NOTES ON

MACHINE DESIGN-II

UNIT-1

TRANSMISSION DRIVES

Belt and rope drives

The belts and ropes are used to transmit power from one shaft to another by means of pulleys which rotate at the same speed or at different speeds. The amount of power transmitted depends upon the following factors :

1. The velocity of the belt.

2. The tension under which the belt is placed on the pulleys.

3. The arc of contact between the belt and the smaller pulley.

4. The conditions under which the belt is used.

Types of Belt Drives

The belt drives are usually classified into the following three groups:

1. ***Light drives.*** These are used to transmit small powers at belt speeds upto about 10 m/s as in agricultural machines and small machine tools.

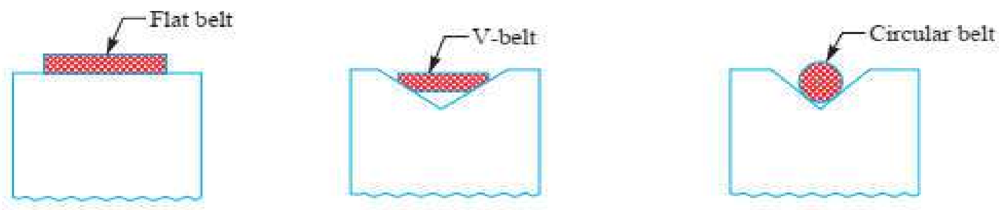
2. ***Medium drives.*** These are used to transmit medium powers at belt speeds over 10 m/s but up to 22 m/s, as in machine tools.

3. ***Heavy drives.*** These are used to transmit large powers at belt speeds above 22 m/s as in compressors and generators

Types of Belts

Though there are many types of belts used these days, yet the following are important from the subject point of view:

1. ***Flat belt.*** The flat belt as shown in Fig. (*a*), is mostly used in the factories and workshops, where a moderate amount of power is to be transmitted, from one pulley to another when the two pulleys are not more than 8 metres apart



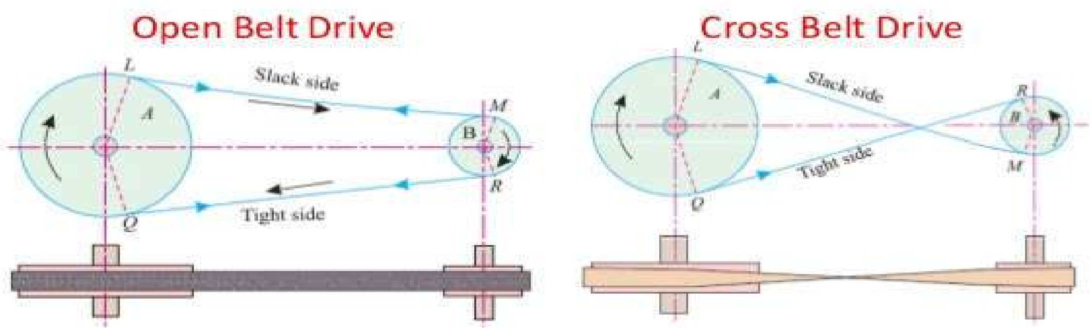
(*a*) Flat belt. (*b*) V-belt. (*c*) Circular belt.

2. ***V- belt.*** The V-belt as shown in Fig.(*b*), is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are very near to each other.

3. *Circular belt or rope.* The circular belt or rope as shown in Fig. (*c*) is mostly used in the factories and workshops, where a great amount of power is to be transmitted, from one pulley to another, when the two pulleys are more than 8 metres apart.

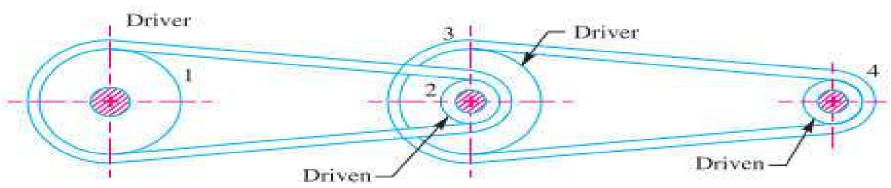
Types of Flat Belt Drives

The power from one pulley to another may be transmitted by any of the following types of belt drives.



An open belt drive is used to rotate the driven pulley in the same direction of driving pulley. In the motion of belt drive, power transmission results make one side of pulley more tightened compared to the other side. In horizontal drives, tightened side is always kept on the lower side of two pulleys because the sag of the upper side slightly increases the angle of folding of the belt on the two pulleys.

A crossed belt drive is used to rotate driven pulley in the opposite direction of driving pulley. Higher the value of wrap enables more power can be transmitted than an open belt drive. However, bending and wear of the belt are important concerns.



A compound belt drive, is used when power is transmitted from one shaft to another through a number of pulleys.

Slip of the Belt

In the previous articles we have discussed the motion of belts and pulleys assuming a firm frictional grip between the belts and the pulleys. But sometimes, the frictional grip becomes insufficient. This may cause some forward motion of the driver without carrying the belt with it. This is called ***slip of the belt*** and is generally expressed as a percentage

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1}{100} - \frac{s_2}{100}\right) \quad \ldots \left(\text{Neglecting } \frac{s_1 \times s_2}{100 \times 100}\right)$$

$$= \frac{d_1}{d_2}\left[1 - \left(\frac{s_1 + s_2}{100}\right)\right] = \frac{d_1}{d_2}\left(1 - \frac{s}{100}\right)$$

...(where $s = s_1 + s_2$ *i.e.* total percentage of slip)

Creep of Belt

When the belt passes from the slack side to the tight side, a certain portion of the belt extends and it contracts again when the belt passes from the tight side to the slack side. Due to these changes of length, there is a relative motion between the belt and the pulley surfaces. This relative motion is termed as ***creep.***

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

σ_1 and σ_2 = Stress in the belt on the tight and slack side respectively, and

E = Young's modulus for the material of the belt.

Selection of a Belt Drive

Following are the various important factors upon which the selection of a belt drive depends:

1. Speed of the driving and driven shafts,

2. Speed reduction ratio,

3. Power to be transmitted,

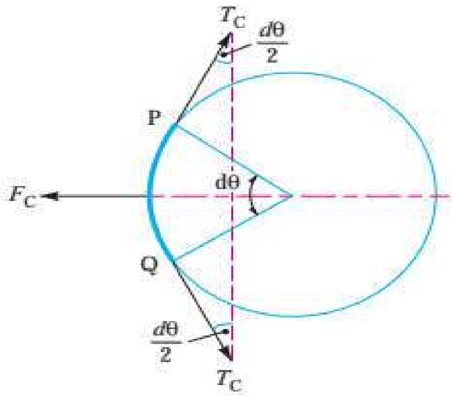
4. Centre distance between the shafts,

5. Positive drive requirements,

6. Shafts layout,

7. Space available,

Design of flat belt



Length of an Open Belt Drive

$$L = \pi(r_1 + r_2) + 2 \times \frac{(r_1 - r_2)}{x}(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

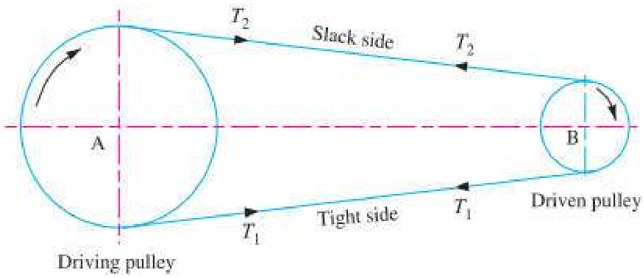
Length of a Cross Belt Drive

$$L = \pi(r_1 + r_2) + 2 \times \frac{(r_1 + r_2)}{x}(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + \frac{2(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x}$$

Power Transmitted by a Belt

The driving pulley (or driver) *A* and the driven pulley (or follower) *B*. As already discussed, the driving pulley pulls the belt from one side and delivers it to the other side. It is thus obvious that the tension on the former side (*i.e.* tight side) will be greater than the latter side



Let T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively in newtons,

r_1 and r_2 = Radii of the driving and driven pulleys respectively in metres,

and v = Velocity of the belt in m/s.

The effective turning (driving) force at the circumference of the driven pulley or follower is the difference between the two tensions (*i.e.* $T_1 - T_2$).

$\therefore$ Work done per second = $(T_1 - T_2)\,v$ N-m/s

and power transmitted = $(T_1 - T_2)\,v$ W ... ($\because$ 1 N-m/s = 1W)

Ratio of Driving Tensions for Flat Belt Drive

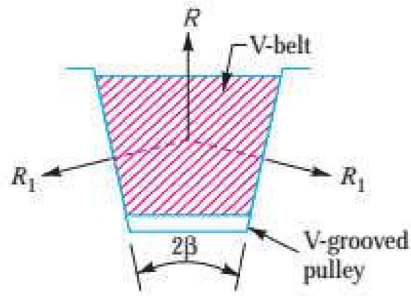
$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

Centrifugal Tension

Since the belt continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both the tight as well as the slack sides. The tension caused by centrifugal force is called ***centrifugal tension***.

$$2T_C\left(\frac{d\theta}{2}\right) = m.d\theta.v^2$$
$$T_C = m.v^2$$

Maximum Tension in the Belt



Let σ = Maximum safe stress, b = Width of the belt, and
t = Thickness of the belt.
We know that the maximum tension in the belt, T
= Maximum stress × Cross-sectional area of belt = $\sigma.b.t$
When centrifugal tension is neglected, then
T (or T_{t1}) = T_1, *i.e.* Tension in the tight side of the belt.
When centrifugal tension is considered, then T (or T_{t1}) = $T_1 + T_C$

Condition for the Transmission of Maximum Power

$$\therefore \quad T - 3T_C = 0 \quad \text{or} \quad T = 3T_C \qquad \ldots (\because m.v^2 = T_C)$$

It shows that when the power transmitted is maximum, 1/3rd of the maximum tension is absorbed as centrifugal tension.

i. Arc of contact

Open belt drive $\theta = 180° - \frac{(D-d)}{c} 60°$

Cross belt drive $\theta = 180° + \frac{(D+d)}{C} 60°$

DESIGN PROCEDURE

From the given conditions like power, type of working conditions, diameters of pulleys, speed ratio etc, determine maximum power

1.Design power = rated power x service factor x arc of contact factor

Select service factor based on nature of load and applications from PSG data book

2.Decide the type of belt

3.Then calculate the belt rating

4.Find the reqired width by design power by belt capacity and adopt the standard available

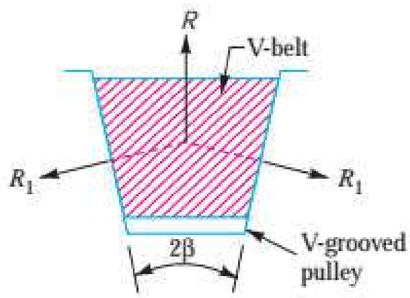
5.Determine the length of belt based on type of drive and reduce certain amount length

6.Find out the pulley dimension and draw the arrangement of belt drive.

V BELTS

V-belt is mostly used in factories and workshops where a great amount of power is to be transmitted from one pulley to another when the two pulleys are very near to each other. The *V*-belts are made of fabric and cords moulded in rubber and covered with fabric and rubber

Ratio of Driving Tensions for V-belt



Let R_1 = Normal reactions between belts and sides of the groove.
R = Total reaction in the plane of the groove.
μ = Coefficient of friction between the belt and sides of the groove.
Resolving the reactions vertically to the groove, we have
$R = R_1 \sin\beta + R_1 \sin\beta = 2R_1 \sin\beta$

Rope Drives

The rope drives are widely used where a large amount of power is to be transmitted, from one pulley to another, over a considerable distance. It may be noted that the use of flat belts is limited for the transmission of moderate power from one pulley to another when the two pulleys are not more than 8 metres apart. If large amounts of power are to be transmitted, by the flat belt, then it would result in excessive belt cross-section.The ropes drives use the following two types of ropes :

1. Fibre ropes, and **2.** Wire ropes.

Fibre Ropes

The ropes for transmitting power are usually made from fibrous materials such as hemp, manila and cotton. Since the hemp and manila fibres are rough, therefore the ropes made from these fibres are not very flexible and possesses poor mechanical properties. The hemp ropes have less strength as compared to manila ropes. When the hemp and manila ropes are bent over the sheave, there is some sliding of the fibres, causing the rope to wear and chafe internally. In order to minimise this defect, the rope fibres are lubricated with a tar, tallow or graphite.

Ratio of Driving Tensions for Fibre Rope

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta \operatorname{cosec}\beta$$

where μ, θ and β have usual meanings.

Wire Ropes

When a large amount of power is to be transmitted over long distances from one pulley to another (*i.e.* when the pulleys are upto 150 metres apart), then wire ropes are used. The wire ropes are widely used in elevators, mine hoists, cranes, conveyors, hauling devices and suspension bridges.

Designation of Wire Ropes

The wire ropes are designated by the number of strands and the number of wires in each strand. For example, a wire rope having six strands and seven wires in each strand is designated by 6 × 7

Designing a Wire Rope

The following procedure may be followed while designing a wire rope.

1. First of all, select a suitable type of rope from Tables 20.6, 20.7, 20.8 and 20.9 for the given application.

2. Find the design load by assuming a factor of safety 2 to 2.5 times the factor of safety given in Table.

3. Find the diameter of wire rope (*d*) by equating the tensile strength of the rope selected to the design load.

4. Find the diameter of the wire (*dw*) and area of the rope (*A*) from.

5. Find the various stresses (or loads) in the rope.

6. Find the effective stresses (or loads) during normal working, during starting and during acceleration of the load.

7. Now find the actual factor of safety and compare with the factor of safety given in Table. If the actual factor of safety is within permissible limits, then the design is safe.

Stresses in Wire Ropes

1. ***Direct stress due to axial load lifted and weight of the rope***

Let W = Load lifted,

w = Weight of the rope, and

A = Net cross-sectional area of the rope.

∴ Direct stress,
$$\sigma_d = \frac{W + w}{A}$$

2. ***Bending stress when the rope winds round the sheave or drum.*** When a wire rope is wound over the sheave, then the bending stresses are induced in the wire which is tensile at the top and compressive at the lower side of the wire. The bending stress induced depends upon many factors such as construction of rope, size of wire, type of centre and the amount of restraint in the grooves. The approximate value of the bending stress in the wire as proposed by Reuleaux, is

$$\sigma_b = \frac{E_r \times d_w}{D}$$

and equivalent bending load on the rope,

$$W_b = \sigma_b \times A = \frac{E_r \times d_w \times A}{D}$$

where E_r = Modulus of elasticity of the wire rope,

d_w = Diameter of the wire,

D = Diameter of the sheave or drum, and

A = Net cross-sectional area of the rope.

3. ***Stresses during starting and stopping.*** During starting and stopping, the rope and the supported load are to be accelerated. This induces additional load in the rope which is given by

$$W_a = \frac{W + w}{g} \times a \qquad \text{...(}W\text{ and }w\text{ are in newton)}$$

and the corresponding stress,

$$\sigma_a = \frac{W + w}{g} \times \frac{a}{A}$$

4. *Stress due to change in speed.* The additional stress due to change in speed may be obtained in the similar way as discussed above in which the acceleration is given by

$$a = (v_2 - v_1) / t$$

where $(v_2 - v_1)$ is the change in speed in m/s and t is the time in seconds.

Design of Pulleys

1. *Dimensions of pulley*

(i) The diameter of the pulley (D) may be obtained either from velocity ratio consideration or centrifugal stress consideration. We know that the centrifugal stress induced in the rim of the pulley,

$$\sigma_t = \rho . v^2$$

where ρ = Density of the rim material

= 7200 kg/m³ for cast iron

v = Velocity of the rim = $\pi DN / 60$, D being the diameter of pulley and N is speed of the pulley.

The following are the diameter of pulleys in mm for flat and V-belts.

20, 22, 25, 28, 32, 36, 40, 45, 50, 56, 63, 71, 80, 90, 100, 112, 125, 140, 160, 180, 200, 224, 250, 280, 315, 355, 400, 450, 500, 560, 630, 710, 800, 900, 1000, 1120, 1250, 1400, 1600, 1800, 2000, 2240, 2500, 2800, 3150, 3550, 4000, 5000, 5400.

The first six sizes (20 to 36 mm) are used for V-belts only.

(ii) If the width of the belt is known, then width of the pulley or face of the pulley (B) is taken 25% greater than the width of belt.

$\therefore$ $B = 1.25\, b$; where b = Width of belt.

According to Indian Standards, IS : 2122 (Part I) – 1973 (Reaffirmed 1990), the width of pulley

The following are the width of flat cast iron and mild steel pulleys in mm :

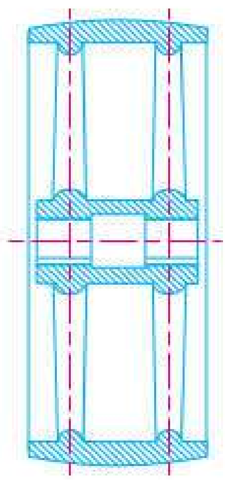
16, 20, 25, 32, 40, 50, 63, 71, 80, 90, 100, 112, 125, 140, 160, 180, 200, 224, 250, 315, 355, 400, 450, 560, 630.

(iii) The thickness of the pulley rim (t) varies from $\dfrac{D}{300} + 2$ mm to $\dfrac{D}{200} + 3$ mm for single belt and $\dfrac{D}{200} + 6$ mm for double belt. The diameter of the pulley (D) is in mm.

2. *Dimensions of arms*

(i) The number of arms may be taken as 4 for pulley diameter from 200 mm to 600 mm and 6 for diameter from 600 mm to 1500 mm.

(ii) The cross-section of the arms is usually elliptical with major axis (a_1) equal to twice the minor axis (b_1). The cross-section of the arm is obtained by considering the arm as cantilever *i.e.* fixed at the hub end and carrying a concentrated load at the rim end. The length of the cantilever is taken equal to the radius of the pulley. It is further assumed that at any given time, the power is transmitted from the hub to the rim or ***vice versa***, through only half the total number of arms.



Let T = Torque transmitted,

R = Radius of pulley, and

n = Number of arms,

$\therefore$ Tangential load per arm,

$$W_T = \frac{T}{R \times n / 2} = \frac{2\,T}{R \cdot n}$$

Maximum bending moment on the arm at the hub end,

$$M = \frac{2\,T}{R \times n} \times R = \frac{2\,T}{n}$$

and section modulus,

$$Z = \frac{\pi}{32} \times b_1\,(a_1)^2$$

Now using the relation,

σ_b or $\sigma_t = M/Z$, the cross-section of the arms is obtained.

(iii) The arms are tapered from hub to rim. The taper is usually 1/48 to 1/32.

(iv) When the width of the pulley exceeds the diameter of the pulley, then two rows of arms are provided, as shown in Fig. This is done to avoid heavy arms in one row.

3. *Dimensions of hub*

(i) The diameter of the hub (d_1) in terms of shaft diameter (d) may be fixed by the following relation :

$$d_1 = 1.5\, d + 25 \text{ mm}$$

The diameter of the hub should not be greater than $2\, d$.

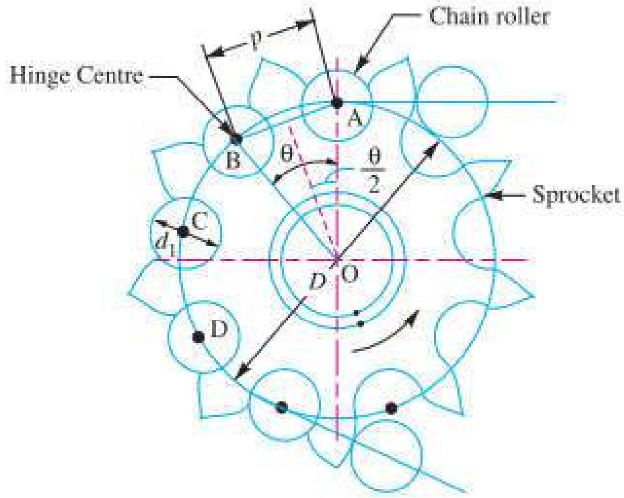
(ii) The length of the hub,

$$L = \frac{\pi}{2} \times d$$

The minimum length of the hub is $\frac{2}{3}$ B but it should not be more than width of the pulley (B).

Chain Drives

To avoid slipping, steel chains are used. The chains are made up of number of rigid links which are hinged together by pin joints in order to provide the necessary flexibility for wraping round the driving and driven wheels. These wheels have projecting teeth of special profile and fit into the corresponding recesses in the links of the chain. The toothed wheels are known as ****sprocket wheels or simply sprockets***. The sprockets and the chain are thus constrained to move together without slipping and ensures perfect velocity ratio



Relation Between Pitch and Pitch Circle Diameter

A chain wrapped round the sprocket is shown in Fig.. Since the links of the chain are rigid, therefore pitch of the chain does not lie on the arc of the pitch circle. The pitch length becomes a chord. Consider one pitch length *AB* of the chain subtending an angle θ at the centre of sprocket (or pitch circle),

Let D = Diameter of the pitch circle, and

T = Number of teeth on the sprocket.

From Fig. 21.2, we find that pitch of the chain,

$$p = AB = 2\,AO\sin\left(\frac{\theta}{2}\right) = 2 \times \left(\frac{D}{2}\right)\sin\left(\frac{\theta}{2}\right) = D\sin\left(\frac{\theta}{2}\right)$$

We know that $\theta = \dfrac{360°}{T}$

$$\therefore \quad p = D\sin\left(\frac{360°}{2\,T}\right) = D\sin\left(\frac{180°}{T}\right)$$

or

$$D = p\,\text{cosec}\left(\frac{180°}{T}\right)$$

The sprocket outside diameter (D_o), for satisfactory operation is given by

$$D_o = D + 0.8\,d_1$$

Velocity Ratio of Chain Drives

The velocity ratio of a chain drive is given by

$$V.R. = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

where N_1 = Speed of rotation of smaller sprocket in r.p.m.,

N_2 = Speed of rotation of larger sprocket in r.p.m.,

T_1 = Number of teeth on the smaller sprocket, and

T_2 = Number of teeth on the larger sprocket.

The average velocity of the chain is given by

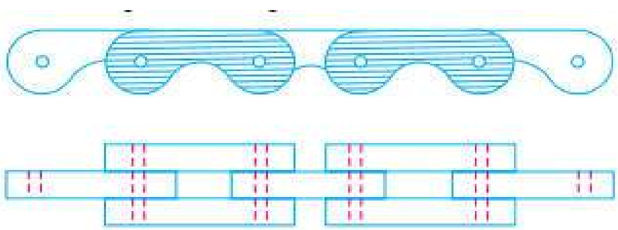
$$v = \frac{\pi D N}{60} = \frac{T\ p\ N}{60}$$

where D = Pitch circle diameter of the sprocket in metres, and

p = Pitch of the chain in metres.

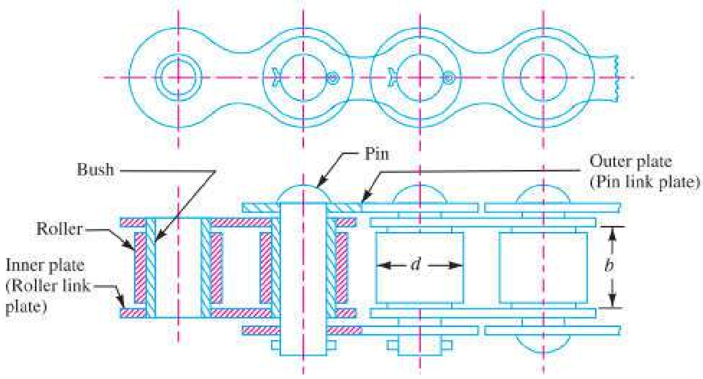
These chains are used for transmission of power, when the distance between the centres of shafts is short. These chains have provision for efficient lubrication. The power transmitting chains are of the following three types.

1. *Block or bush chain.* A block or bush chain is shown in Fig.. This type of chain was used in the early stages of development in the power transmission.



It produces noise when approaching or leaving the teeth of the sprocket because of rubbing between the teeth and the links. Such type of chains are used to some extent as conveyor chain at small speed.

2. *Bush roller chain.* A bush roller chain as shown in Fig. 21.7, consists of outer plates or pin link plates, inner plates or roller link plates, pins, bushes and rollers. A pin passes through the bush which is secured in the holes of the roller between the two sides of the chain. The rollers are free to rotate on the bush which protect the sprocket wheel teeth against wear. The pins, bushes and rollers are made of alloy steel.



Factor of Safety for Chain Drives

The factor of safety for chain drives is defined as the ratio of the breaking strength (W_B) of the chain to the total load on the driving side of the chain (W). Mathematically,

$$\text{Factor of safety} = \frac{W_B}{W}$$

The breaking strength of the chain may be obtained by the following empirical relations, *i.e.*

$$W_B = 106\, p^2 \text{ (in newtons) for roller chains}$$

$$= 106\, p \text{ (in newtons) per mm width of chain for silent chains.}$$

We know that the tangential driving force acting on the chain,

$$F_T = \frac{\text{Power transmitted (in watts)}}{\text{Speed of chain in m/s}} = \frac{P}{v} \quad \text{(in newtons)}$$

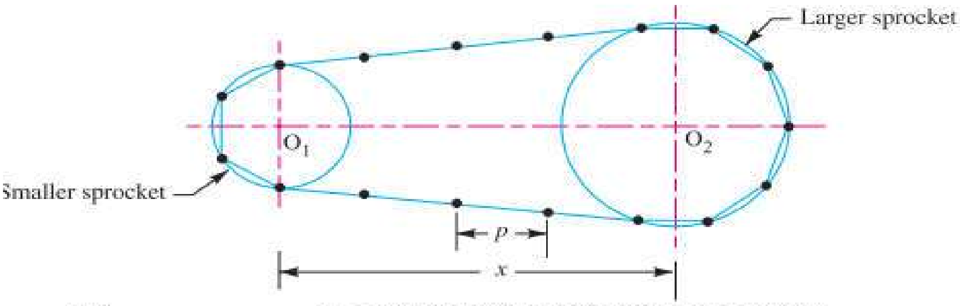
Centrifugal tension in the chain,

$$F_C = m.v^2 \text{ (in newtons)}$$

l tension in the chain due to sagging,

$$F_S = k.mg.x \text{ (in newtons)}$$

Length of Chain and Centre Distance



Let T_1 = Number of teeth on the smaller sprocket,

T_2 = Number of teeth on the larger sprocket,

p = Pitch of the chain, and

x = Centre distance.

The length of the chain (L) must be equal to the product of the number of chain links (K) and the pitch of the chain (p). Mathematically,

$$L = K.p$$

The number of chain links may be obtained from the following expression, *i.e.*

$$K = \frac{T_1 + T_2}{2} + \frac{2\,x}{p} + \left[\frac{T_2 - T_1}{2\,\pi}\right]^2 \frac{p}{x}$$

The value of K as obtained from the above expression must be approximated to the nearest even number.

The centre distance is given by

$$x = \frac{p}{4}\left[K - \frac{T_1 + T_2}{2} + \sqrt{\left(K - \frac{T_1 + T_2}{2}\right)^2 - 8\left(\frac{T_2 - T_1}{2\,\pi}\right)^2}\,\right]$$

Power Transmitted by Chains

The power transmitted by the chain on the basis of breaking load is given by

$$P = \frac{W_B \times v}{n \times K_S} \quad \text{(in watts)}$$

where W_b = Breaking load in newtons,

v = Velocity of chain in m/s

n = Factor of safety, and

K_S = Service factor = $K_1.K_2.K_3$

The power transmitted by the chain on the basis of bearing stress is given by

$$P = \frac{\sigma_b \times A \times v}{K_S}$$

The service factor (K_S) is the product of various factors, such as load factor (K_1), lubrication factor (K_2) and rating factor (K_3). The values of these factors are taken as follows:

1. Load factor (K_1) = 1, for constant load
   = 1.25, for variable load with mild shock
   = 1.5, for heavy shock loads
2. Lubrication factor (K_2) = 0.8, for continuous lubrication
   = 1, for drop lubrication
   = 1.5, for periodic lubrication
3. Rating factor (K_3) = 1, for 8 hours per day
   = 1.25, for 16 hours per day
   = 1.5, for continuous service

Principal Dimensions of Tooth Profile

1. Tooth flank radius (r_e)

$$= 0.008\, d_1\, (T^2 + 180) \quad \text{...(Maximum)}$$

$$= 0.12\, d_1\, (T + 2) \quad \text{...(Minimum)}$$

where d_1 = Roller diameter, and
T = Number of teeth.

2. Roller seating radius (r_i)

$$= 0.505\, d_1 + 0.069\, \sqrt[3]{d_1} \quad \text{...(Maximum)}$$

$$= 0.505\, d_1 \quad \text{...(Minimum)}$$

3. Roller seating angle (α)

$$= 140° - \frac{90°}{T} \quad \text{...(Maximum)}$$

$$= 120° - \frac{90°}{T} \quad \text{...(Minimum)}$$

4. Tooth height above the pitch polygon (h_a)

$$= 0.625\, p - 0.5\, d_1 + \frac{0.8\, p}{T} \quad \text{...(Maximum)}$$

$$= 0.5\, (p - d_1) \quad \text{...(Minimum)}$$

5. Pitch circle diameter (D)

$$= \frac{p}{\sin\left(\frac{180}{T}\right)} = p \operatorname{cosec}\left(\frac{180}{T}\right)$$

6. Top diameter (D_a)

$$= D + 1.25\, p - d_1 \quad \text{...(Maximum)}$$

$$= D + p\left(1 - \frac{1.6}{T}\right) - d_1 \quad \text{...(Minimum)}$$

7. Root diameter (D_f)

$$= D - 2\, r_i$$

8. Tooth width (b_{f1})

$$= 0.93\, b_1 \text{ when } p \le 12.7 \text{ mm}$$

$$= 0.95\, b_1 \text{ when } p > 12.7 \text{ mm}$$

Design Procedure of Chain Drive

The chain drive is designed as discussed below:

1. First of all, determine the velocity ratio of the chain drive.
2. Select the minimum number of teeth on the smaller sprocket or pinion from Table
3. Find the number of teeth on the larger sprocket.
4. Determine the design power by using the service factor, such that

   Design power = Rated power × Service factor
5. Choose the type of chain, number of strands for the design power and r.p.m. of the smaller sprocket from Table
6. Note down the parameters of the chain, such as pitch, roller diameter, minimum width of roller etc. from Table
7. Find pitch circle diameters and pitch line velocity of the smaller sprocket.
8. Determine the load (W) on the chain by using the following relation, *i.e.*

$$W = \frac{\text{Rated power}}{\text{Pitch line velocity}}$$

9. Calculate the factor of safety by dividing the breaking load (W_B) to the load on the chain (W). This value of factor of safety should be greater than the value given in Table
10. Fix the centre distance between the sprockets.
11. Determine the length of the chain.

Gear Drives

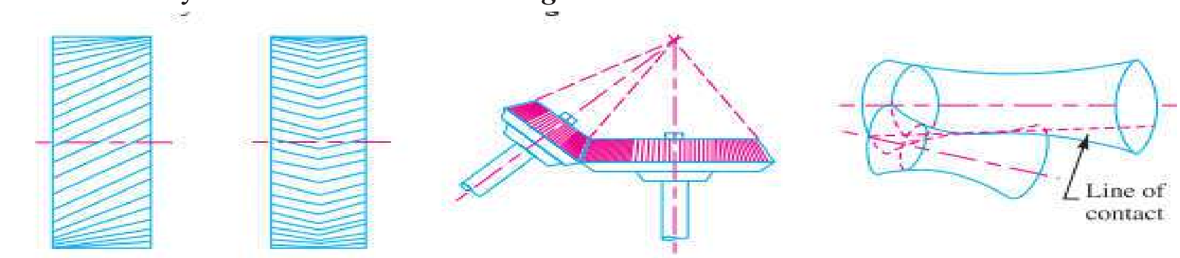
The slipping of a belt or rope is a common phenomenon, in the transmission of motion or power between two shafts. The effect of slipping is to reduce the velocity ratio of the system. In precision machines, in which a definite velocity ratio is of importance (as in watch mechanism), the only positive drive is by ***gears*** or ***toothed wheels.*** A gear drive is also provided, when the distance between the driver and the follower is very small

Classification of Gears

The gears or toothed wheels may be classified as follows :

1. ***According to the position of axes of the shafts.*** The axes of the two shafts between which the motion is to be transmitted, may be **(*a*)** Parallel, **(*b*)** Intersecting, and **(*c*)** Non-intersecting and non-parallel.

The two parallel and co-planar shafts connected by the gears is shown in Fig. These gears are called ***spur gears*** and the arrangement is known as ***spur gearing.*** These gears have teeth parallel to the axis of the wheel as shown in Fig. Another name given to the spur gearing is ***helical gearing***, in which the teeth are inclined to the axis. The ***single*** and ***double helical gears*** connecting parallel shafts are shown in Fig. (*a*) and (*b*) respectively. The object of the double helical gear is to balance out the end thrusts that are induced in single helical gears when transmitting load. The double helical gears are known as ***herringbone gears.*** A pair of spur gears are kinematically equivalent to a pair of cylindrical discs, keyed to a parallel shaft having line contact. The two non-parallel or intersecting, but coplaner shafts connected by gears is shown in Fig. (*c*). These gears are called ***bevel gears*** and the arrangement is known as ***bevel gearing.*** The ***bevel gears***, like spur gears may also have their teeth inclined to the face of the bevel, in which case they are known as ***helical bevel gears.***



(*a*) Single helical gear.   (*b*) Double helical gear.   (*c*) Bevel gear.   (*d*) Spiral gear.

The two non-intersecting and non-parallel *i.e.* non-coplanar shafts connected by gears is shown in Fig. (*d*). These gears are called ***skew bevel gears*** or ***spiral gears*** and the arrangement is known as ***skew bevel gearing*** or ***spiral gearing.*** This type of gearing also have a line contact, the rotation of which about the axes generates the two pitch surfaces known as ***hyperboloids.***

Terms used in Gears

1. ***Pitch circle.*** It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

2. *Pitch circle diameter.* It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also called as *pitch diameter*.

3. *Pitch point.* It is a common point of contact between two pitch circles.

4. *Pitch surface.* It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

5. *Pressure angle or angle of obliquity.* It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ϕ. The standard pressure angles are $14\frac{1}{2}°$ and $20°$.

6. *Addendum.* It is the radial distance of a tooth from the pitch circle to the top of the tooth.

7. *Dedendum.* It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

8. *Addendum circle.* It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. *Dedendum circle.* It is the circle drawn through the bottom of the teeth. It is also called ***root circle.***

Note : Root circle diameter = Pitch circle diameter × cos ϕ, where ϕ is the pressure angle.

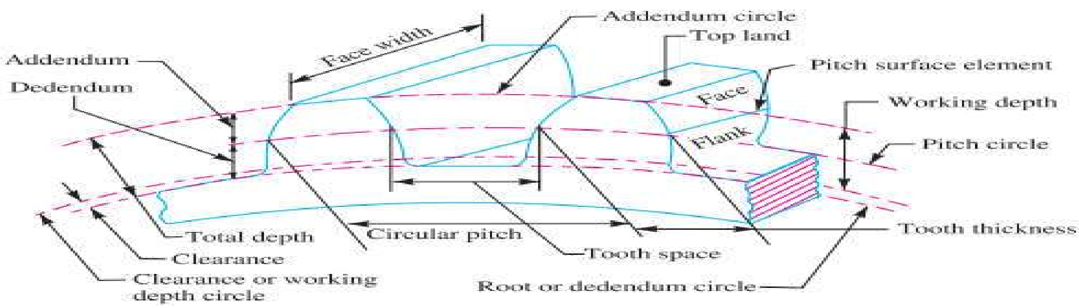
10. *Circular pitch.* It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c. Mathematically,

Circular pitch, $p_c = \pi D/T$

where D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.



11. ***Diametral pitch.*** It is the ratio of number of teeth to the pitch circle diameter in millimetres. It denoted by p_d. Mathematically,

$$\text{Diametral pitch,} \quad p_d = \frac{T}{D} = \frac{\pi}{p_c} \qquad \ldots \left(\because p_c = \frac{\pi D}{T} \right)$$

where T = Number of teeth, and

D = Pitch circle diameter.

12. ***Module.*** It is the ratio of the pitch circle diameter in millimetres to the number of teeth. It is usually denoted by m. Mathematically,

$$\text{Module,} \quad m = D / T$$

Note : The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40 and 50.

The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5,5.5, 7, 9, 11, 14, 18, 22, 28, 36 and 45 are of second choice.

13. ***Clearance.*** It is the radial distance from the top of the tooth to the bottom of the tooth, in a meshing gear. A circle passing through the top of the meshing gear is known as ***clearance circle.***

14. ***Total depth.*** It is the radial distance between the addendum and the dedendum circle of a gear. It is equal to the sum of the addendum and dedendum.

15. ***Working depth.*** It is radial distance from the addendum circle to the clearance circle. It is equal to the sum of the addendum of the two meshing gears.

16. ***Tooth thickness.*** It is the width of the tooth measured along the pitch circle.

17. ***Tooth space.*** It is the width of space between the two adjacent teeth measured along the pitch circle.

18. ***Backlash.*** It is the difference between the tooth space and the tooth thickness, as measured on the pitch circle.

Dynamic Tooth Load

In the previous article, the velocity factor was used to make approximate allowance for the effect of dynamic loading. The dynamic loads are due to the following reasons :

1. Inaccuracies of tooth spacing,

2. Irregularities in tooth profiles, and

3. Deflections of teeth under load

A closer approximation to the actual conditions may be made by the use of equations based on extensive series of tests, as follows :

$$W_D = W_T + W_I$$

where W_D = Total dynamic load,

W_T = Steady load due to transmitted torque, and

W_I = Increment load due to dynamic action.

The increment load (W_I) depends upon the pitch line velocity, the face width, material of the gears, the accuracy of cut and the tangential load. For average conditions, the dynamic load is determined by using the following Buckingham equation, *i.e.*

$$W_D = W_T + W_I = W_T + \frac{21\,v\,(b.C + W_T)}{21\,v + \sqrt{b.C + W_T}} \qquad \text{...}(i)$$

where W_D = Total dynamic load in newtons,

W_T = Steady transmitted load in newtons,

v = Pitch line velocity in m/s,

b = Face width of gears in mm, and

C = A deformation or dynamic factor in N/mm.

Static Tooth Load

The ***static tooth load*** (also called ***beam strength*** or ***endurance strength*** of the tooth) is obtained by Lewis formula by substituting flexural endurance limit or elastic limit stress (σ_e) in place of permissible working stress (σ_w).

∴ Static tooth load or beam strength of the tooth,

$$W_S = \sigma_e.b.p_c.y = \sigma_e.b.\pi\,m.y$$

Wear Tooth Load

The maximum load that gear teeth can carry, without premature wear, depends upon the radii of curvature of the tooth profiles and on the elasticity and surface fatigue limits of the materials. The maximum or the limiting load for satisfactory wear of gear teeth, is obtained by using the following Buckingham equation, *i.e.*

$$W_w = D_P.b.Q.K$$

where W_w = Maximum or limiting load for wear in newtons,

D_P = Pitch circle diameter of the pinion in mm,

b = Face width of the pinion in mm,

Q = Ratio factor

$$= \frac{2 \times V.R.}{V.R. + 1} = \frac{2\,T_G}{T_G + T_P}, \text{ for external gears}$$

$$= \frac{2 \times V.R.}{V.R. - 1} = \frac{2\,T_G}{T_G - T_P}, \text{ for internal gears.}$$

$V.R.$ = Velocity ratio = T_G / T_P,

K = Load-stress factor (also known as material combination factor) in N/mm².

The load stress factor depends upon the maximum fatigue limit of compressive stress, the pressure angle and the modulus of elasticity of the materials of the gears. According to Buckingham, the load stress factor is given by the following relation :

$$K = \frac{(\sigma_{es})^2 \sin\phi}{1.4}\left(\frac{1}{E_P} + \frac{1}{E_G}\right)$$

where σ_{es} = Surface endurance limit in MPa or N/mm²,

ϕ = Pressure angle,

E_P = Young's modulus for the material of the pinion in N/mm², and

E_G = Young's modulus for the material of the gear in N/mm².

Gear Tooth Failure The different modes of failure of gear teeth and their possible remedies to avoid the failure, are as follows :

1. ***Bending failure.*** Every gear tooth acts as a cantilever. If the total repetitive dynamic load acting on the gear tooth is greater than the beam strength of the gear tooth, then the gear tooth will fail in bending, *i.e*. the gear tooth will break. In order to avoid such failure, the module and face width of the gear is adjusted so that the beam strength is greater than the dynamic load.

2. ***Pitting.*** It is the surface fatigue failure which occurs due to many repetition of Hertz contact stresses. The failure occurs when the surface contact stresses are higher than the endurance limit of the material. The failure starts with the formation of pits which continue to grow resulting in the rupture of the tooth surface. In order to avoid the pitting, the dynamic load between the gear tooth should be less than the wear strength of the gear tooth.

3. ***Scoring.*** The excessive heat is generated when there is an excessive surface pressure, high speed or supply of lubricant fails. It is a stick-slip phenomenon in which alternate shearing and welding takes place rapidly at high spots. This type of failure can be avoided by properly designing the parameters such as speed, pressure and proper flow of the lubricant, so that the temperature at the rubbing faces is within the permissible limits.

4. ***Abrasive wear.*** The foreign particles in the lubricants such as dirt, dust or burr enter between the tooth and damage the form of tooth. This type of failure can be avoided by providing filters for the lubricating oil or by using high viscosity lubricant oil which enables the formation of thicker oil film and hence permits easy passage of such particles without damaging the gear surface.

5. ***Corrosive wear.*** The corrosion of the tooth surfaces is mainly caused due to the presence of corrosive elements such as additives present in the lubricating oils. In order to avoid this type of wear, proper anti-corrosive additives should be used.

Design Procedure for Spur Gears

1. First of all, the design tangential tooth load is obtained from the power transmitted and the pitch line velocity by using the following relation :

$$W_T = \frac{P}{v} \times C_S \qquad \ldots(i)$$

where
W_T = Permissible tangential tooth load in newtons,
P = Power transmitted in watts,
*v = Pitch line velocity in m / s $= \frac{\pi D N}{60}$,
D = Pitch circle diameter in metres.

2. Apply the Lewis equation as follows :

$$W_T = \sigma_w . b . p_c . y = \sigma_w . b . \pi\, m . y$$
$$= (\sigma_o . C_v)\, b . \pi\, m . y \qquad \ldots(\because \sigma_w = \sigma_o . C_v)$$

(iv) The product $(\sigma_w \times y)$ is called ***strength factor*** of the gear.

(v) The face width (b) may be taken as 3 p_c to 4 p_c (or 9.5 m to 12.5 m) for cut teeth and 2 p_c to 3 p_c (or 6.5 m to 9.5 m) for cast teeth.

3. Calculate the dynamic load (W_D) on the tooth by using Buckingham equation, *i.e.*

$$W_D = W_T + W_I$$
$$= W_T + \frac{21v\,(b.C + W_T)}{21v + \sqrt{b.C + W_T}}$$

In calculating the dynamic load (W_D), the value of tangential load (W_T) may be calculated by neglecting the service factor (C_S) *i.e.*

$$W_T = P / v, \text{ where } P \text{ is in watts and } v \text{ in m / s.}$$

4. Find the static tooth load (*i.e.* beam strength or the endurance strength of the tooth) by using the relation,

$$W_S = \sigma_e . b . p_c . y = \sigma_e . b . \pi\, m . y$$

For safety against breakage, W_S should be greater than W_D.

5. Finally, find the wear tooth load by using the relation,

$$W_w = D_P . b . Q . K$$

The wear load (W_w) should not be less than the dynamic load (W_D).

A helical gear has teeth in form of helix around the gear. Two such gears may be used to connect two parallel shafts in place of spur gears. The helixes may be right handed on one gear and left handed on the other

Terms used in Helical Gears

1. ***Helix angle.*** It is a constant angle made by the helices with the axis of rotation.

2. ***Axial pitch***. It is the distance, parallel to the axis, between similar faces of adjacent teeth. It is the same as circular pitch and is therefore denoted by *pc*. The axial pitch may also be defined as the circular pitch in the plane of rotation or the diametral plane.

3. ***Normal pitch.*** It is the distance between similar faces of adjacent teeth along a helix on the pitch cylinders normal to the teeth. It is denoted by *p*N. The normal pitch may also be defined as the circular pitch in the normal plane which is a plane perpendicular to the teeth. Mathematically, normal pitch, $p_N = p_c \cos \alpha$

Proportions for Helical Gears

Though the proportions for helical gears are not standardised, yet the following are recommended by American Gear Manufacturer's Association (AGMA).

Pressure angle in the plane of rotation,

$$\phi = 15° \text{ to } 25°$$

Helix angle, $\alpha = 20°$ to $45°$

Addendum $= 0.8\, m$ (Maximum)

Dedendum $= 1\, m$ (Minimum)

Minimum total depth $= 1.8\, m$

Minimum clearance $= 0.2\, m$

Thickness of tooth $= 1.5708\, m$

Strength of Helical Gears

In helical gears, the contact between mating teeth is gradual, starting at one end and moving along the teeth so that at any instant the line of contact runs diagonally across the teeth. Therefore in order to find the strength of helical gears, a modified Lewis equation is used. It is given by

$$W_T = (\sigma_o \times C_v)\, b.\pi\, m.y'$$

where W_T = Tangential tooth load,

σ_o = Allowable static stress,

C_v = Velocity factor,

b = Face width,

m = Module, and

y' = Tooth form factor or Lewis factor corresponding to the formative or virtual or equivalent number of teeth.

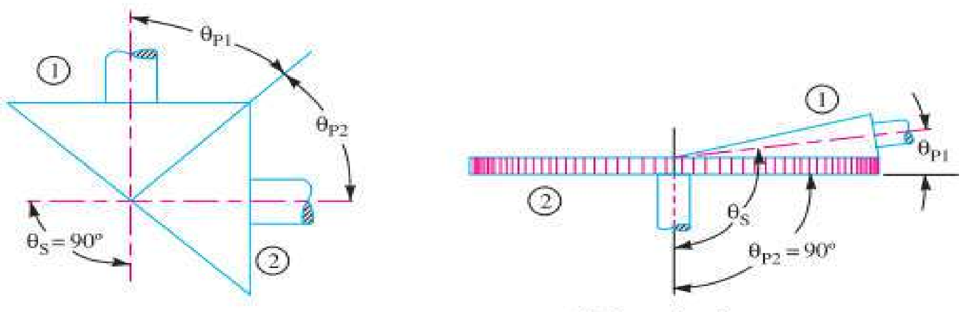
Bevel Gears

The bevel gears are used for transmitting power at a constant velocity ratio between two shafts whose axes intersect at a certain angle.

Classification of Bevel Gears

The bevel gears may be classified into the following types, depending upon the angles between the shafts and the pitch surfaces.

1. *Mitre gears.* When equal bevel gears (having equal teeth and equal pitch angles) connect two shafts whose axes intersect at right angle, as shown in Fig. 30.2 (*a*), then they are known as ***mitre gears.***



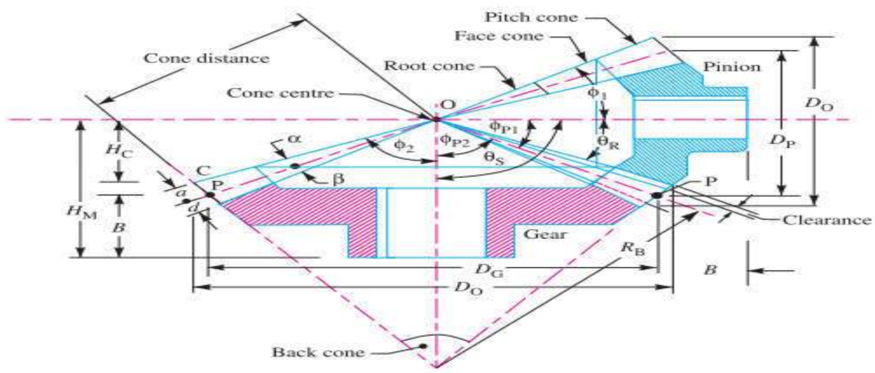
(*a*) Mitre gears. (*b*) Crown bevel gear.

2. *Angular bevel gears.* When the bevel gears connect two shafts whose axes intersect at an angle other than a right angle, then they are known as ***angular bevel gears.***

3. *Crown bevel gears.* When the bevel gears connect two shafts whose axes intersect at an angle greater than a right angle and one of the bevel gears has a pitch angle of 90º, then it is known as a crown gear. The crown gear corresponds to a rack in spur gearing

4. *Internal bevel gears.* When the teeth on the bevel gear are cut on the inside of the pitch cone, then they are known as ***internal bevel gears.***

Terms used in Bevel Gears



1. ***Pitch cone.*** It is a cone containing the pitch elements of the teeth.

2. ***Cone centre.*** It is the apex of the pitch cone. It may be defined as that point where the axes of two mating gears intersect each other.

3. ***Pitch angle.*** It is the angle made by the pitch line with the axis of the shaft. It is denoted by 'θ_P'.

4. ***Cone distance.*** It is the length of the pitch cone element. It is also called as a ***pitch cone radius.*** It is denoted by 'OP'. Mathematically, cone distance or pitch cone radius,

$$OP = \frac{\text{Pitch radius}}{\sin \theta_P} = \frac{D_P / 2}{\sin \theta_{P1}} = \frac{D_G / 2}{\sin \theta_{P2}}$$

5. ***Addendum angle.*** It is the angle subtended by the addendum of the tooth at the cone centre. It is denoted by 'α' Mathematically, addendum angle,

$$\alpha = \tan^{-1}\left(\frac{a}{OP}\right)$$

where a = Addendum, and OP = Cone distance.

6. ***Dedendum angle.*** It is the angle subtended by the dedendum of the tooth at the cone centre. It is denoted by 'β'. Mathematically, dedendum angle,

$$\beta = \tan^{-1}\left(\frac{d}{OP}\right)$$

where d = Dedendum, and OP = Cone distance.

7. ***Face angle.*** It is the angle subtended by the face of the tooth at the cone centre. It is denoted by 'ϕ'. The face angle is equal to the pitch angle *plus* addendum angle.

8. ***Root angle***. It is the angle subtended by the root of the tooth at the cone centre. It is denoted by 'θ_R'. It is equal to the pitch angle *minus* dedendum angle.

9. ***Back (or normal) cone***. It is an imaginary cone, perpendicular to the pitch cone at the end of the tooth.

10. Back cone distance. It is the length of the back cone. It is denoted by 'R_B'. It is also called back cone radius.

11. ***Backing.*** It is the distance of the pitch point (P) from the back of the boss, parallel to the pitch point of the gear. It is denoted by 'B'.

12. ***Crown height.*** It is the distance of the crown point (C) from the cone centre (O), parallel to the axis of the gear. It is denoted by 'H_C'.

13. ***Mounting height.*** It is the distance of the back of the boss from the cone centre. It is denoted by 'H_M'.

14. ***Pitch diameter.*** It is the diameter of the largest pitch circle.

15. ***Outside or addendum cone diameter***. It is the maximum diameter of the teeth of the gear. It is equal to the diameter of the blank from which the gear can be cut. Mathematically, outside diameter,

$$D_O = D_P + 2\,a \cos \theta_P$$

where D_P = Pitch circle diameter,

a = Addendum, and

θ_P = Pitch angle.

16. ***Inside or dedendum cone diameter***. The inside or the dedendum cone diameter is given by

$$D_d = D_P - 2d \cos \theta_P$$

where D_d = Inside diameter, and

Proportions for Bevel Gear

The proportions for the bevel gears may be taken as follows :

1. Addendum, $a = 1\ m$
2. Dedendum, $d = 1.2\ m$
3. Clearance = $0.2\ m$
4. Working depth = $2\ m$
5. *Thickness of tooth* $= 1.5708\ m$

Strength of Bevel Gears

The strength of a bevel gear tooth is obtained in a similar way as discussed in the previous articles. The modified form of the Lewis equation for the tangential tooth load is given as follows:

$$W_T = (\sigma_o \times C_v)\, b.\pi\, m.y' \left(\frac{L - b}{L}\right)$$

where

σ_o = Allowable static stress,

C_v = Velocity factor,

$= \dfrac{3}{3 + v}$, for teeth cut by form cutters,

$= \dfrac{6}{6 + v}$, for teeth generated with precision machines,

v = Peripheral speed in m / s,

b = Face width,

m = Module,

y' = Tooth form factor (or Lewis factor) for the equivalent number of teeth,

L = Slant height of pitch cone (or cone distance),

$$= \sqrt{\left(\frac{D_G}{2}\right)^2 + \left(\frac{D_P}{2}\right)^2}$$

Design of a Shaft for Bevel Gears

In designing a pinion shaft, the following procedure may be adopted :

1. First of all, find the torque acting on the pinion. It is given by

$$T = \frac{P \times 60}{2\pi N_P} \text{ N-m}$$

where P = Power transmitted in watts, and

N_P = Speed of the pinion in r.p.m.

2. Find the tangential force (W_T) acting at the mean radius (R_m) of the pinion. We know that

$$W_T = T / R_m$$

3. Now find the axial and radial forces (*i.e.* W_{RH} and W_{RV}) acting on the pinion shaft as discussed above.

4. Find resultant bending moment on the pinion shaft as follows :

The bending moment due to W_{RH} and W_{RV} is given by

$$M_1 = W_{RV} \times \text{Overhang} - W_{RH} \times R_m$$

and bending moment due to W_T,

$$M_2 = W_T \times \text{Overhang}$$

∴ Resultant bending moment,

$$M = \sqrt{(M_1)^2 + (M_2)^2}$$

5. Since the shaft is subjected to twisting moment (T) and resultant bending moment (M), therefore equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2}$$

6. Now the diameter of the pinion shaft may be obtained by using the torsion equation. We know that

$$T_e = \frac{\pi}{16} \times \tau (d_P)^3$$

where d_P = Diameter of the pinion shaft, and

τ = Shear stress for the material of the pinion shaft.

7. The same procedure may be adopted to find the diameter of the gear shaft.

Worm Gears

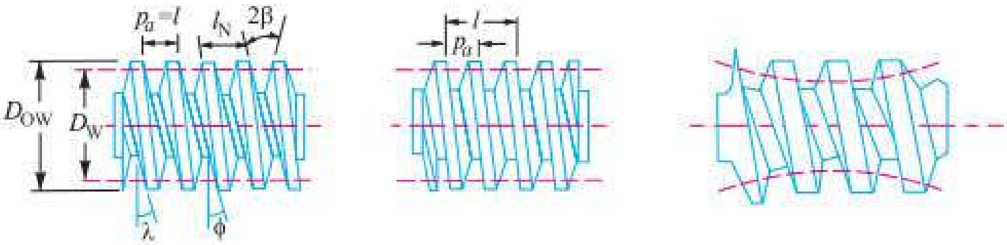
The worm gears are widely used for transmitting power at high velocity ratios between non-intersecting shafts that are generally, but not necessarily, at right angles. It can give velocity ratios as high as 300 : 1 or more in a single step in a minimum of space, but it has a lower efficiency. The worm gearing is mostly used as a speed reducer, which consists of worm and a worm wheel or gear. The worm (which is the driving member) is usually of a cylindrical form having threads of the same shape as that of an involute rack. The threads of the worm may be left handed or right handed and single or multiple threads. The worm wheel or gear (which is the driven member) is similar to a helical gear with a face curved to conform to the shape of the worm.

Types of Worms

The following are the two types of worms :

1. Cylindrical or straight worm, and

2. Cone or double enveloping worm.

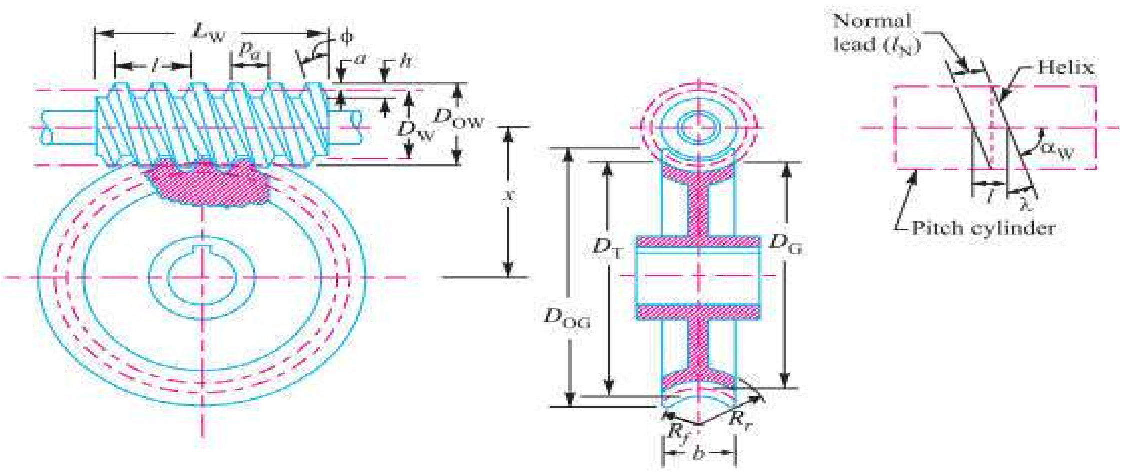


Single threaded. Double threaded.

(*a*) Cylindrical or straight worm. (*b*) Cone or double enveloping worm.

The ***cylindrical*** or ***straight worm****,* as shown in Fig.(*a*), is most commonly used. The shape of the thread is involute helicoid of pressure angle 14 ½° for single and double threaded worms and 20° for triple and quadruple threaded worms. The worm threads are cut by a straight sided milling cutter having its diameter not less than the outside diameter of worm or greater than 1.25 times the outside diameter of worm. The ***cone*** or ***double enveloping worm****,* as shown in Fig. (*b*), is used to some extent, but it requires extremely accurate alignment

Terms used in Worm Gearing



1. *Axial pitch.* It is also known as ***linear pitch*** of a worm. It is the distance measured axially (*i.e.* parallel to the axis of worm) from a point on one thread to the corresponding point on the adjacent thread on the worm, as shown in Fig. 31.3. It may be noted that the axial pitch (p_a) of a worm is equal to the circular pitch (p_c) of the mating worm gear, when the shafts are at right angles.

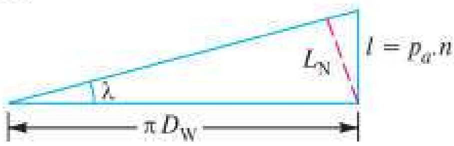
2. *Lead.* It is the linear distance through which a point on a thread moves ahead in one revolution of the worm. For single start threads, lead is equal to the axial pitch, but for multiple start threads, lead is equal to the product of axial pitch and number of starts. Mathematically,

Lead, $l = p_a \cdot n$

where p_a = Axial pitch ; and n = Number of starts.

3. *Lead angle.* It is the angle between the tangent to the thread helix on the pitch cylinder and the plane normal to the axis of the worm. It is denoted by λ.

A little consideration will show that if one complete turn of a worm thread be imagined to be unwound from the body of the worm, it will form an inclined plane whose base is equal to the pitch circumference of the worm and



Strength of Worm Gear Teeth

In finding the tooth size and strength, it is safe to assume that the teeth of worm gear are always weaker than the threads of the worm. In worm gearing, two or more teeth are usually in contact, but due to uncertainty of load distribution among themselves it is assumed that the load is transmitted by one tooth only. We know that according to Lewis equation,

$$W_T = (\sigma_o \cdot C_v)\, b \cdot \pi m \cdot y$$

where W_T = Permissible tangential tooth load or beam strength of gear tooth,

σ_o = Allowable static stress,

C_v = Velocity factor,

b = Face width,

m = Module, and

y = Tooth form factor or Lewis factor.

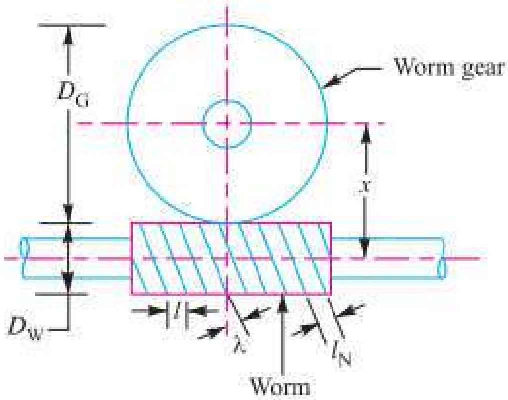
Wear Tooth Load for Worm Gear

The limiting or maximum load for wear (WW) is given by

$$W_W = D_G \cdot b \cdot K$$

Design of Worm Gearing

In designing a worm and worm gear, the quantities like the power transmitted, speed, velocity ratio and the centre distance between the shafts are usually given and the quantities such as lead angle, lead and number of threads on the worm are to be determined. In order to determine the satisfactory combination of lead angle, lead and centre distance, the following method may be used:



we find that the centre distance,

$$x = \frac{D_W + D_G}{2}$$

The centre distance may be expressed in terms of the axial lead (l), lead angle (λ) and velocity ratio ($V.R.$), as follows :

$$x = \frac{l}{2\pi}(\cot\lambda + V.R.)$$

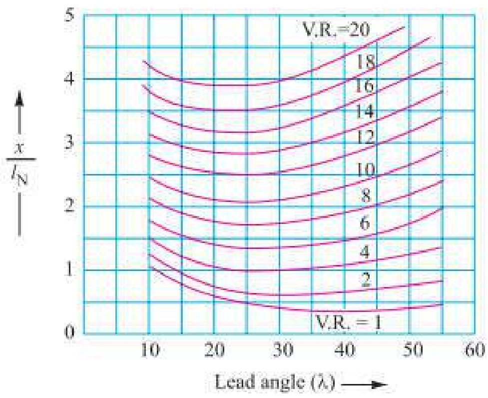
In terms of normal lead ($l_N = l\cos\lambda$), the above expression may be written as :

$$x = \frac{l_N}{2\pi}\left(\frac{1}{\sin\lambda} + \frac{V.R.}{\cos\lambda}\right)$$

or

$$\frac{x}{l_N} = \frac{1}{2\pi}\left(\frac{1}{\sin\lambda} + \frac{V.R.}{\cos\lambda}\right) \quad \text{...}(i)$$

Since the velocity ratio ($V.R.$) is usually given, therefore the equation **(*i*)** contains three variables *i.e.* x, l_N and λ. The right hand side of the above expression may be calculated for various values of velocity ratios and the curves



This minimum value represents the minimum centre distance that can be used with a given lead or inversely the maximum leadthat can be used with a given centre distance.

Chapter 2

BEARINGS

BEARINGS

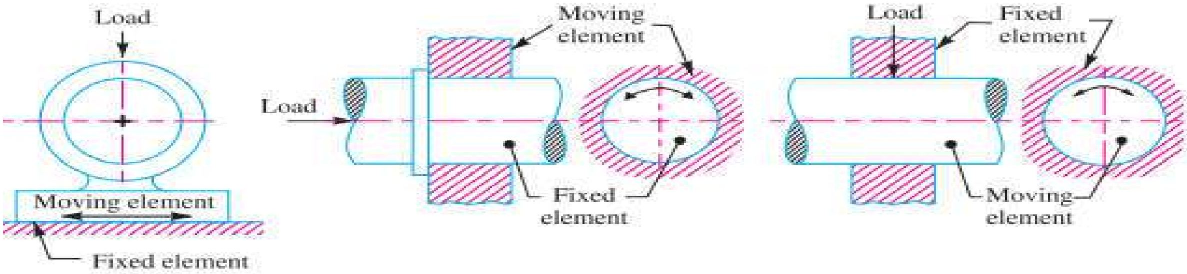
A bearing is a machine element which support another moving machine element (known as journal). It permits a relative motion between the contact surfaces of the members, while carrying the load

Classification of Bearing

Depending upon the direction of load to be supported

In ***radial bearings***, the load acts perpendicular to the direction of motion of the moving element as shown in Fig (*a*) and (*b*).

In ***thrust bearings,*** the load acts along the axis of rotation as shown in Fig (*c*).



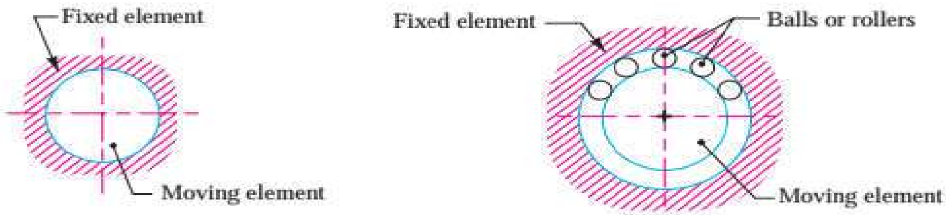
(*a*) Radial bearing. (*b*) Radial bearing. (*c*) Thrust bearing.

Depending upon the nature of contact. The bearings under this group are classified as :

(a) Sliding contact bearings, and ***(b)*** Rolling contact bearings.

In ***sliding contact bearings,*** as shown in Fig. (*a*), the sliding takes place along the surfaces of contact between the moving element and the fixed element. The sliding contact bearings are also known as ***plain bearings***

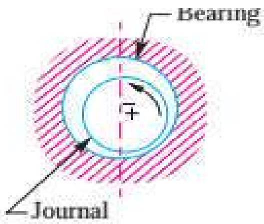
In ***rolling contact bearings,*** as shown in Fig. (*b*), the steel balls or rollers, are interposed between the moving and fixed elements. The balls offer rolling friction at two points for each ball or roller



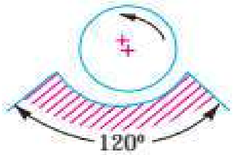
(*a*) Sliding contact bearing. (*b*) Rolling contact bearings.

Types of Sliding Contact Bearings

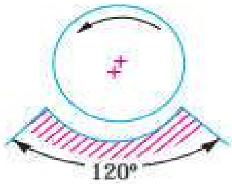
The sliding contact bearings in which the sliding action is guided in a straight line and carrying radial loads,



(a) Full journal bearing.



(b) Partial journal bearing.



(c) Fitted journal bearing.

The sliding contact bearings in which the sliding action is along the circumference of a circle or an arc of a circle and carrying radial loads are known as ***journal*** or ***sleeve bearings.*** When the angle of contact of the bearing with the journal is 360° as shown in Fig.(*a*), then the bearing is called a ***full journal bearing.*** This type of bearing is commonly used in industrial machinery to accommodate bearing loads in any radial direction.

When the angle of contact of the bearing with the journal is 120°, as shown in Fig. (*b*), then the bearing is said to be ***partial journal bearing.*** This type of bearing has less friction than ful journal bearing, but it can be used only where the load is always in one direction. The most common application of the partial journal bearings is found in rail road car axles. The full and partial journal bearings may be called as ***clearance bearings*** because the diameter of the journal is less than that of bearing.

When a partial journal bearing has no clearance *i.e.* the diameters of the journal and bearing are equal, then the bearing is called a ***fitted bearing,*** as shown in Fig. (*c*)

The sliding contact bearings, according to the thickness of layer of the lubricant between the bearing and the journal, may also be classified as follows :

1. *Thick film bearings.* The thick film bearings are those in which the working surfaces are completely separated from each other by the lubricant. Such type of bearings are also called as ***hydrodynamic lubricated bearings.***

2. *Thin film bearings.* The thin film bearings are those in which, although lubricant is present, the working surfaces partially contact each other atleast part of the time. Such type of bearings are also called ***boundary lubricated bearings.***

3. *Zero film bearings.* The zero film bearings are those which operate without any lubricant present.

4. *Hydrostatic or externally pressurized lubricated bearings.* The hydrostatic bearings are those which can support steady loads without any relative motion between the journal and the bearing. This is achieved by forcing externally pressurized lubricant between the members

Hydrodynamic Lubricated Bearings

Hydrodynamic lubricated bearings, there is a thick film of lubricant between the journal and the bearing. A little consideration will show that when the bearing is supplied with sufficient lubricant, a pressure is build up in the clearance space when the journal is rotating about an axis that is eccentric with the bearing axis. The load can be supported by this fluid pressure without any actual contact between the journal and bearing. The load carrying ability of a hydrodynamic bearing arises simply because a viscous fluid resists being pushed around. Under the proper conditions, this resistance to motion will develop a pressure distribution in the lubricant film that can support a useful load. The load supporting pressure in hydrodynamic bearings arises from either

1. the flow of a viscous fluid in a converging channel (known as ***wedge film lubrication***),
2. the resistance of a viscous fluid to being squeezed out from between approaching surfaces(known as ***squeeze film lubrication***).

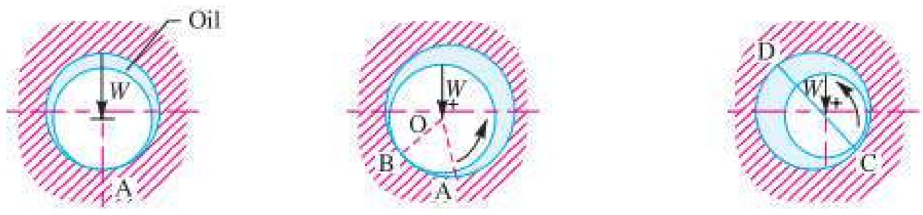
Assumptions in Hydrodynamic Lubricated Bearings

The following are the basic assumptions used in the theory of hydrodynamic lubricated bearings:

1. The lubricant obeys Newton's law of viscous flow.
2. The pressure is assumed to be constant throughout the film thickness.
3. The lubricant is assumed to be incompressible.
4. The viscosity is assumed to be constant throughout the film.
5. The flow is one dimensional, *i.e.* the side leakage is neglected.

Wedge Film Journal Bearings

The load carrying ability of a wedge-film journal bearing results when the journal and/or the bearing rotates relative to the load. The most common case is that of a steady load, a fixed (nonrotating) bearing and a rotating journal. Fig. (*a*) shows a journal at rest with metal to metal contact at *A* on the line of action of the supported load. When the journal rotates slowly in the anticlockwise direction, as shown in Fig. (*b*), the point of contact will move to *B*, so that the angle *AOB* is the angle of sliding friction of the surfaces in contact at *B*.



(*a*) At rest. (*b*) Slow speed. (*c*) High speed.

When the speed of the journal is increased, a continuous fluid film is established as in Fig.(*c*). The centre of the journal has moved so that the minimum film thickness is at *C*. It may be noted that from *D* to *C* in the direction of motion, the film is continually narrowing and hence is a converging film.

Squeeze Film Journal Bearing

We have seen in the previous article that in a wedge film journal bearing, the bearing carries a steady load and the journal rotates relative to the bearing. But in certain cases, the bearings oscillate or rotate so slowly that the wedge film cannot provide a satisfactory film thickness. If the load is uniform or varying in magnitude while acting in a constant direction, this becomes a thin film or possibly a zero film problem. But if the load reverses its direction, the squeeze film may develop sufficient capacity to carry the dynamic loads without contact between the journal and the bearing. Such bearings are known as ***squeeze film journal bearing.***

Modes of Lubrications

The lubricants are used in bearings to reduce friction between the rubbing surfaces and to carry away the heat generated by friction. It also protects the bearing against corrosion. All lubricants are classified into the following three groups

1. Liquid, **2.** Semi-liquid, and **3.** Solid.

The ***liquid lubricants*** usually used in bearings are mineral oils and synthetic oils. The mineral oils are most commonly used because of their cheapness and stability. The liquid lubricants are usually preferred where they may be retained. A grease is a ***semi-liquid lubricant*** having higher viscosity than oils. The greases are employed where slow speed and heavy pressure exist and where oil drip from the bearing is undesirable. The ***solid lubricants*** are useful in reducing friction where oil films cannot be maintained because of pressures or temperatures. They should be softer than materials being lubricated. A graphite is the most common of the solid lubricants either alone or mixed with oil or grease

Properties of Lubricants

1. ***Viscosity.*** It is the measure of degree of fluidity of a liquid. It is a physical property by virtue of which an oil is able to form, retain and offer resistance to shearing a buffer film-under heat and pressure.

According to Newton's law of viscous flow, the magnitude of this shear stress varies directly with the velocity gradient (dV / dy). It is assumed that

(*a*) the lubricant completely fills the space between the two surfaces,

(*b*) the velocity of the lubricant at each surface is same as that of the surface, and

(*c*) any flow of the lubricant perpendicular to the velocity of the plate is negligible.

$$\therefore \qquad \tau = \frac{P}{A} \propto \frac{dV}{dy} \quad \text{or} \quad \tau = Z \times \frac{dV}{dy}$$

where Z is a constant of proportionality and is known as ***absolute viscosity*** (or simply viscosity) of the lubricant.

2. ***Oiliness.*** It is a joint property of the lubricant and the bearing surfaces in contact. It is a measure of the lubricating qualities under boundary conditions where base metal to metal is prevented only by absorbed film. There is no absolute measure of oiliness
3. ***Viscosity index.*** The term viscosity index is used to denote the degree of variation of viscosity
with temperature.
4. ***Flash point.*** It is the lowest temperature at which an oil gives off sufficient vapour to support a momentary flash without actually setting fire to the oil when a flame is brought within 6 mm at the surface of the oil.
5. ***Fire point.*** It is the temperature at which an oil gives off sufficient vapour to burn it continuously when ignited.
6. ***Pour point or freezing point.*** It is the temperature at which an oil will cease to flow when cooled.
7. ***Density.*** This property has no relation to lubricating value but is useful in changing the kinematic viscosity to absolute viscosity. Mathematically
Absolute viscosity = $\rho \times$ Kinematic viscosity (in m^2/s)
where ρ = Density of the lubricating oil.

Reynolds Equation

A theoretical analysis of hydrodynamic lubrication was carried out by Osborne Reynolds. The equations resulted from the analysis has served a basis for designing hydrodynamically lubricated bearings. The following assumptions were made by Reynolds in the analysis

Bearing Characteristic Number and Bearing Modulus for Journal Bearings

The coefficient of friction in design of bearings is of great importance, because it affords a means for determining the loss of power due to bearing friction. It has been shown by experiments that the coefficient of friction for a full lubricated journal bearing is a function of three variables, *i.e*

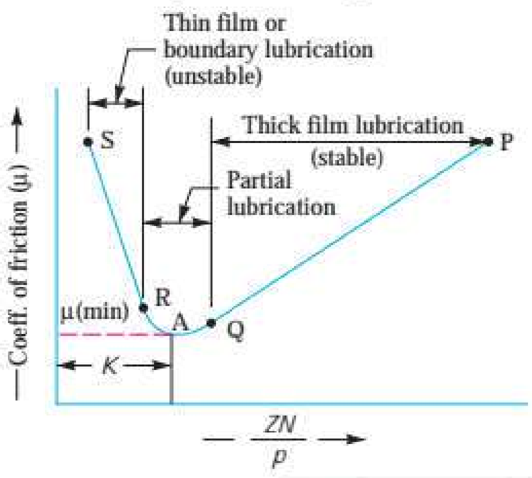
(*i*) $\frac{ZN}{p}$; (*ii*) $\frac{d}{c}$; and (*iii*) $\frac{l}{d}$

Therefore the coefficient of friction may be expressed as

$$\mu = \phi\left(\frac{ZN}{p}, \frac{d}{c}, \frac{l}{d}\right)$$

where
μ = Coefficient of friction,
ϕ = A functional relationship,
Z = Absolute viscosity of the lubricant, in kg / m-s,
N = Speed of the journal in r.p.m.,
p = Bearing pressure on the projected bearing area in N/mm^2,
= Load on the journal ÷ $l \times d$
d = Diameter of the journal,
l = Length of the bearing, and
c = Diametral clearance.

The factor ZN/p is termed as ***bearing characteristic number*** and is a dimensionless number. The variation of coefficient of friction with the operating values of bearing characteristic number we see that the minimum amount of friction occurs at A and at this point the value of ZN/p is known as ***bearing modulus*** which is denoted by K. The bearing should not be operated at this value of bearing modulus, because a slight decrease in speed or slight increase in pressure will break the oil film and make the journal to operate with metal to metal contact. This will result in high friction, wear and heating. In order to prevent such conditions, the bearing should be designed for a value of ZN/p at least three times the minimum value of bearing modulus (K). If the bearing is subjected to large fluctuations of load and heavy impacts, the value of $ZN/p = 15\,K$ may be used.



From above, it is concluded that when the value of ZN/p is greater than K, then the bearing will operate with thick film lubrication or under hydrodynamic conditions.

Coefficient of Friction for Journal Bearings

In order to determine the coefficient of friction for well lubricated full journal bearings, the following empirical relation established

*Coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k \quad \text{... (when Z is in kg / m-s and } p \text{ is in N / mm}^2\text{)}$$

where Z, N, p, d and c have usual meanings as discussed in previous article, and

k = Factor to correct for end leakage. It depends upon the ratio of length to the diameter of the bearing (*i.e.* l / d).

= 0.002 for l / d ratios of 0.75 to 2.8.

Critical Pressure of the Journal Bearing

The pressure at which the oil film breaks down so that metal to metal contact begins, is known as **critical pressure** or the **minimum operating pressure** of the bearing. It may be obtained by the following empirical relation, *i.e.*

Critical pressure or minimum operating pressure,

$$p = \frac{ZN}{4.75 \times 10^6}\left(\frac{d}{c}\right)^2\left(\frac{l}{d+l}\right) \text{ N/mm}^2 \quad \text{...(when Z is in kg / m-s)}$$

Sommerfeld Number

The Sommerfeld number is also a dimensionless parameter used extensively in the design of journal bearings. Mathematically,

$$\text{Sommerfeld number} = \frac{ZN}{p}\left(\frac{d}{c}\right)^2$$

For design purposes, its value is taken as follows :

$$\frac{ZN}{p}\left(\frac{d}{c}\right)^2 = 14.3 \times 10^6 \quad \text{... (when } Z \text{ is in kg / m-s and } p \text{ is in N / mm}^2\text{)}$$

Heat Generated in a Journal Bearing

The heat generated in a bearing is due to the fluid friction and friction of the parts having relative motion. Mathematically, heat generated in a bearing,

$$Q_g = \mu.W.V \text{ N-m/s or J/s or watts} \quad \text{...(}i\text{)}$$

where μ = Coefficient of friction,

W = Load on the bearing in N,

Heat dissipated by the bearing,

$$Q_d = C.A\,(t_b - t_a) \text{ J/s or W} \quad \text{... } (\because \text{ 1 J/s = 1 W}) \text{ ...(}ii\text{)}$$

where C = Heat dissipation coefficient in W/m^2/°C,

A = Projected area of the bearing in m^2 = $l \times d$,

t_b = Temperature of the bearing surface in °C, and

t_a = Temperature of the surrounding air in °C.

Design Procedure for Sliding Bearing

The following procedure may be adopted in designing journal bearings, when the bearing load, the diameter and the speed of the shaft are known. Sliding Contact Bearings „

1. Determine the bearing length by choosing a ratio ofl / d from Table from data book.

2. Check the bearing pressure, $p = W / l.d$ from Table from data book. for probable satisfactory value.
3. Assume a lubricant from Table from data book and its operating temperature ($t0$). This temperature should be between 26.5°C and 60°C with 82°C as a maximum for high temperature installations such as steam turbines.
4. Determine the operating value of ZN / p for the assumed bearing temperature and check this value with corresponding values in Table from data book. to determine the possibility of maintaining fluid film operation.
5. Assume a clearance ratio c / d from Table from data book..
6. Determine the coefficient of friction (μ) by using the relation as discussed above.
7. Determine the heat generated by using the relation as discussed above.
8. Determine the heat dissipated by using the relation as discussed above.
9. Determine the thermal equilibrium to see that the heat dissipated becomes atleast equal to the heat generated. In case the heat generated is more than the heat dissipated then either the bearing is redesigned or it is artificially cooled by water.

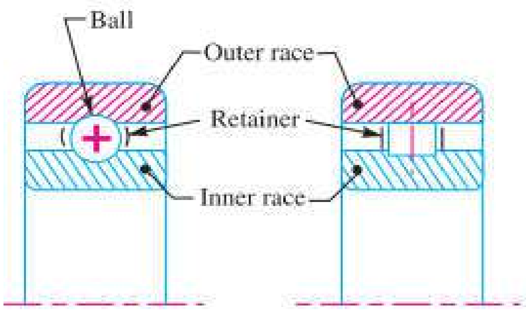
Rolling Contact Bearings

In rolling contact bearings, the contact between the bearing surfaces is rolling instead of sliding as in sliding contact bearings. Due to this low friction offered by rolling contact bearings, these are called ***antifriction bearings.***

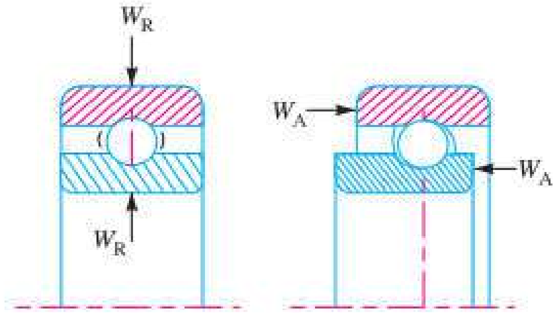
Types of Rolling Contact Bearings

Following are the two types of rolling contact bearings:

1. Ball bearings; and **2.** Roller bearings.



(*a*) Ball bearing. (*b*) Roller bearing.



(*a*) Radial ball bearing. (*b*) Thrust ball bearing.

The ***ball and roller bearings*** consist of an inner race which is mounted on the shaft or journal and an outer race which is carried by the housing or casing. In between the inner and outer race, there are balls or rollers as shown in Figure. A number of balls or rollers are used and these are held at proper distances by retainers so that they do not touch each other. The retainers are thin strips and is usually in two parts which are assembled after the balls have been properly spaced. The ball bearings are used for light loads and the roller bearings are used for heavier loads. The rolling contact bearings, depending upon the load to be carried, are classified as :

(*a*) Radial bearings, and **(*b*)** Thrust bearings.

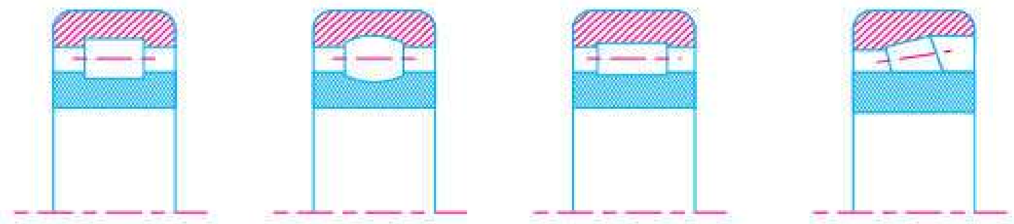
The radial and thrust ball bearings are shown in Fig. (*a*) and (*b*) respectively. When a ball bearing supports only a radial load (*W*R), the plane of rotation of the ball is normal to the centre line of the bearing, as shown in Fig. (*a*). The action of thrust load (*W*A) is to shift the plane of rotation of the balls, as shown in Fig. (*b*). The radial and thrust loads both may be carried simultaneously.

Types of Roller Bearings

Following are the principal types of roller bearings :

1. Cylindrical roller bearings. A cylindrical roller bearing is shown in Fig.(a). These bearings have short rollers guided in a cage. These bearings are relatively rigid against radial motion and have the lowest coefficient of friction of any form of heavy duty rolling-contact bearings. Such type of bearings are used in high speed service.

2. Spherical roller bearings. A spherical roller bearing is shown in Fig. (b). These bearings are self-aligning bearings. The self-aligning feature is achieved by grinding one of the races in the form of sphere. These bearings can normally tolerate angular misalignment in the order of
and when used with a double row of rollers, these can carry thrust loads in either direction.



(*a*) Cylindrical roller. (*b*) Spherical roller. (*c*) Needle roller. (*d*) Tapered roller.

3. Needle roller bearings. A needle roller bearing is shown in Fig. (c). These bearings are relatively slender and completely fill the space so that neither a cage nor a retainer is needed. These bearings are used when heavy loads are to be carried with an oscillatory motion, e.g. piston pin bearings in heavy duty diesel engines, where the reversal of motion tends to keep the rollers in correct alignment.

4. Tapered roller bearings. A tapered roller bearing is shown in Fig.(d). The rollers and race ways of these bearings are truncated cones whose elements intersect at a common point. Such type of bearings can carry both radial and thrust loads. These bearings are available in various combinations as double row bearings and with different cone angles for use with different relative magnitudes of radial and thrust loads

SELECTION GUIDELINES

1. For radial ball bearings, the basic static radial load rating (C_0) is given by

$$C_0 = f_0 . i . Z . D^2 \cos \alpha$$

where i = Number of rows of balls in any one bearing,

Z = Number of ball per row,

D = Diameter of balls, in mm,

α = Nominal angle of contact *i.e.* the nominal angle between the line of action of the ball load and a plane perpendicular to the axis of bearing, and

f_0 = A factor depending upon the type of bearing.

The value of factor (f_0) for bearings made of hardened steel are taken as follows :

f_0 = 3.33, for self-aligning ball bearings

= 12.3, for radial contact and angular contact groove ball bearings.

2. For radial roller bearings, the basic static radial load rating is given by

$$C_0 = f_0 . i . Z . l_e . D \cos \alpha$$

where i = Number of rows of rollers in the bearing,

Z = Number of rollers per row,

l_e = Effective length of contact between one roller and that ring (or washer) where the contact is the shortest (in mm). It is equal to the overall length of roller ***minus*** roller chamfers or grinding undercuts,

D = Diameter of roller in mm. It is the mean diameter in case of tapered rollers,

α = Nominal angle of contact. It is the angle between the line of action of the roller resultant load and a plane perpendicular to the axis of the bearing, and

f_0 = 21.6, for bearings made of hardened steel.

3. For thrust ball bearings, the basic static axial load rating is given by

$$C_0 = f_0 . Z . D^2 \sin \alpha$$

where Z = Number of balls carrying thrust in one direction, and

f_0 = 49, for bearings made of hardened steel.

4. For thrust roller bearings, the basic static axial load rating is given by

$$C_0 = f_0 . Z . l_e . D . \sin \alpha$$

where Z = Number of rollers carrying thrust in one direction, and

f_0 = 98.1, for bearings made of hardened steel.

Static Equivalent Load

The static equivalent load may be defined as the static radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller bearings) which, if applied, would cause the same total permanent deformation at the most heavily stressed ball (or roller) and race contact as that which occurs under the actual conditions of loading.

The static equivalent radial load (W_{0R}) for radial or roller bearings under combined radial and axial or thrust loads is given by the greater magnitude of those obtained by the following two equations, *i.e.*

1. $W_{0R} = X_0 . W_R + Y_0 . W_A$; and **2.** $W_{0R} = W_R$

where W_R = Radial load,

W_A = Axial or thrust load,

X_0 = Radial load factor, and

Y_0 = Axial or thrust load factor.

Basic Dynamic Load Rating of Rolling Contact Bearings

The basic dynamic load rating (C) in newtons for ball and roller bearings may be obtained as discussed below :

1. According to IS: 3824 (Part 1)– 1983, the basic dynamic radial load rating for radial and angular contact ball bearings, except the filling slot type, with balls not larger than 25.4 mm in diameter, is given by

$$C = f_c\,(i \cos \alpha)^{0.7}\,Z^{2/3}\,.\,D^{1.8}$$

and for balls larger than 25.4 mm in diameter,

$$C = 3.647\,f_c\,(i \cos \alpha)^{0.7}\,Z^{2/3}\,.\,D^{1.4}$$

2. According to IS: 3824 (Part 2)–1983, the basic dynamic radial load rating for radial roller bearings is given by

$$C = f_c\,(i . l_e \cos \alpha)^{7/9}\,Z^{3/4} . D^{29/27}$$

3. According to IS: 3824 (Part 3)–1983, the basic dynamic axial load rating for single row, single or double direction thrust ball bearings is given as follows :

(*a*) For balls not larger than 25.4 mm in diameter and $\alpha = 90°$,

$$C = f_c\,.\,Z^{2/3}\,.\,D^{1.8}$$

(*b*) For balls not larger than 25.4 mm in diameter and $\alpha \neq 90°$,

$$C = f_c\,(\cos \alpha)^{0.7} \tan \alpha . Z^{2/3}\,.\,D^{1.8}$$

(*c*) For balls larger than 25.4 mm in diameter and $\alpha = 90°$

$$C = 3.647\,f_c\,.\,Z^{2/3}\,.\,D^{1.4}$$

(*d*) For balls larger than 25.4 mm in diameter and $\alpha \neq 90°$,

$$C = 3.647\,f_c\,(\cos \alpha)^{0.7} \tan \alpha\,.\,Z^{2/3}\,.\,D^{1.4}$$

4. According to IS: 3824 (Part 4)–1983, the basic dynamic axial load rating for single row, single or double direction thrust roller bearings is given by

$$C = f_c\,.\,l_e^{7/9}\,.\,Z^{3/4}\,.\,D^{29/27} \qquad \text{... (when } \alpha = 90°\text{)}$$

$$= f_c\,(l_e \cos \alpha)^{7/9} \tan \alpha . Z^{3/4} . D^{29/27} \qquad \text{... (when } \alpha \neq 90°\text{)}$$

Dynamic Equivalent Load for Rolling Contact Bearings

The dynamic equivalent load may be defined as the constant stationary radial load (in case of radial ball or roller bearings) or axial load (in case of thrust ball or roller bearings) which, if applied to a bearing with rotating inner ring and stationary outer ring, would give the same life as that which the bearing will attain under the actual conditions of load and rotation.

The dynamic equivalent radial load (W) for radial and angular contact bearings, except the filling slot types, under combined constant radial load (W_R) and constant axial or thrust load (W_A) is given by

$$W = X \cdot V \cdot W_R + Y \cdot W_A$$

where V = A rotation factor,

= 1, for all types of bearings when the inner race is rotating,

= 1, for self-aligning bearings when inner race is stationary,

= 1.2, for all types of bearings except self-aligning, when inner race is stationary.

Reliability of a Bearing

We have already discussed in the previous article that the rating life is the life that 90 per cent of a group of identical bearings will complete or exceed before the first evidence of fatigue develops. The reliability (R) is defined as the ratio of the number of bearings which have successfully completed L million revolutions to the total number of bearings under test. Sometimes, it becomes necessary to select a bearing having a reliability of more than 90%. According to Wiebull, the relation between the bearing life and the reliability is given as

$$\log_e\left(\frac{1}{R}\right) = \left(\frac{L}{a}\right)^b \quad \text{or} \quad \frac{L}{a} = \left[\log_e\left(\frac{1}{R}\right)\right]^{1/b} \qquad ...(i)$$

where L is the life of the bearing corresponding to the desired reliability R and a and b are constants whose values are

$$a = 6.84, \text{ and } \quad b = 1.17$$

If L_{90} is the life of a bearing corresponding to a reliability of 90% (*i.e.* R_{90}), then

$$\frac{L_{90}}{a} = \left[\log_e\left(\frac{1}{R_{90}}\right)\right]^{1/b} \qquad ...(ii)$$

Dividing equation (*i*) by equation (*ii*), we have

$$\frac{L}{L_{90}} = \left[\frac{\log_e(1/R)}{\log_e(1/R_{90})}\right]^{1/b} = {}^*6.85\,[\log_e(1/R)]^{1/1.17} \qquad ...(\because b = 1.17)$$

Selection of Radial Ball Bearings

In order to select a most suitable ball bearing, first of all, the basic dynamic radial load is calculated. It is then multiplied by the service factor (KS) to get the design basic dynamic radial load capacity. The service factor for the ball bearings is shown in the following table

S.No.	Type of service	Service factor (K_S) for radial ball bearings
1.	Uniform and steady load	1.0
2.	Light shock load	1.5
3.	Moderate shock load	2.0
4.	Heavy shock load	2.5
5.	Extreme shock load	3.0

Comparison of Rolling Contact Bearings Over Sliding Contact Bearings

The following are some advantages and disadvantages of rolling contact bearings over sliding contact bearings.

Advantages

1. Low starting and running friction except at very high speeds.
2. Ability to withstand momentary shock loads.
3. Accuracy of shaft alignment.
4. Low cost of maintenance, as no lubrication is required while in service
5. Small overall dimensions.
6. Reliability of service.
7. Easy to mount and erect.
8. Cleanliness.

Disadvantages

1. More noisy at very high speeds.
2. Low resistance to shock loading.
3. More initial cost.
4. Design of bearing housing complicated

UNIT-3

DESIGN OF FLYWHEEL

Introduction

A flywheel used in machines serves as a reservoir which stores energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than supply. A flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed

The flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed. In other words, a flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation. It does not control the speed variations caused by the varying load.

Coefficient of Fluctuation of Speed

The difference between the maximum and minimum speeds during a cycle is called the ***maximum fluctuation of speed.*** The ratio of the maximum fluctuation of speed to the mean speed is called ***coefficient of fluctuation of speed***

∴ Coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{N} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \quad \text{...(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \quad \text{...(In terms of linear speeds)}$$

Coefficient of Fluctuation of Energy

It is defined as the ratio of the maximum fluctuation of energy to the work done per cycle. It is usually denoted by C_E. Mathematically, coefficient of fluctuation of energy

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

$$\text{Work done/cycle} = T_{mean} \times \theta$$

The mean torque (*Tmean*) in N-m may be obtained by using the following relation *i.e*

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

The workdone per cycle may also be obtained by using the following relation

$$\text{Workdone / cycle} = \frac{P \times 60}{n}$$

Energy Stored in a Flywheel

A flywheel is shown in Figure. We have already discussed that when a flywheel absorbs energy its speed increases and when it gives up energy its speed decreases.

Let m = Mass of the flywheel in kg

,k = Radius of gyration of the flywheel in metres,

I = Mass moment of inertia of the flywheel about the axis of rotation in kg-m^2 = $m.k^2$,

N_1 and N_2 = Maximum and minimum speeds during the cycle in r.p.m.,

ω_1 and ω_2 = Maximum and minimum angular speeds during the cycle in rad / s,

N = Mean speed during the cycle in r.p.m. = $\dfrac{N_1 + N_2}{2}$,

ω = Mean angular speed during the cycle in rad / s = $\dfrac{\omega_1 + \omega_2}{2}$

C_S = Coefficient of fluctuation of speed = $\dfrac{N_1 - N_2}{N}$ or $\dfrac{\omega_1 - \omega_2}{\omega}$

We know that mean kinetic energy of the flywheel,

$$E = \frac{1}{2} \times I.\omega^2 = \frac{1}{2} \times m.k^2.\omega^2 \text{ (in N-m or joules)}$$

As the speed of the flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,

$$\Delta E = \text{Maximum K.E.} - \text{Minimum K.E.} = \frac{1}{2} \times I(\omega_1)^2 - \frac{1}{2} \times I(\omega_2)^2$$

$$= \frac{1}{2} \times I \left[(\omega_1)^2 - (\omega_2)^2\right] = \frac{1}{2} \times I\,(\omega_1 + \omega_2)(\omega_1 - \omega_2)$$

$$= I.\omega\,(\omega_1 - \omega_2) \qquad \ldots\left(\because \omega = \frac{\omega_1 + \omega_2}{2}\right) \ldots(i)$$

$$= I.\omega^2 \left(\frac{\omega_1 - \omega_2}{\omega}\right) \qquad \ldots\text{[Multiplying and dividing by } \omega]$$

$$= I.\omega^2.C_S = m.k^2.\omega^2.C_S \qquad \ldots(\because I = m.k^2) \ldots(ii)$$

$$= 2\,E.C_S \qquad \ldots\left(\because E = \frac{1}{2} \times I.\omega^2\right) \ldots(iii)$$

The radius of gyration (k) may be taken equal to the mean radius of the rim (R), because the thickness of rim is very small as compared to the diameter of rim. Therefore substituting $k = R$ in equation (ii), we have

$$\Delta E = m.R^2.\omega^2.C_S = m.v^2.C_S \qquad \ldots(\because v = \omega.R)$$

From this expression, the mass of the flywheel rim may be determined.

Stresses in a Flywheel Rim

A flywheel, as shown in Figure below, consists of a rim at which the major portion of the mass or weight of flywheel is concentrated, a boss or hub for fixing the flywheel on to the shaft and a number of arms for supporting the rim on the hub.

The following types of stresses are induced in the rim of a flywheel:

1. Tensile stress due to centrifugal force,

2. Tensile bending stress caused by the restraint of the arms, and

3. The shrinkage stresses due to unequal rate of cooling of casting. These stresses may be very high but there is no easy method of determining. This stress is taken care of by a factor of safety.

We shall now discuss the first two types of stresses as follows:

Tensile stress due to the centrifugal force

The tensile stress in the rim due to the centrifugal force, assuming that the rim is unstrained by the arms, is determined in a similar way as a thin cylinder subjected to internal pressure.

Let b = Width of rim,

t = Thickness of rim

A = Cross-sectional area of rim = $b \times t$,

D = Mean diameter of flywheel

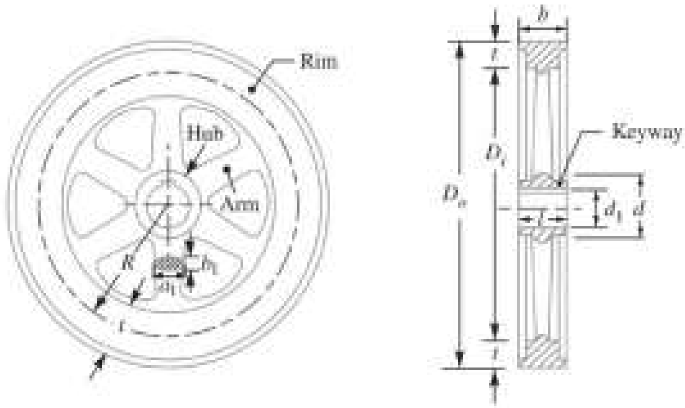
R = Mean radius of flywheel,

ρ = Density of flywheel material,

ω = Angular speed of flywheel,

v = Linear velocity of flywheel, and

σt = Tensile or hoop stress.



Consider a small element of the rim as shown shaded in Fig. below. Let it subtends an angle $\delta\theta$ at the centre of the flywheel.

Volume of the small element

$$= A.R.\delta\theta$$

$\therefore$ Mass of the small element,

$$dm = \text{Volume} \times \text{Density} = A.R.\delta\theta.\rho = \rho.A.R.\delta\theta$$

and centrifugal force on the element,

$$dF = dm.\omega^2.R = \rho.A.R.\delta\theta.\omega^2.R = \rho.A.R^2.\omega^2.\delta\theta$$

Vertical component of dF

$$= dF.\sin\theta = \rho.A.R^2.\omega^2.\delta\theta \sin\theta$$

∴ Total vertical bursting force across the rim diameter X-Y,

$$= \rho.A\,R^2.\omega^2 \int_0^\pi \sin\theta\,d\theta$$

$$= \rho.A.R^2.\omega^2\left[-\cos\theta\right]_0^\pi = 2\,\rho.A.R^2.\omega^2 \qquad \ldots(i)$$

This vertical force is resisted by a force of $2P$, such that

$$2P = 2\sigma_t \times A \qquad \ldots(ii)$$

From equations (i) and (ii), we have

$$2\rho A.R^2.\omega^2 = 2\,\sigma_t \times A$$

$$\therefore \qquad \sigma_t = \rho.R^2.\omega^2 = \rho.v^2 \qquad \ldots(\because v = \omega.R) \qquad \ldots(iii)$$

2. ***Tensile bending stress caused by restraint of the arms***

The tensile bending stress in the rim due to the restraint of the arms is based on the assumption that each portion of the rim between a pair of arms behaves like a beam fixed at both ends and uniformly loaded, as shown in Fig. 22.11, such that length between fixed ends,

$$l = \frac{\pi D}{n} = \frac{2\,\pi\,R}{n}, \text{ where } n = \text{Number of arms.}$$

The uniformly distributed load (w) per metre length will be equal to the centrifugal force between a pair of arms.

$$\therefore \qquad w = b.t.\rho.\omega^2.R \text{ N/m}$$

We know that maximum bending moment,

$$M = \frac{w.l^2}{12} = \frac{b.t\,\rho.\omega^2.R}{12}\left(\frac{2\,\pi\,R}{n}\right)^2$$

and section modulus, $$Z = \frac{1}{6}\,b \times t^2$$

∴ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{b.t\,\rho.\omega^2.R}{12}\left(\frac{2\,\pi\,R}{n}\right)^2 \times \frac{6}{b \times t^2}$$

$$= \frac{19.74\,\rho.\omega^2.R^3}{n^2.t} = \frac{19.74\,\rho.v^2.R}{n^2.t} \qquad \ldots(iv)$$

...(Substituting $\omega = v/R$)

Now total stress in the rim,

$$\sigma = \sigma_t + \sigma_b$$

If arms of flywheel do not stretch and are placed close together, then centrifugal force will not set up in rim. σ_t will be zero. On the other hand, if arms are stretched enough to allow free expansion of the rim due to centrifugal action, there will be no restraint due to arms, *i.e.* σ_b will be zero

It has been shown by G. Lanza that the arms of a flywheel stretch about $\frac{3}{4}$ th of the amount necessary for free expansion. Therefore the total stress in the rim,

$$= \frac{3}{4}\,\sigma_t + \frac{1}{4}\,\sigma_b = \frac{3}{4}\,\rho.v^2 + \frac{1}{4} \times \frac{19.74\,\rho.v^2.R}{n^2.t} \qquad \ldots(v)$$

$$= \rho.v^2\left(0.75 + \frac{4.935\,R}{n^2.t}\right)$$

UNIT-4

SPRINGS

SPRINGS

- A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed.
- Spring act as a flexible joint in between two parts or bodies
- Mechanical springs have varied use in different types of machines. In this chapter We shall briefly discuss here about some applications, followed by design of springs.

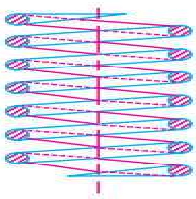
OBJECTIVES OF SPRING

1. To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, shock absorbers and vibration dampers.
2. To apply forces, as in brakes, clutches and spring loaded valves.
3. To control motion by maintaining contact between two elements as in cams and followers.
4. To measure forces, as in spring balances and engine indicators.
5. To store energy, as in watches, toys, etc

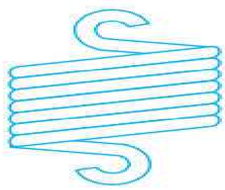
TYPES OF SPRINGS

- According to their shape:

  ***Helical springs*:**The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads.The two forms of helical springs are *Compression helical spring* as shown in Fig. (*a*) and *tension helical spring (b)*



(*a*) Compression helical spring.



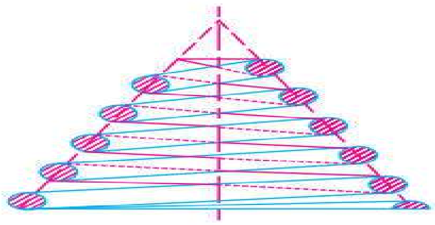
(*b*) Tension helical sprin

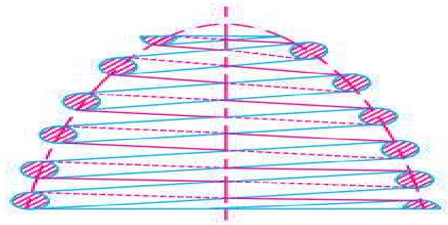
CLOSELY COILED & OPEN COILED HELICAL SPRINGS

- The helical springs are said to be ***closely coiled*** when the spring wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix and the wire is subjected to torsion. In other words, in a closely coiled helical spring, the helix angle is very small, it is usually less than 10°.
- In ***open coiled helical springs*,** the spring wire is coiled in such a way that there is a gap between the two consecutive turns, as a result of which the helix angle is large. Since the application of open coiled helical springs are limited, therefore our discussion shall confine to closely coiled helical springs only.

CONICAL AND VOLUTE SPRINGS

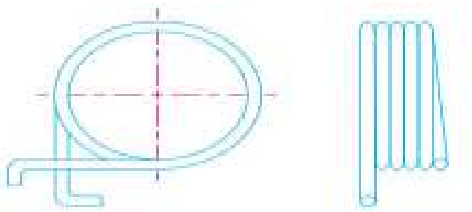
The conical and volute springs, as shown in Figure, are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired. The conical spring, as shown in Fig. (*a*), is wound with a uniform pitch whereas the volute springs, as shown in Fig.(*b*), are wound in the form of paraboloid with constant pitch

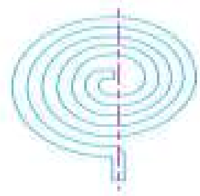

(*a*) Conical spring.

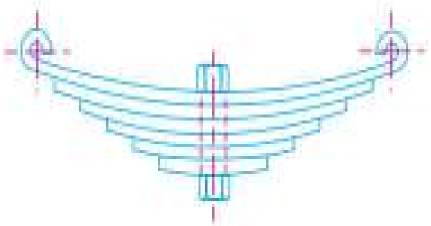

(*b*) Volute spring.

TORSION SPRING

- These springs may be of ***helical*** or ***spiral*** type as shown in Figure.
- The **helical type** may be used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms.
- The **spiral type** is also used where the load tends to increase the number of coils and when made of flat strip are used in watches and clocks


(*a*) Helical torsion spring.


(*b*) Spiral torsion spring.



LAMINATED OR LEAF SPRINGS

- The laminated or leaf spring (also known as ***flat spring*** or ***carriage spring***) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts

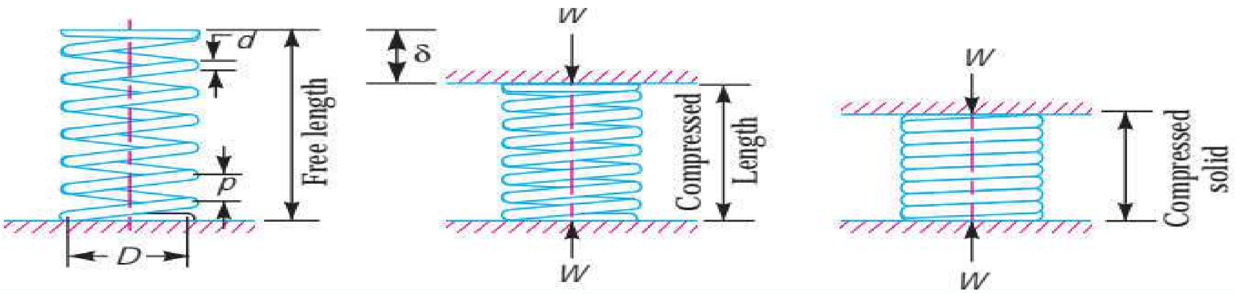
MATERIAL FOR HELICAL SPRINGS

- The material of the spring should have high fatigue strength, high ductility, high resilience and it should be creep resistant.
- The springs are mostly made from oil-tempered carbon steel wires containing 0.60 to 0.70 per cent carbon and 0.60 to 1.0 per cent manganese.

- Non-ferrous materials like phosphor bronze, beryllium copper, monel metal, brass etc., may be used in special cases to increase fatigue resistance, temperature resistance and corrosion resistance.
- The helical springs are either cold formed or hot formed depending upon the size of the wire. Wires of small sizes (less than 10 mm diameter) are usually wound cold whereas larger size wires are wound hot.

TERMS USED IN SPRINGS

1. *Solid length*. When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be *solid*. The solid length of a spring is the product of total number of coils and the diameter of the wire.
   Solid length of the spring,
   $L_S = n'.d$ where n' = Total number of coils, and d = Diameter of the wire.



2. *Free length*. The free length of a compression spring, is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils

Free length of the spring,

L_F = Solid length + Maximum compression + Clearance between adjacent coils (or clash allowance)

$= n'.d + \delta_{max} + 0.15\ \delta_{max}$

The following relation may also be used to find the free length of the spring, *i.e.*

$L_F = n'.d + \delta max + (n' - 1) \times 1$ mm

3. *Spring index*. The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire. Mathematically,

Spring index, $C = D / d$ where D = Mean diameter of the coil, and d = Diameter of the wire.

***4.Spring rate*.** The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring. Mathematically,

Spring rate, $k = W / \delta$ where W = Load, and δ = Deflection of the spring

5. *Pitch*. The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state. Mathematically,

Pitch of the coil, p =Free length/($n' - 1$)

Pitch of the coil, $p = (L_F - L_S / n) + d$

L_F = Free length of the spring, L_S = Solid length of the spring,

n' = Total number of coils, and d = Diameter of the wire

END CONNECTIONS FOR COMPRESSION HELICAL SPRINGS

The end connections for compression helical springs are suitably formed in order to apply the load.

In all springs, the end coils produce an eccentric application of load, increasing stress on one side of the spring. When the number of coils is small, this effect must be taken into account. The part of the coil which is in contact with the seat/element does not contribute to spring action and hence are termed as ***inactive coils***. The turns which impart spring action are known as ***active turns*.**

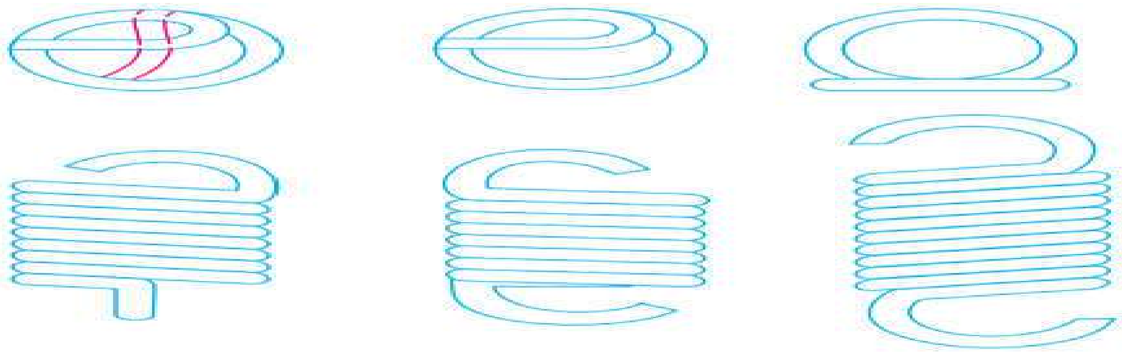
As the load increases, number of inactive coils also increases due to seating of the end coils and the amount of increase varies from 0.5 to 1 turn at the usual working loads.

Type of end	*Total number of turns (n')*	*Solid length*	*Free length*
1. Plain ends	n	$(n+1)\,d$	$p \times n + d$
2. Ground ends	n	$n \times d$	$p \times n$
3. Squared ends	$n+2$	$(n+3)\,d$	$p \times n + 3d$
4. Squared and ground ends	$n+2$	$(n+2)\,d$	$p \times n + 2d$

END CONNECTIONS FOR TENSION HELICAL SPRINGS

The tensile springs are provided with hooks or loops which may be made by turning whole coil or half of the coil. In a tension spring, large stress concentration is produced at the loop or other attaching device of tension spring.

The main disadvantage of tension spring is the failure of the spring when the wire breaks. A compression spring used for carrying a tensile load



For a spring having loops on both ends, the total number of active turns, $n' = n + 1$

STRESSES IN HELICAL SPRINGS OF CIRCULAR WIRE

Consider a helical compression spring made of circular wire and Subjected to an axial load W

Let D = Mean diameter of the spring coil,

d = Diameter of the spring wire,

n = Number of active coils,

G = Modulus of rigidity for the spring material,

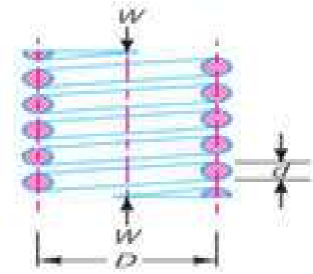
W = Axial load on the spring,

τ = Maximum shear stress induced in the wire,

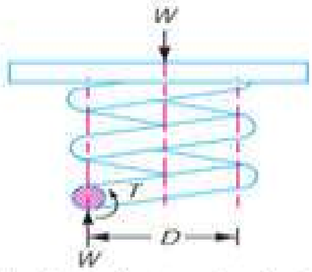
C = Spring index = D/d,

p = Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load W



(a) Axially loaded helical spring



(b) Free body diagram showing that wi is subjected to torsional shear and a direct shear.

The load W tends to rotate the wire due to the twisting moment (T) set up in the wire. Thus torsional shear stress is induced in the wire. A little consideration will show that part of the spring, as shown in Fig. (*b*), is in equilibrium under the action of two forces W and the twisting moment T.

We know that the twisting moment

$$T = W \times \frac{D}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$\tau_1 = \frac{8W.D}{\pi d^3} \qquad \text{...(i)}$$

In addition to the torsional shear stress (τ_1) induced in the wire, the following stresses also act on the wire :

1. Direct shear stress due to the load W, and

2. Stress due to curvature of wire

Direct shear stress due to the load W

$$\tau_2 = \frac{\text{Load}}{\text{Cross-sectional area of the wire}}$$

$$= \frac{W}{\frac{\pi}{4} \times d^2} = \frac{4W}{\pi d^2} \qquad \text{...(ii)}$$

The resultant shear stress induced in the wire

$$\tau = \tau_1 \pm \tau_2 = \frac{8W.D}{\pi d^3} \pm \frac{4W}{\pi d^2}$$

The ***positive*** sign is used for the inner edge of the wire and ***negative*** sign is used for the outer edge of the wire

Maximum shear stress induced in the wire, = *Torsional shear stress* + *Direct shear stress*

$$= \frac{8W.D}{\pi d^3} + \frac{4W}{\pi d^2} = \frac{8W.D}{\pi d^3}\left(1 + \frac{d}{2D}\right)$$

$$= \frac{8W.D}{\pi d^3}\left(1 + \frac{1}{2C}\right) = K_S \times \frac{8W.D}{\pi d^3} \quad ...(iii)$$

... (Substituting $D/d = C$)

where K_S = Shear stress factor $= 1 + \frac{1}{2C}$

From the above equation, it can be observed that the effect of direct shear $\left(\frac{8WD}{\pi d^3} \times \frac{1}{2C}\right)$ is appreciable for springs of small spring index C. Also we have neglected the effect of wire curvature in equation *(iii)*. It may be noted that when the springs are subjected to static loads, the effect of wire curvature may be neglected, because yielding of the material will relieve the stresses.

In order to consider the effects of both direct shear as well as curvature of the wire, a Wahl's stress factor (K) introduced by A.M. Wahl may be used. The resultant diagram of torsional shear, direct shear and curvature shear stress

∴ Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8W.D}{\pi d^3} = K \times \frac{8W.C}{\pi d^2} \quad ...(iv)$$

where $K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$

DEFLECTION OF HELICAL SPRINGS OF CIRCULAR WIRE

We Know,

Maximum shear stress induced in the wire,

$$\tau = K \times \frac{8W.D}{\pi d^3} = K \times \frac{8W.C}{\pi d^2}$$

Total active length of the wire,

l = Length of one coil × No. of active coils = $\pi D \times n$

Let θ = Angular deflection of the wire when acted upon by the torque T.

$$\therefore \quad \theta = \frac{T.l}{J.G} \quad ...\left(\text{considering } \frac{T}{J} = \frac{G.\theta}{l}\right)$$

where J = Polar moment of inertia of the spring wire

$= \frac{\pi}{32} \times d^4$, d being the diameter of spring wire.

and G = Modulus of rigidity for the material of the spring wire.

∴ Axial deflection of the spring,

$$\delta = \theta \times D/2 \quad ...(i)$$

We also know that

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G.\theta}{l}$$

Now substituting the values of l and J in the above equation, we have

$$\theta = \frac{T.l}{J.G} = \frac{\left(W \times \frac{D}{2}\right)\pi D.n}{\frac{\pi}{32} \times d^4 G} = \frac{16W.D^2.n}{G.d^4} \quad ...(ii)$$

Substituting this value of θ in equation *(i)*, we have

$$\delta = \frac{16W.D^2.n}{G.d^4} \times \frac{D}{2} = \frac{8W.D^3.n}{G.d^4} = \frac{8W.C^3.n}{G.d} \quad ... (\because C = D/d)$$

and the stiffness of the spring or spring rate,

$$\frac{W}{\delta} = \frac{G.d^4}{8\,D^3.n} = \frac{G.d}{8\,C^3.n} = \text{constant}$$

BUCKLING OF COMPRESSION SPRINGS

When the free length of the spring (L_F) is more than four times the mean or pitch diameter (D), then the spring behaves like a column and may fail by buckling at a comparatively low load

The critical axial load (W_{cr}) that causes buckling may be calculated by using the following relation, i.e. $W_{cr} = k \times K_B \times L_F$

where k = Spring rate or stiffness of the spring = W/δ,

L_F = Free length of the spring, and

K_B = Buckling factor depending upon the ratio LF / D

ECCENTRIC LOADING OF SPRINGS

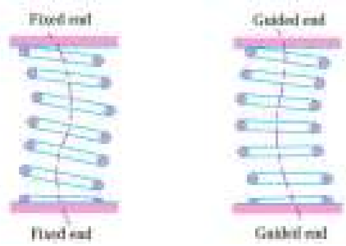
Sometimes, the load on the springs does not coincide with the axis of the spring, i.e. the spring is subjected to an eccentric load. The eccentric load on the spring increases the stress on one side of the spring and decreases on the other side. When the load is offset by a distance e from the spring axis, then the safe load on the spring may be obtained by multiplying the axial load by the factor:

$\dfrac{D}{2\,e + D}$ *D is the mean diameter of the spring*

SURGE IN SPRINGS

- *When one end of a helical spring is resting on a rigid support and the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire.*

- *End coils of the spring in contact with the applied load takes deflection and transmits its deflection to adjacent coils. In this way, a wave of compression propagates through the coils to the supported end.*



- *This wave of compression travels along the spring .If the applied load is of fluctuating type and if the time interval between the load applications is equal to the time required for the wave to travel from one end to the other end, then resonance will occur. This results in very large deflections of the coils and high stresses & it is just possible that the spring may fail. This phenomenon is called* ***surge.***
- *The natural frequency for springs clamped between two plates is given by*

$$f_n = \frac{d}{2\pi D^2 n}\sqrt{\frac{6\,G.g}{\rho}} \text{ cycles/s}$$

The surge in springs may be eliminated by using the following methods :

1. By using friction dampers on the centre coils so that the wave propagation dies out.
2. By using springs of high natural frequency.
3. By using springs having pitch of the coils near the ends different than at the centre to have different natural frequencies.

ENERGY STORED IN HELICAL SPRINGS OF CIRCULAR WIRE

We know that the springs are used for storing energy which is equal to the work done on it by some external load.

Let W = Load applied on the spring, and

δ = Deflection produced in the spring due to the load W.

Assuming that the load is applied gradually, the energy stored in a spring is,

$$U = \frac{1}{2} W.\delta \qquad \text{...}(i)$$

We have already discussed that the maximum shear stress induced in the spring wire,

$$\tau = K \times \frac{8\,W.D}{\pi\,d^3} \text{ or } W = \frac{\pi\,d^3.\tau}{8K.D}$$

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4} = \frac{8 \times \pi\,d^3.\tau}{8K.D} \times \frac{D^3.n}{G.d^4} = \frac{\pi\,\tau.D^2.n}{K.d.G}$$

Substituting the values of W and δ in equation (i), we have

$$U = \frac{1}{2} \times \frac{\pi\,d^3.\tau}{8\,K.D} \times \frac{\pi\,\tau.D^2.n}{K.d.G}$$

$$= \frac{\tau^2}{4\,K^2.G}(\pi\,D.n)\left(\frac{\pi}{4} \times d^2\right) = \frac{\tau^2}{4\,K^2.G} \times V$$

where V = Volume of the spring wire

= Length of spring wire × Cross-sectional area of spring wire

$$= (\pi\,D.n)\left(\frac{\pi}{4} \times d^2\right)$$

PROBLEM 1 : *A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm², find the axial load which the spring can carry and the deflection per active turn.*

Solution. Given : $d = 6$ mm ; $D_o = 75$ mm ; $\tau = 350$ MPa $= 350$ N/mm² ; $G = 84$ kN/mm $= 84 \times 10^3$ N/mm²

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

$\therefore$ Spring index, $$C = \frac{D}{d} = \frac{69}{6} = 11.5$$

Let W = Axial load, and

δ / n = Deflection per active turn.

1. *Neglecting the effect of curvature*

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire (τ),

$$350 = K_S \times \frac{8\, W.D}{\pi\, d^3} = 1.043 \times \frac{8\, W \times 69}{\pi \times 6^3} = 0.848\, W$$

$\therefore$ $W = 350 / 0.848 = 412.7$ N **Ans.**

We know that deflection of the spring,

$$\delta = \frac{8\, W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\, W.D^3}{G.d^4} = \frac{8 \times 412.7\, (69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm}$$ **Ans.**

2. *Considering the effect of curvature*

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire (τ),

$$350 = K \times \frac{8W.C}{\pi\, d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\, W$$

$\therefore$ $W = 350 / 0.913 = 383.4$ N **Ans.**

and deflection of the spring,

$$\delta = \frac{8\, W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\, W.D^3}{G.d^4} = \frac{8 \times 383.4\, (69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm}$$ **Ans.**

PROBLEM 2 *Design a helical compression spring for a maximum load of 1000 N for a deflection of 25 mm using the value of spring index as 5. The maximum permissible shear stress for spring wire is 420 MPa and modulus of rigidity is 84 kN/mm²*
Take Wahl's factor,

Solution. Given : $W = 1000$ N ; $\delta = 25$ mm ; $C = D/d = 5$; $\tau = 420$ MPa $= 420$ N/mm²
$G = 84$ kN/mm² $= 84 \times 10^3$ N/mm²

1. *Mean diameter of the spring coil*

Let D = Mean diameter of the spring coil, and

d = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

and maximum shear stress (τ),

$$420 = K \times \frac{8\,W.C}{\pi\, d^2} = 1.31 \times \frac{8 \times 1000 \times 5}{\pi\, d^2} = \frac{16\,677}{d^2}$$

$$\therefore \quad d^2 = 16\,677 / 420 = 39.7 \quad \text{or} \quad d = 6.3 \text{ mm}$$

we shall take a standard wire of size *SWG* 3 having diameter (d) = 6.401 mm.

$\therefore$ Mean diameter of the spring coil,

$$D = C.d = 5\,d = 5 \times 6.401 = 32.005 \text{ mm } \textbf{Ans.} \qquad \ldots (\because C = D/d = 5)$$

and outer diameter of the spring coil,

$$D_o = D + d = 32.005 + 6.401 = 38.406 \text{ mm } \textbf{Ans.}$$

2. *Number of turns of the coils*

Let n = Number of active turns of the coils.

We know that compression of the spring (δ),

$$25 = \frac{8W.C^3.n}{G.d} = \frac{8 \times 1000\,(5)^3 n}{84 \times 10^3 \times 6.401} = 1.86\,n$$

$$\therefore \quad n = 25 / 1.86 = 13.44 \text{ say } 14 \textbf{ Ans.}$$

For squared and ground ends, the total number of turns,

$$n' = n + 2 = 14 + 2 = 16 \textbf{ Ans.}$$

3. *Free length of the spring*

We know that free length of the spring

$$= n'.d + \delta + 0.15\,\delta = 16 \times 6.401 + 25 + 0.15 \times 25$$

$$= 131.2 \text{ mm } \textbf{Ans.}$$

4. *Pitch of the coil*

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{131.2}{16 - 1} = 8.75 \text{ mm } \textbf{Ans.}$$

PROBLEM 3 *A closely coiled helical spring is made of 10 mm diameter steel wire, the coil consisting of 10 complete turns with a mean diameter of 120 mm. The spring carries an axial pull of 200 N. Determine the shear stress induced in the spring neglecting the effect of stress concentration. Determine also the deflection in the spring, its stiffness and strain energy stored by it if the modulus of rigidity of the material is 80 kN/mm².*

Solution. Given : $d = 10$ mm ; $n = 10$; $D = 120$ mm ; $W = 200$ N ; $G = 80$ kN/mm² $= 80 \times 10^3$ N/mm²

Shear stress induced in the spring neglecting the effect of stress concentration

We know that shear stress induced in the spring neglecting the effect of stress concentration is,

$$\tau = \frac{8\,W.D}{\pi\,d^3}\left(1 + \frac{d}{2D}\right) = \frac{8 \times 200 \times 120}{\pi\,(10)^3}\left[1 + \frac{10}{2 \times 120}\right] \text{N/mm}^2$$

$$= 61.1 \times 1.04 = 63.54 \text{ N/mm}^2 = 63.54 \text{ MPa } \textbf{Ans.}$$

Deflection in the spring

We know that deflection in the spring,

$$\delta = \frac{8\,W.D^3 n}{G.d^4} = \frac{8 \times 200\,(120)^3\,10}{80 \times 10^3\,(10)^4} = 34.56 \text{ mm } \textbf{Ans.}$$

Stiffness of the spring

We know that stiffness of the spring

$$= \frac{W}{\delta} = \frac{200}{34.56} = 5.8 \text{ N/mm}$$

Strain energy stored in the spring

We know that strain energy stored in the spring,

$$U = \frac{1}{2}\,W.\delta = \frac{1}{2} \times 200 \times 34.56 = 3456 \text{ N-mm} = 3.456 \text{ N-m } \textbf{Ans.}$$

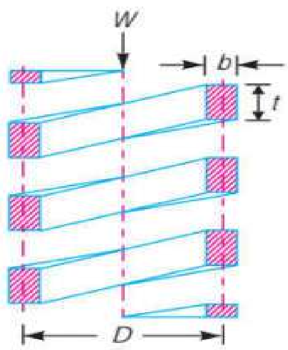
STRESS AND DEFLECTION IN HELICAL SPRINGS OF NON-CIRCULAR WIRE

The helical springs may be made of non-circular wire such as rectangular or square wire, in order to provide greater resilience in a given space. However these springs have the following main disadvantages :

1. The quality of material used for springs is not so good.

2. The shape of the wire does not remain square or rectangular while forming helix which reduces the energy absorbing capacity of the spring.

3. The stress distribution is not as favourable as for circular wires. But this effect is negligible where loading is of static nature.



shear stress is given by

$$\tau = K \times \frac{W.D\,(1.5\,t + 0.9\,b)}{b^2.t^2}$$

This expression is applicable when the longer side (*i.e.* $t > b$) is parallel to the axis of the spring. But when the shorter side (*i.e.* $t < b$) is parallel to the axis of the spring, then maximum shear stress,

$$\tau = K \times \frac{W.D\,(1.5\,b + 0.9\,t)}{b^2.t^2}$$

and deflection of the spring,

$$\delta = \frac{2.45\,W.D^3.n}{G.b^3\,(t - 0.56\,b)}$$

For springs made of square wire, the dimensions b and t are equal. Therefore, the maximum shear stress is given by

$$\tau = K \times \frac{2.4\,W.D}{b^3}$$

and deflection of the spring,

$$\delta = \frac{5.568\,W.D^3.n}{G.b^4} = \frac{5.568\,W.C^3.n}{G.b} \qquad \ldots\left(\because C = \frac{D}{b}\right)$$

where b = Side of the square.

SPRINGS IN SERIES

Consider two springs connected in series as shown in Fig.

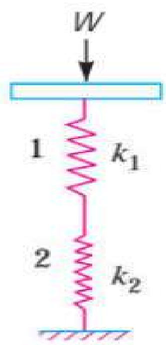
Let W = Load carried by the springs,

δ_1 = Deflection of spring 1,

δ_2 = Deflection of spring 2,

k_1 = Stiffness of spring 1 = W / δ_1, and

k_2 = Stiffness of spring 2 = W / δ_2



A little consideration will show that when the springs are connected in series, then the total deflection produced by the springs is equal to the sum of the deflections of the individual springs.

∴ Total deflection of the springs,

$$\delta = \delta_1 + \delta_2$$

or
$$\frac{W}{k} = \frac{W}{k_1} + \frac{W}{k_2}$$

∴
$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

where k = Combined stiffness of the springs.

SPRINGS IN PARALLEL

Consider two springs connected in parallel as shown in Fig

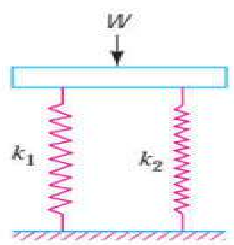
Let W = Load carried by the springs,

W_1 = Load shared by spring 1,

W_2 = Load shared by spring 2,

k_1 = Stiffness of spring 1, and

k_2 = Stiffness of spring 2.



A little consideration will show that when the springs are connected in parallel, then the total deflection produced by the springs is same as the deflection of the individual springs.

We know that $W = W_1 + W_2$

or $\delta.k = \delta.k_1 + \delta.k_2$

∴ $k = k_1 + k_2$

where k = Combined stiffness of the springs, and

δ = Deflection produced.

A loaded narrow-gauge car of mass 1800 kg and moving at a velocity 72 m/min., is brought to rest by a bumper consisting of two helical steel springs of square section. The mean diameter of the coil is six times the side of the square section. In bringing the car to rest, the springs are to be compressed 200 mm. Assuming the allowable shear stress as 365 MPa and spring index of 6, find

1. Maximum load on each spring, *2. Side of the square section of the wire*

3. Mean diameter of coils, and *4. Number of active coils.*

Take modulus of rigidity as 80 kN/mm².

Solution. Given : m = 1800 kg ; v = 72 m/min = 1.2 m/s ; δ = 200 mm ; τ = 365 MPa = 365 N/mm² ; C = 6 ; G = 80 kN/mm² = 80 × 10³ N/mm²

1. *Maximum load on each spring,*

Let W = Maximum load on each spring.

We know that kinetic energy of the car

$$= \frac{1}{2} m.v^2 = \frac{1}{2} \times 1800\,(1.2)^2 = 1296 \text{ N-m} = 1296 \times 10^3 \text{ N-mm}$$

This energy is absorbed in the two springs when compressed to 200 mm. If the springs are loaded gradually from 0 to W, then

$$\left(\frac{0+W}{2}\right) 2 \times 200 = 1296 \times 10^3$$

$$\therefore \quad W = 1296 \times 10^3 / 200 = 6480 \text{ N} \quad \textbf{Ans.}$$

2. *Side of the square section of the wire*

Let b = Side of the square section of the wire, and

D = Mean diameter of the coil = 6 b ... ($\because C = D/b = 6$)

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 6 - 1}{4 \times 6 - 4} + \frac{0.615}{6} = 1.2525$$

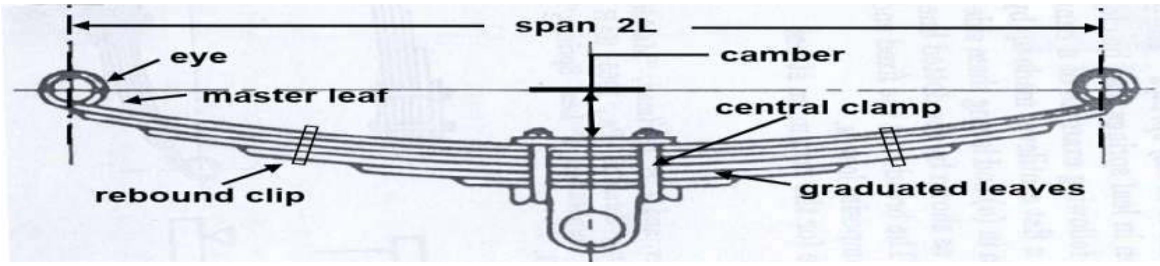
and maximum shear stres (τ),

$$365 = K \times \frac{2.4\,W.D}{b^3} = 1.2525 \times \frac{2.4 \times 6480 \times 6\,b}{b^3} = \frac{116\,870}{b^2}$$

$$\therefore \quad b^2 = 116\,870 / 365 = 320 \quad \text{or} \quad b = 17.89 \text{ say } 18 \text{ mm} \quad \textbf{Ans.}$$

LEAF SPRING

- The laminated or leaf spring (also known as ***flat spring*** or ***carriage spring***) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts.
- These are mostly used in automobiles
- The major stresses produced in leaf springs are tensile and compressive stresses
- The advantage of leaf spring over helical spring is that the ends of the spring may be guided along a definite path as it deflects to act as a structural member in addition to energy absorbing device.
- Thus the leaf springs may carry lateral loads, brake torque, driving torque etc., in addition to shocks.



CONSTRUCTION

- A leaf spring is of semi-elliptical form and consists of number of leaves.
- The longest leaf is called as master leaf and has its ends formed in the shape of an eye, through which bolts are passes to secure the spring to its supports.
- The other leaves are called as graduated leaves, which are arranged in the order of decreasing length and then clamped to the master leaf with the help of strips.
- Since master leaf has to withstand vertical bending loads as well as loads due to sideways of vehicle, therefore, due to presence of stresses caused by these loads, it is usual to provide two full length leaves and rest as graduated leaves.

MATERIALS FOR LEAF SPRING

Oil hardened and tempered alloy steels such as, 50Cr1, 50Cr1V23 for automobiles & 55Si2Mn90 For rail road springs

ADVANTAGES OF LEAF SPRING:

1) Leaf spring is acts as a structural member rather than energy absorbing device.
2) Leaf springs can take up the lateral loads, brake torque and driving torque in addition to shock.

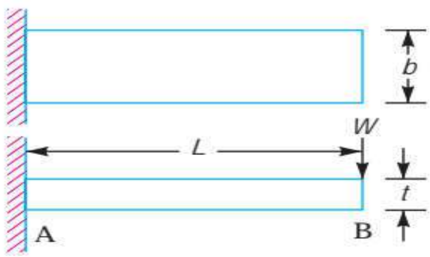
APPLICATIONS OF LEAF SPRING:

1) Automobiles like heavy loaded trucks, jeep, trailers etc.
2) In machines to absorb the shock loads.

STRESS IN LEAF SPRING

Consider a single plate fixed at one end and loaded at the other end & plate may be used as a flat spring

Let t = Thickness of plate,
b = Width of plate, and
L = Length of plate or distance of the load W from the cantilever end.



We know that the maximum bending moment at the cantilever end A,

$$M = WL$$

and section modulus,

$$Z = \frac{I}{y} = \frac{b\,t^3/12}{t/2} = \frac{1}{6} \times b.t^2$$

∴ Bending stress in such a spring,

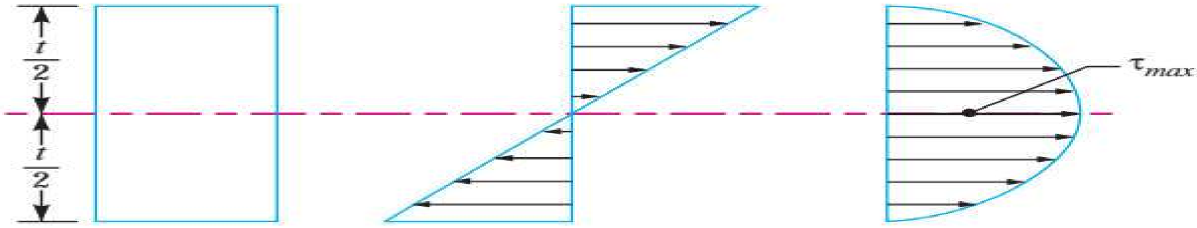
$$\sigma = \frac{M}{Z} = \frac{W.L}{\frac{1}{6} \times b.t^2} = \frac{6\,W.L}{b.t^2} \qquad ...(i)$$

maximum deflection for a cantilever with concentrated load at the free end is

$$\delta = \frac{W.L^3}{3E.I} = \frac{W.L^3}{3E \times b.t^3/12} = \frac{4\,W.L^3}{E.b.t^3} \qquad ...(ii)$$

$$= \frac{2\,\sigma.L^2}{3\,E.t} \qquad ...\left(\because \sigma = \frac{6W.L}{b.t^2}\right)$$

It may be noted that due to bending moment, top fibres will be in tension and the bottom fibres are in compression, but the shear stress is zero at the extreme fibres and maximum at the centre.



(a) Cross-section of plate. (b) Bending stress diagram. (c) Shear stress diagram.

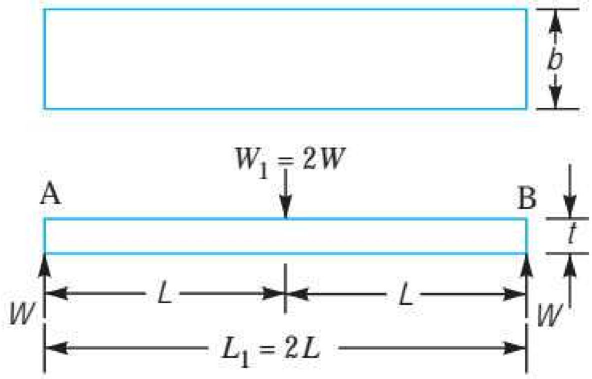
If the spring is not of cantilever type but it is like a simply supported beam, with length $2L$ and load $2W$ in the centre,

Maximum bending moment in the centre,

$$M = W.L$$

Section modulus, $Z = b.t^2/6$

∴ Bending stress, $$\sigma = \frac{M}{Z} = \frac{W.L}{b.t^2/6} = \frac{6\,W.L}{b.t^2}$$



We know that maximum deflection of a simply supported beam loaded in the centre is given by

$$\delta = \frac{W_1 (L_1)^3}{48\ E.I} = \frac{(2W)(2L)^3}{48\ E.I} = \frac{W.L^3}{3\ E.I}$$

...($\because$ In this case, $W_1 = 2W$, and $L_1 = 2L$)

Spring such as automobile spring (semi-elliptical spring) with length 2L and loaded in the centre by a load 2W, may be treated as a double cantilever.

If the plate of cantilever is cut into a series of n strips of width b and these are placed

$$\sigma = \frac{6\ W.L}{n.b.t^2} \quad ...(iii)$$

$$\delta = \frac{4\ W.L^3}{n.E.b.t^3} = \frac{2\ \sigma.L^2}{3\ E.t} \quad ...(iv)$$

If a triangular plate is used ,the stress will be uniform throughout. If this triangular plate is cut into strips of uniform width and placed one below the other, to form a graduated or laminated leaf spring, then

$$\sigma = \frac{6\ W.L}{n.b.t^2} \quad ...(v)$$

$$\delta = \frac{6\ W.L^3}{n.E.b.t^3} = \frac{\sigma.L^2}{E.t} \quad ...(vi)$$

n = Number of graduated leaves.

When bending stress alone is considered, the graduated leaves may have zero width at the loaded end. Therefore, it becomes necessary to have one or more leaves of uniform cross-section extending clear to the end.

We see from equations ***(iv)*** and ***(vi)*** that for the same deflection, the stress in the uniform cross-section leaves (*i.e.* full length leaves) is 50% greater than in the graduated leaves, assuming that each spring element deflects according to its own elastic curve. If the suffixes F and G are used to indicate the full length (or uniform cross-section) and graduated leaves, then

$$\sigma_F = \frac{3}{2}\sigma_G$$

$$\frac{6 W_F.L}{n_F.b.t^2} = \frac{3}{2}\left[\frac{6\ W_G.L}{n_G.b.t^2}\right] \quad \text{or} \quad \frac{W_F}{n_F} = \frac{3}{2} \times \frac{W_G}{n_G}$$

$$\frac{W_F}{W_G} = \frac{3\ n_F}{2\ n_G} \quad ...(vii)$$

$$\therefore \quad W_G = \left(\frac{2\ n_G}{3\ n_F + 2\ n_G}\right)(W_F + W_G) = \left(\frac{2\ n_G}{3\ n_F + 2\ n_G}\right) W \quad ...(viii)$$

where W = Total load on the spring = $W_G + W_F$

W_G = Load taken up by graduated leaves, and

W_F = Load taken up by full length leaves.

$$\therefore \quad W_F = \left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right)(W_G + W_F) = \left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right)W \quad \ldots(ix)$$

$\therefore$ Bending stress for full length leaves,

$$\sigma_F = \frac{6\,W_F.L}{n_F.b\,t^2} = \frac{6\,L}{n_F.b.t^2}\left(\frac{3\,n_F}{2\,n_G + 3\,n_F}\right)W = \frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}$$

Since $\sigma_F = \frac{3}{2}\sigma_G$, therefore

$$\sigma_G = \frac{2}{3}\sigma_F = \frac{2}{3} \times \frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)} = \frac{12\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}$$

The deflection in full length and graduated leaves is given by equation (*iv*), *i.e.*

$$\delta = \frac{2\,\sigma_F \times L^2}{3\,E.t} = \frac{2\,L^2}{3\,E.t}\left[\frac{18\,W.L}{b.t^2\,(2\,n_G + 3\,n_F)}\right] = \frac{12\,W.L^3}{E.b.t^3\,(2\,n_G + 3\,n_F)}$$

NIPPING IN LEAVE SPRING

- The stresses in extra full-length leaves are 50% more than the stresses in graduated-length leaves.
- One of the methods of equalising the stresses in different leaves is to pre-stress the spring.
- The pre-stressing is achieved by bending the leaves to different radii of curvature, before they are assembled with the centre clip.
- As shown in Fig, the full-length leaf is given a greater radius of curvature than the adjacent leaf.
- The radius of curvature decreases with shorter leaves. The initial gap C between the extra full-length leaf and the graduated-length leaf before the assembly is called a 'nip'. Such pre-stressing, achieved by a difference in radii of curvature, is known as 'nipping'.

PROBLEM Design a leaf spring for the following specifications :Total load = 140 kN ; Number of springs supporting the load = 4 ; Maximum number of leaves = 10; Span of the spring = 1000 mm ; Permissible deflection = 80 mm. Take Young's modulus, E = 200 kN/mm² and allowable stress in spring material as 600 MPa.

Solution. Given : Total load = 140 kN ; No. of springs = 4; n = 10 ; $2L$ = 1000 mm or L = 500 mm ; δ = 80 mm ; E = 200 kN/mm² = 200 × 10³ N/mm² ; σ = 600 MPa = 600 N/mm²

We know that load on each spring,

$$2W = \frac{\text{Total load}}{\text{No. of springs}} = \frac{140}{4} = 35 \text{ kN}$$

$$\therefore \quad W = 35/2 = 17.5 \text{ kN} = 17\,500 \text{ N}$$

Let t = Thickness of the leaves, and

b = Width of the leaves.

We know that bending stress (σ),

$$600 = \frac{6\,W.L}{n.b.t^2} = \frac{6 \times 17\,500 \times 500}{n.b.t^2} = \frac{52.5 \times 10^6}{n.b.t^2}$$

$$\therefore \quad n.b.t^2 = 52.5 \times 10^6 / 600 = 87.5 \times 10^3 \quad \ldots(i)$$

and deflection of the spring (δ),

$$80 = \frac{6\,W.L^3}{n.E.bt^3} = \frac{6 \times 17\,500\,(500)^3}{n \times 200 \times 10^3 \times b \times t^3} = \frac{65.6 \times 10^6}{n.bt^3}$$

$\therefore$ $\quad n.b.t^3 = 65.6 \times 10^6 / 80 = 0.82 \times 10^6$ ...(*ii*)

Dividing equation (*ii*) by equation (*i*), we have

$$\frac{n.bt^3}{n.bt^2} = \frac{0.82 \times 10^6}{87.5 \times 10^3} \quad \text{or} \quad t = 9.37 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

Now from equation (*i*), we have

$$b = \frac{87.5 \times 10^3}{n.t^2} = \frac{87.5 \times 10^3}{10\,(10)^2} = 87.5 \text{ mm}$$

and from equation (*ii*), we have

$$b = \frac{0.82 \times 10^6}{n.t^3} = \frac{0.82 \times 10^6}{10\,(10)^3} = 82 \text{ mm}$$

Taking larger of the two values, we have width of leaves,

$$b = 87.5 \text{ say } 90 \text{ mm } \textbf{Ans.}$$

PROBLEM A locomotive semi-elliptical laminated spring has an overall length of 1 m and sustains a load of 70 kN at its centre. The spring has 3 full length leaves and 15 graduated leaves with a central band of 100 mm width. All the leaves are to be stressed to 400 MPa, when fully loaded. The ratio of the total spring depth to that of width is 2. E = 210 kN/mm^2. Determine :

1. The thickness and width of the leaves.

2. The initial gap that should be provided between the full length and graduated leaves before the band

load is applied.3. The load exerted on the band after the spring is assembled

Solution. Given : $2L_1 = 1$ m $= 1000$ mm ; $2W = 70$ kN or $W = 35$ kN $= 35 \times 10^3$ N ; $n_F = 3$; $n_G = 15$; $l = 100$ mm ; $\sigma = 400$ MPa $= 400$ N/mm^2 ; $E = 210$ kN/mm$^2 = 210 \times 10^3$ N/mm^2

1. ***Thickness and width of leaves***

Let $\quad t$ = Thickness of leaves, and

$\quad b$ = Width of leaves.

We know that the total number of leaves,

$$n = n_F + n_G = 3 + 15 = 18$$

Since it is given that ratio of the total spring depth ($n \times t$) and width of leaves is 2, therefore

$$\frac{n \times t}{b} = 2 \quad \text{or} \quad b = n \times t / 2 = 18 \times t / 2 = 9\,t$$

We know that the effective length of the leaves,

$$2L = 2L_1 - l = 1000 - 100 = 900 \text{ mm} \quad \text{or} \quad L = 900 / 2 = 450 \text{ mm}$$

Since all the leaves are equally stressed, therefore final stress (σ),

$$400 = \frac{6\,W.L}{n.bt^2} = \frac{6 \times 35 \times 10^3 \times 450}{18 \times 9\,t \times t^2} = \frac{583 \times 10^3}{t^3}$$

$\therefore$ $\quad t^3 = 583 \times 10^3 / 400 = 1458 \quad$ or $\quad t = 11.34$ say 12 mm **Ans.**

$\quad b = 9\,t = 9 \times 12 = 108$ mm **Ans.**

2. *Initial gap*

We know that the initial gap (C) that should be provided between the full length and graduated leaves before the band load is applied, is given by

$$C = \frac{2\,W.L^3}{n.E.b.t^3} = \frac{2 \times 35 \times 10^3\ (450)^3}{18 \times 210 \times 10^3 \times 108\ (12)^3} = 9.04 \text{ mm } \textbf{Ans.}$$

3. *Load exerted on the band after the spring is assembled*

We know that the load exerted on the band after the spring is assembled,

$$W_b = \frac{2\,n_F.n_G.W}{n(2n_G + 3n_F)} = \frac{2 \times 3 \times 15 \times 35 \times 10^3}{18\ (2 \times 15 + 3 \times 3)} = 4487 \text{ N } \textbf{Ans.}$$

PROBLEM A *truck spring has 12 number of leaves, two of which are full length leaves. The spring supports are 1.05 m apart and the central band is 85 mm wide. The central load is to be 5.4 kN with a permissible stress of 280 MPa. Determine the thickness and width of the steel spring leaves. The ratio of the total depth to the width of the spring is 3. Also determine the deflection of the spring*

Solution. Given : $n = 12$; $n_F = 2$; $2L_1 = 1.05$ m $= 1050$ mm ; $l = 85$ mm ; $2W = 5.4$ kN $= 5400$ N or $W = 2700$ N ; $\sigma_F = 280$ MPa $= 280$ N/mm^2

Thickness and width of the spring leaves

Let t = Thickness of the leaves, and

b = Width of the leaves.

Since it is given that the ratio of the total depth of the spring ($n \times t$) and width of the spring (b) is 3, therefore

$$\frac{n \times t}{b} = 3 \quad \text{or} \quad b = n \times t / 3 = 12 \times t / 3 = 4\,t$$

We know that the effective length of the spring,

$$2L = 2L_1 - l = 1050 - 85 = 965 \text{ mm}$$

$$\therefore \quad L = 965 / 2 = 482.5 \text{ mm}$$

and number of graduated leaves,

$$n_G = n - n_F = 12 - 2 = 10$$

Assuming that the leaves are not initially stressed, therefore maximum stress or bending stress for full length leaves (σ_F),

$$280 = \frac{18\,W.L}{b.t^2\ (2n_G + 3n_F)} = \frac{18 \times 2700 \times 482.5}{4\,t \times t^2\,(2 \times 10 + 3 \times 2)} = \frac{225\,476}{t^3}$$

$\therefore$ $t^3 = 225\,476 / 280 = 805.3$ or $t = 9.3$ say 10 mm **Ans.**

and $b = 4\,t = 4 \times 10 = 40$ mm **Ans.**

Deflection of the spring

We know that deflection of the spring,

$$\delta = \frac{12\,W.L^3}{E.b.t^3\ (2n_G + 3n_F)}$$

$$= \frac{12 \times 2700 \times (482.5)^3}{210 \times 10^3 \times 40 \times 10^3\ (2 \times 10 + 3 \times 2)} \text{ mm}$$

$= 16.7$ mm **Ans.** ... (Taking $E = 210 \times 10^3$ N/mm^2)

UNIT-5

CLUTCHES

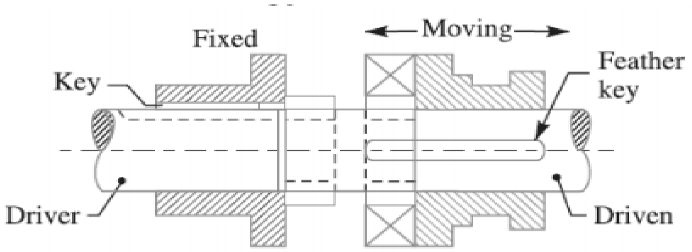
Clutches

A clutch is a machine member used to connect a driving shaft to a driven shaft so that the driven shaft may be started or stopped at will, without stopping the driving shaft. The use of a clutch is mostly found in automobiles.Clutch is a device used in the transmission system of a motor vehicle to engage and disengage the engine to the transmission

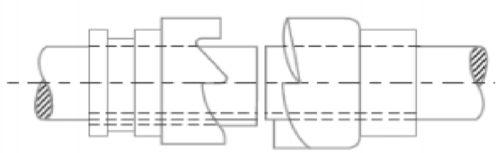
Types of Clutches
1. Positive clutches, 2. Friction clutches.

POSITIVE CLUTCH

The positive clutches are used when a positive drive is required. The simplest type of a positive clutch is a jaw or claw clutch. The jaw clutch permits one shaft to drive another through a direct contact of interlocking jaws. The jaws of the clutch may be of square type or of spiral type. A square jaw type is used where transmission of power in either direction of rotation. The spiral jaws may be left-hand or right-hand, because power transmitted by them is in one direction only.



(*a*) Square jaw clutch.



(*b*) Spiral jaw clutch.

FRICTION CLUTCHES

A friction clutch has its principal application in the transmission of power of shafts and machines which must be started and stopped frequently. The force of friction is used to start the driven shaft from rest and gradually brings it up to the proper speed without excessive slipping of the friction surfaces.

MATERIAL FOR FRICTION SURFACES

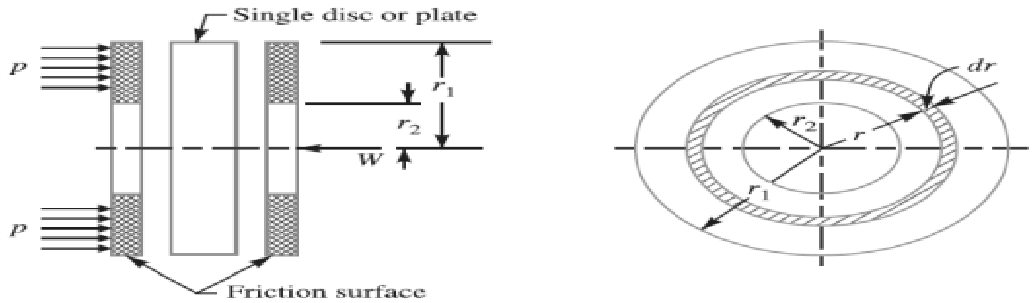
Material used for friction surfaces of a clutch should have the following characteristics
1. High And Uniform Coefficient Of Friction.
2. Not Be Affected By Moisture And Oil
3. Ability To Withstand High Temperatures Caused By Slippage.
4. High Heat Conductivity.
5. High Resistance To Wear And Scoring.

PLATE CLUTCH

A single disc or plate clutch, consists of a clutch plate whose both sides are faced with a frictional material (usually of Ferrodo). It is mounted on the hub which is free to move axially along driven shaft. The pressure plate is mounted inside the clutch body which is bolted to the flywheel. Both the pressure plate and the flywheel rotate with the engine crankshaft or the driving shaft. The pressure plate pushes the clutch plate towards the flywheel by a set of strong springs which are arranged radially inside the body. The levers are arraged in a manner so that the pressure plate moves away from the flywheel by the inward movement of a thrust bearing. The bearing is mounted upon a forked shaft and moves forward when the clutch pedal is pressed.

DESIGN OF A DISC OR PLATE CLUTCH

Consider two friction surfaces maintained in contact by an axial thrust (*W*).



Let T = *Torque transmitted by the clutch,*

p = *Intensity of axial pressure with which the contact surfaces are* held together,

r1 and r2 = External and internal radii of friction faces,

r = *Mean radius of the friction face, and*

μ = Coefficient of friction.

Consider an elementary ring of radius *r and thickness dr.* We know that area of contact surface or

friction surface= $2\pi\, r.dr$

∴ Normal or axial force on the ring,

$\delta W = Pressure \times Area = p \times 2\pi\, r.dr$ and frictional force on the ring acting tangentially at radius r,

$Fr = \mu \times \delta W = \mu.p \times 2\pi\, r.dr$

∴ Frictional torque acting on the ring,

$Tr = Fr \times r = \mu.p \times 2\pi\, r.dr \times r = 2\, \pi\, \mu\, p.\, r2.dr$

Two cases: When there is a uniform pressure, and when there is a uniform axial wear.

CONSIDERING UNIFORM PRESSURE

When the pressure is uniformly distributed over the entire area of the friction face *then the intensity of pressure,*

$$p = \frac{W}{\pi\left[(r_1)^2 - (r_2)^2\right]}$$

W = Axial thrust with which the friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius r and thickness dr is

$$T_r = 2\pi\, \mu.p.r^2.dr$$

Integrating this equation within the limits from r_2 to r_1 for the total friction torque. ∴ Total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\, \mu.p.r^2.dr = 2\pi\mu.p\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\, \mu.p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right] = 2\pi\, \mu \times \frac{W}{\pi\,[(r_1)^2 - (r_2)^2]}\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

... (Substituting the value of p)

$$= \frac{2}{3}\mu.\, W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \mu\, .W.R$$

$$R = \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \text{Mean radius of the friction surface.}$$

WHEN THERE IS A UNIFORM AXIAL WEAR

The basic principle in designing machine parts that are subjected to wear due to sliding friction is that the normal wear is proportional to the work of friction. The work of friction is proportional to the product of normal pressure (*p) and the sliding velocity* (*V).*

Therefore, Normal wear ∝ Work of friction ∝ $p.V$ or

$p.V = K$ *(a constant) or* $p = K/V$

This wearing-in process continues until the product $p.V$ *is constant over the entire surface. After this, the wear will* be uniform.

Let p *be the normal intensity of pressure at a distance* r from the axis of the clutch. Since the intensity of pressure varies inversely with the distance, therefore

$p.r = C$ *(a constant) or* $p = C / r$ and the normal force on the ring

$$\delta W = p.2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

∴ Total force acing on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi\, C\, dr = 2\pi\, C\, [r]_{r_2}^{r_1} = 2\,\pi\, C\, (r_1 - r_2)$$

$$C = \frac{W}{2\,\pi\, (r_1 - r_2)}$$

Frictional torque acting on the ring,

$$T_r = 2\,\pi\, \mu.p.r^2.dr = 2\pi\, \mu \times \frac{C}{r} \times r^2.dr = 2\pi\mu.C.r.dr$$

Total frictional torque acting on the friction surface (or on the clutch),

$$T = \int_{r_2}^{r_1} 2\pi\, \mu\, C.r.dr = 2\pi\, \mu\, C\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

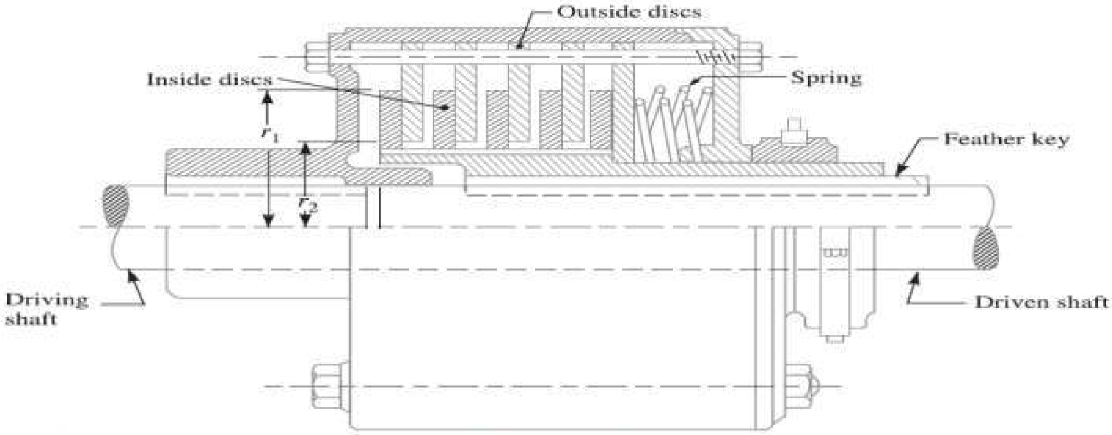
$$= 2\pi\mu\, .C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right] = \pi\, \mu.C\, [(r_1)^2 - (r_2)^2]$$

$$= \pi\mu \times \frac{W}{2\pi\, (r_1 - r_2)}[(r_1)^2 - (r_2)^2] = \frac{1}{2} \times \mu.W\, (r_1 + r_2) = \mu.W.R$$

$$R = \frac{r_1 + r_2}{2} = \text{Mean radius of the friction surface.}$$

MULTIPLE DISC CLUTCH

It is used when a large torque is to be transmitted. The inside discs (usually of steel) are fastened to the driven shaft to permit axial motion (except for the last disc). The outside discs (usually of bronze) are held by bolts and are fastened to the housing which is keyed to the driving shaft.



Let n_1 = Number of discs on the driving shaft, and

n_2 = Number of discs on the driven shaft.

$\therefore$ Number of pairs of contact surfaces,

$$n = n_1 + n_2 - 1$$

and total frictional torque acting on the friction surfaces or on the clutch,

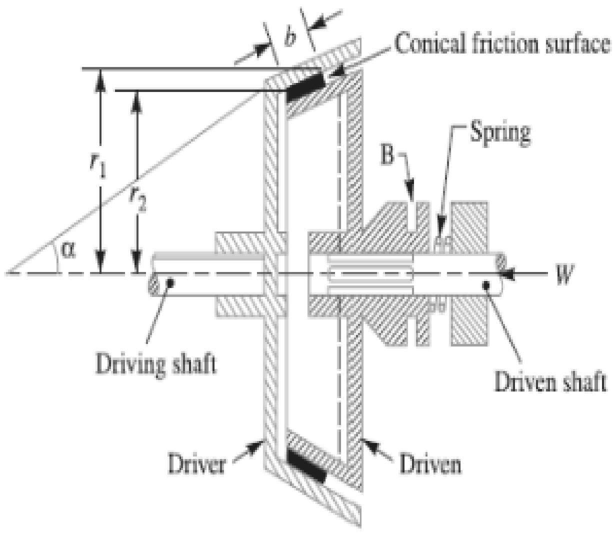
$$T = n.\mu.W.R$$

where R = Mean radius of friction surfaces

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \quad \text{... (For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \quad \text{... (For uniform wear)}$$

CONE CLUTCH



It consists of one pair of friction surface only. In a cone clutch, the driver is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits into the outside conical surface of the driven. The driven member resting on the feather key in the driven shaft, may be shifted along the shaft by a forked lever provided at *B, in* order to engage the clutch by bringing the two conical surfaces in contact. Due to the frictional resistance set up at this contact surface, the torque is transmitted from one shaft to another.

Due to the frictional resistance set up at this contact surface, the torque is transmitted from one shaft to another

DESIGN OF A CONE CLUTCH

Consider a pair of friction surfaces of a cone clutch. A little consideration will show that the area of contact of a pair of friction surface is a frustrum of a cone. Let p_n = *Intensity of pressure with which the conical friction surfaces are held* together (*i.e. normal pressure between the contact surfaces),*

r_1 = *Outer radius of friction surface,* r_2 = *Inner radius of friction surface,*

R = Mean radius = $(r_1+r_2)/2$

α = Semi-angle of the cone (also called face angle of the cone) or angle of the friction surface with the axis of the clutch,

μ = Coefficient of friction between the contact surfaces, and

b = Width of the friction surfaces (also known as face width or cone face).

Consider a small ring of radius *r and thickness dr .Let dl is the length of* ring of the friction surface, such that, $dl = dr \operatorname{cosec} \alpha$

∴ Area of ring = $2\pi r.\, dl = 2\pi r.dr \operatorname{cosec} \alpha$

It has two cases a) Uniform b) Non Uniform

1. *Considering uniform pressure*

We know that the normal force acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_n \times 2\pi r.dr \operatorname{cosec} \alpha$$

and the axial force acting on the ring,

$$\delta W = \text{Horizontal component of } \delta W_n \text{ (i.e. in the direction of } W)$$
$$= \delta W_n \times \sin\alpha = p_n \times 2\pi r.dr \operatorname{cosec}\alpha \times \sin\alpha = 2\pi \times p_n.r.dr$$

∴ Total axial load transmitted to the clutch or the axial spring force required,

$$W = \int_{r_2}^{r_1} 2\pi \times p_n.r.dr = 2\pi\, p_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi p_n \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi p_n [(r_1)^2 - (r_2)^2]$$

and $$p_n = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \quad \ldots(i)$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_n \times 2\pi r.dr \text{ cosec } \alpha$$

$\therefore$ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_n \times 2\pi r.dr \text{ cosec } \alpha \times r$$
$$= 2\pi\, \mu.p_n \text{ cosec } \alpha.r^2\, dr$$

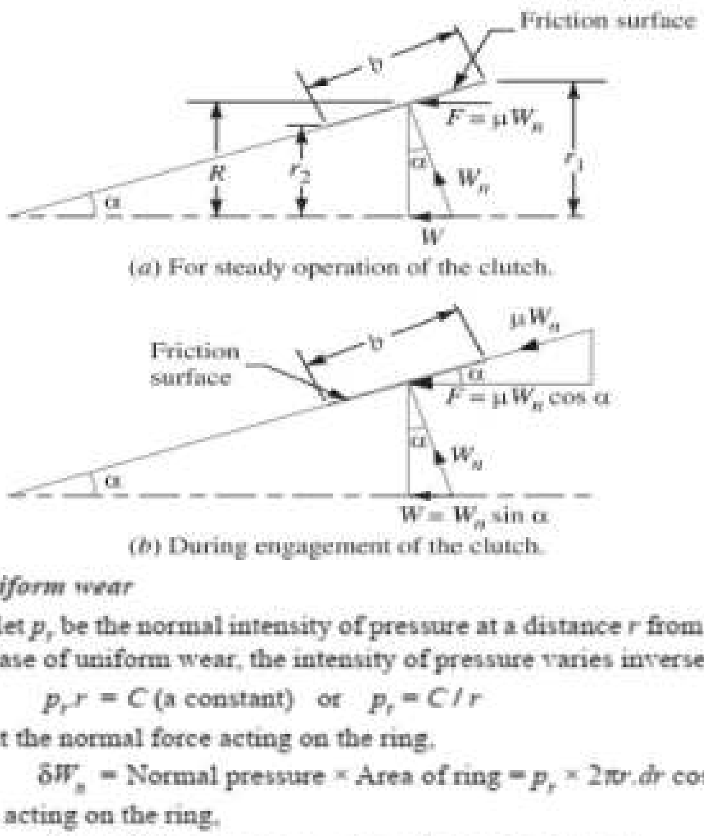
Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch.

$\therefore$ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p_n.\text{ cosec } \alpha.r^2\, dr = 2\pi\, \mu.p_n \text{ cosec } \alpha \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$
$$= 2\pi\, \mu.p_n \text{ cosec } \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p_n from equation (i), we get

$$T = 2\pi\, \mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \text{cosec } \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$
$$= \frac{2}{3} \times \mu.W \text{ cosec } \alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] \quad \ldots(ii)$$



(a) For steady operation of the clutch.

(b) During engagement of the clutch.

2. *Considering uniform wear*

let p_r be the normal intensity of pressure at a distance r from the axis of the clutch. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

$\therefore$ $$p_r.r = C \text{ (a constant)} \quad \text{or} \quad p_r = C/r$$

We know that the normal force acting on the ring,

$$\delta W_n = \text{Normal pressure} \times \text{Area of ring} = p_r \times 2\pi r.dr \text{ cosec } \alpha$$

and the axial force acting on the ring,

$$\delta W = \delta W_n \times \sin \alpha = p_r \times 2\pi\, r.dr \text{ cosec } \alpha \times \sin \alpha$$

$$= 2\pi \times p_r . r\ dr$$

$$= 2\pi \times \frac{C}{r} \times r.\ dr = 2\pi\ C.dr \qquad \ldots\left(\because p_r = \frac{C}{r}\right)$$

$\therefore$ Total axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi\ C.dr = 2\pi\ C\ [r]_{r_2}^{r_1} = 2\pi\ C\ (r_1 - r_2)$$

or
$$C = \frac{W}{2\pi\ (r_1 - r_2)} \qquad \ldots (iii)$$

We know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu.\delta W_n = \mu.p_r \times 2\pi\ r.dr \text{ cosec } \alpha$$

$\therefore$ Frictional torque acting on the ring,

$$T_r = F_r \times r = \mu.p_r \times 2\pi\ r.dr \text{ cosec } \alpha \times r$$

$$= \mu \times \frac{C}{r} \times 2\pi\ r.dr \text{ cosec } \alpha \times r = 2\pi\ \mu.C \text{ cosec } \alpha \times r\ dr$$

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch.

$\therefore$ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\ \mu.C \text{ cosec } \alpha \times r\ dr = 2\pi\mu.C \text{ cosec } \alpha \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\ \mu.C \text{ cosec } \alpha \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

Substituting the value of C from equation (iii), we have

$$T = 2\pi\mu \times \frac{W}{2\pi\ (r_1 - r_2)} \times \text{cosec } \alpha \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \mu.W \text{ cosec } \alpha \left[\frac{r_1 + r_2}{2}\right] = \mu\ WR \text{ cosec } \alpha \qquad \ldots (iv)$$

where
$$R = \frac{r_1 + r_2}{2} = \text{Mean radius of friction surface.}$$

Since the normal force acting on the friction surface, $W_n = W \text{ cosec } \alpha$, therefore the equation (iv) may be written as

$$T = \mu\ W_n\ R \qquad \ldots (v)$$

$$r_1 - r_2 = b \sin \alpha \qquad \text{and} \qquad R = \frac{r_1 + r_2}{2} \qquad \text{or} \qquad r_1 + r_2 = 2R$$

$\therefore$ From equation (i), normal pressure acting on the friction surface,

$$p_n = \frac{W}{\pi\ [(r_1)^2 - (r_2)^2]} = \frac{W}{\pi\ (r_1 + r_2)\ (r_1 - r_2)} = \frac{W}{2\pi\ Rb \sin \alpha}$$

or
$$W = p_n \times 2\pi\ R.b \sin \alpha = W_n \sin \alpha$$

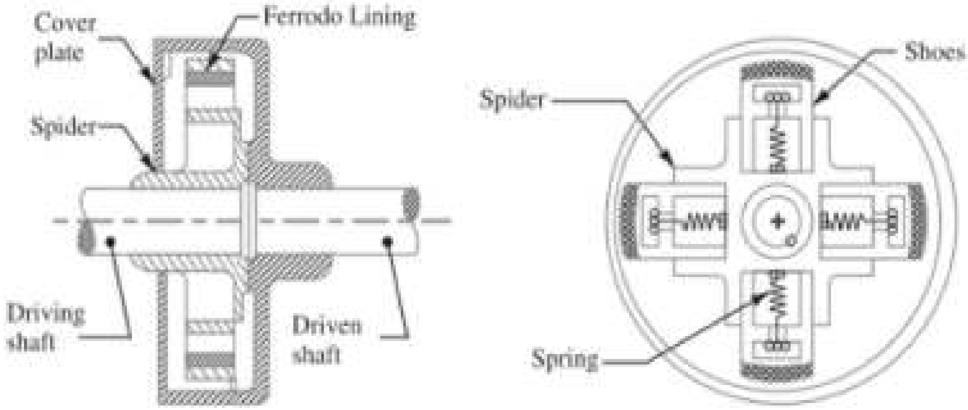
where
$$W_n = \text{Normal load acting on the friction surface} = p_n \times 2\pi R.b$$

Now the equation (iv) may be written as

$$T = \mu\ (p_n \times 2\pi\ R.\ b \sin \alpha)\ R \text{ cosec } \alpha = 2\pi\ \mu.p_n\ R^2.b$$

CENTRIFUGAL CLUTCH

- It consists of a number of shoes on the inside of a rim of the pulley, as shown in Figure. The outer surfaces of the shoes are covered with a friction material. These shoes, which can move radially in guides, are held against the boss (or spider) on the driving shaft by means of springs.
- The springs exert a radially inward force which is assumed constant. The weight of the shoe, when revolving causes it to exert a radially outward force (i.e. centrifugal force). The magnitude of this centrifugal force depends upon the speed at which the shoe is revolving.



When the centrifugal force is less than the spring force, the shoe remains in the same position as when the driving shaft was stationary, but when the centrifugal force is equal to the spring force, the shoe is just floating.

When the centrifugal force exceeds the spring force, the shoe moves outward and comes into contact with the driven member and presses against it The increase of speed causes the shoe to press harder and enables more torque to be transmitted.

DESIGN OF A CENTRIFUGAL CLUTCH

- In designing a centrifugal clutch, it is required to determine
- ✓ the weight of the shoe,
- ✓ size of the shoe and
- ✓ dimensions of the spring.

Mass of the shoes

Consider one shoe of a centrifugal clutch as shown in Figure.

Let m = Mass of each shoe,

n = Number of shoes,

r = Distance of centre of gravity of the shoe from the centre of the spider,

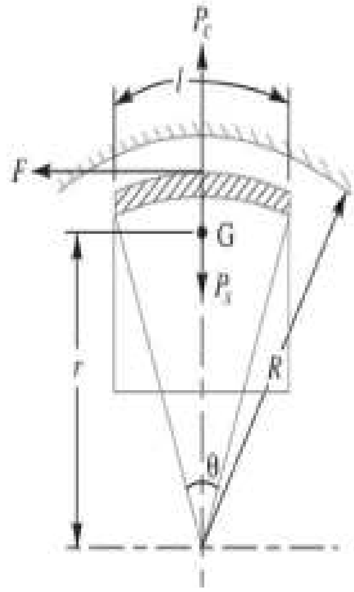
R = Inside radius of the pulley rim,

N = Running speed of the pulley in r.p.m.,

ω = Angular running speed of the pulley in rad / s $= 2\pi N / 60$ rad/s,

ω_1 = Angular speed at which the engagement begins to take place, and

μ = Coefficient of friction between the shoe and rim.



We know that the centrifugal force acting on each shoe at the running speed,

$$^*P_c = m.\omega^2.r$$

Since the speed at which the engagement begins to take place is generally taken as 3/4th of the running speed, therefore the inward force on each shoe exerted by the spring is given by

$$P_s = m\,(\omega_1)^2\,r = m\left(\frac{3}{4}\omega\right)^2 r = \frac{9}{16}\,m.\omega^2.r$$

$\therefore$ Net outward radial force (*i.e.* centrifugal force) with which the shoe presses against the rim at the running speed

$$= P_c - P_s = m.\omega^2.r - \frac{9}{16}\,m.\omega^2.r = \frac{7}{16}\,m.\omega^2.r$$

and the frictional force acting tangentially on each shoe,

$$F = \mu\,(P_c - P_s)$$

$\therefore$ Frictional torque acting on each shoe

$$= F \times R = \mu\,(P_c - P_s)\,R$$

and total frictional torque transmitted,

$$T = \mu\,(P_c - P_s)\,R \times n = n.F.R$$

From this expression, the mass of the shoes (m) may be evaluated.

2. *Size of the shoes*

Let l = Contact length of the shoes,

b = Width of the shoes,

R = Contact radius of the shoes. It is same as the inside radius of the rim of the pulley,

θ = Angle subtended by the shoes at the centre of the spider in radians, and

p = Intensity of pressure exerted on the shoe. In order to ensure reasonable life, it may be taken as 0.1 N/mm^2.

We know that $\theta = \frac{l}{R}$ or $l = \theta.R = \frac{\pi}{3}\,R$ ...(Assuming $\theta = 60° = \pi/3$ rad)

$\therefore$ Area of contact of the shoe

$$= l\,.\,b$$

and the force with which the shoe presses against the rim

$$= A \times p = l.b.p$$

Since the force with which the shoe presses against the rim at the running speed is $(P_c - P_s)$,

$$l.b.p = P_c - P_s$$

From this expression, the width of shoe (b) may be obtained.

3. *Dimensions of the spring*

We have discussed above that the load on the spring is given by

$$P_s = \frac{9}{16} \times m.\omega^2.r$$

The dimensions of the spring may be obtained as usual.

UNIT-6

BRAKES

BRAKES

A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. The brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc. The energy absorbed by brakes is dissipated in the form of heat. This heat is dissipated in the surrounding air so that excessive heating of the brake lining does not take place

MATERIALS FOR BRAKE LINING

The material used for the brake lining should have the following characteristics :

- It should have high coefficient of friction with minimum fading.
- It should have low wear rate.
- It should have high heat resistance.
- It should have high heat dissipation capacity.
- It should have low coefficient of thermal expansion.
- It should have adequate mechanical strength

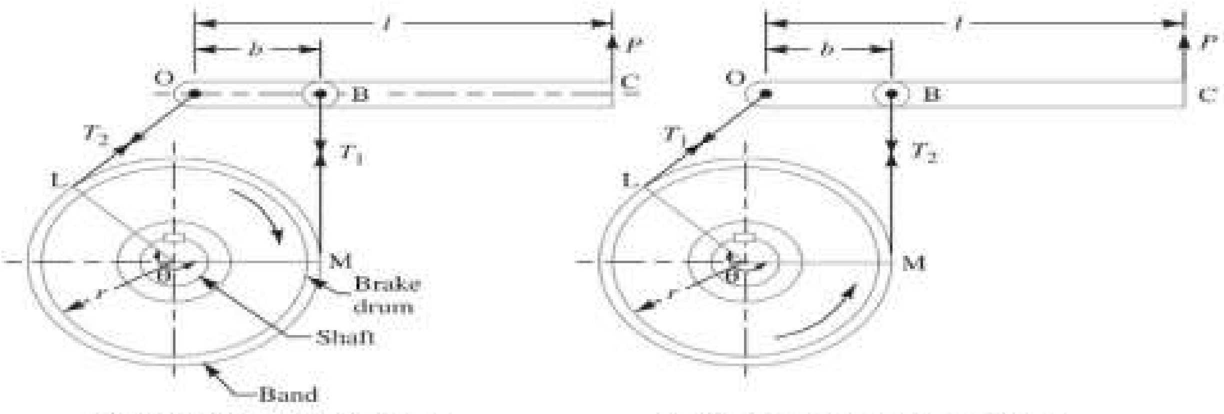
TYPES OF BRAKES

The brakes, according to the means used for transforming the energy by the braking element, are classified as :

1) Hydraulic brakes *e.g*. pumps or hydrodynamic brake and fluid agitator,
2) Electric brakes *e.g*. generators and eddy current brakes, and
3) Mechanical brakes.

BAND BRAKE

A band brake consists of a flexible band of leather, one or more ropes, or a steel lined with friction material, which embraces a part of the circumference of the drum. A band brake, as shown in Figure below, is called a ***simple band brake*** in which one end of the band is attached to a fixed pin or fulcrum of the lever while the other end is attached to the lever at a distance b from the fulcrum.



(*a*) Clockwise rotation of drum. (*b*) Anticlockwise rotation of drum.

When a force P is applied to the lever at C, the lever turns about the fulcrum pin O and tightens the band on the drum and hence the brakes are applied. The friction between the band and the drum

provides the braking force. The force P on the lever at C may be determined as discussed below:

Let T_1 = Tension in the tight side of the band,

T_2 = Tension in the slack side of the band,

θ = Angle of lap (or embrace) of the band on the drum,

μ = Coefficient of friction between the band and the drum,

r = Radius of the drum,

t = Thickness of the band, and

r_e = Effective radius of the drum = $r + t / 2$.

We know that limiting ratio of the tensions is given by the relation,

$$\frac{T_1}{T_2} = e^{\mu.\theta} \quad \text{or} \quad 2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

and braking force on the drum

$$= T_1 - T_2$$

$\therefore$ Braking torque on the drum,

$$T_B = (T_1 - T_2)\, r \qquad \text{...(Neglecting thickness of band)}$$

$$= (T_1 - T_2)\, r_e \qquad \text{...(Considering thickness of band)}$$

Now considering the equilibrium of the lever OBC. It may be noted that when the drum rotates in the clockwise direction as shown in Fig. (a), the end of the band attached to the fulcrum O will be slack with tension T_2 and end of the band attached to B will be tight with tension T_1. On the other hand, when the drum rotates in the anticlockwise direction as shown in Fig.(b), the tensions in the band will reverse, i.e. the end of the band attached to the fulcrum O will be tight with tension T_1 and the end of the band attached to B will be slack with tension T_2. Now taking moments about the fulcrum O, we have

$$P.l = T_1.b \qquad \text{...(for clockwise rotation of the drum)}$$

and

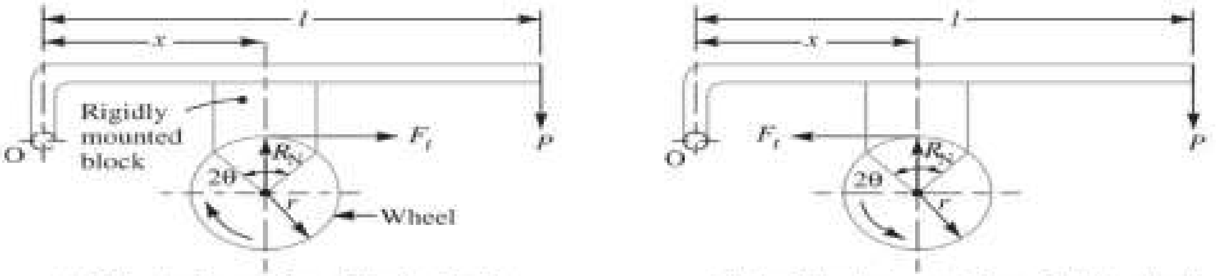
$$P.l = T_2.b \qquad \text{...(for anticlockwise rotation of the drum)}$$

where l = Length of the lever from the fulcrum (OC), and

b = Perpendicular distance from O to the line of action of T_1 or T_2.

SHOE BRAKE

A single block or shoe brake is shown in Figure. It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. This type of a brake is commonly used on railway trains and tram cars.



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel. The block is pressed against the wheel by a force applied to one end of a lever to which the block is rigidly fixed

Let P = Force applied at the end of the lever,

R_N = Normal force pressing the brake block on the wheel,

r = Radius of the wheel,

2θ = Angle of contact surface of the block,

μ = Coefficient of friction, and

F_t = Tangential braking force or the frictional force acting at the contact surface of the block and the wheel.

If the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. In such cases, tangential braking force on the wheel,

$$F_t = \mu . R_N \qquad \text{...}(i)$$

and the braking torque, $T_B = F_t \cdot r = \mu\, R_N \cdot r$ ...(*ii*)

Let us now consider the following three cases :

Case 1. When the line of action of tangential braking force (*Ft*) passes through the fulcrum *O* of the lever, and the brake wheel rotates clockwise as shown in Fig. (*a*), then for equilibrium, taking moments about the fulcrum *O*, we have

$$R_N \times x = P \times l \qquad \text{or} \qquad R_N = \frac{P \times l}{x}$$

$$\therefore \quad \text{Braking torque,} \qquad T_B = \mu . R_N . r = \mu \times \frac{Pl}{x} \times r = \frac{\mu . P l . r}{x}$$

It may be noted that when the brake wheel rotates anticlockwise as shown in Fig.(*b*), then the braking torque is same, *i.e.* $T_B = \mu . R_N . r = \dfrac{\mu . P . l . r}{x}$

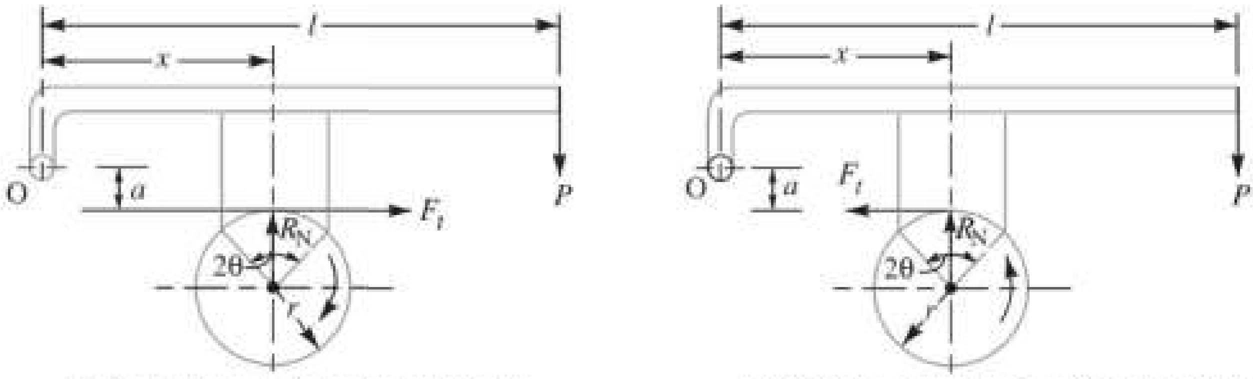
Case 2. When the line of action of the tangential braking force (*Ft*) passes through a distance '*a*' below the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig. 25.2 (*a*), then

for
equilibrium, taking moments about the fulcrum O

$$R_N \times x + F_t \times a = P.l$$

or $\quad R_N \times x + \mu\, R_N \times a = P.l \quad$ or $\quad R_N = \dfrac{P.l}{x + \mu.a}$

and braking torque, $\quad T_B = \mu\, R_N.r = \dfrac{\mu.P.l.r}{x + \mu.a}$



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

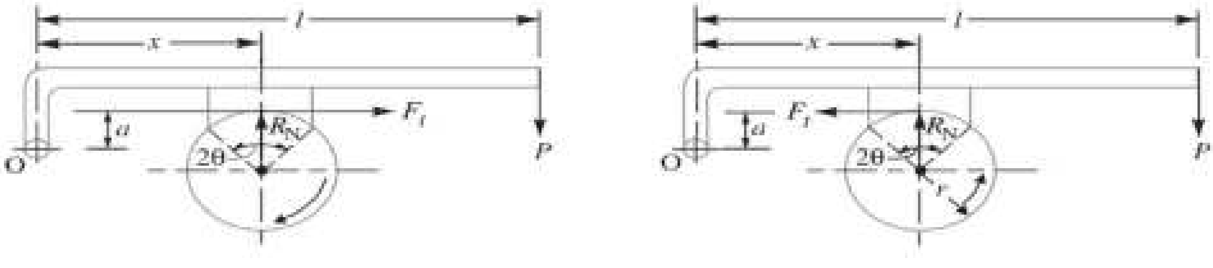
When the brake wheel rotates anticlockwise, as shown in Fig. 25.2 (*b*), then for equilibrium,

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a \qquad \text{...}(i)$$

or $\quad R_N\,(x - \mu.a) = P.l \quad$ or $\quad R_N = \dfrac{P.l}{x - \mu.a}$

and braking torque, $\quad T_B = \mu.R_N.r = \dfrac{\mu.P.l.r}{x - \mu.a}$

Case 3. When the line of action of the tangential braking force passes through a distance '*a*' above the fulcrum, and the brake wheel rotates clockwise as shown in Fig.(*a*), then for equilibrium, taking moments about the fulcrum O, we have



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

$$R_N . x = P.l + F_t . a = P.l + \mu . R_N . a$$

or $\quad R_N (x - \mu.a) = P.l \quad$ or $\quad R_N = \dfrac{P.l}{x - \mu.a}$

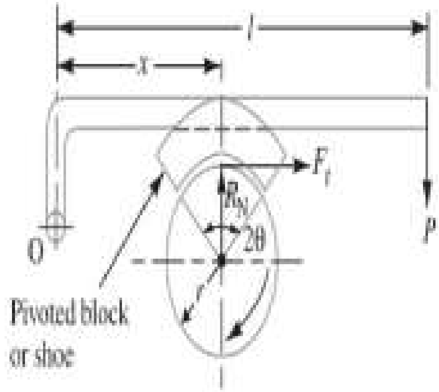
and braking torque, $\quad T_B = \mu . R_N . r = \dfrac{\mu . P . l . r}{x - \mu . a}$

When the brake wheel rotates anticlockwise as shown in Fig. taking moments about the fulcrum O, we have

$$R_N \times x + F_t \times a = P.l$$

or $\quad R_N \times x + \mu . R_N \times a = P.l \quad$ or $\quad R_N = \dfrac{P.l}{x + \mu.a}$

and braking torque, $\quad T_B = \mu . R_N . r = \dfrac{\mu . P . l . r}{x + \mu . a}$



Pivoted Block or Shoe Brake

When the angle of contact is less than 60°, then it is assumed that the normal pressure between block and wheel is uniform. But when angle of contact is greater than 60°, then unit pressure normal to surface of contact is less at ends than at centre. In such cases, block or shoe is pivoted to lever as shown in Figure, instead of being rigidly attached to lever. This gives uniform wear of brake lining in direction of applied force. The braking torque for a pivoted block or shoe brake (*i.e.* when $2\theta > 60°$)

$$T_B = F_t \times r = \mu' . R_N . r$$

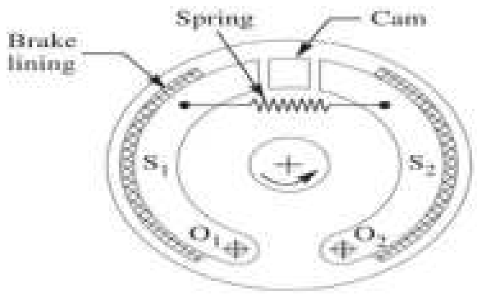
where $\quad \mu' = $ Equivalent coefficient of friction $= \dfrac{4 \mu \sin \theta}{2\theta + \sin 2\theta}$, and

$\quad \mu = $ Actual coefficient of friction.

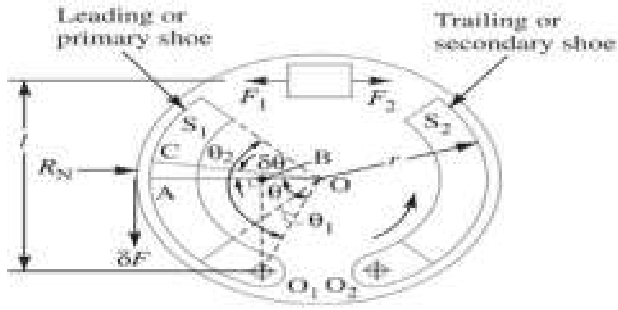
These brakes have more life and may provide a higher braking torque.

Internal Expanding Brake

An internal expanding brake consists of two shoes S_1 and S_2 as shown in Fig.(*a*). The outer surface of the shoes are lined with some friction material to increase the coefficient of friction and to prevent wearing away of the metal. Each shoe is pivoted at one end about a fixed fulcrum O_1 and O_2 and made to contact a cam at other end. When cam rotates, shoes are pushed outwards against the rim of the drum. Friction between the shoes and the drum produces the braking torque and hence reduces the speed of the drum. Shoes are normally held in off position by a spring as shown in Fig. (*a*).The drum encloses the entire mechanism to keep out dust and moisture. This type of brake is commonly used in motor cars and light trucks.



(*a*) Internal expanding brake.



(*b*) Forces on an internal expanding brake.

We shall now consider the forces acting on such a brake, when the drum rotates in the anticlockwise direction as shown in Fig.(*b*). It may be noted that for the anticlockwise direction, the left hand shoe is known as ***leading*** **or** ***primary shoe*** while right hand shoe is known as ***trailing*** **or** ***secondary shoe***

Consider a small element of the brake lining *AC* subtending an angle δθ at the centre. Let *OA* makes an angle θ with *OO*

Let r = Internal radius of the wheel rim.

b = Width of the brake lining.

p_1 = Maximum intensity of normal pressure,

p_N = Normal pressure,

F_1 = Force exerted by the cam on the leading shoe, and

F_2 = Force exerted by the cam on the trailing shoe.

Shoe turns about O_1, therefore the rate of wear of the shoe lining at *A* will be proportional to the radial displacement of that point. The rate of wear of the shoe lining varies directly as the perpendicular distance from O_1 to *OA*, *i.e.* O_1B. From the geometry of the figure,

$$O_1B = OO_1 \sin\theta$$

and normal pressure at A, $p_N \propto \sin\theta$ or $p_N = p_1 \sin\theta$

∴ Normal force acting on the element,

$$\delta R_N = \text{Normal pressure} \times \text{Area of the element}$$

$$= p_N\,(b\,.\,r\,.\,\delta\theta) = p_1 \sin\theta\,(b\,.\,r\,.\,\delta\theta)$$

and braking or friction force on the element,

$$O_1B = OO_1 \sin\theta$$

and normal pressure at A, $p_N \propto \sin\theta$ or $p_N = p_1 \sin\theta$

$\therefore$ Normal force acting on the element,

$$\delta R_N = \text{Normal pressure} \times \text{Area of the element}$$

$$= p_N\,(b\,.\,r\,.\,\delta\theta) = p_1 \sin\theta\,(b\,.\,r\,.\,\delta\theta)$$

and braking or friction force on the element,

$$\delta F = \mu.\delta R_N = \mu\, p_1 \sin\theta\,(b\,.\,r\,.\,\delta\theta)$$

$\therefore$ Braking torque due to the element about O,

$$\delta T_B = \delta F\,.\,r = \mu\, p_1 \sin\theta\,(b\,.\,r\,.\,\delta\theta)\, r = \mu\, p_1\, b\, r^2 (\sin\theta.\delta\theta)$$

and total braking torque about O for whole of one shoe,

$$T_B = \mu\ p_1\ b\, r^2 \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = \mu\ p_1\ b\ r^2\ [-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= \mu\, p_1\, b\, r^2 (\cos\theta_1 - \cos\theta_2)$$

Moment of normal force δR_N of the element about the fulcrum O_1,

$$\delta M_N = \delta R_N \times O_1B = \delta R_N\,(OO_1 \sin\theta)$$

$$= p_1 \sin\theta\,(b\,.\,r\,.\,\delta\theta)\,(OO_1 \sin\theta) = p_1 \sin^2\theta\,(b\,.\,r\,.\,\delta\theta)\, OO_1$$

Total moment of normal forces about the fulcrum O_1,

$$M_N = \int_{\theta_1}^{\theta_2} p_1 \sin^2\theta\,(b\,.\,r\,\delta\theta)\, OO_1 = p_1\,.\,b\,.\,r\,.\,OO_1 \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta$$

$$= p_1.b.r.OO_1 \int_{\theta_1}^{\theta_2} \tfrac{1}{2}(1-\cos 2\theta)\, d\theta \qquad \ldots\left[\because \sin^2\theta = \tfrac{1}{2}(1-\cos 2\theta)\right]$$

$$= \tfrac{1}{2}\, p_1\,.\,b\,.\,r\,.\,OO_1 \left[\theta - \frac{\sin 2\theta}{2}\right]_{\theta_1}^{\theta_2}$$

$$= \tfrac{1}{2}\, p_1\,.\,b\,.\,r\,.\,OO_1 \left[\theta_2 - \frac{\sin 2\theta_2}{2} - \theta_1 + \frac{\sin 2\theta_1}{2}\right]$$

$$= \tfrac{1}{2}\, p_1\,.\,b\,.\,r\,.\,OO_1\, [(\theta_2 - \theta_1) + \tfrac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2)]$$

Moment of frictional force δF about the fulcrum O_1,

$$\delta M_F = \delta F \times AB = \delta F\,(r - OO_1 \cos\theta) \qquad \ldots(\because AB = r - OO_1 \cos\theta)$$

$$= \mu\,.\,p_1 \sin\theta\,(b\,.\,r\,.\,\delta\theta)\,(r - OO_1 \cos\theta)$$

$$= \mu\,.\,p_1\,.\,b\,.\,r\,(r \sin\theta - OO_1 \sin\theta\cos\theta)\,\delta\theta$$

$$= \mu\,.\,p_1\,.\,b\,.\,r \left(r\sin\theta - \frac{OO_1}{2}\sin 2\theta\right)\delta\theta \qquad \ldots(\because 2\sin\theta\cos\theta = \sin 2\theta)$$

$\therefore$ Total moment of frictional force about the fulcrum O_1,

$$M_F = \mu\,.\,p_1\,.\,b\,.\,r \int_{\theta_1}^{\theta_2} \left(r\sin\theta - \frac{OO_1}{2}\sin 2\theta\right) d\theta$$

$$= \mu \cdot p_1 \cdot b \cdot r \left[-r\cos\theta + \frac{OO_1}{4}\cos 2\theta \right]_{\theta_1}^{\theta_2}$$

$$= \mu \cdot p_1 \cdot b \cdot r \left[-r\cos\theta_2 + \frac{OO_1}{4}\cos 2\theta_2 + r\cos\theta_1 - \frac{OO_1}{4}\cos 2\theta_1 \right]$$

$$= \mu \cdot p_1 \cdot b \cdot r \left[r\,(\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4}\,(\cos 2\theta_2 - \cos 2\theta_1) \right]$$

Now for leading shoe, taking moments about the fulcrum O_1,

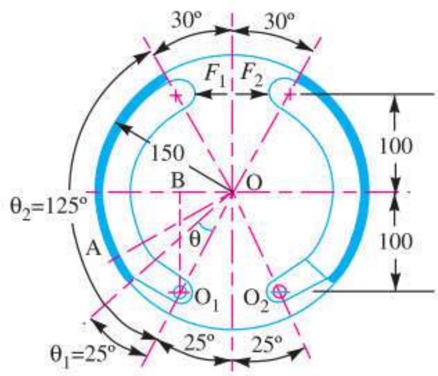
$$F_1 \times l = M_N - M_F$$

and for trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

NUMERICALS

Figure shows the arrangement of two brake shoes which act on the internal surface of a cylindrical brake drum. The braking force F1 and F2 are applied as shown and each shoe pivots on its fulcrum O1 and O2. The width of the brake lining is 35 mm. The intensity of pressure at any point A is 0.4 sin θ N/mm2, where θ is measured as shown from either pivot. The coefficient of friction is 0.4. Determine the braking torque and the magnitude of the forces F1 and F2



All dimensions in mm.

Solution. Given : $b = 35$ mm ; $\mu = 0.4$; $r = 150$ mm ; $l = 200$ mm ; $\theta_1 = 25°$; $\theta_2 = 125°$

Since the intensity of normal pressure at any point is 0.4 sin θ N/mm², therefore maximum intensity of normal pressure,

$$p_1 = 0.4 \text{ N/mm}^2$$

We know that the braking torque for one shoe,

$$= \mu \,.\, p_1 \,.\, b \,.\, r^2 (\cos\theta_1 - \cos\theta_2)$$
$$= 0.4 \times 0.4 \times 35\,(150)^2 (\cos 25° - \cos 125°)$$
$$= 126\,000\,(0.9063 + 0.5736) = 186\,470 \text{ N-mm}$$

∴ Total braking torque for two shoes,

$$T_B = 2 \times 186\,470 = 372\,940 \text{ N-mm}$$

Magnitude of the forces F_1 and F_2

From the geometry of the figure, we find that

$$OO_1 = \frac{O_1B}{\cos 25°} = \frac{100}{0.9063} = 110.3 \text{ mm}$$
$$\theta_1 = 25° = 25 \times \pi / 180 = 0.436 \text{ rad}$$

and $$\theta_2 = 125° = 125 \times \pi / 180 = 2.18 \text{ rad}$$

We know that the total moment of normal forces about the fulcrum O_1,

$$M_N = \tfrac{1}{2}\, p_1 \,.\, b \,.\, r \,.\, OO_1 \left[(\theta_2 - \theta_1) + \tfrac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2)\right]$$
$$= \tfrac{1}{2} \times 0.4 \times 35 \times 150 \times 110.3 \left[(2.18 - 0.436) + \tfrac{1}{2}(\sin 50° - \sin 250°)\right]$$
$$= 115\,815\left[1.744 + \frac{1}{2}(0.766 + 0.9397)\right] = 300\,754 \text{ N-mm}$$

and total moment of friction force about the fulcrum O_1,

$$M_F = \mu \,.\, p_1 \,.\, b \,.\, r\left[r(\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right]$$
$$= 0.4 \times 0.4 \times 35 \times 150\left[150(\cos 25° - \cos 125°) + \frac{110.3}{4}(\cos 250° - \cos 50°)\right]$$
$$= 840\,[150\,(0.9063 + 0.5736) + 27.6\,(-0.342 - 0.6428)]$$
$$= 840\,(222 - 27) = 163\,800 \text{ N-mm}$$

For the leading shoe, taking moments about the fulcrum O_1,

$$F_1 \times l = M_N - M_F$$

or $$F_1 \times 200 = 300\,754 - 163\,800 = 136\,954$$

∴ $$F_1 = 136\,954 / 200 = 685 \text{ N}$$ **Ans.**

For the trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

or $$F_2 \times 200 = 300\,754 + 163\,800 = 464\,554$$

∴ $$F_2 = 464\,554 / 200 = 2323 \text{ N}$$ **Ans.**