FIG. 11w Bending moment diagram

Curved Beams

Derivation of stress equations



M
t
h
ZC
YC
O
XC
r_i
M
r_o


a
b'
b
Centroidal Axis
Neutral Axis
y
c_o
c_i
M
M
e
dA
d
c
c'
r_o
r_i
dϕ
ϕ
$\bar{r}$
ρ
r
$\bar{r}$
O
Note that y is measured positive inward from the neutral axis.

CURVED MEMBERS IN FLEXURE

The distribution of stress in a curved flexural member is determined by using the following assumptions.

1 The cross section has an axis of symmetry in a plane along the length of the beam.
2 Plane cross sections remain plane after bending.
3 The modulus of elasticity is the same in tension as in compression.

It will be found that the neutral axis and the centroidal axis of a curved beam, unlike a straight beam, are not coincident and also that the stress does not vary linearly from the neutral axis. The notation shown in the above figures is defined as follows:

r_o = radius of outer fiber
r_i = radius of inner fiber
h = depth of section
c_o = distance from neutral axis to outer fiber
c_i = distance from neutral axis to inner fiber
r = radius of neutral axis
$\bar{r}$ = radius of centroidal axis
e = distance from centroidal axis to neutral axis

To begin, we define the element *abcd* by the angle ϕ. A bending moment *M* causes section *bc* to rotate through $d\phi$ to *b'c'*. The strain on any fiber at distance ρ from the center *0* is

$$\varepsilon = \frac{\delta l}{l} = \frac{(r-\rho)\,d\phi}{\rho\phi}$$

The normal stress corresponding to this strain is

$$\sigma = \varepsilon E = \frac{E(r-\rho)\,d\phi}{\rho\phi} \qquad (1)$$

Since there are no axial external forces acting on the beam, the sum of the normal forces acting on the section must be zero. Therefore

$$\int \sigma\, dA = E\frac{d\phi}{\phi}\int \frac{(r-\rho)dA}{\rho} = 0 \qquad (2)$$

Now arrange Eq. (2) in the form

$$E\frac{d\phi}{\phi}\left(r\int \frac{dA}{\rho} - \int dA\right) = 0 \qquad (3)$$

and solve the expression in parentheses. This gives

$$r\int \frac{dA}{\rho} - A = 0 \qquad \text{or} \qquad r = \frac{A}{\int \frac{dA}{\rho}} \qquad (4)$$

This important equation is used to find the location of the neutral axis with respect to the center of curvature *0* of the cross section. The equation indicates that the neutral and the centroidal axes are not coincident.

Our next problem is to determine the stress distribution. We do this by balancing the external applied moment against the internal resisting moment. Thus, from Eq. (2),

$$\int (r-\rho)(\sigma dA) = E\frac{d\phi}{\phi}\int \frac{(r-\rho)^2 dA}{\rho} = M \qquad (5)$$

Since $(r-\rho)^2 = r^2 - 2\rho r + \rho^2$, Eq. (5) can be written in the form

$$M = E\frac{d\phi}{\phi}\left(r^2\int \frac{dA}{\rho} - r\int dA - r\int dA + \int \rho dA \right) \qquad (6)$$

Note that *r* is a constant; then compare the first two terms in parentheses with Eq. (4). These terms vanish, and we have left

$$M = E\frac{d\phi}{\phi}\left(-r\int dA + \int \rho dA\right)$$

The first integral in this expression is the area *A,* and the second is the product *rA.* Therefore

$$M = E\frac{d\phi}{\phi}(\bar{r} - r)A = E\frac{d\phi}{\phi}eA$$

Now, using Eq. (1) once more, and rearranging, we finally obtain $\sigma = \dfrac{My}{Ae(r-y)}$

This equation shows that the stress distribution is hyperbolic. The algebraic *maximum* stresses occur at the inner and outer fibers and are

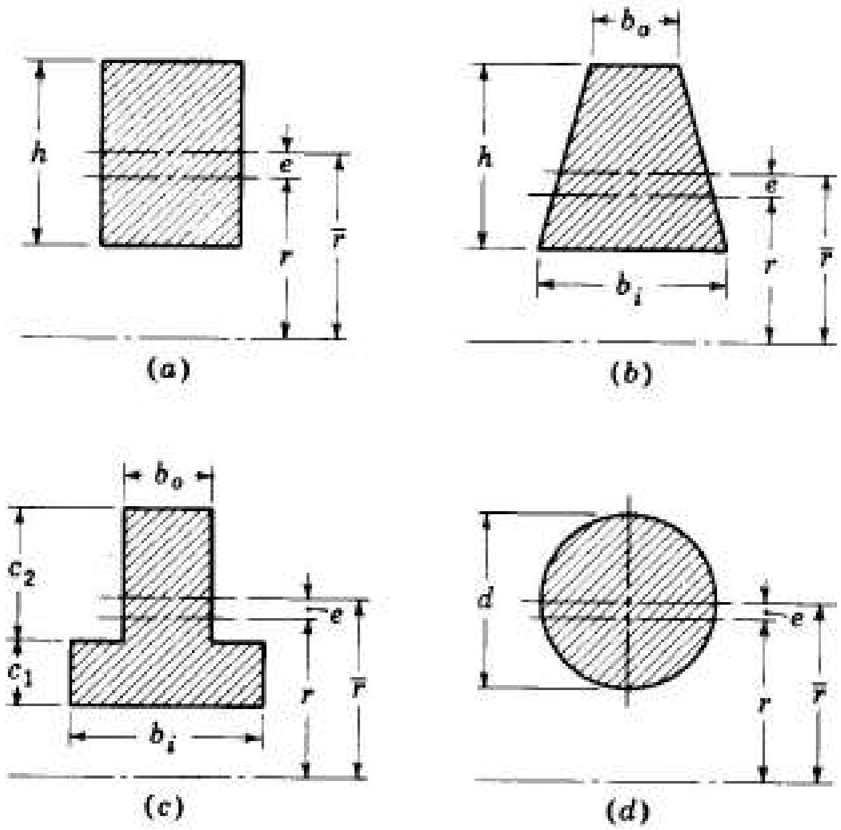
$$\sigma_i = \frac{Mc_i}{Aer_i} \qquad\qquad \sigma_o = \frac{Mc_o}{Aer_o} \qquad\qquad (7)$$

The sign convention used is that M is positive if it acts to straighten on the beam. The distance y is positive inwards to the center of curvature and is measured from the neutral axis. It follows that c_i is positive and c_o is negative.

These equations are valid for pure bending. In the usual and more general case such as a crane hook, the U frame of a press, or the frame of a clamp, the bending moment is due to forces acting to one side of the cross section under consideration. In this case the bending moment is computed about the *centroidal axis, not* the neutral axis. Also, an additional axial tensile (P/A) or compressive ($-P/A$) stress must be added to the bending stress given by Eq. (7) to obtain the resultant stress acting on the section.

Formulas for Some Common Sections

Sections most frequently encountered in the stress analysis of curved beams are shown below.



For the rectangular section shown in (a), the formulae are

$$\bar{r} = r_i + \frac{h}{2} \qquad and \qquad r = \frac{h}{\ln(r_o / r_i)}$$

For the trapezoidal section in (b), the formulae are

$$\bar{r} = r_i + \frac{h}{3}\frac{b_i + 2b_o}{b_i + b_o}$$

$$r = \frac{A}{b_o - b_i + [(b_i r_o - b_o r_i)/h]\ln(r_o / r_i)}$$

For the T section in we have

$$\bar{r} = r_i + \frac{b_i c_1^2 + 2b_o c_1 c_2 + b_o c_2^2}{2(b_o c_2 + b_i c_1)}$$

$$r = \frac{b_i c_1 + b_o c_2}{b_i \ln[(r_i + c_1)/r_i] + b_o \ln[r_o /(r_i + c_1)]}$$

The equations for the solid round section of Fig. *(d)* are

$$\bar{r} = r_i + \frac{d}{2}$$

$$r = \frac{d^2}{4\left(2\bar{r} - \sqrt{4\bar{r}^2 - d^2}\right)}$$

§ 36.21. DOMES

A dome consists of a shell which is generated by the revolution of a geometrical curve about an axis. If a segment of a circle is revolved about the vertical diameter, we get a spherical dome. A conical dome is obtained by revolving a triangle round a central pivot. Domes are provided to roof large circular areas. They are commonly used over temples, gurudwaras and mosques. They are also provided to roof over large circular auditoriums. Though domes can cover large areas, they need relatively very small thickness. In most cases the thickness may be 75 *mm* to 150 *mm*. From this point of view domes are economical. But economy may be offset by the costly curved shattering required. Domes are most efficient structurally since they are subjected to compressive stresses only. Domes provided for water tanks have a rise equal to 1/4 to 1/6 of the span.

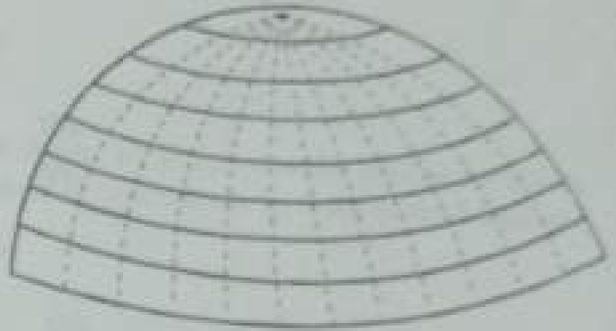


Fig. 36.62. Dome formed by placing a series of concentric rings of decreasing diameters, one above the other.

Stresses in a spherical dome. Two types of stresses are caused in a dome slab. They are:

(*i*) Meridional thrust, (*ii*) Hoop stress.

Meridional thrust. For purposes of analysis let us consider the dome as formed by a series of horizontal rings of decreasing diameters placed one above the other (See Fig. 36.63). Hence if a load is applied on the dome it gets resisted by these horizontal rings. There will thus be a thrust of one ring on the other. This thrust is called the meridional thrust.

Consider one such ring *DEFG*. Let the radius of this ring be r and let the radius of the sphere be R. Let this ring be corresponding to the radius vector at angle θ with the vertical. Consider unit length of this ring. The meridional thrust on this ring can now be determined as follows.

Case 1. *Uniformly distributed load of w per unit area of the dome surface.*

Area of the dome surface above ring *DEFG*

$$= 2\pi Rh = 2\pi R\,[R - R\cos\theta] = 2\pi R^2 (1-\cos\theta)$$

$\therefore$ Load above the ring *DEFG* $= 2\pi R^2 (1-\cos\theta)\,w$

This load is now transferred as a thrust on the circumference of the ring *DEFG*. Let the thrust per unit run of this ring be T.

$\therefore$ Total vertical component of the thrust on the ring

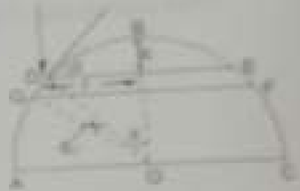
$$= T\sin\theta \times 2\pi r = 2\pi R^2 (1-\cos\theta)\,w$$

But $\quad r = R\sin\theta$

$$\therefore \quad T\sin\theta \cdot 2\pi R\sin\theta = 2\pi R^2 (1-\cos\theta)\,w$$

$$\therefore \quad T = \frac{wR\,(1-\cos\theta)}{\sin^2\theta} = \frac{wR\,(1-\cos\theta)}{1-\cos^2\theta}$$

$$\therefore \quad T = \frac{wR}{1+\cos\theta}$$



Hoop Stress. Let T be the thrust per unit run on the ring *DEFG*. The horizontal component of this thrust will produce a hoop tension of

$$H = T\cos\theta \cdot R\sin\theta$$

$$\therefore \quad H = \frac{wR^2}{1+\cos\theta}\sin\theta\cos\theta$$

Similarly the thrust on this ring just on its lower face will produce a hoop compression of $H + dH$

Hence the net hoop compression on the ring

$$= dH$$

Area of the section of the ring $\quad = R\,d\theta\,t$

$$\therefore \text{ Hoop stress} \quad = f = \frac{dH}{R\,d\theta\,t}$$

$$\therefore \quad f = \frac{1}{Rt}\cdot\frac{dH}{d\theta}$$

Fig. 36.63.

But from $\quad H = wR^2\,\frac{\sin\theta\cos\theta}{1+\cos\theta}$

$$\therefore \quad \frac{dH}{d\theta} = wR^2\,\frac{(1+\cos\theta)(-\sin^2\theta+\cos^2\theta)+\sin^2\theta\cos\theta}{(1+\cos\theta)^2}$$

$$= wR^2\cdot\frac{(1+\cos\theta)(\cos^2\theta-\sin^2\theta)+\cos\theta(1-\sin^2\theta)}{(1+\cos\theta)^2}$$

$$\frac{dH}{d\theta} = wR^2\,\frac{\cos\theta-\sin^2\theta}{1+\cos\theta} = wR^2\,\frac{\cos\theta-(1-\cos^2\theta)}{1+\cos\theta}$$

$$= wR^2\,\frac{\cos^2\theta+\cos\theta-1}{1+\cos\theta}$$

$$\frac{dH}{d\theta} = wR^2\left[\cos\theta - \frac{1}{1+\cos\theta}\right]$$

$$\therefore \text{ Hoop stress} \quad = f = \frac{1}{Rt}\cdot\frac{dH}{d\theta}$$

$$\therefore \quad f = \frac{wR}{t}\left[\cos\theta - \frac{1}{1+\cos\theta}\right]$$

If the above expression simplifies to a positive quantity the hoop stress is compressive. If the above expression simplifies to a negative quantity the hoop stress is tensile.

It is possible for a certain value of θ the hoop stress is zero. This is obtained from the relation

$$f = \frac{wR}{t}\left[\cos\theta - \frac{1}{1+\cos\theta}\right] = 0$$

$$\cos\theta = \frac{1}{1+\cos\theta}$$

$\cos^2\theta + \cos\theta - 1 = 0$

Solving as a quadratic in cos θ, we have

$$\cos\theta = \frac{-1+\sqrt{1+4}}{2} = \frac{-1+\sqrt{5}}{2} = 0.618$$

$$\theta = 51^\circ 51'$$

Case 2. *Concentrated load on the crown of the dome.*

If now there is a concentrated load W on the dome, considering the ring $CDEF$ the load transferred per unit run of the circumference on the ring

$$= \frac{W}{2\pi R\sin\theta}$$

If the thrust per unit run on this ring be T, we have, resolving vertically,

$$T\sin\theta = \frac{W}{2\pi R\sin\theta}$$

$$T = \frac{W}{2\pi R\sin^2\theta}$$

The horizontal component of this thrust will produce a hoop tension of

$$H = T\cos\theta\, R\sin\theta = \frac{W}{2\pi}\cot\theta$$

Similarly the thrust on the lower face of the ring $CDEF$ will produce a hoop compression $H + dH$.

Net hoop load $= dH$

Hoop stress $= \dfrac{dH}{R\,d\theta\, t} = \dfrac{1}{Rt}\dfrac{dH}{d\theta}$

But $H = \dfrac{W}{2\pi}\cot\theta$

$$\frac{dH}{d\theta} = -\frac{W}{2\pi}\operatorname{cosec}^2\theta = -\frac{W}{2\pi\sin^2\theta}$$

Hoop stress $= -\dfrac{W}{2\pi Rt\sin^2\theta}$

The negative sign indicates that the hoop stress induced is a tensile stress. We find from the above expressions for thrust and hoop stress, at θ = 0 that is at the crown of the dome.

$$T = \frac{W}{2\pi R\sin^2(0)} = \infty$$

and

$$f = -\frac{W}{2\pi Rt\sin^2\theta} = \infty$$

Hence strictly a truly point load should not be applied on the crown of the dome. In practice it the load at the crown should be distributed over a sufficient area so as not to cause excessive stresses at the crown. In order to reach this aim, sometimes an opening is provided at the crown.

Summary of Formulae

Concentrated load on the dome $= W$

Distributed load on the dome $= w$ per unit area

Meridional thrust $= T = \dfrac{wR}{1+\cos\theta} + \dfrac{W}{2\pi R\sin^2\theta}$

Hoop stress $= f = \dfrac{wR}{t}\left[\cos\theta - \dfrac{1}{1+\cos\theta}\right] - \dfrac{W}{2\pi Rt\sin^2\theta}$

Design of a dome. From load considerations the thickness requirement for a dome is very small. The thickness practically adopted is never less than 100 *mm*. The domes provided for water tanks are usually spherical with a rise equal to 1/4 to 1/6 of the span. Often the stresses in domes are compressive and in dome constructions the compressive stresses adopted are 1/5 to 2/5 of the direct stresses. If the direct compressive stress be taken at 4 N/mm^2 the compressive stress in concrete in dome construction may be taken at 0.8 N/mm^2 to 1.6 N/mm^2. The safe compressive stress in steel shall be taken at 150 N/mm^2.

Ring beam for a dome. Very often the domes constructed are segmental and not hemispherical. Hence the meridional thrust at the base of the dome will be at an inclination with the horizontal. The horizontal component of the meridional thrust at the base will induce an outward push of the cylindrical wall carrying the dome. In order to prevent this, a ring beam is provided at the base of the dome. The horizontal component of the meridionalthrust will be resisted by the ring beam by hoop action. Often the ring beam at the base is made by thickening the edges and providing adequate hoop steel to resist the hoop tension. Sometimes no thickening of the edges may become necessary and the requisite amount of hoop steel will be sufficient. Often a ring beam section may be based on an architectural consideration. A minimum of 0.3% of gross area shall be provided as the reinforcement in each principal direction for the dome section.

Design 36.26. *A spherical dome of a water tank of span 6 metres has a rise of 1.20 metres. It carries an all-inclusive distributed load of 600 N/metre² and a lantern load of 800 N at the crown. Design the dome. Use M 20 concrete and Fe 415 steel.*

Solution. Let the radius of the dome be *R metres*

$$1.2\,(2R-1.2)=3^2 \quad \therefore \quad R = 4.35 \text{ metres}$$

$$\sin\theta = \frac{3}{4.35}$$

$$\theta = 43°\ 36'$$

$$\sin\theta = 0.6896 \text{ and } \cos\theta = 0.7242$$

Meridional thrust per metre run at springing level

$$= T = \frac{wR}{1+\cos\theta} + \frac{W}{2\pi R\sin^2\theta}$$

$$= \frac{6000\times 4.35}{1.7242} + \frac{8000}{2\pi\times 4.35\,(0.6896)^2} = 15753\ N$$

Let the shell be 100 *mm* thick.

Compressive stress due to meridional thrust

$$= \frac{1573}{1000\times 100} = 0.16\ N/mm^2$$

Hoop stress

$$= f = \frac{wR}{t}\left[\cos\theta - \frac{1}{1+\cos\theta}\right] - \frac{W}{2\pi R t\sin^2\theta}$$

$$= \frac{6000 \times 4.34}{0.15}\left[0.7242 - \frac{1}{1.7242}\right] - \frac{8500}{2\pi \times 4.35 \times 0.15 \times (0.8896)^2}$$

$$= 25084 - 4103 = 20981\ N/metre^2 = 0.0209\ N/mm^2$$

Similarly these stresses may be calculated at other levels also.

These stresses are very low and hence a minimum of 0.3% steel will be provided, in each principal direction.

Minimum steel requirement $= \frac{0.3}{100} \times 1000 \times 100 = 300\ mm^2$

Spacing of 10 mm ϕ bars $= \frac{78 \times 1000}{300} = 260\ mm$ say $250\ mm$

Ring Beam

Horizontal component of meridional thrust

$$= T \cos\theta = 15753 \times 0.7242 = 11408\ N$$

Hoop tension in the ring beam $= T\cos\theta \cdot r = 11408 \times 3 = 34224\ N$

Safe stress in steel $= 150\ N/mm^2$

$$A_{st} = \frac{34224}{150} = 229\ mm^2$$

Provide 4 bars of 12 mm ϕ

Size of the ring beam. The tensile stress on the equivalent concrete area shall not exceed $1.20\ N/mm^2$.

Let A be the minimum area of the ring beam section.

∴ Equivalent concrete area

$$= A + (13.33 - 1)\,452$$
$$= A + 5573\ mm^2$$

Limiting the tensile stress in concrete to $1.20\ N/mm^2$, we have,

$$\frac{34224}{A + 5573} = 1.20,\ A = 22947\ mm^2$$

Let us provide a section of $200\ mm \times 150\ mm$

Fig. 36.64

The ring beam reinforcement should be bound by ties of 10 mm dia. bars at

P 87480

Design. 29.2 A cantilever type retaining wall has a 5.5 m tall stem. It retains earth level with its top. The soil weighs 18500 N/m³ and has an angle of repose of 30°. The safe bearing capacity of the soil is 200 kN/m². Design the wall. Use M 20 concrete and Fe 250 steel.

Solution.

Wall proportions

Thickness of the stem at the top = 200 *mm*

Thickness of the stem at the bottom

Consider one *metre* run of the wall

Maximum bending moment per *metre* run of the wall = M

$$= C_p \frac{wh^3}{6} = \frac{1}{3} \times 18500 \times \frac{5.5^3}{6} = 170996.53\ Nm$$

Ultimate moment $M_u = 1.5 \times 170996.53 = 256494.79\ Nm$

$$0.149 f_{ck}\, bd^2 = 0.149 \times 20 \times 1000\, d^2 = 256494.79 \times 10^3$$

$$d = 293.4\ mm$$

Effective cover to stem reinforcement = 40 *mm*

Overall thickness of the stem = 293.4 + 40 = 333.4 *mm*

The thickness may be increased by 30% to 35% for an economical design.

Provide a thickness of 450 mm at the bottom of the stem.

The base slab also will be made 450 *mm* thick.

Total height of the wall = H = 5.5 + 0.45 = 5.95 *m*

Width of the base slab $b = 0.5H$ to $0.6H$

$$0.5\,H = 0.5 \times 5.95 = 2.975\ m$$

$$0.6\,H = 0.6 \times 5.95 = 3.570\ m$$

Provide a base width of 3.50 m

Toe projection = About one – third the base width

$$= \frac{3.50}{3} = 1.17\ m$$

Provide a toe projection of 1 m

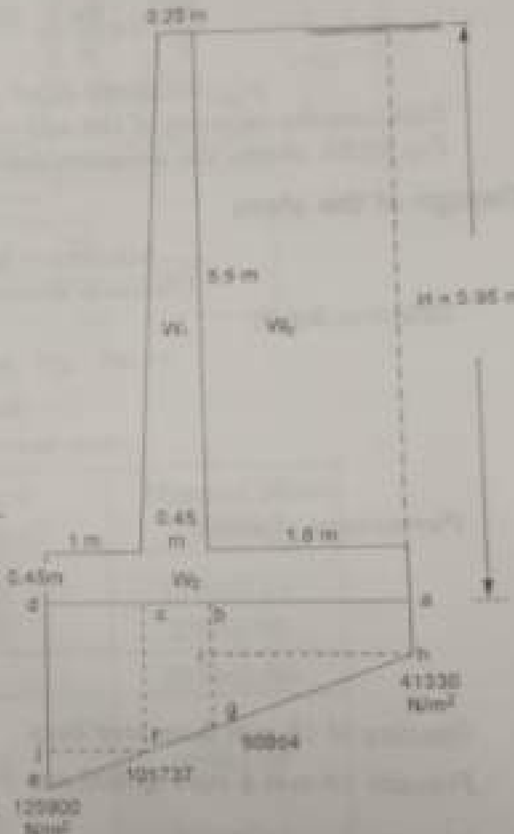


Fig. 29.26

Stability Calculations

See table below for Stability Calculations

Load due to	Magnitude of the load (N)	Distance from a (m)	Moment about a (Nm)
W_1 0.2 × 5.5 × 25000	27500	2.15	59125
$\frac{1}{2} \times 0.25 \times 5.5 \times 25000$	17187.5	$\frac{7}{3}$	40104.17
W_2 3.5 × 0.45 × 25000	39375	1.75	68906.25
W_3 2.05 × 5.5 × 18500	208587.5	1.025	213802.19
Moment of lateral pressure $C_p \frac{wH^3}{6} = \frac{1}{3} \times 18500 \times \frac{5.95^3}{6}$			216496.12
Total	292650		598433.73

Distance of the point of application of the resultant force from the heel end a

$$= z = \frac{598433.73}{292650} = 2.045 \ m$$

Eccentricity

$$e = z - \frac{b}{2} = 2.045 - 1.750 = 0.295$$

$$\frac{b}{6} = \frac{3.5}{6} = 0.583$$

$$\therefore \quad e < \frac{b}{6}$$

Extreme pressure intensity at the base

$$\frac{W}{b}\left[1 \pm \frac{6e}{b}\right] = \frac{292650}{3.5}\left[1 \pm \frac{6 \times 0.295}{3.5}\right] N/m^2$$

$$p_{max} = 125900 \ N/m^2 \text{ and } p_{min} = 41330 \ N/m^2$$

Safe bearing capacity of the soil = 200 kN/m^2 = 200000 N/m^2

Fig.29.28. shows the pressure distribution at the base.

Design of the stem

$$\text{Maximum B.M} = M = 170996.53 \ Nm$$

$$\text{Ultimate moment} = M_u = 1.5 \times 170996.53 = 256494.79 \ Nm$$

Effective depth

$$d = 450 - 40 = 410 \ mm$$

$$\frac{M_u}{bd^2} = \frac{256494.79 \times 10^3}{1000 \times 410^2} = 1.526$$

Percentage of steel

$$p_t = 50\left[\frac{1 - \sqrt{1 - \frac{4.6 \times 1.526}{20}}}{\frac{250}{20}}\right] = 0.778\%$$

$$\therefore \quad A_{st} = \frac{0.778}{100} \times (1000 \times 310) = 2412 \ mm^2$$

Spacing of 18 *mm* diameter bars $= \frac{254 \times 1000}{2412} = 105 \ mm$

Provide 18 mm ϕ bars @ 100 mm c/c

Distribution steel $= \frac{0.15}{100} \times 1000 \times 450 = 675 \ mm^2$

Spacing of 8 *mm* diameter bars $= \frac{50 \times 1000}{675} = 74 \ mm$ say 70 *mm*

Provide 8 mm ϕ bars @ 140 mm c/c near each face

Design of the toe slab

The bending moment calculations for a 1 *metre* wide strip of the toe slab are shown in the table below.

B.M. Calculations for a 1 metre wide strip of the toe slab

Load due to	*Magnitude of the load (N)*	*Distance from c (m)*	*Moment about c (Nm)*
Upward pressure *cdjf* 101737 × 1101737 *jfe* $\frac{1}{2} \times 1 \times 24163$	 12081.5	0.5 $\frac{2}{3}$	50868.50 8054.33
			58922.83
Deduct for self weight of the toe slab 1 × 0.45 × 25000	11250	0.5	5625
B.M. for toe slab			53297.83

B.M. for the toe slab $= M = 53297.83\ Nm$

Ultimate moment $M_u = 1.5 \times 53297.83 = 79946.745\ Nm$

Effective depth $= d = 450 - 60 = 390\ mm$

[Effective cover to reinforcement for base slab = 60 mm]

$$\frac{M_u}{bd^2} = \frac{79946.745 \times 10^3}{1000 \times 390^2} = 0.526$$

Percentage of steel

$$p_t = 50\left[\frac{1-\sqrt{1-\frac{4.6 \times 0.526}{20}}}{\frac{250}{20}}\right] = 0.25\%$$

$$A_{st} = \frac{0.25}{100}(1000 \times 390) = 975\ mm^2$$

Spacing of 12 mm diameter bars $= \frac{113 \times 1000}{975} = 115\ mm$

Provide 12 mm φ bars @ 110 mm c/c

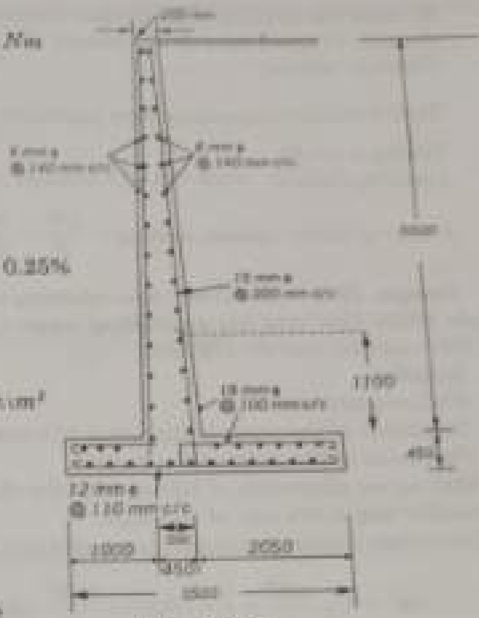


Fig. 29.27.

Design of the heel slab

The bending moment calculations for a 1 *metre* wide strip of the heel slab are shown in the table below.

B.M. Calculations for a 1 metre wide strip of the heel slab

Load due to	*Magnitude of the load (N)*	*Distance from b (m)*	*Moment about b (Nm)*
Backing 2.05 × 5.5 × 18500 DL of heel slab 2.05 × 0.45 × 25000	208587.5 23062.5	1.025 1.025	213802.19 23639.06
			237441.25
Deduct for upward pressure abih 41330 × 2.05 igh $\frac{1}{2} \times 2.05 \times 49534$	 84726.5 50772.35	 1.025 $\frac{2.05}{3}$	 86844.66 34694.44
Total duduction			121539.10
BM for Heel slab			115902.15

B.M. for the heel slab $= 115902.15\ Nm$

Ultimate moment $M_u = 1.5 \times 115902.15 = 173853.23\ Nm$

$$\frac{M_u}{bd^2} = \frac{173853.23 \times 10^3}{1000 \times 390^2} = 1.143$$

Percentage of steel

$$p_t = 50\left[\frac{1-\sqrt{1-\frac{4.6 \times 1.143}{20}}}{\frac{250}{20}}\right] = 0.566\%$$

$$A_{st} = \frac{0.566}{100}(1000 \times 390) = 2207\ mm^2$$

Spacing of 18 mm diameter bars $= \frac{254 \times 1000}{2207} = 115\ mm$

In order the spacing of these bars may match with the spacing of stem reinforcement, we will therefore

Provide 18 mm ϕ bars @ 100 mm c/c

Check for sliding

Total horizontal pressure force per *metre* run of the wall $= P = C_p \dfrac{wH^2}{2} = \dfrac{1}{3} \times 18500 \times \dfrac{5.95^2}{2} = 109157.71$

Taking $\mu = 0.65$,

Limiting friction $= \mu W = 0.65 \times 292650 = 190222.5\ N$

Factor of safety against sliding $= \dfrac{\mu W}{P} = \dfrac{190222.5}{109157.71} = 1.74$

20.2 A cantilever type retaining wall has a total height of 4.25 m and retains soil level with its top. The so

Lateral pressure intensity at the bottom of the upright slab

$= p = C_p wh = \frac{1}{3} \times 16500 \times 6.55 = 36025\ N/m^2$

Consider the bottom 1 m deep strip of the upright slab.

Maximum bending moment for the strip $= M = \frac{pL^2}{12} = \frac{36025 \times 3^2}{12} = 27018.75\ Nm$

Factored moment $= M_u = 1.5 \times 27018.75 = 40528.125\ Nm$

Equating $M_{u,lim}$ to M_u

$0.149 f_{ck} bd^2 = 0.149 \times 20 \times 1000\ d^2 = 40528.125 \times 1000$

$d = 117\ mm$

Provide a thickness of 200 mm

Actual effective depth $= d = 200 - 40 = 160\ mm$

Fig. 29.54 shows the proposed section of the wall.

Stability Calculations per metre run of the wall. See table below.

Load due to	*Magnitude of the load (n)*	*Distance from a (m)*	*Moment about a n/m*
W_1 $0.20 \times 6.55 \times 25000$	32750	3.4	111350
W_2 $4.5 \times 0.45 \times 25000$	50625	2.25	113906.25
W_3 $3.3 \times 6.55 \times 16500$	356647.5	1.65	588468.38
Moment of lateral pressure $C_p w \frac{H^3}{6} = \frac{1}{3} \times 16500 \times \frac{7^3}{6}$			314416.67
	440022.5		1128141.3

Distance of the point of application of the resultant force from the heel end a

$= z = \frac{1128141.3}{440022.5} = 2.56\ m$

Eccentricity $e = z - \frac{b}{2} = 2.56 - 2.25 = 0.31\ m$

$\frac{b}{6} = \frac{4.50}{6} = 0.75\ m, e < \frac{b}{6}$

Extreme pressure intensity at the base $= p = \frac{W}{b}\left(1 \pm \frac{6e}{b}\right) = \frac{440022.5}{4.50}\left(1 \pm \frac{6 \times 0.31}{4.50}\right)$

$p_{max} = 138200 N/m^2$ and $p_{min} = 57366\ N/m^2$

Design of the upright slab

Maximum B.M. for a 1 m deep strip of the slab $= M_u = 40528.125\ Nm$

$\frac{M_u}{bd^2} = \frac{40528.125 \times 1000}{1000 \times 160^2} = 1.583$

Percentage of steel $P_t = 50\left[\frac{1 - \sqrt{1 - \frac{4.6 \times 1.583}{20}}}{\frac{250}{20}}\right] = 0.81\%$

$A_{st} = \frac{0.81}{100}(1000 \times 160) = 1296\ mm^2$

Spacing of 16 mm φ bars $= \frac{201 \times 1000}{1296} = 155\ mm$

Provide 16 mm φ bars @ 150 mm c/c

Distribution steel $= \frac{0.15}{100}(1000 \times 200) = 300\ mm^2$

Spacing of 8 mm φ bars $= \frac{50 \times 1000}{300} = 166\ mm$

Provide 8 mm φ bars @ 200 mm c/c near each face

Design of the toe slab

B.M. Calculations for a 1 m wide strip of the toe slab are shown in the table below. (see also Fig. 29.54)

Load due to	Magnitude of of the load (N)	Distance from c (m)	Moment about c (Nm)
Upward pressure cdef 116644 × 1 cfg $\frac{1}{2}$ × 1 × 21556	116644 10778	0.5 $\frac{2}{3}$	58322 7185
			65507
Deduct for weight of toe slab 1 × 0.45 × 25000	11250	0·5	5625
B.M. for Toe slab			59882

Maximum B.M. for the toe slab = 59882 *Nm*

Factored moment $M_u = 1.50 \times 59882 = 89823\ Nm$

Effective cover to steel = 60 *mm*

Effective depth $d = 450 - 60 = 390$ mm

$$\frac{M_u}{bd^2} = \frac{89823 \times 1000}{1000 \times 390^2} = 0.591$$

percentage of steel $P_t = 50\left[\dfrac{1 - \sqrt{1 - \dfrac{4\cdot6 \times 0\cdot591}{20}}}{\dfrac{250}{20}}\right] = 0\cdot282\%$

$\therefore A_{st} = \frac{0\cdot282}{100}(1000 \times 390) = 1100\ mm^2$

Spacing of 16 *mm* φ bars $= \frac{201 \times 1000}{1100} = 182\ mm$

Provide 16 mm φ bars @ 180 mm c/c

Distribution steel $= \frac{0\cdot15}{100}(1000 \times 450) = 675\ mm^2$

Spacing of 8 *mm* φ bars $= \frac{50 \times 1000}{675} = 74\ mm$

Provide 8 mm φ bars @ 140 mm c/c near each face.

Design of the heel slab

The heel slab is designed as a continuous slab spanning horizontaly between the counterforts. Consider a 1 m wide strip of the heel slab near the heel end. The loading on this strip consists of the following.

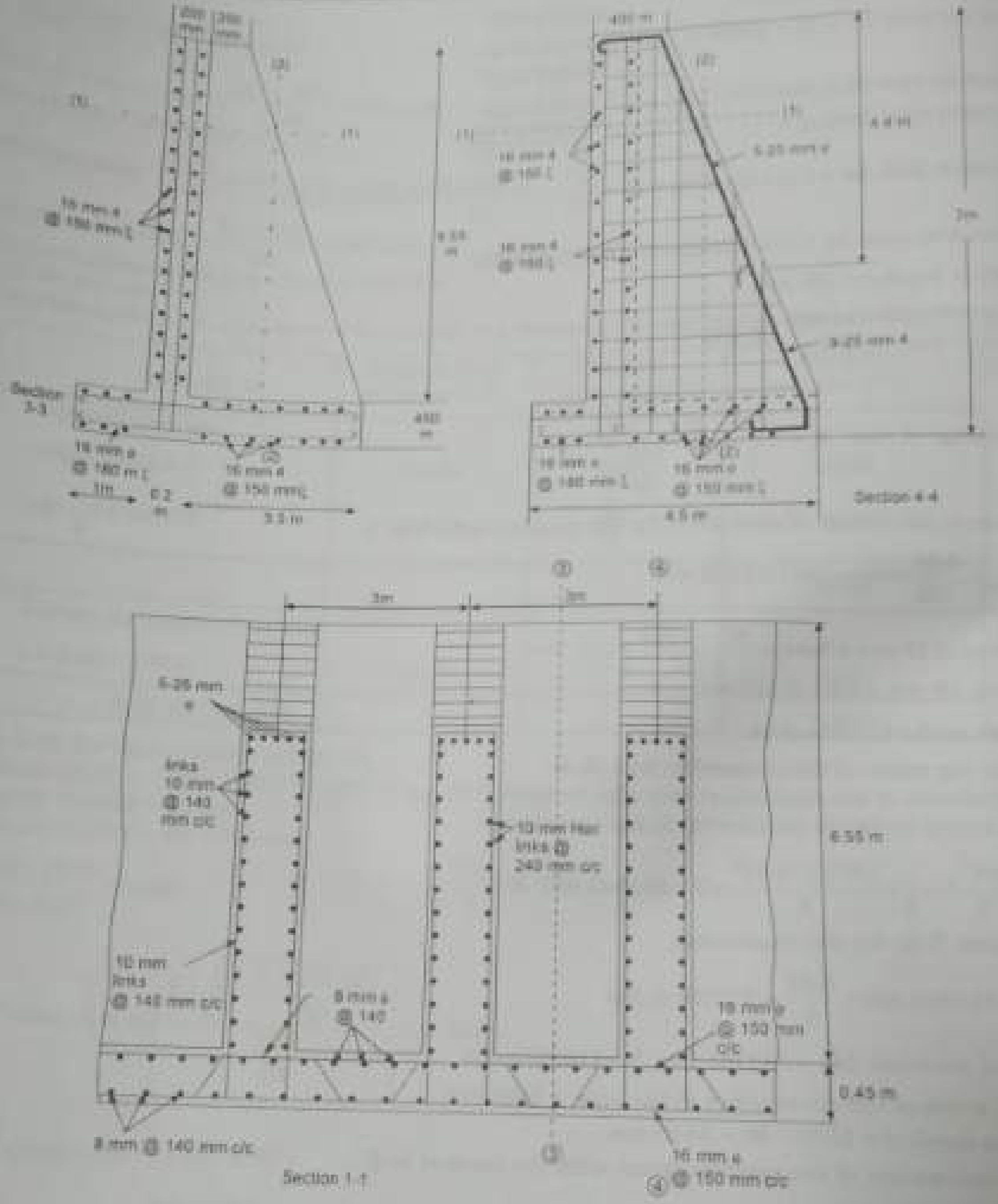


Fig. 29.55

$$A_{st} = \frac{0.34}{100}(400 \times 3440 \times 0.9038) = 4228.34\ mm^2$$

Provide 9 bars of 25 mm φ (4418 mm²)

Let 4 bars be curtailed at a depth y from the top.

$$\frac{A'_{st}}{A_{st}} = \frac{4}{9} = \frac{y^2}{6.55^2} \therefore y = 4.4\ m \text{ from the top.}$$

Connection between the counterfort and the upright slab.

Consider the botton 1*m* deep strip of the upright slab.

Pressure intensity on the strip = 36025 N/m^2

Tension transmitted to the counterfort by the strip = 36025 (3 − 0.40) = 93665 N

Factored tension = 1.5 × 93665 = 140497.5 N

$$A_{st} = \frac{\text{Factored tension}}{0.87 f_y} = \frac{140497.5}{0.87 \times 250} = 646\ mm^2$$

$$\text{Spacing of } 10\ mm\ \phi \text{ bars} = \frac{79 \times 1000}{646} = 122\ mm$$

Provide 2 - legged 10 mm ϕ horizontal links @ 240 mm c/c

Connection between the counterfort and the heel slab

Consider a 1 m wide strip of the heel slab near the heel end.

Net downward loading on the strip = 61959 N/m^2

Tension transmitted to the counterfort by this strip = 61959 (3 − 0.40) = 161093.4 N

Factored tension = 1.5 × 161093.4 = 241640.1 N

$$A_{st} = \frac{241640.1}{0.87 \times 250} = 1110.99\ mm^2$$

$$\text{Spacing of } 10\ mm\ \phi \text{ bars} = \frac{79 \times 1000}{1110.99} = 71\ mm$$

Provide 2 legged 10 mm ϕ vertical links @ 140 mm c/c

...taining wall to the following particulars.

Design 36.4. A circular water tank has an internal diameter of 12 metres, the maximum depth of water being 4 metres. The walls of the tank are restrained at the base. The tank rests on ground. Design the tank. Use M 20 concrete and grade I mild steel.

Solution.

$$H = 4 \text{ metres}, D = 12 \text{ metres}$$

Thickness of wall. This shall be not less than

(i) 150 *mm*

(ii) 30 *mm* per *metre* depth + 50 *mm* = 30 × 4 + 50 = 170 *mm*

Hence let us provide a thickness of 170 *mm*

$$K = \frac{12H^4}{\left(\frac{D}{2}\right)^2 t^2} = \frac{12 \times 4^4}{6^2 \times (0.17)^2} = 2953$$

From table 1, we find

For

$$K = 1000 \qquad M = 0.024\, pH^3$$

$$K = 10{,}000 \qquad M = 0.0085\, pH^3$$

(Adopting logarithmic interpolation),

For

$$K = 2953, \qquad M = \alpha\, pH^3$$

where,

$$\alpha = 0.0085 + (0.0024 - 0.0085)\frac{\log 10{,}000 - \log 2953}{\log 10{,}000 - \log 1000}$$

$$= 0.0085 + (0.0024 - 0.0085) \times 0.5298 = 0.0167$$

$$M = 0.0167\, pH^3$$

$$M = 0.0167\,(9810 \times 4)\, 4^2\ Nm = 10485\ Nm$$

Adopting

$$c = 7\ N/mm^2,\ t = 115\ N/mm^2 \text{ and } m = 13.33$$
$$n = 0.448d,\ a = 0.85d,\ Q = 1.333$$

Equating the moment of resistance to the bending moment, we have,

$$1.333 \times 1000\ d^2 = 10485 \times 1000, \quad d = 89\ mm$$

But, effective depth available to the centre of 12 *mm* ϕ bars

$$= 170 - 25 - 6 = 139\ mm$$

$$A_{st} = \frac{10485 \times 1000}{115 \times 0.85 \times 139} = 772\ mm^2$$

$$\text{Spacing of 12 } mm\ \phi \text{ bars} = \frac{113 \times 1000}{772} = 146\ mm$$

Provide 12 mm ϕ bars @ 140 mm c/c.

Hoop tension. From table 2, we have,

For

$$K = 1000, \text{ max. hoop tension}$$
$$T = 0.47\,p\left(\frac{D}{2}\right) \text{ at } 0.47\,H \text{ from the base.}$$

Following a logarithmic interpolation.

Max. hoop tension for

$$K = 2953 \text{ will be equal to } \beta p\left(\frac{D}{2}\right)$$

where,

$$\beta = 0.47 + (0.67 - 0.47)\frac{\log 2953 - \log 1000}{\log 10000 - \log 1000}$$
$$= 0.47 + 0.20 \times 0.4702 = 0.47 + 0.094 = 0.564$$

$$\text{Max. hoop tension} = 0.564\,p\left(\frac{D}{2}\right) = 0.564\,(9810 \times 4)\frac{12}{2}\,N = 132788\,N$$

This occurs at γH from the base

$$\gamma = 0.47 - (0.47 - 0.31) \times 0.4702 = 0.40$$

Maximum hoop tension will occur at 0.4 *H* from the base, *i.e.*, at a height of 0.4 × 4 = 1.6 *metres* from the base

$$\text{Steel for hoop tension} = \frac{132788}{115} = 1155\ mm^2$$

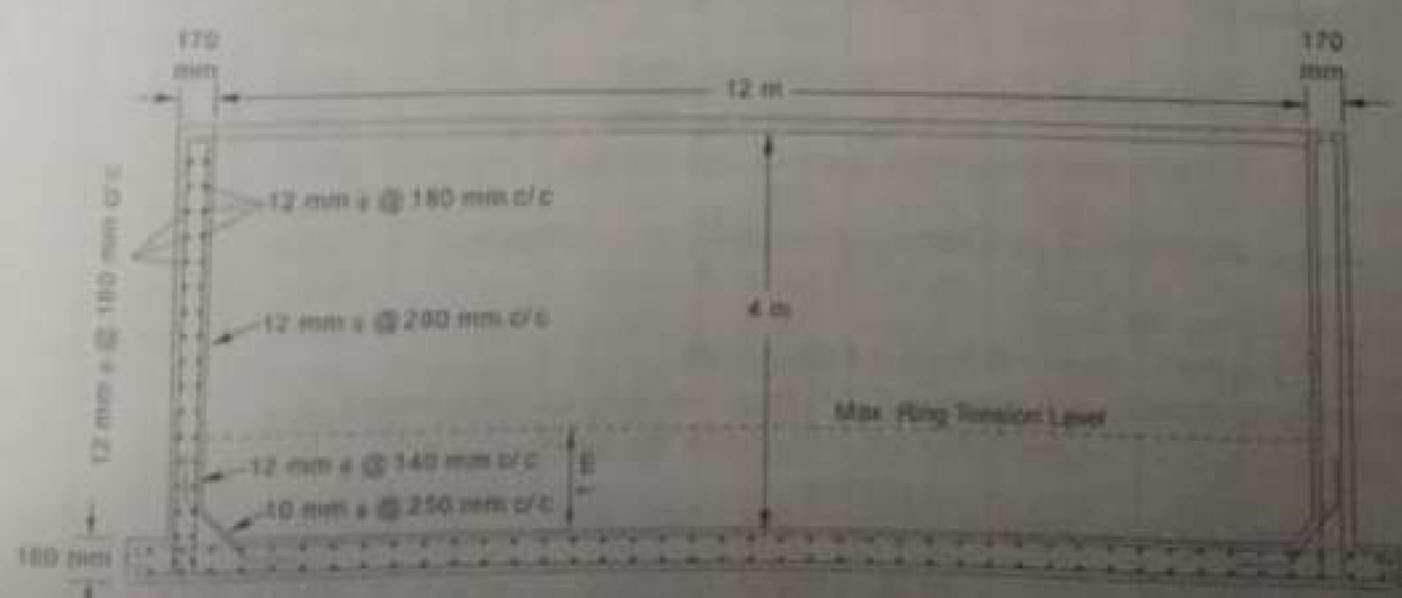
$$\text{Spacing of 12 } mm\ \phi \text{ bars} = \frac{113 \times 1000}{1155} = 97\ mm \text{ say } 90\ mm\ c/c.$$

Since this steel is provided in the form of rings near both the faces the spacing of the rings will be **@ 180 mm c/c** *near each face. Fig. 36.9 shows the details of reinforcement.*

Check for tensile stress in concrete

$$\text{Tensile stress} = \frac{132788}{1000 \times 170 + (13.33 - 1)1155} = 0.72\ N/mm^2 \text{ (less than } 1.20\ N/mm^2\text{)}$$

Base slab. This slab may be 150 *mm* thick with a top mesh and a bottom mesh of steel with 10 *mm* ϕ bars @ 250 *mm c/c*. It is also usual to provide 150 *mm* × 150 *mm* haunches at the junction of wall and base. A haunch reinforcement of 10 *mm* ϕ @ 250 *mm c/c* may provided.



200 mm

Design 36.13. An open square tank 5 m × 5 m × 3 m deep rests on firm ground. Design the tank. Use M 20 concrete and mild steel reinforcement.

Solution. **Thickness of wall.** This will be not less than,

(*i*) 150 *mm*

(*ii*) 30 *mm* per *metre* depth + 50 *mm* = (30 × 3) + 50 = 140 *mm*

(*iii*) 60 *mm* per *metre* length of side = 60 × 5 = 300 *mm*

Provide a thickness of 300 *mm*

Effective span of the slab = 5 + 0.30 = 5.30 *metre*

Consider a level 1 *metre* above the floor.

Water pressure intensity at this level

$$= 9810 \times 2 = 19620\ N/metre^2$$

Bending moment per *metre* height at this level at corners

$$= \frac{19620 \times 5.3^2}{12}\ Nm = 45927\ Nm$$

$$\text{Bending moment at mid span} = \frac{19621 \times 5.3^2}{16}\ Nm = 34445\ Nm$$

Pull in the wall per *metre* height at this level

$$= T = \frac{19620 \times 5}{2}\ N = 49050\ N$$

Design of the corner section. Providing a cover of 25 *mm* and using 18 *mm* ϕ bars,

Effective depth = 300 − 34 = 266 *mm*

Effective bending moment at corners

$$= M - Tx = 45927 \times 1000 - 49050\,(266 - 150) = 40237200\ Nmm$$

Since this bending moment produces tension near the water face the stresses and the design coefficients to be adopted are,

$t = 115\ N/mm^2$, $c = 7\ N/mm^2$, $m = 13.33$, $Q = 1.333$, $a = 0.85d$

$$\therefore \quad \text{Steel for bending moment} = A_{st1} = \frac{40237200}{115 \times 0.85 \times 266}\ mm^2 = 1548\ mm^2$$

$$\text{Steel for pull} = A_{st2} = \frac{49050}{115} = 427\ mm^2$$

$$\text{Total steel required} = A_{st} = A_{st1} + A_{st2} = 1548 + 427 = 1975\ mm^2$$

$$\text{Spacing of 18 } mm \text{ bars} = \frac{254 \times 1000}{1975} = 126\ mm$$

Provide 18 mm ϕ @ 120 mm c/c.

Design of the mid span section

Bending moment = 34445 *Nm*

Pull = 49050 *N*

$$\therefore \quad \text{Effective bending moment} = M - Tx$$

$$= 34445 \times 1000 - 49050\,(266 - 150) = 28755200\ Nmm$$

Since this bending moment produces tension away from the water face, the stresses and tee design coefficients to be adopted are,

$t = 125\ N/mm^2$, $c = 7\ N/mm^2$, $M = 13.33$ and $a = 0.86\ d$

$$\text{Steel for B.M.} = A_{st1} = \frac{28755200}{125 \times 0.86 \times 266} = 1006\ mm^2$$

$$\text{Steel for pull} = A_{st2} = \frac{49050}{125} = 392\ mm^2$$

$$\therefore \quad \text{Total steel required} = A_{st} = A_{t1} + A_{t2} = 1006 + 392 = 1398\ mm^2$$

Provide 18 mm ϕ @ 170 mm c/c.

Design of the bottom 1 *metre* height of wall

This will be designed as a cantilever 1 *metre* high and subjected to a triangular load ranging from zero at the top of the catilever to $9810 \times 3 = 29430\ N/metre^2$ at the bottom.

∴ Maximum pressure force on the cantilever (for 1 *metre* width)

$$= \frac{1}{2} \times 29430 \times 1 = 14715\,N \text{ acting at } \frac{1}{3}\ metre \text{ from the base.}$$

∴ B.M. for the cantilever

$$14715 \times \frac{1}{3} = 4905\ Nm$$

This bending moment produces tension near the water face.

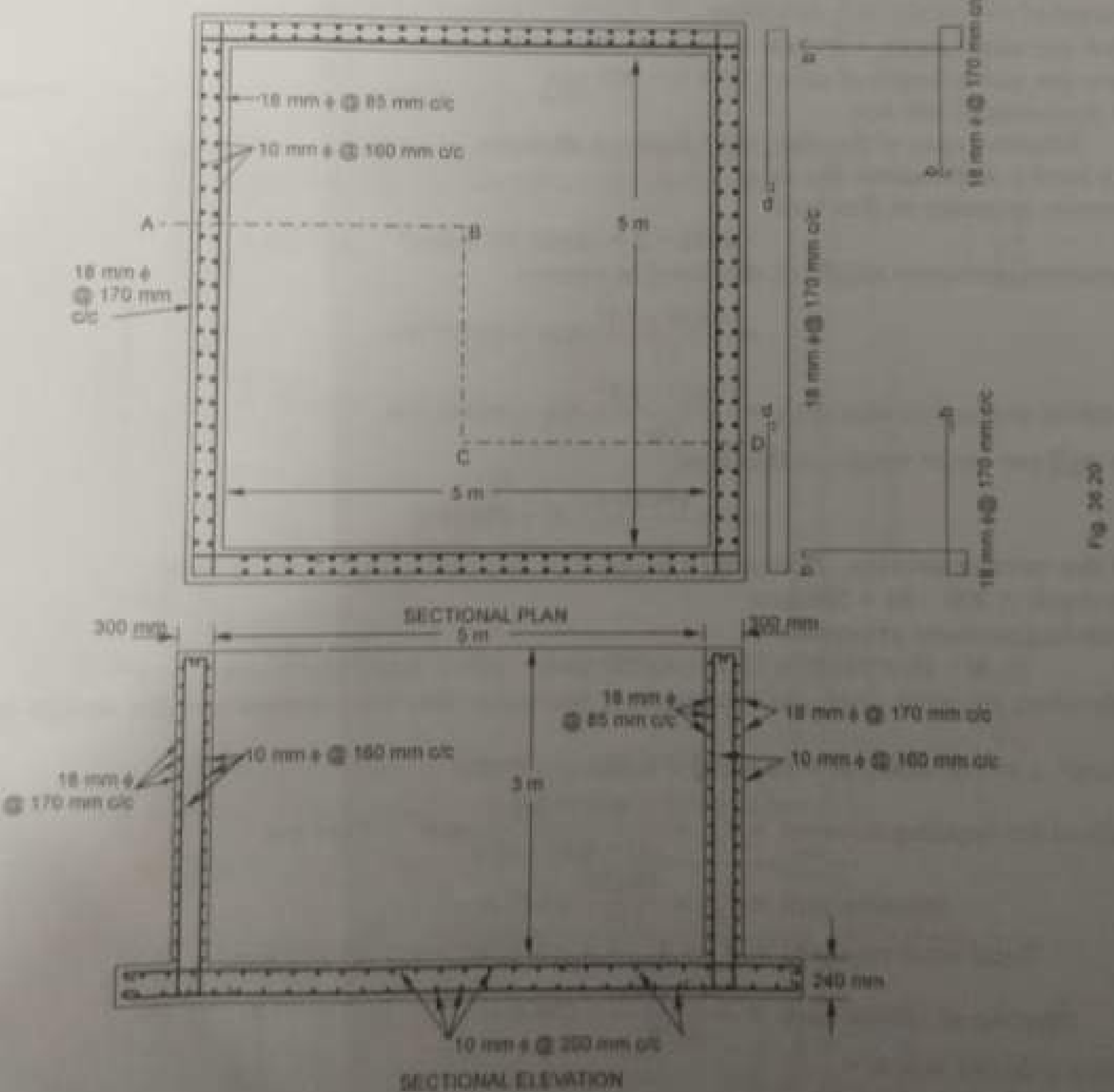


Fig. 36.22.

Effective depth to centre of 10 *mm* ϕ bars = 300 − 25 − 18 − 5 = 252 *mm*

$$\text{Steel for B.M.} = \frac{4905000}{115 \times 0.85 \times 252} = 192\ mm^2$$

$$\text{But 0.3\% of gross area} = \frac{0.3}{100} = 300 \times 100 = 900\ mm^2$$

$$\text{Spacing of } 10\ mm\ \phi \text{ bars} = \frac{79 \times 1000}{900} = 88\ mm\ say, 10\ mm\ \phi\ @80\ mm\ c/c.$$

Provide 10 *mm* ϕ vertical bars @ 160 *mm c/c* near each face. These vertical bars near the water face are enough to resist cantilever moment.

Base slab. The base slab will be 250 *mm* thick with a top mesh and a bottom mesh of reinforcement of 10 *mm bars at 200 mm centres.*

Design 36.4. A circular water tank has an internal diameter of 12 metres, the maximum depth of water being 4 metres. The walls of the tank are restrained at the base. The tank rests on ground. Design the tank. Use M 20 concrete and grade I mild steel.

Solution.

$$H = 4 \text{ metres}, D = 12 \text{ metres}$$

Thickness of wall. This shall be not less than

(i) 150 *mm*

(ii) 30 *mm* per *metre* depth + 50 *mm* = 30 × 4 + 50 = 170 *mm*

Hence let us provide a thickness of 170 *mm*

$$K = \frac{12 H^4}{\left(\frac{D}{2}\right)^2 t^2} = \frac{12 \times 4^4}{6^2 \times (0.17)^2} = 2953$$

From table 1, we find

For

$$K = 1000 \qquad M = 0.024\, pH^3$$

$$K = 10{,}000 \qquad M = 0.0085\, pH^3$$

(Adopting logarithmic interpolation),

For

$$K = 2953, \qquad M = \alpha\, pH^3$$

where,

$$\alpha = 0.0085 + (0.0024 - 0.0085) \frac{\log 10{,}000 - \log 2953}{\log 10{,}000 - \log 1000}$$

$$= 0.0085 + (0.0024 - 0.0085) \times 0.5298 = 0.0167$$

$$M = 0.0167\, pH^3$$

$$M = 0.0167\,(9810 \times 4)\, 4^2\, Nm = 10485\, Nm$$

Adopting

$$c = 7\ N/mm^2,\ t = 115\ N/mm^2 \text{ and } m = 13.33$$
$$n = 0.448d,\ a = 0.85d,\ Q = 1.333$$

Equating the moment of resistance to the bending moment, we have,

$$1.333 \times 1000\, d^2 = 10485 \times 1000, \quad d = 89\ mm$$

But, effective depth available to the centre of 12 mm ϕ bars

$$= 170 - 25 - 6 = 139\ mm$$

$$A_{st} = \frac{10485 \times 1000}{115 \times 0.85 \times 139} = 772\ mm^2$$

$$\text{Spacing of 12 } mm\ \phi \text{ bars} = \frac{113 \times 1000}{772} = 146\ mm$$

Provide 12 mm ϕ bars @ 140 mm c/c.

Hoop tension. From table 2, we have,

For $K = 1000$, max. hoop tension

$$T = 0.47\, p\left(\frac{D}{2}\right) \text{ at } 0.47\ H \text{ from the base.}$$

Following a logarithmic interpolation.

Max. hoop tension for $K = 2953$ will be equal to $\beta p\left(\frac{D}{2}\right)$

where,

$$\beta = 0.47 + (0.67 - 0.47)\frac{\log 2953 - \log 1000}{\log 10000 - \log 1000}$$
$$= 0.47 + 0.20 \times 0.4702 = 0.47 + 0.094 = 0.564$$

$$\text{Max. hoop tension} = 0.564\, p\left(\frac{D}{2}\right) = 0.564\,(9810 \times 4)\frac{12}{2}\ N = 132788\ N$$

This occurs at γH from the base

$$\gamma = 0.47 - (0.47 - 0.31) \times 0.4702 = 0.40$$

Maximum hoop tension will occur at 0.4 H from the base, *i.e.*, at a height of $0.4 \times 4 = 1.6$ *metres* from the base

$$\text{Steel for hoop tension} = \frac{132788}{115} = 1155\ mm^2$$

$$\text{Spacing of 12 } mm\ \phi \text{ bars} = \frac{113 \times 1000}{1155} = 97\ mm \text{ say } 90\ mm\ c/c.$$

Since this steel is provided in the form of rings near both the faces the spacing of the rings will be **@ 180 mm c/c near** *each face. Fig. 36.9 shows the details of reinforcement.*

Check for tensile stress in concrete

$$\text{Tensile stress} = \frac{132788}{1000 \times 170 + (13.33 - 1)1155} = 0.72\ N/mm^2 \text{ (less than } 1.20\ N/mm^2)$$

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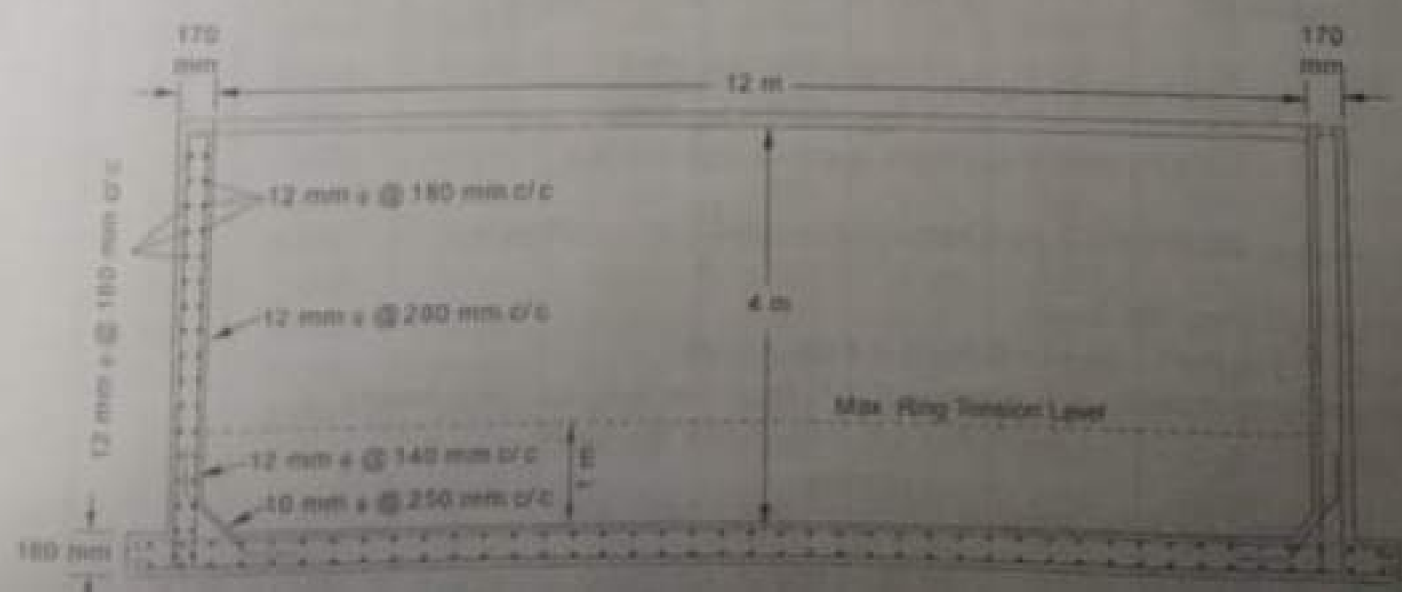

200 mm ...

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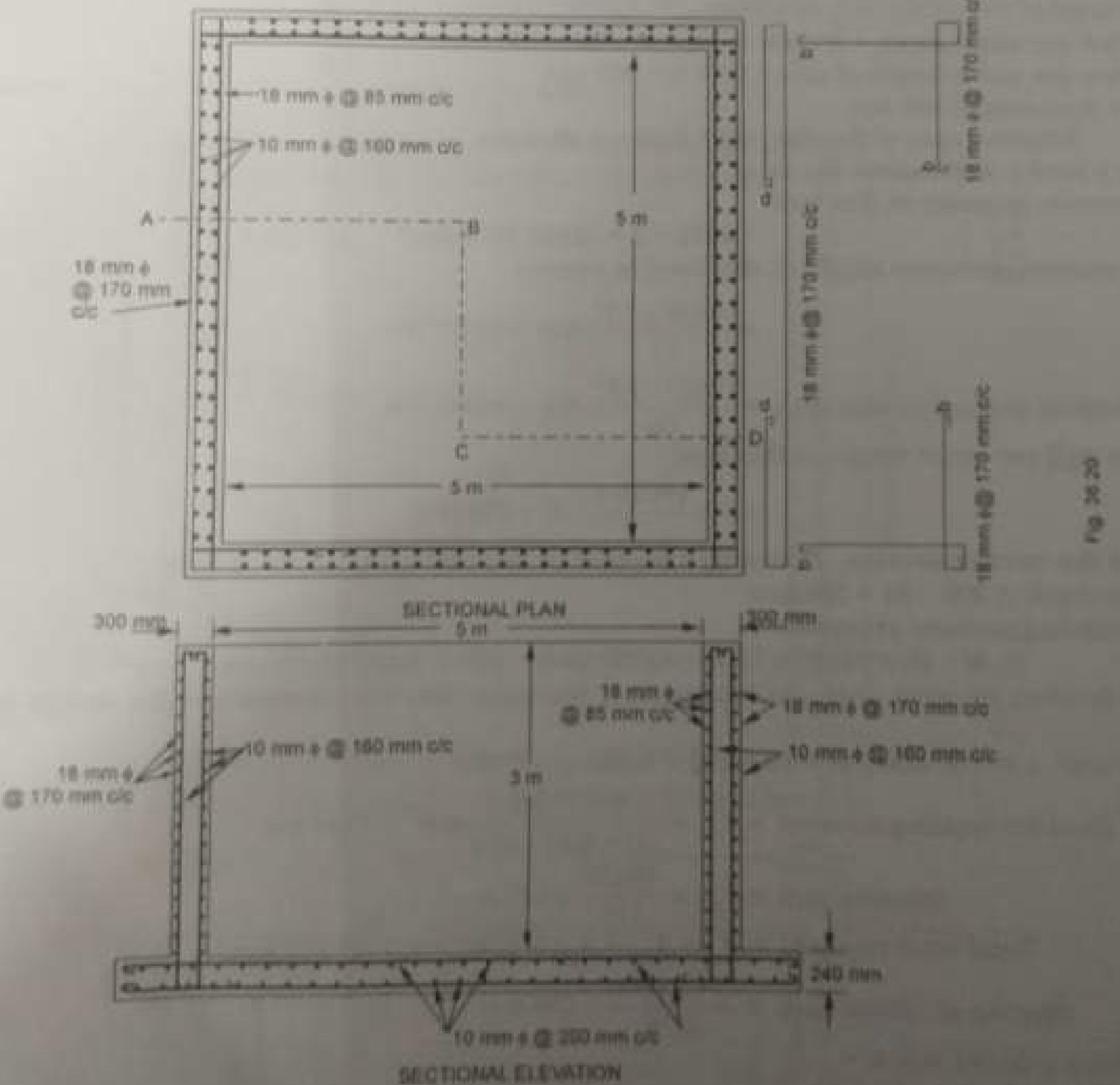

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