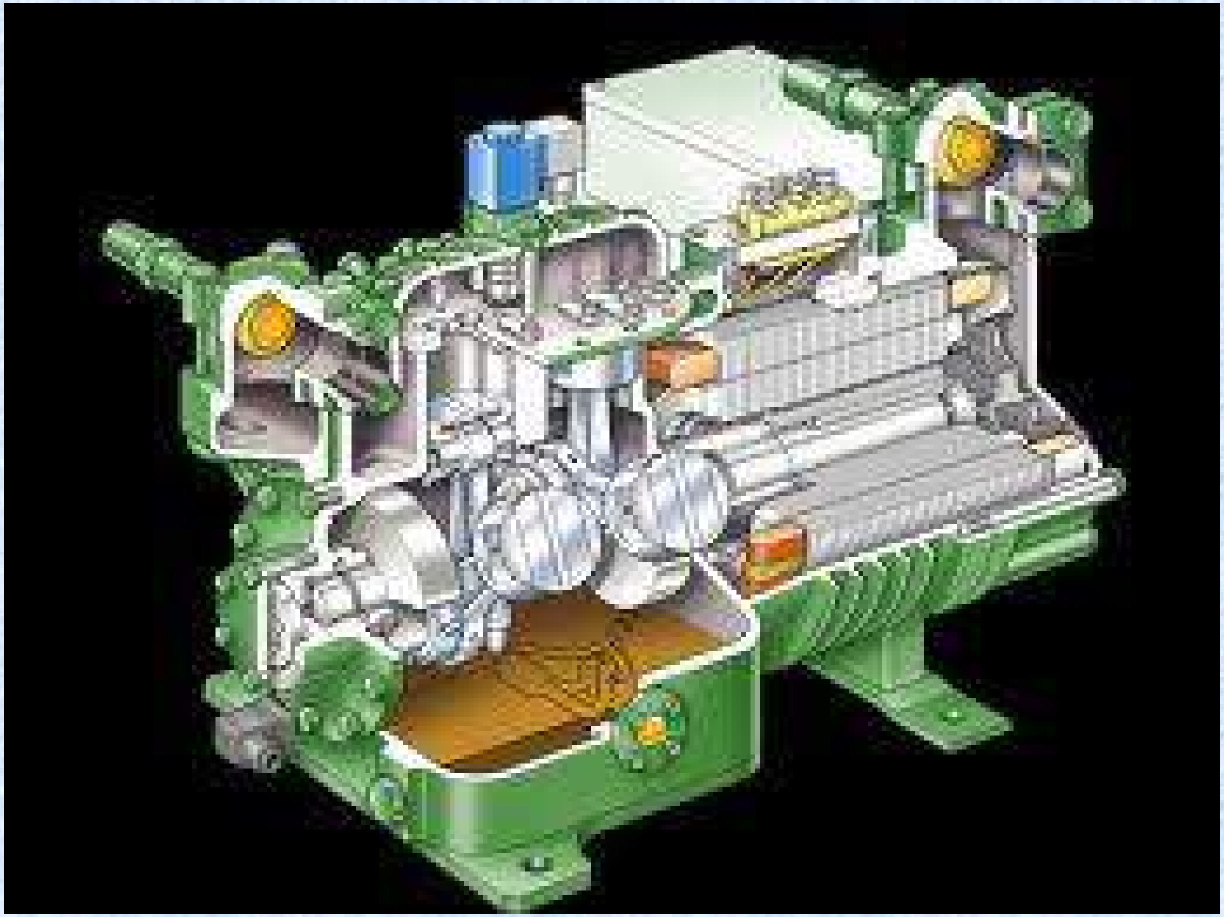
Compressor

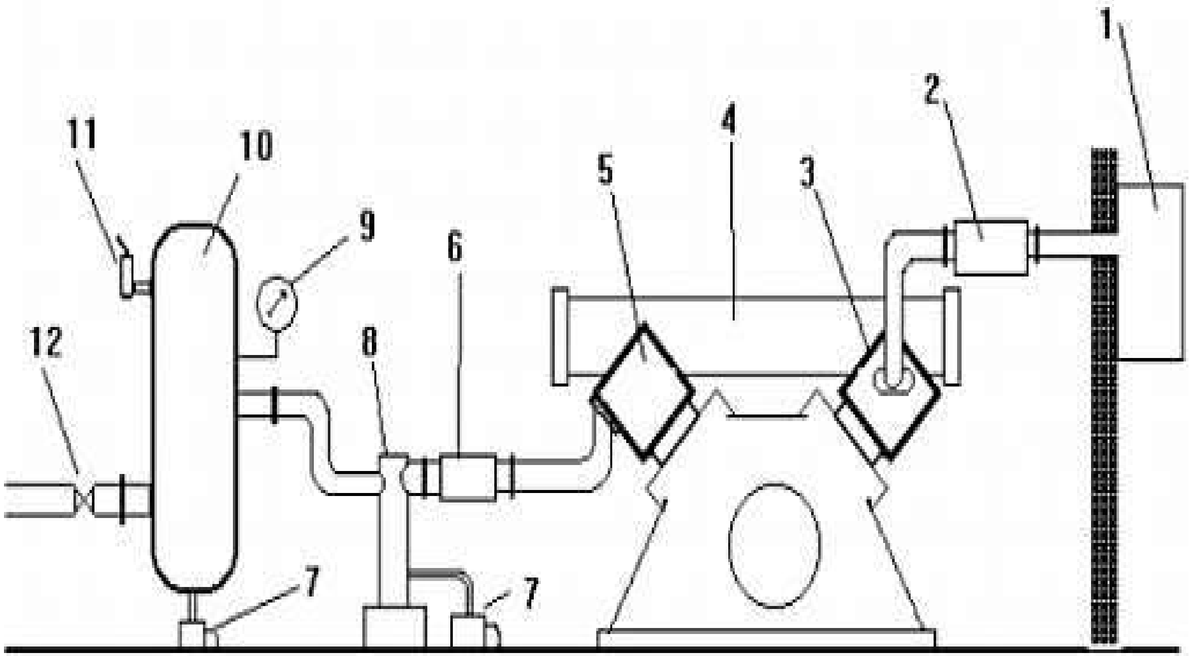


CAS

COMPONENT

- ***Intake Air Filters :*** *Prevent dust and atmospheric impurities from entering compressor. Dust causes sticking valves, scored cylinders, excessive wear etc.*
- **Compressor :** Pressurizes the air
- ***Inter-stage Coolers :*** *Reduce the temperature of the air (gas) before it enters the next stage to reduce the work of compression and increase efficiency. They can be water-or air-cooled.*
- ***After Coolers :*** *Reduce the temperature of the discharge air, and thereby reduce the moisture carrying capacity of air.*
- ***Air-dryers :*** *Air dryers are used to remove moisture, as air for instrument and pneumatic equipment needs to be relatively free of any moisture. The moisture is removed by suing adsorbents or refrigerant dryers, or state of the art heatless dryers.*
- ***Moisture Traps :*** *Air traps are used for removal of moisture in the compressed air distribution lines. They resemble steam traps wherein the air is trapped and moisture is removed.*
- ***Receivers :*** *Depending on the system requirements, one or more air receivers are generally provided to reduce output pulsations and pressure variations.*

Parts of reciprocating Compressor



1. Induction box and silencer
2. Induction filter.
3. Low pressure stage.
4. Intercooler.
5. High pressure stage.
6. Silencer.
7. Drain trap.
8. After cooler
9. Pressure gauge.
10. Air receiver.
11. Safety pressure relief valve.
12. Stop valve

COMPRESSOR

What is Compressor?

A compressor is a device that pressurize a working fluid, one of the basic aim of compressor is to compress the fluid and deliver it to a pressure which is higher than its original pressure.

PURPOSE

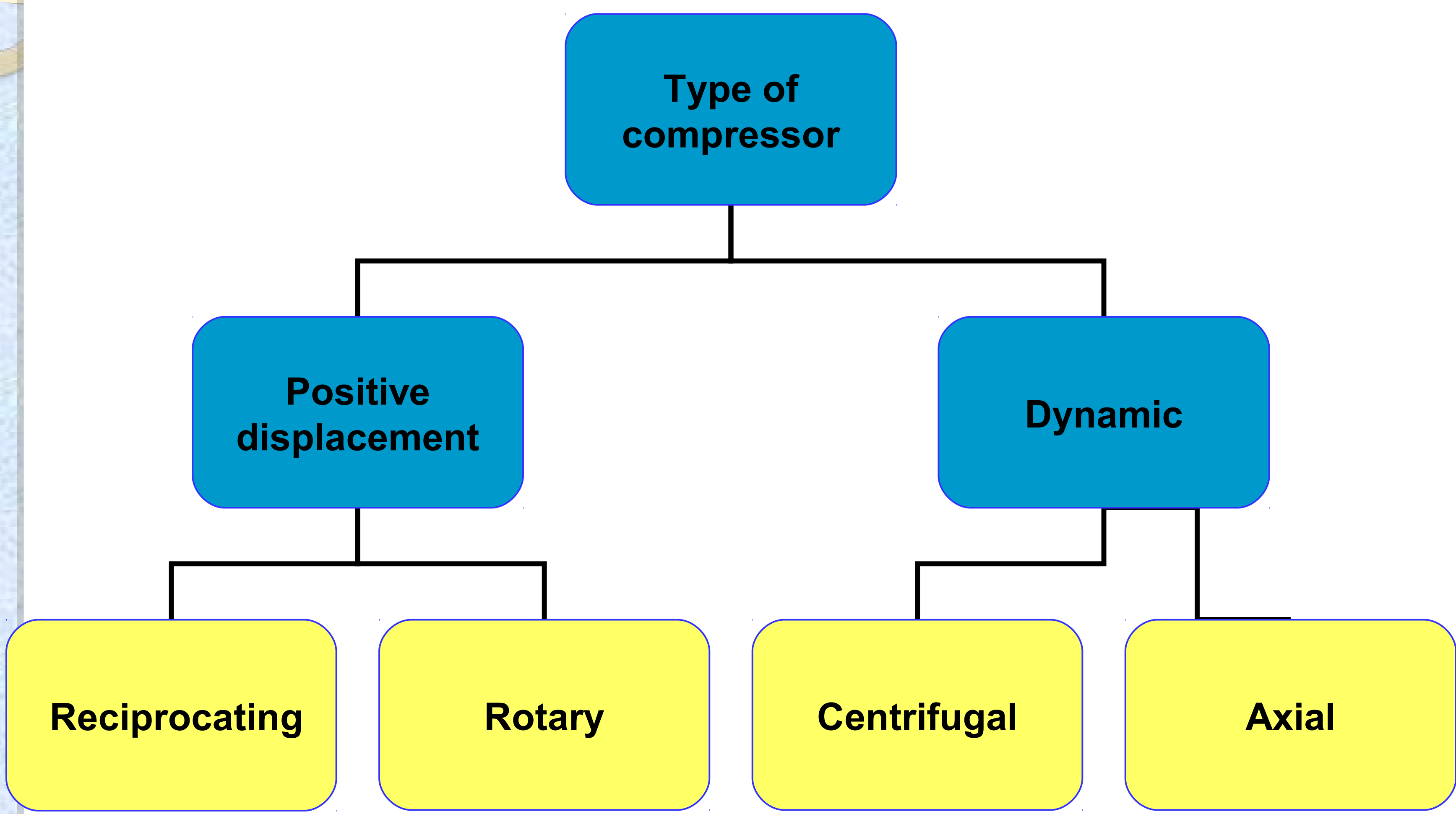
To provide air for combustion

To transport process fluid through pipeline

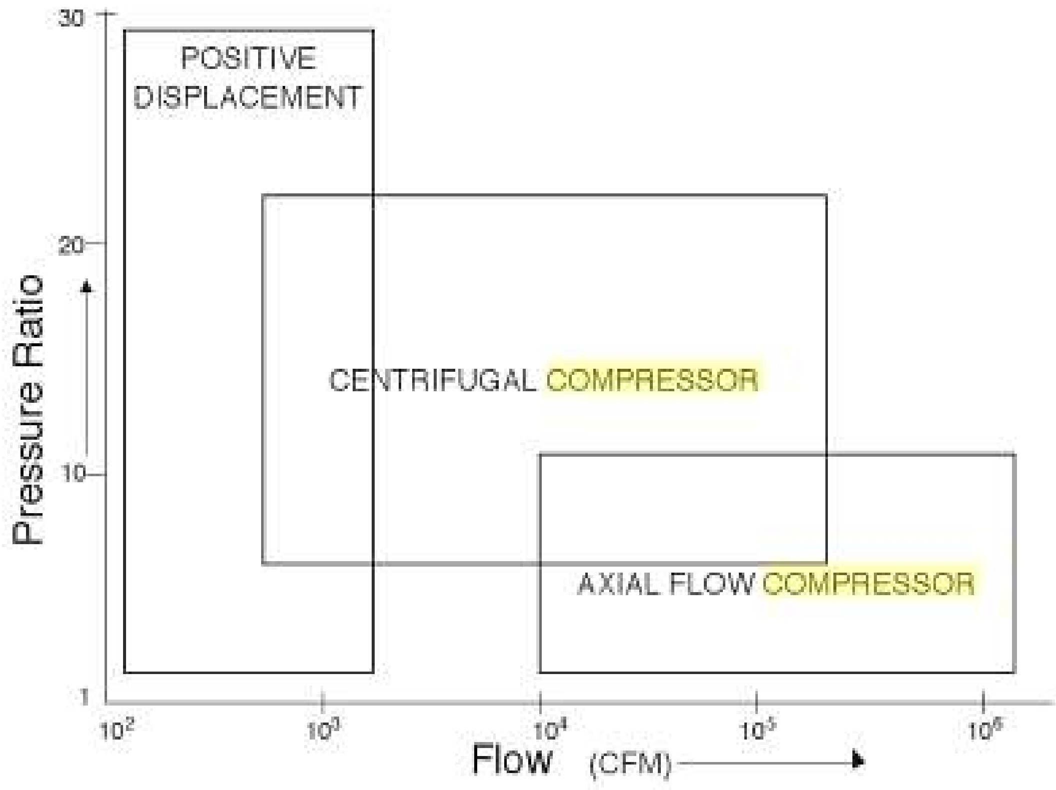
To provide compressed air for diving pneumatic tools

To circulate process fluid through certain process

Types of compressor



Compressor selection



Capacity of compressor

- Capacity of Compressor basically indicated by following two parameter

1. Pressure
2. FAD

What is FAD- Capacity of a Compressor?

- The FAD is the volume of air drawn into a compressor from the atmosphere. After compression and cooling the air is returned to the original temperature but it is at high pressure
- Suppose atmospheric condition are Pa Ta and Va(the FAD) and the compressed condition are p , V and T

$$\frac{pV}{T}=\frac{paVa}{Ta}$$

$$Va=\frac{pVTa}{paT}$$

Some definations

- Bore = Cylinder diameter.
- Stroke = Distance through which the piston moves.
- The two extreme positions of the piston are known as head-end and crank-end dead centers.
- Clearance Volume (Cl) : Volume occupied by the fluid when the piston is
- at head-end dead centre.
- Piston Displacement (PD) : Volume, a piston sweeps through.
- Compression Ratio (rv) : Ratio of cylinder volume with the piston at crank-end dead centre to the cylinder volume with the piston at head-end dead centre.
- Mechanical Efficiency : $\frac{\text{Brake work}}{\text{Indicated work}}$, which gives an indication of the

  losses occurring between the piston and driving shaft.

Compressor Efficiency Definitions

Isothermal Efficiency

$$\text{Isothermal Efficiency} = \frac{\textit{IsothermalPower}}{\text{Actual measured input power}}$$

$$\text{Isothermal power(kW)} = P_1 \text{ x } Q_1 \text{ x } \log_e r/36.7$$

P_1 = Absolute intake pressure $kg/\ cm^2$

Q_1 = Free air delivered m^3/hr.

r = Pressure ratio P_2/P_1

Compressor Efficiency Definitions

Volumetric Efficiency

$$\text{Volumetric efficiency} = \frac{\text{Free air delivered m}^3/\text{min}}{\text{Compressor displacement}}$$

$$\text{Compressor Displacement} = \frac{\pi}{4} \times D^2 \times L \times S \times \chi \times n$$

D	=	Cylinder bore, metre
L	=	Cylinder stroke, metre
S	=	Compressor speed rpm
χ	=	1 for single acting and 2 for double acting cylinders
n	=	No. of cylinders

Reciprocating Compressors

- Types

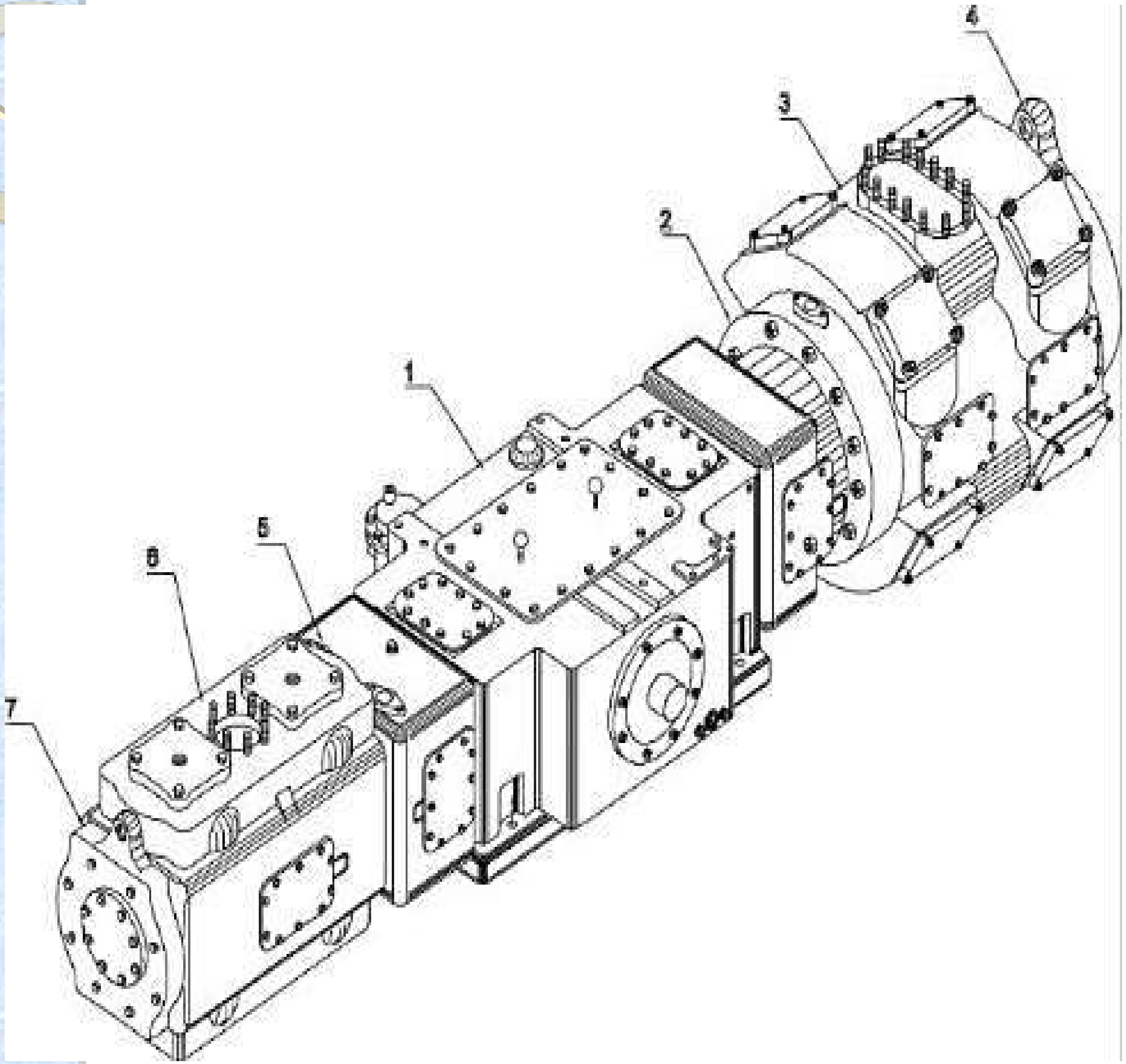
1. Single acting

   The working fluid compressed at only one side of the piston

2. Double acting

   The working fluid compressed alternately on both sides of the piston.

Frame HN2T - 150NP



1 Frame Assly.

2 Inner Head Assly. (LP)

3 Cylinder Assly. (LP)

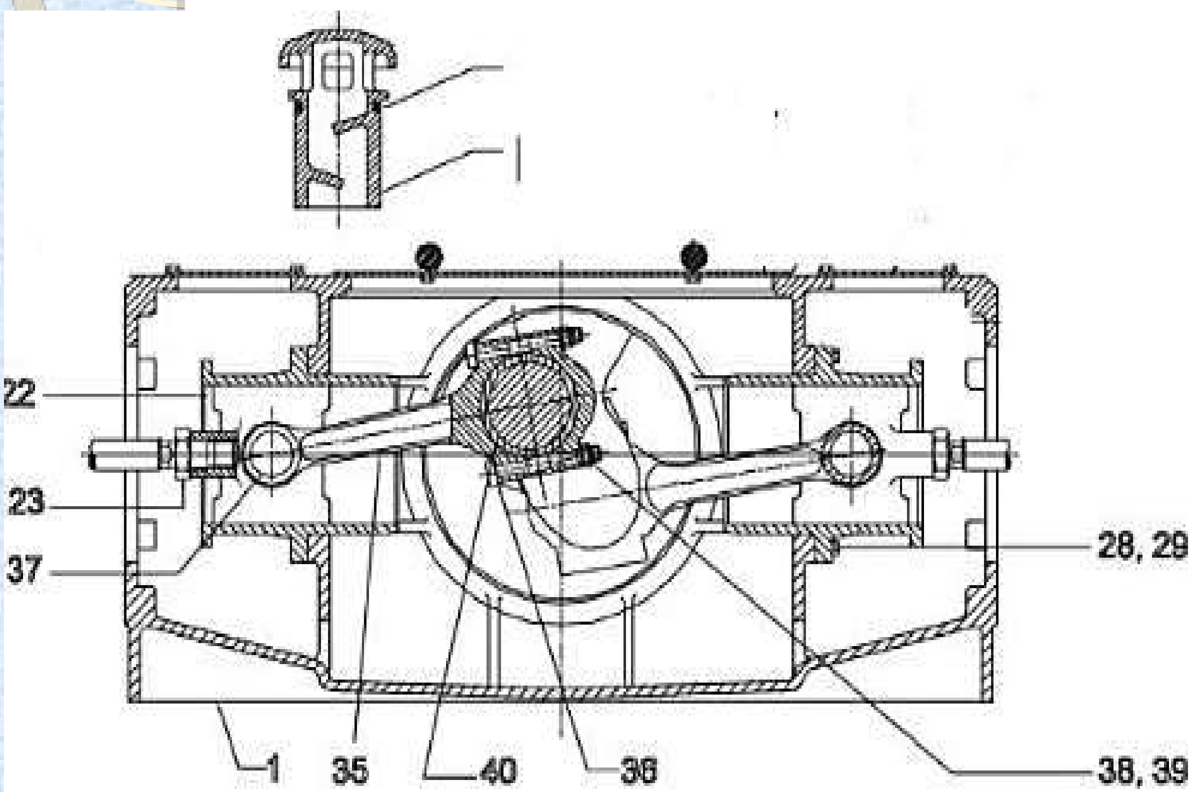
4 Outer Head Assly. (LP)

5 Inner Head Assly. (HP)

6 Cylinder Assly. (HP)

7 Outer Head Assly. (HP)

Frame, Cross Slide, Crank shaft and Connecting rod assembly

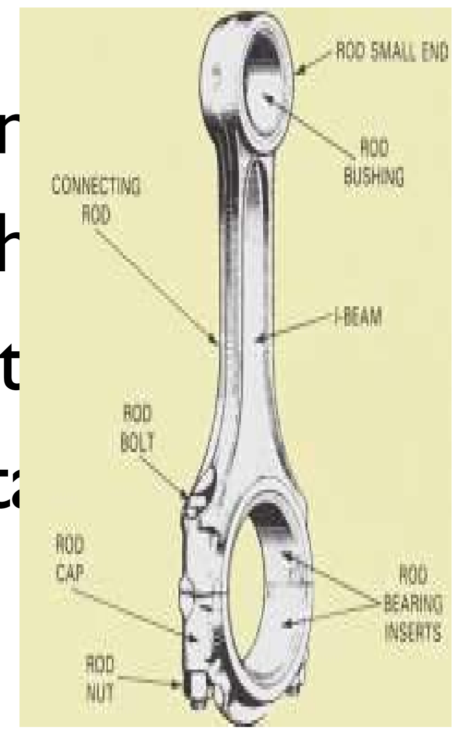


1. Breather
22. Crosshead
23. Cross Head Nut
35. Connecting Rod
40. Big End Bearing
36. Connecting rod Bolt
28,29. Stud,Nut

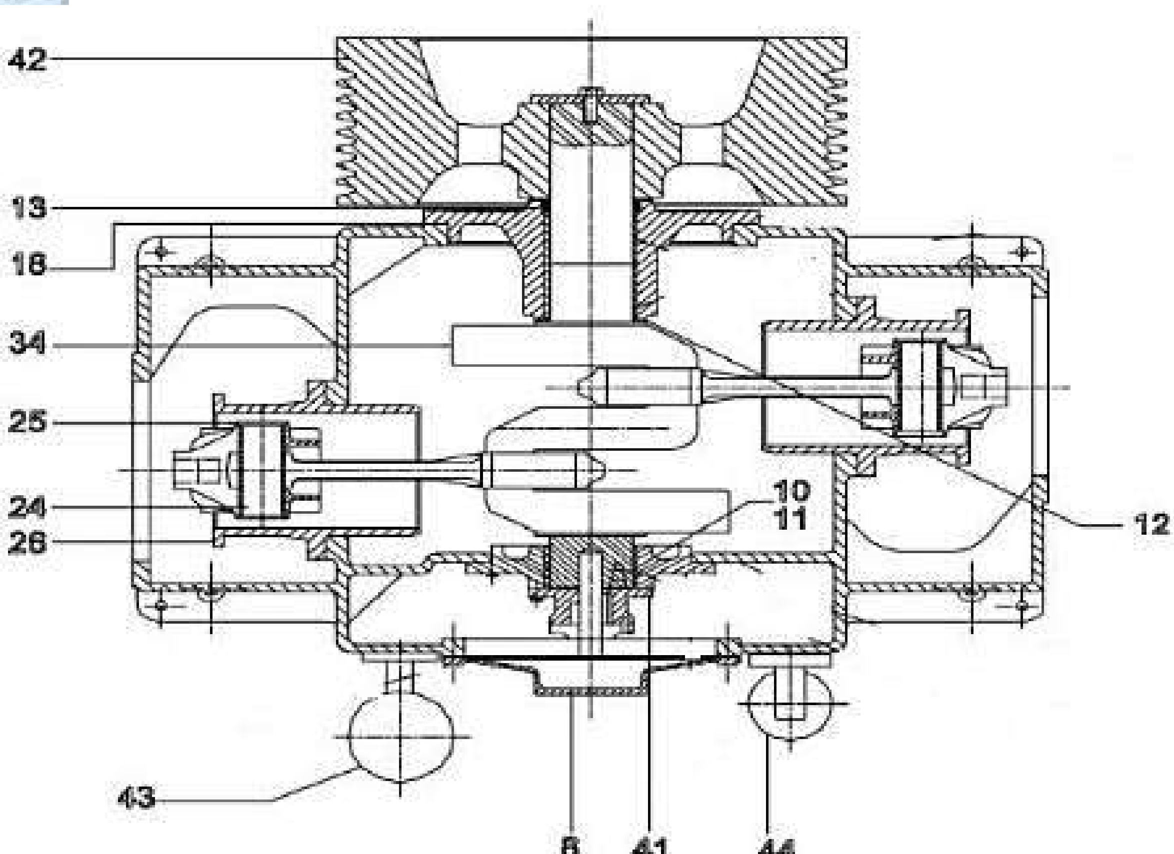
Breather: A vent or valve to release pressure or to allow air to move freely around something.

Crosshead: Is a mechanism used in large and reciprocating compressors to eliminate sideways pressure on the piston.

Connecting Rod: connects the piston the crank or crankshaft. Together with crank, they form a simple mechanism t converts reciprocating motion into rota motion.



Crank Case



42. Belt wheel
13. Oil Seal Ring
18. Gasket for Cover Flywheel end
34. Crank Shaft
25. Internal Circlip
24. Cross Head Pin
26. Cross Head Pin
43. Oil Cooler
8. Cover for Oil Pump end
41. Oil Pump Assembly
44. Oil filter
12. Thrust washer

Oil Seal Ring :It prevent the oil the oil to flow further

Gasket: is a mechanical seal which fills the space between two or more mating surfaces, generally to prevent leakage from or into the joined objects while under compression.

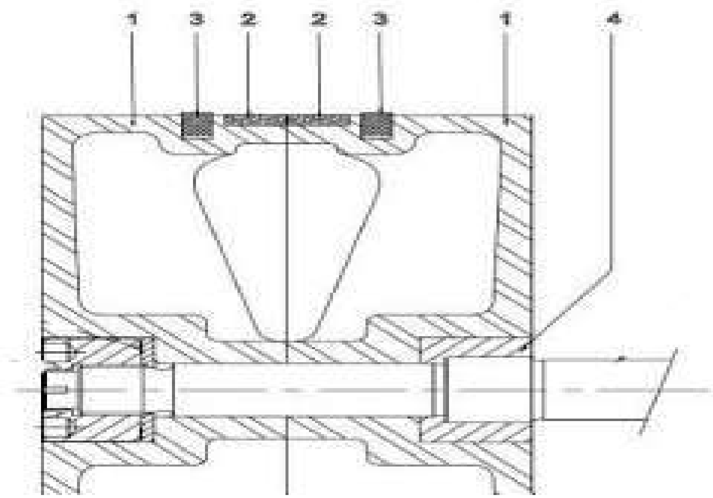
Circlip: It is a type of fastener or retaining ring consisting of a semiflexible metal ring with open ends which can be snapped into place, into a machined groove on a dowel pin or other part to permit rotation but to prevent lateral movement. There are two basic types: I nternal and external, referring to whether they are fitted into a bore or over a shaft.

Cross Head Pin : It connects the piston to the connecting rod and provides a bearing for the connecting rod to pivot upon as the piston moves.

Thrust washer: Thrust washers are long-wearing flat bearings in the shape of a washer that transmit and resolve axial forces in rotating mechanisms to keep components aligned along a shaft.

Crank Pin/Gudgeon Pin: Connects the piston to the connecting rod and provides a bearing for the connecting rod to pivot upon as the piston moves

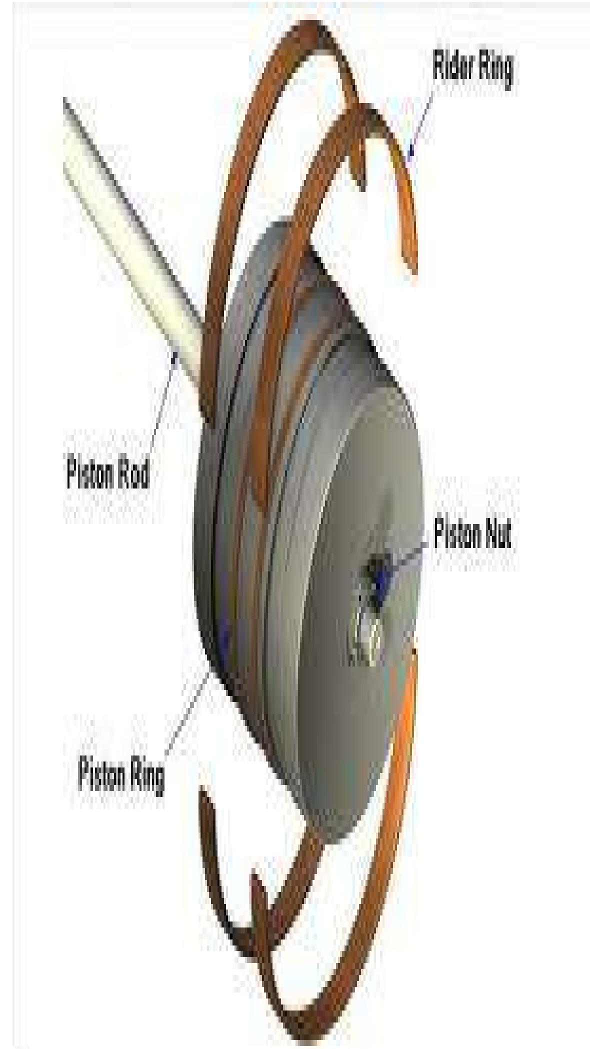
Piston Parts



1.Piston Assembly
2.Rider Ring
3.Piston Ring
4.Sleeve for piston

Piston Ring: Piston rings, mounted on the pistons of lubricated or non-lube (oil free) compressors, are designed to ensure that the gas is compressed and to provide a seal between the piston and the cylinder.

Rider Ring: The function of rider rings, used mainly in oil free or mini-lube compressors, is to support or guide the piston and rod assembly and prevent contact between the piston and the cylinder (risk of seizure).



Working:

- Reciprocating compressors generally, employ piston-cylinder arrangement where displacement of piston in cylinder causes rise in pressure.

Sequence of operation

Intake

Between d and a gas flows into the cylinder at a pressure lower than p_1 by the amount of pressure loss through the valve.

Compression

Starting at maximum cylinder volume, point a, slightly below the inlet pressure p, as the volume decreases the pressure rises until it reaches p_2 at b; the discharge valve does not open until the pressure in the cylinder exceeds p_2 by enough to overcome the valve spring force.

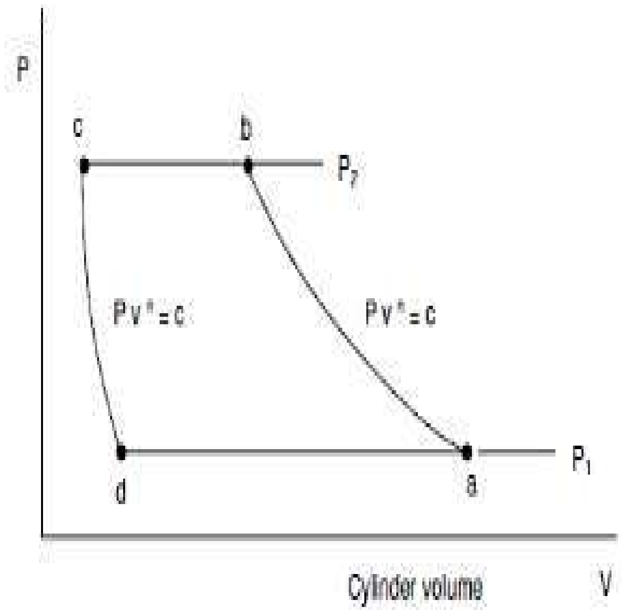
Discharge

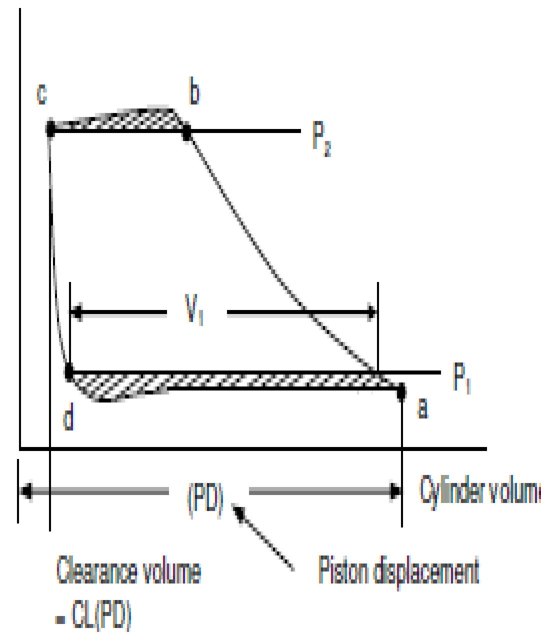
Between b and c gas flows out a pressure higher than p_2 by the amount of the pressure loss through the valves; at C, the point of minimum volume, the discharge valve is closed by its spring.

Expansion

From c to d, as the volume increases, the gas remaining in the clearance volume expands and its pressure falls; the suction valve does not open until the pressure falls sufficiently below p_1 to overcome the sprig force.

Ideal indicator diagram





The total work interaction per cycle :

$$w = w_{a-b} + w_{b-c} + w_{c-d} + w_{d-a}$$

$$= \frac{p_b v_b - p_a v_a}{1-n} + p_2 (v_c - v_b) + \frac{p_d v_d - p_c v_c}{1-n} + p_1 (v_a - v_d)$$

$$= \frac{p_2 (v_b - v_c)}{1-n} - p_2 (v_b - v_c) + \frac{p_1 (v_d - v_a)}{1-n} - p_1 (v_d - v_a)$$

$$= \frac{n}{1-n} [p_2 (v_b - v_c) + p_1 (v_d - v_a)]$$

$$= \frac{n}{1-n} [p_2 m_f v_2 - p_1 m_f v_1]$$

$$= m_f \frac{n}{1-n} [p_2 v_2 - p_1 v_1]$$

$$= m_f \frac{n}{1-n} p_1 v_1 \left[\frac{p_2 v_2}{p_1 v_1} - 1 \right]$$

Since pv^n = constant

$$\frac{p_2 v_2}{p_1 v_1} = \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}}$$

substituting this in the above expression

$$w = m_f \cdot \frac{n}{1-n} p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

Chicago Pneumatic: For over a century Chicago Pneumatic has represented tough tools designed to make tough jobs easier.

Way back in **1889** John W. Duntley realized that construction workers in particular had a need for many tools that weren't yet available. He founded Chicago Pneumatic Tool Company and set out on a lifelong mission to provide all types of industries and companies the tools necessary for their success.

Over the years Duntley grew the company through product innovation, always insisting on product quality and reliability.

Manufactures of air & gas compressors & pneumatic portable tools like grinders demolition tools, pumps vibrators, rammers hammers, etc.

Decades of innovation

1901 Chicago Pneumatic Tool Company is incorporated, after Duntley persuades young **steel magnate Charles M. Schwab**to invest in the company

1925 CP seals an agreement to manufacture the **Benz diesel engine** , used in various racing cars in Europe at the time.

1930s Chicago Pneumatic construction and mining equipment is used in the building of the

Lincoln Tunnel, New York
Triborough Bridge, New York
Chicago subway system
Boulder Dam, Arizona
Grand Coulee Dam, Washington
Eight dams comprising the Tennessee Valley Authority flood control and power generation project
Golden Gate suspension bridge, San Francisco

1940s In response to war effort demands, CP develops **the "hot dimpling machine,"** a device that heats rivets to 1,000 degrees Fahrenheit
1960s Chicago Pneumatic customizes tools for the production of new aircraft designs: the **Boeing 737 and 747,**

1987 Atlas Copco acquires Chicago Pneumatic Tool Company

Chicago Pneumatic Competition

- Elgi Equipment
- Ingersoll rand
- Revathi Cp

16

Reciprocating and Rotary Compressor

16.1 INTRODUCTION

Compressors are work absorbing devices which are used for increasing pressure of fluid at the expense of work done on fluid.

The compressors used for compressing air are called air compressors. Compressors are invariably used for all applications requiring high pressure air. Some of popular applications of compressor are, for driving pneumatic tools and air operated equipments, spray painting, compressed air engine, supercharging in internal combustion engines, material handling (for transfer of material), surface cleaning, refrigeration and air conditioning, chemical industry etc. Compressors are supplied with low pressure air (or any fluid) at inlet which comes out as high pressure air (or any fluid) at outlet, Fig. 16.1. Work required for increasing pressure of air is available from the prime mover driving the compressor. Generally, electric motor, internal combustion engine or steam engine, turbine etc. are used as prime movers. Compressors are similar to fans and blowers but differ in terms of pressure ratios. Fan is said to have pressure ratio up to 1.1 and blowers have pressure ratio between 1.1 and 4 while compressors have pressure ratios more than 4.

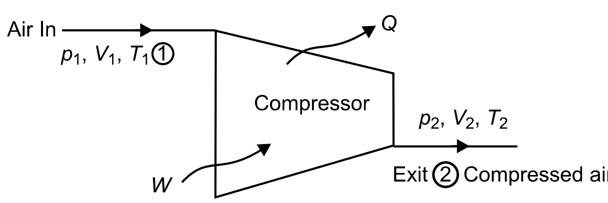


Fig. 16.1 *Compressor*

Compressors can be classified in the following different ways.

(a) ***Based on principle of operation:*** Based on the principle of operation compressors can be classified as,

(i) Positive displacement compressors

(ii) Non-positive displacement compressors

In positive displacement compressors the compression is realized by displacement of solid boundary and preventing fluid by solid boundary from flowing back in the direction of pressure gradient. Due to solid wall displacement these are capable of providing quite large pressure ratios. Positive displacement compressors can be further classified based on the type of mechanism used for compression. These can be

(i) Reciprocating type positive displacement compressors
(ii) Rotary type positive displacement compressors

Reciprocating compressors generally, employ piston-cylinder arrangement where displacement of piston in cylinder causes rise in pressure. Reciprocating compressors are capable of giving large pressure ratios but the mass handling capacity is limited or small. Reciprocating compressors may also be single acting compressor or double acting compressor. Single acting compressor has one delivery stroke per revolution while in double acting there are two delivery strokes per revolution of crank shaft. Rotary compressors employing positive displacement have a rotary part whose boundary causes positive displacement of fluid and thereby compression. Rotary compressors of this type are available in the names as given below;

(i) Roots blower
(ii) Vaned type compressors

Rotary compressors of above type are capable of running at higher speed and can handle large mass flow rate than reciprocating compressors of positive displacement type.

Non-positive displacement compressors, also called as steady flow compressors use dynamic action of solid boundary for realizing pressure rise. Here fluid is not contained in definite volume and subsequent volume reduction does not occur as in case of positive displacement compressors. Non-positive displacement compressor may be of 'axial flow type' or 'centrifugal type' depending upon type of flow in compressor.

(b) ***Based on number of stages:*** Compressors may also be classified on the basis of number of stages. Generally, the number of stages depend upon the maximum delivery pressure. Compressors can be single stage or multistage. Normally maximum compression ratio of 5 is realized in single stage compressors. For compression ratio more than 5 the multi-stage compressors are used.

Typical values of maximum delivery pressures generally available from different types of compressor are,

(i) Single stage compressor, for delivery pressure up to 5 bar
(ii) Two stage compressor, for delivery pressure between 5 and 35 bar
(iii) Three stage compressor, for delivery pressure between 35 and 85 bar
(iv) Four stage compressor, for delivery pressure more than 85 bar

(c) ***Based on capacity of compressors:*** Compressors can also be classified depending upon the capacity of compressor or air delivered per unit time. Typical values of capacity for different compressors are given as;

(i) Low capacity compressors, having air delivery capacity of 0.15 m^3/s or less
(ii) Medium capacity compressors, having air delivery capacity between 0.15 and 5 m^3/s.
(iii) High capacity compressors, having air delivery capacity more than 5 m^3/s.

(d) ***Based on highest pressure developed:*** Depending upon the maximum pressure available from compressor they can be classified as low pressure, medium pressure, high pressure and super high pressure compressors. Typical values of maximum pressure developed for different compressors are as under;

(i) Low pressure compressor, having maximum pressure up to 1 bar
(ii) Medium pressure compressor, having maximum pressure from 1 to 8 bar
(iii) High pressure compressor, having maximum pressure from 8 to 10 bar
(iv) Super high pressure compressor, having maximum pressure more than 10 bar.

16.2 RECIPROCATING COMPRESSORS

Reciprocating compressor has piston cylinder arrangement as shown in Fig. 16.2.

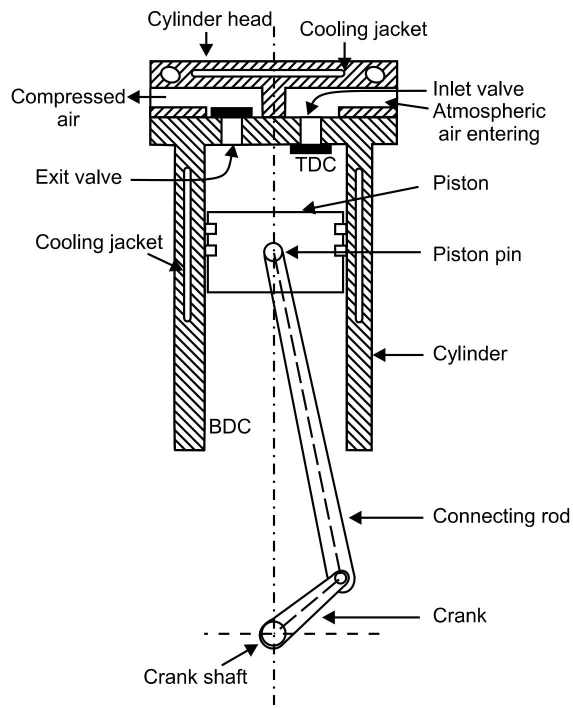


Fig. 16.2 *Line diagram of reciprocating compressor*

Reciprocating compressor has piston, cylinder, inlet valve, exit valve, connecting rod, crank, piston pin, crank pin and crank shaft. Inlet valve and exit valves may be of spring loaded type which get opened and closed due to pressure differential across them. Let us consider piston to be at top dead centre (TDC) and move towards bottom dead centre (BDC). Due to this piston movement from TDC to BDC suction pressure is created causing opening of inlet valve. With this opening of inlet valve and suction pressure the atmospheric air enters the cylinder.

Air gets into cylinder during this stroke and is subsequently compressed in next stroke with both inlet valve and exit valve closed. Both inlet valve and exit valves are of plate type and spring loaded so as to operate automatically as and when sufficient pressure difference is available to cause deflection in spring of valve plates to open them. After piston reaching BDC it reverses its motion and compresses the air inducted in previous stroke. Compression is continued till the pressure of air inside becomes sufficient to cause deflection in exit valve. At the moment when exit valve plate gets lifted the exhaust of compressed air takes place. This piston again reaches TDC from where downward piston movement is again accompanied by suction. This is how reciprocating compressor keeps on working as flow device. In order to counter for the heating of piston-cylinder arrangement during compression the provision of cooling the cylinder is there in the form of cooling jackets in the body. Reciprocating compressor described above has suction, compression and discharge as three prominent processes getting completed in two strokes of piston or one revolution of crank shaft.

16.3 THERMODYNAMIC ANALYSIS

Compression of air in compressor may be carried out following number of thermodynamic processes such as isothermal compression, polytropic compression or adiabatic compression. Figure 16.3 shows the thermodynamic cycle involved in compression. Theoretical cycle is shown neglecting clearance volume but in actual cycle clearance volume can not be negligible. Clearance volume is necessary in order to prevent collision of piston with cylinder head, accommodating valve mechanism etc. Compression process is shown by process 1–2, 1–2′, 1–2″ following adiabatic, polytropic and isothermal processes.

On *p-V* diagram process 4–1 shows the suction process followed by compression during 1–2 and discharge through compressor is shown by process 2–3.

Air enters compressor at pressure p_1 and is compressed up to p_2. Compression work requirement can be estimated from the area below the each compression process. Area on *p–V* diagram shows that work requirement shall be minimum with isothermal process 1–2″. Work requirement is maximum with process 1–2 i.e. adiabatic process. As a designer one shall be interested in a compressor having minimum compression work requirement. Therefore, ideally compression should occur isothermally for minimum work input. In practice it is not possible to have isothermal compression because constancy of temperature

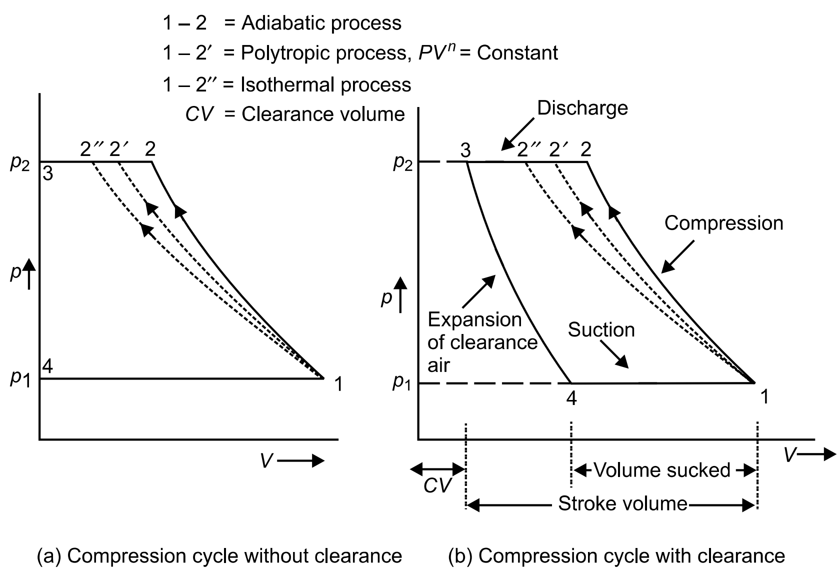


Fig. 16.3 *Compression cycle on* p-V *diagram*

during compression can not be realized. Generally, compressors run at substantially high speed while isothermal compression requires compressor to run at very slow speed so that heat evolved during compression is dissipated out and temperature remains constant. Actually due to high speed running of compressor the compression process may be assumed to be near adiabatic or polytropic process following law of compression as $pV^n = C$ with value of 'n' varying between 1.25 and 1.35 for air. Compression process following three processes is also shown on *T-s* diagram in Fig. 16.4. It is thus obvious that actual compression process should be compared with isothermal compression process. A mathematical parameter called isothermal efficiency is defined for quantifying the degree of deviation of actual compression process from ideal compression process. Isothermal efficiency is defined by the ratio of isothermal work and actual indicated work in reciprocating compressor.

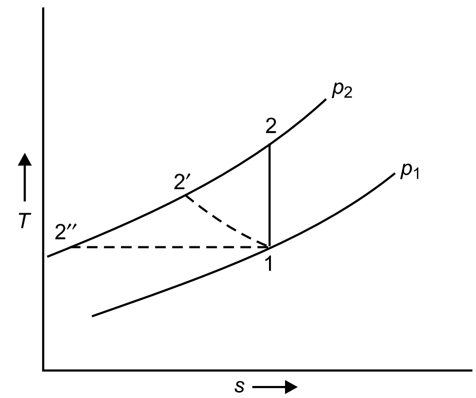


Fig. 16.4 *Compression process on* T-s *diagram*

$$\text{Isothermal efficiency} = \frac{\text{Isothermal work}}{\text{Actual indicated work}}$$

Practically, compression process is attempted to be closed to isothermal process by air/water cooling, spraying cold water during compression process. In case of multi-stage compression process the compression in different stages is accompanied by intercooling in between the stages.

Mathematically, for the compression work following polytropic process, $pV^n = C$. Assuming negligible clearance volume the cycle work done,

$$W_c = \text{Area on } p\text{-}V \text{ diagram}$$

$$= \left[p_2V_2 + \left(\frac{p_2V_2 - p_1V_1}{n-1}\right)\right] - p_1V_1$$

$$= \left(\frac{n}{n-1}\right)[p_2V_2 - p_1V_1]$$

$$= \left(\frac{n}{n-1}\right)(p_1V_1)\left[\frac{p_2V_2}{p_1V_1} - 1\right]$$

$$W_c = \left(\frac{n}{n-1}\right)(p_1V_1)\left[\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right]$$

$$\boxed{W_c = \left(\frac{n}{n-1}\right)(mRT_1)\left[\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right]}$$

or, $$\boxed{W_c = \left(\frac{n}{n-1}\right) mR\,(T_2 - T_1)}$$

In case of compressor having isothermal compression process, $n = 1$, i.e. $p_1V_1 = p_1V_2$

$$W_{c,\ \text{iso}} = p_2V_2 + p_1V_1 \ln r - p_1V_1$$

$$W_{c,\ \text{iso}} = p_1V_1 \ln r, \text{ where } r = \frac{V_1}{V_2}$$

In case of compressor having adiabatic compression process, $n = \gamma$

$$\boxed{W_{c,\ \text{adiabatic}} = \left(\frac{\gamma}{\gamma - 1}\right) mR\ (T_2 - T_1)}$$

or,

$$W_{c,\ \text{adiabatic}} = mC_p\ (T_2 - T_1)$$

$$\boxed{W_{c,\ \text{adiabatic}} = m\ (h_2 - h_1)}$$

Hence, isothermal efficiency, $$\boxed{\eta_{\text{iso}} = \frac{p_1V_1 \ln r}{\left(\frac{n}{n-1}\right)(p_1V_1)\left[\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right]}}$$

The isothermal efficiency of a compressor should be close to 100% which means that actual compression should occur following a process close to isothermal process. For this the mechanism be derived to maintain constant temperature during compression process. Different arrangements which can be used are:

(i) Faster heat dissipation from inside of compressor to outside by use of fins over cylinder. Fins facilitate quick heat transfer from air being compressed to atmosphere so that temperature rise during compression can be minimized.

(ii) Water jacket may be provided around compressor cylinder so that heat can be picked by cooling water circulating through water jacket. Cooling water circulation around compressor regulates rise in temperature to great extent.

(iii) The water may also be injected at the end of compression process in order to cool the air being compressed. This water injection near the end of compression process requires special arrangement in compressor and also the air gets mixed with water and needs to be separated out before being used. Water injection also contaminates the lubricant film on inner surface of cylinder and may initiate corrosion etc. The water injection is not popularly used.

(iv) In case of multistage compression in different compressors operating serially, the air leaving one compressor may be cooled up to ambient state or somewhat high temperature before being injected into subsequent compressor. This cooling of fluid being compressed between two consecutive compressors is called intercooling and is frequently used in case of multistage compressors.

Considering clearance volume: With clearance volume the cycle is represented on Fig. 16.3 (b). The work done for compression of air polytropically can be given by the area enclosed in cycle 1–2–3–4. Clearance volume in compressors varies from 1.5% to 35% depending upon type of compressor.

$$W_{c,\ \text{with CV}} = \text{Area1234}$$

$$= \left(\frac{n}{n-1}\right) (p_1V_1) \left[\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right] - \left(\frac{n}{n-1}\right) (p_4V_4) \left[\left(\frac{p_3}{p_4}\right)^{\frac{(n-1)}{n}} - 1\right]$$

Here $p_1 = p_4,\ p_2 = p_3$

$$W_{c,\text{ with }CV} = \left(\frac{n}{n-1}\right)(p_1V_1)\left[\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right] - \left(\frac{n}{n-1}\right)(p_1V_4)\cdot\left[\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right]$$

$$\boxed{W_{c,\text{ with }CV} = \left(\frac{n}{n-1}\right)p_1\cdot\left[\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right]\cdot(V_1 - V_4)}$$

In the cylinder of reciprocating compressor $(V_1 - V_4)$ shall be the actual volume of air delivered per cycle. $V_d = V_1 - V_4$. This $(V_1 - V_4)$ is actually the volume of air inhaled in the cycle and delivered subsequently.

$$\boxed{W_{c,\text{ with }CV} = \left(\frac{n}{n-1}\right)p_1V_d\left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]}$$

If air is considered to behave as perfect gas then pressure, temperature, volume and mass can be inter related using perfect gas equation. The mass at state 1 may be given as m_1, mass at state 2 shall be m_1, but at state 3 after delivery mass reduces to m_2 and at state 4 it shall be m_2.

So, at state 1, $p_1V_1 = m_1RT_1$

at state 2, $p_2V_2 = m_1RT_2$

at state 3, $p_3V_3 = m_2RT_3$ or, $p_2V_3 = m_2RT_3$

at state 4, $p_4V_4 = m_2RT_4$, or $p_1V_4 = m_2RT_4$

Ideally there shall be no change in temperature during suction and delivery i.e., $T_4 = T_1$ and $T_2 = T_3$. From earlier equation,

$$W_{c,\text{ with }CV} = \left(\frac{n}{n-1}\right)p_1(V_1 - V_4)\left[\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right]$$

Temperature and pressure can be related as,

$$\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} = \frac{T_2}{T_1}$$

and

$$\left(\frac{p_4}{p_3}\right)^{\frac{(n-1)}{n}} = \frac{T_4}{T_3} \Rightarrow \left(\frac{p_1}{p_2}\right)^{\frac{(n-1)}{n}} = \frac{T_4}{T_3}$$

Substituting,

$$W_{c,\text{ with }CV} = \left(\frac{n}{n-1}\right)(m_1RT_1 - m_2RT_4)\left[\frac{T_2}{T_1} - 1\right]$$

Substituting for constancy of temperature during suction and delivery.

$$W_{c,\text{ with }CV} = \left(\frac{n}{n-1}\right)(m_1RT_1 - m_2RT_1)\left[\frac{T_2 - T_1}{T_1}\right]$$

or,

$$\boxed{W_{c,\text{ with }CV} = \left(\frac{n}{n-1}\right)(m_1 - m_2)R(T_2 - T_1)}$$

Thus, $(m_1 - m_2)$ denotes the mass of air sucked or delivered. For unit mass of air delivered the work done per kg of air can be given as,

$$\boxed{W_{c,\text{ with } CV} = \left(\frac{n}{n-1}\right) R(T_2 - T_1), \text{ per kg of air}}$$

Thus from above expressions it is obvious that the clearance volume reduces the effective swept volume i.e. the mass of air handled but the work done per kg of air delivered remains unaffected.

From the cycle work estimated as above the theoretical power required for running compressor shall be as given ahead.

For single acting compressor running with N rpm, power input required, assuming clearance volume.

$$\text{Power required} = \left[\left(\frac{n}{n-1}\right) p_1 (V_1 - V_4) \left\{\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right\}\right] \times N$$

$$\text{for double acting compressor, power} = \left[\left(\frac{n}{n-1}\right) p_1 (V_1 - V_4) \left\{\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right\}\right] \times 2N$$

Volumetric efficiency: Volumetric efficiency of compressor is the measure of the deviation from volume handling capacity of compressor. Mathematically, the volumetric efficiency is given by the ratio of actual volume of air sucked and swept volume of cylinder. Ideally the volume of air sucked should be equal to the swept volume of cylinder, but it is not so in actual case. Practically the volumetric efficiency lies between 60 and 90%.

Volumetric efficiency can be overall volumetric efficiency and absolute volumetric efficiency as given below:

$$\text{Overall volumetric efficiency} = \frac{\text{Volume of free air sucked into cylinder}}{\text{Swept volume of LP cylinder}}$$

or

$$\begin{pmatrix}\text{Volumetric efficiency} \\ \text{referred to free air conditions}\end{pmatrix} = \frac{(\text{Mass of air delivered per unit time})}{\begin{pmatrix}\text{Mass of air corresponding to swept} \\ \text{volume of LP cylinder per unit time} \\ \text{for free air conditions}\end{pmatrix}}$$

Here free air condition refers to the standard conditions. Free air condition may be taken as 1 atm or 1.01325 bar and 15°C or 288 K. Consideration for free air is necessary as otherwise the different compressors can not be compared using volumetric efficiency because specific volume or density of air varies with altitude. It may be seen that a compressor at datum level (sea level) shall deliver large mass than the same compressor at high altitude.

This concept is used for giving the capacity of compressor in terms of 'free air delivery' (FAD). "Free air delivery is the volume of air delivered being reduced to free air conditions." In case of air the free air delivery can be obtained using perfect gas equation as,

$$\frac{p_a \cdot V_a}{T_a} = \frac{p_1 (V_1 - V_4)}{T_1} = \frac{p_2 (V_2 - V_3)}{T_2}$$

where subscript a or p_a, V_a, T_a denote properties at free air conditions

or, $$V_a = \frac{p_1 \cdot T_a \cdot (V_1 - V_4)}{p_a \cdot T_1} = \text{FAD per cycle.}$$

This volume V_a gives 'free air delivered' per cycle by the compressor.

Absolute volumetric efficiency can be defined, using NTP conditions in place of free air conditions.

$$\text{Absolute volumetric efficiency} = \frac{\text{Volume of air sucked into cylinder at NTP}}{\text{Swept volume of LP cylinder}}$$

$$= \frac{(\text{Mass of air delivered per unit time})}{\left(\begin{array}{l}\text{Mass of air corresponding to swept volume}\\ \text{of LP cylinder per unit time estimated at NTP}\end{array}\right)}$$

Thus, volumetric efficiency referred to free air conditions.

$$\eta_{\text{vol.}} = \frac{\text{Volume of air sucked referred to free air conditions (FAD)}}{\text{Swept volume}}$$

$$= \frac{V_a}{(V_1 - V_3)}$$

$$= \frac{p_1 T_a (V_1 - V_4)}{p_a T_1 (V_1 - V_3)}$$

$$\eta_{\text{vol.}} = \left(\frac{p_1 T_a}{p_a T_1}\right) \cdot \left\{\frac{(V_s + V_c) - V_4}{V_s}\right\}.$$

Here, V_s is swept volume, $V_s = V_1 - V_3$

and V_c is clearance volume, $V_c = V_3$

$$\eta_{\text{vol.}} = \left(\frac{p_1 T_a}{p_a T_1}\right) \left\{1 + \left(\frac{V_c}{V_s}\right) - \left(\frac{V_4}{V_s}\right)\right\}$$

Here,

$$\frac{V_4}{V_s} = \frac{V_4}{V_c} \cdot \frac{V_c}{V_s}$$

$$= \left(\frac{V_4}{V_3} \cdot \frac{V_c}{V_s}\right)$$

Let the ratio of clearance volume to swept volume be given by C. i.e. $\frac{V_c}{V_s} = C$.

$$\eta_{\text{vol.}} = \left(\frac{p_1 T_a}{p_a T_1}\right) \cdot \left\{1 + C - C \cdot \left(\frac{V_4}{V_3}\right)\right\}$$

or, $$\boxed{\eta_{\text{vol.}} = \left(\frac{p_1 T_a}{p_a T_1}\right) \cdot \left\{1 + C - C \cdot \left(\frac{p_2}{p_1}\right)^{1/n}\right\}}$$

Volumetric efficiency depends on ambient pressure and temperature, suction pressure and temperature, ratio of clearance to swept volume, and pressure limits. Volumetric efficiency increases with decrease in pressure ratio in compressor.

16.4 ACTUAL INDICATOR DIAGRAM

Theoretical indicator diagram of reciprocating compressor as shown in earlier discussion refers to the ideal state of operation of compressor. The practical limitations, when considered in the indicator diagram yield actual indicator diagram as shown in Fig. 16.5.

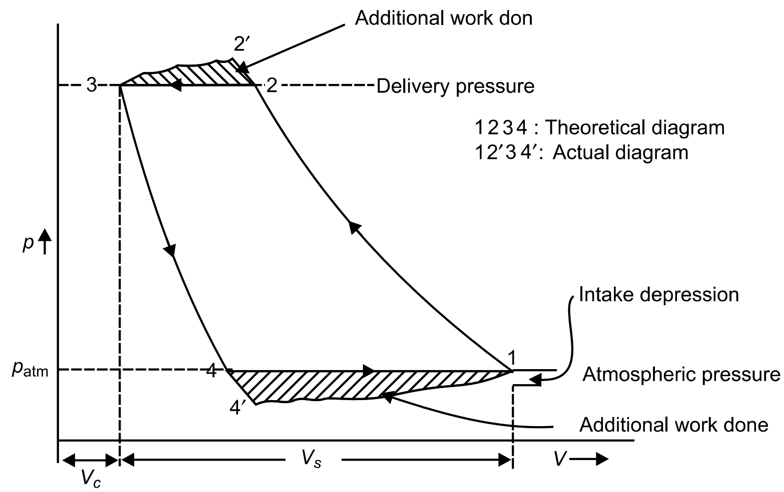


Fig. 16.5 *Actual indicator diagram*

Actual *p-V* diagram varies from theoretical *p-V* diagram due to following: Compressor has mechanical types of valves so the instantaneous opening and closing of valves can never be achieved. Also during suction and discharge there occurs throttling due to reduction in area of flow across inlet and exit valve. 1234 shows theoretical indicator diagram and actual indicator diagram is shown by 12′34′ on *p-V* diagram. Compression process 1–2 ends at state 2. At state 2 exit valve should open instantaneously which does not occur and also due to restricted opening there shall be throttling causing drop in pressure. Due to time lag in opening of exit valve compression process is continued up to 2′. Thus, additional work is done during delivery from compressor as shown by hatched area 22′3.

After delivery stroke the inlet valve should theoretically open at 4 but does not open at this point instead is opened fully at 4′. Shift from state 4 to 4′ is there due to inertia in opening of valve throttling, gradual opening, and friction losses etc. Thus it is seen that during suction there occurs intake depression as shown in actual indicator diagram. Work required as shown in actual indicator diagram is more than theoretical diagram. In order to have compressor close to ideal compressor with minimum losses it shall be desired to have actual indicator diagram close to theoretical diagram, which requires less inertia and efficient operation of valves. Friction losses in pipings and across valves should be minimized.

16.5 MULTISTAGE COMPRESSION

Multistage compression refers to the compression process completed in more than one stage i.e. a part of compression occurs in one cylinder and subsequently compressed air is sent to subsequent cylinders for further compression. In case it is desired to increase the compression ratio of compressor then multi-stage compression becomes inevitable. If we look at the expression for volumetric efficiency then it shows that the volumetric efficiency decreases with increase in pressure ratio. This aspect can also be explained using *p*-*V* representation shown in Fig. 16.6.

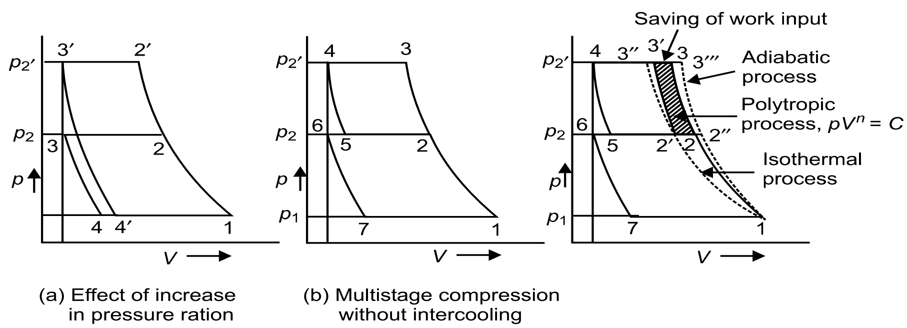


Fig. 16.6 *Multistage compression*

Figure 16.6 a, shows that by increasing pressure ratio i.e. increasing delivery pressure the volume of air being sucked goes on reducing as evident from cycles 1234 and 12′3′4′. Let us increase pressure from p_2 to $p_{2'}$ and this shall cause the suction process to get modified from 4–1 to 4′–1. Thus volume sucked reduces from $(V_1 - V_4)$ to $(V_1 - V_{4'})$ with increased pressure ratio from $\left(\frac{p_2}{p_1}\right)$ to $\left(\frac{p_2{}'}{p_1}\right)$, thereby reducing the free air delivery while swept volume remains same.

Therefore, the volumetric efficiency reduces with increasing pressure ratio in compressor with single stage compression. Also for getting the same amount of free air delivery the size of cylinder is to be increased with increasing pressure ratio. The increase in pressure ratio also requires sturdy structure from mechanical strength point of view for withstanding large pressure difference.

The solution to number of difficulties discussed above lies in using the multistage compression where compression occurs in parts in different cylinders one after the other. Figure 16.6 b, shows the multistage compression occurring in two stages. Here first stage of compression occurs in cycle 12671 and after first stage compression partly compressed air enters second stage of compression and occurs in cycle 2345. In case of multistage compression the compression in first stage occurs at low temperature and subsequent compression in following stages occurs at higher temperature. The compression work requirement depends largely upon the average temperature during compression. Higher average temperature during compression has larger work requirement compared to low temperature so it is always desired to keep the low average temperature during compression.

Apart from the cooling during compression the temperature of air at inlet to compressor can be reduced so as to reduce compression work. In multistage compression the partly compressed air leaving first stage is cooled up to ambient air temperature in intercooler and then sent to subsequent cylinder (stage) for compression. Thus, intercoolers when put between the stages reduce the compression work and compression is called intercooled compression. Intercooling is called perfect when temperature at inlet to subsequent stages of compression is reduced to ambient temperature. Figure 16.6 c, shows multistage (two stage) intercooled compression. Intercooling between two stages causes temperature drop from 2 to 2′ i.e. discharge from first stage (at 2) is cooled up to the ambient temperature state (at 2′) which lies on isothermal compression process 1–2′–3″. In the absence of intercooling the discharge from first stage shall enter at 2. Final discharge from second stage occurs at 3′ in case of intercooled compression compared to discharge at 3 in case of non-intercooled compression. Thus, intercooling offers reduced work requirement by the amount shown by area 22′3′3 on *p-V* diagram. If the inter-cooling is not perfect then the inlet state to second/subsequent stage shall not lie on the isothermal compression process line and this state shall lie between actual discharge state from first stage and isothermal compression line.

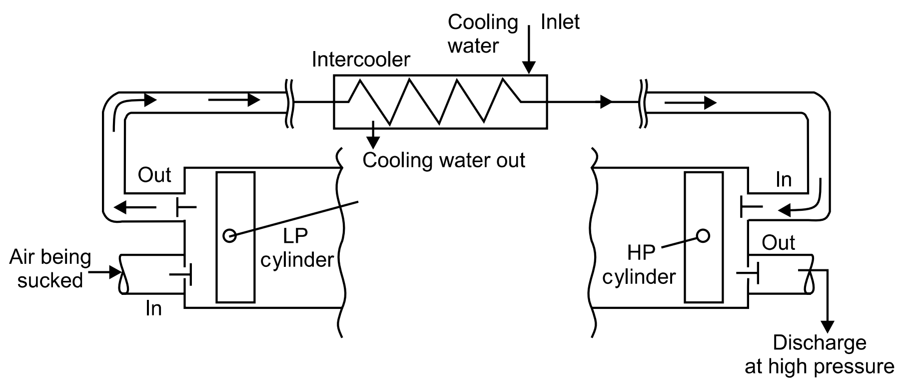


Fig. 16.7 *Schematic for two-stage compression (Multistage compressor)*

Figure 16.7 shows the schematic of multi stage compressor (double stage) with intercooler between stages. *T–s* representation is shown in Fig. 16.8. The total work requirement for running this shall be algebraic summation of work required for low pressure (LP) and high pressure (HP) stages. The size of HP cylinder is smaller than LP cylinder as HP cylinder handles high pressure air having smaller specific volume.

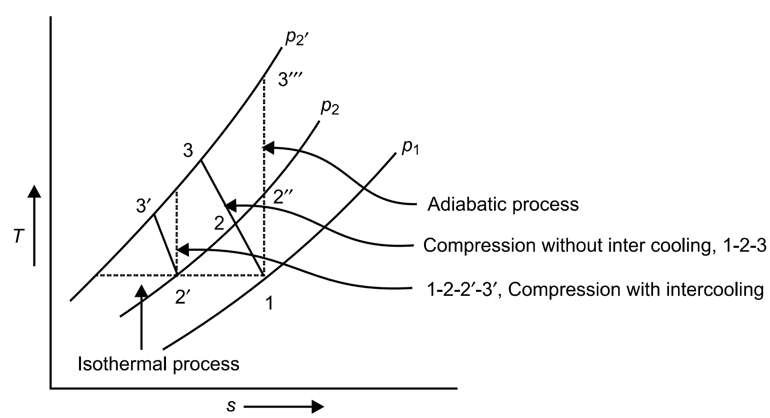


Fig. 16.8 T-s *representation of multistage compression*

Mathematical analysis of multistage compressor is done with following assumptions:

(i) Compression in all the stages is done following same index of compression and there is no pressure drop in suction and delivery pressures in each stage. Suction and delivery pressure remains constant in the stages.

(ii) There is perfect intercooling between compression stages.
(iii) Mass handled in different stages is same i.e. mass of air in LP and HP stages are same.
(iv) Air behaves as perfect gas during compression.

From combined *p-V* diagram the compressor work requirement can be given as,

Work requirement in LP cylinder, $W_{LP} = \left(\frac{n}{n-1}\right) p_1 V_1 \left\{\left(\frac{p_2}{p_1}\right)^{\frac{(n-1)}{n}} - 1\right\}$

Work requirement in HP cylinder, $W_{HP} = \left(\frac{n}{n-1}\right) p_2 V_2 \left\{\left(\frac{p_{2'}}{p_2}\right)^{\frac{(n-1)}{n}} - 1\right\}$

For perfect intercooling, $p_1 V_1 = p_2 V_{2'}$ and

$$W_{HP} = \left(\frac{n}{n-1}\right) p_2 V_{2'} \left\{\left(\frac{p_{2'}}{p_2}\right)^{\frac{(n-1)}{n}} - 1\right\}$$

Therefore, total work requirement, $W_c = W_{LP} + W_{HP}$, for perfect intercooling

$$W_c = \left(\frac{n}{n-1}\right)\left[p_1 V_1 \left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} + p_2 V_{2'} \left\{\left(\frac{p_{2'}}{p_2}\right)^{\frac{n-1}{n}} - 1\right\}\right]$$

$$= \left(\frac{n}{n-1}\right)\left[p_1 V_1 \left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right\} + p_1 V_1 \left\{\left(\frac{p_{2'}}{p_2}\right)^{\frac{n-1}{n}} - 1\right\}\right]$$

$$\boxed{W_c = \left(\frac{n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_{2'}}{p_2}\right)^{\frac{n-1}{n}} - 2\right]}$$

Power required can be given in HP as below, considering speed to be *N* rpm.

$$\text{Power} = \frac{W_c \times N}{75 \times 60}, \text{ HP}$$

If we look at compressor work then it shows that with the initial and final pressures p_1 and $p_{2'}$ remaining same the intermediate pressure p_2 may have value floating between p_1 and $p_{2'}$ and change the work requirement W_c. Thus, the compressor work can be optimized with respect to intermediate pressure p_2. Mathematically, it can be differentiated with respect to p_2.

$$\frac{dW_c}{dp_2} = \frac{d}{dp_2}\left[\left(\frac{n}{n-1}\right) p_1 V_1 \left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_{2'}}{p_2}\right)^{\frac{n-1}{n}} - 2\right\}\right]$$

$$\frac{dW_c}{dp_2} = \left(\frac{n}{n-1}\right) p_1 V_1 \cdot \frac{d}{dp_2}\left\{\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_{2'}}{p_2}\right)^{\frac{n-1}{n}} - 2\right\}$$

$$\frac{dW_c}{dp_2} = \left(\frac{n}{n-1}\right) p_1 V_1 \left\{ \left(\frac{n-1}{n}\right) \cdot p_1^{\frac{1-n}{n}} \cdot p_2^{\frac{-1}{n}} - \left(\frac{n-1}{n}\right) \cdot p_{2'}^{\frac{n-1}{n}} \, p_2^{\frac{1-2n}{n}} \right\}$$

Equating, $\dfrac{dW_c}{dp_2} = 0$ yields.

$$p_1^{\frac{1-n}{n}} \cdot p_2^{\frac{-1}{n}} = p_{2'}^{\frac{n-1}{n}} \cdot p_2^{\frac{1-2n}{n}}$$

or $$p_2^{\frac{-2+2n}{n}} = p_{2'}^{\frac{n-1}{n}} \cdot p_1^{\frac{n-1}{n}}$$

or $$p_2^{2\left(\frac{n-1}{n}\right)} = (p_1 \cdot p_{2'})^{\left(\frac{n-1}{n}\right)} \text{ or,} \qquad p_2^2 = p_1 \cdot p_{2'}, \quad p_2 = \sqrt{p_1 \cdot p_{2'}}$$

or $$\boxed{\frac{p_2}{p_1} = \frac{p_{2'}}{p_2}} \text{ or } \boxed{\frac{p_2}{p_1} = \left(\frac{p_{2'}}{p_1}\right)^{1/2}}$$

Pressure ratio in Ist stage = Pressure ratio in IInd stage

Thus, it is established that the compressor work requirement shall be minimum when the pressure ratio in each stage is equal.

In case of multiple stages, say i number of stages, for the delivery and suction pressures of p_{i+1} and p_1 the optimum stage pressure ratio shall be,

Optimum stage pressure ratio = $\left(\dfrac{p_{i+1}}{p_1}\right)^{1/i}$ for pressures at stages being $p_1, p_2, p_3, p_4, p_5, \ldots, p_{i-1}, p_i, p_{i+1}$

Minimum work required in two stage compressor can be given by

$$W_{c,\,\min} = \left(\frac{n}{n-1}\right) p_1 V_1 \cdot 2 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

For i number of stages, minimum work,

$$W_{c,\,\min} = \left(\frac{n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} + \left(\frac{p_3}{p_2}\right)^{\frac{n-1}{n}} + \left(\frac{p_4}{p_3}\right)^{\frac{n-1}{n}} + \cdots + \left(\frac{p_{i+1}}{p_i}\right)^{\frac{n-1}{n}} - i\right]$$

$$W_{c,\,\min} = i \cdot \left(\frac{n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

or,

$$W_{c,\,\min} = i \cdot \left(\frac{n}{n-1}\right) p_1 V_1 \left[\left(\frac{p_{i+1}}{p_i}\right)^{\frac{n-1}{n \cdot i}} - 1\right]$$

It also shows that for optimum pressure ratio the work required in different stages remains same for the assumptions made for present analysis. Due to pressure ratio being equal in all stages the temperature ratios and maximum temperature in each stage remains same for perfect intercooling.

Cylinder dimensions: In case of multistage compressor the dimension of cylinders can be estimated basing upon the fact that the mass flow rate of air across the stages remains same. For perfect intercooling the temperature of air at suction of each stage shall be same.

If the actual volume sucked during suction stroke is V_1, V_2, V_3 …, for different stages then by perfect gas law, $p_1V_1 = RT_1$, $p_2V_2 = RT_2$, $p_3V_3 = RT_3$

For perfect intercooling ($T_1 = T_2 = T_3 = \ldots$) so

$$p_1V_1 = RT_1,\ p_2V_2 = RT_1,\ p_3V_3 = RT_1,$$

or

$$p_1V_1 = p_2V_2 = p_3V_3 = \ldots$$

If the volumetric efficiency of respective stages in $\eta_{V_1}, \eta_{V_2}, \eta_{V_3}, \ldots$

Then theoretical volume of cylinder 1, $V_{1,\,\text{th}} = \dfrac{V_1}{\eta_{V_1}}$; $V_1 = \eta_{V_1} \cdot V_{1,\,\text{th}}$

cylinder 2,

$$V_{2,\,\text{th}} = \frac{V_2}{\eta_{V_2}};\ V_2 = \eta_{V2} \cdot V_{2,\,\text{th}}$$

cylinder 3,

$$V_{3,\,\text{th}} = \frac{V_3}{\eta_{V_3}};\ V_3 = \eta_{V3} \cdot V_{3,\,\text{th}}$$

Substituting,

$$p_1 \cdot \eta_{V1} \cdot V_{1,\,\text{th}} = p_2 \cdot \eta_{V2} \cdot V_{2,\,\text{th}} = p_3 \cdot \eta_{V3} \cdot V_{3,\,\text{th}} = \ldots$$

Theoretical volumes of cylinder can be given using geometrical dimensions of cylinder as diameters D_1, D_2, D_3, … and stroke lengths L_1, L_2, L_3, ….

or

$$V_{1,\,\text{th}} = \frac{\pi}{4} D_1^2 \cdot L_1$$

$$V_{2,\,\text{th}} = \frac{\pi}{4} D_2^2 \cdot L_2$$

$$V_{3,\,\text{th}} = \frac{\pi}{4} D_3^2 \cdot L_3$$

or,

$$p_1 \cdot \eta_{Vi} \cdot \frac{\pi}{4} D_1^2 L_1 = p_2 \cdot \eta_{V2} \cdot \frac{\pi}{4} D_2^2 L_2 = p_3 \cdot \eta_{V3} \cdot \frac{\pi}{4} D_3^2 \cdot L_3 = \ldots$$

$$p_1\ \eta_{V1} \cdot D_1^2 L_1 = p_2 \cdot \eta_{V2} \cdot D_2^2 \cdot L_2 = p_3 \cdot \eta_{V3} \cdot D_3^2 \cdot L_3 = \ldots$$

If the volumetric efficiency is same for all cylinders, i.e. $\eta_{V1} = \eta_{V2} = \eta_{V3} = \ldots$ and stroke for all cylinder is same i.e. $L_1 = L_2 = L_3 = \ldots$

Then,

$$D_1^2\, p_1 = D_2^2\, p_2 = D_3^2\, p_3 = \ldots$$

These generic relations may be used for getting the ratio of diameters of cylinders of multistage compression.

Energy balance: Energy balance may be applied on the different components constituting multistage compression.

For LP stage the steady flow energy equation can be written as below:

$$m \cdot h_1 + W_{LP} = mh_2 + Q_{LP}$$
$$Q_{LP} = W_{LP} - m\,(h_2 - h_1)$$
$$Q_{LP} = W_{LP} - mC_p\,(T_2 - T_1)$$

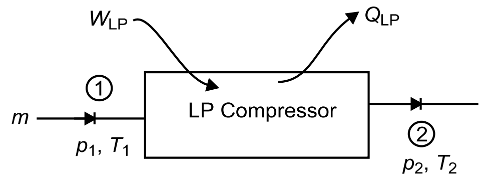


Fig. 16.9

For intercooling (Fig. 16.10) between LP and HP stage steady flow energy equation shall be;

$$mh_2 = mh_{2'} + Q_{Int}$$

or,

$$Q_{Int} = m\,(h_2 - h_{2'})$$
$$Q_{Int} = mC_p\,(T_2 - T_{2'})$$

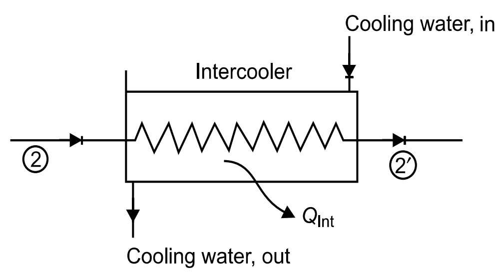


Fig. 16.10

For HP stage (Fig. 16.11) the steady flow energy equation yields.

$$mh_{2'} + W_{HP} = mh_{3'} + Q_{HP}$$
$$Q_{HP} = W_{HP} + m\,(h_{2'} - h_{3'})$$
$$Q_{HP} = W_{HP} + mC_p\,(T_{2'} - T_{3'}) = W_{HP} - mC_p\,(T_3' - T_{2'})$$

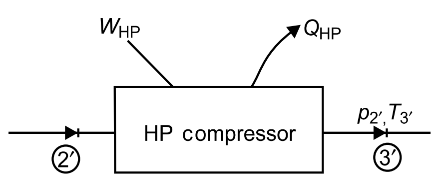


Fig. 16.11

In case of perfect intercooling and optimum pressure ratio, $T_{2'} = T_1$ and $T_2 = T_{3'}$
Hence for these conditions,

$$Q_{LP} = W_{LP} - mC_p\,(T_2 - T_1)$$

$$Q_{\text{Int}} = mC_p\,(T_2 - T_1)$$
$$Q_{\text{HP}} = W_{\text{HP}} - mC_p\,(T_2 - T_1)$$

Total heat rejected during compression shall be the sum of heat rejected during compression and heat extracted in intercooler for perfect intercooling.

$$\text{Heat rejected during compression for polytropic process} = \left(\frac{\gamma - n}{\gamma - 1}\right) \times \text{Work}$$

16.6 CONTROL OF RECIPROCATING COMPRESSORS

Output from the compressors can be controlled by different measures which regulate the compressor output. In practical applications the compressors are fitted with air receiver to store the high pressure air and supply as and when required. Therefore, the compressors are run only for the duration required to maintain the limiting pressure inside receiver. When the pressure inside receiver starts dropping down then the compressor again starts supplying compressed air till the level is restored. Different ways for this control are based on throttle, clearance, blow off control and speed control.

(i) Throttle control has the regulation of opening/closing of inlet valve so that the quantity of air entering can be varied. With partial opening of inlet valve throttling occurs at valve and quantity of air entering is reduced while the pressure ratio gets increased.

(ii) Clearance control is the arrangement in which the clearance volume is increased when pressure ratio exceeds the limit. For this cylinder has clearance openings which are closed by spring loaded valves. Whenever pressure exceeds, then the clearance openings get opened and the increased clearance volume reduces maximum pressure.

(iii) Blow-off control has spring loaded safety valve or by pass valve for blowing out excess air in receiver. After release of excess air automatically the valve gets closed on its own.

(iv) Speed of compressor can also be controlled by regulating the prime mover, thereby regulating compressor output. Thus, compressor is run on variable speed for its' control.

16.7 RECIPROCATING AIR MOTOR

Air motors are actually prime movers run using the compressed air. It is used extensively in the applications where electric motor/IC engine/Gas turbine etc. can not be used due to fire hazard, specially in coal/oil mining applications. Apart from use of air motors in coal/oil mines these are also used for running pneumatic tools in workshops and manufacturing/assembly lines. Air motors have a reciprocating piston-cylinder arrangement where compressed air is admitted in cylinder with inlet valve open for limited period in suction stroke and this causes piston movement to yield shaft work. Thus air motors are reverse of air compressors. Schematic for air motor and *p–V* diagram for cycle used is shown in Fig. 16.12.

Compressed air enters the cylinder up to state 1 and expands up to state 2. Expansion process is generally polytropic and yields expansion work. Expansion is terminated even before the atmospheric pressure is reached because in later part of expansion the positive work is less than negative work. The exhaust valve opens causing drop in pressure from 2 to 3. Exhaust pressure is slightly more or nearly equal to atmospheric pressure. The exhaust of air occurs at constant pressure up to state 4 where the closing of exhaust valve occurs. The inlet valve is opened at state 5 causing sudden rise in pressure upto state 6 after which piston displacement begins and is continued with compressed air inlet up to state 1 after which expansion occurs with inlet valve closed. Here point of cut-off is at state 1 which can be

10

Fuels and Combustion

10.1 INTRODUCTION

Every real life system requires energy input for its' performance. Energy input may be in the form of heat. Now question arises from where shall we get heat? Traditionally heat for energy input can be had from the heat released by fuel during combustion process. Fuels have been provided by nature and the combustion process provides a fluid medium at elevated temperature. During combustion the energy is released by oxidation of fuel elements such as carbon C, hydrogen H_2 and sulphur S, i.e. high temperature chemical reaction of these elements with oxygen O_2 (generally from air) releases energy to produce high temperature gases. These high temperature gases act as heat source.

In this chapter the detailed study of fuels and their combustion is being made.

Air fuel ratio: It refers to the ratio of amount of air in combustion reaction with the amount of fuel. Mathematically, it can be given by the ratio of mass of air and mass of fuel.

$$AF = \frac{\text{Mass of air}}{\text{Mass of fuel}} = \left(\frac{\text{Molecular wt. of air} \times \text{no. of moles of air}}{\text{Molecular wt. of fuel} \times \text{no. of moles of fuel}}\right)$$

Fuel-air ratio is inverse of Air-fuel ratio. Theoretical air-fuel ratio can be estimated from stoichiometric combustion analysis for just complete combustion.

Equivalence ratio: It is the ratio of actual fuel-air ratio to the theoretical fuel-air ratio for complete combustion. Fuel-air mixture will be called lean mixture when equivalence ratio is less than unity while for equivalence ratio value being greater than unity the mixture will be rich mixture.

Theoretical air: Theoretical amount of air refers to the minimum amount of air that is required for providing sufficient oxygen for complete combustion of fuel. Complete combustion means complete reaction of oxygen present in air with C, H_2, S etc. resulting into carbon dioxide, water, sulphur dioxide, nitrogen with air as combustion products. At the end of complete reaction there will be no free oxygen in the products. This theoretical air is also called "stoichiometric air".

Excess air: Any air supplied in excess of "theoretical air" is called excess air. Generally excess air is 25 to 100% to ensure better and complete combustion.

Flash point and Fire point: Flash point refers to that temperature at which vapour is given off from liquid fuel at a sufficient rate to form an inflammable mixture but not at a sufficient rate to support continuous combustion.

Fire point refers to that temperature at which vaporization of liquid fuel is sufficient enough to provide for continuous combustion.

These temperatures depend not only on the fuel characteristics but also on the rate of heating, air movement over fuel surface and means of ignition. These temperatures are specified in reference to certain standard conditions. Although flash point and fire point temperatures are defined in relation with ignition but these temperatures are not measure of ignitability of fuel but of the initial volatility of fuel.

Adiabatic flame temperature: Adiabatic flame temperature refers to the temperature that could be attained by the products of combustion when the combustion reaction is carried out in limit of adiabatic operation of combustion chamber. Limit of adiabatic operation of combustion chamber means that in the absence of work, kinetic and potential energies the energy released during combustion shall be carried by the combustion products with minimum or no heat transfer to surroundings. This is the maximum temperature which can be attained in a combustion chamber and is very useful parameter for designers. Actual temperature shall be less than adiabatic flame temperature due to heat transfer to surroundings, incomplete combustion and dissociation etc.

Wet and dry analysis of combustion: Combustion analysis when carried out considering water vapour into account is called "wet analysis" while the analysis made on the assumption that vapour is removed after condensing it, is called "dry analysis".

Volumetric and gravimetric analysis: Combustion analysis when carried out based upon percentage by volume of constituent reactants and products is called volumetric analysis.

Combustion analysis carried out based upon percentage by mass of reactants and products is called gravimetric analysis.

Pour point: It refers to the lowest temperature at which liquid fuel flows under specified conditions.

Cloud point: When some petroleum fuels are cooled, the oil assumes cloudy appearance. This is due to paraffin wax or other solid substances separating from solution. The temperature at which cloudy appearance is first evident is called cloud point.

Composition of air: Atmospheric air is considered to be comprising of nitrogen, oxygen and other inert gases. For combustion calculations the air is considered to be comprising of nitrogen and oxygen in following proportions. Molecular weight of air is taken as 29.

Composition of air by mass = Oxygen (23.3%) + Nitrogen (76.7%)

Composition of air by volume = Oxygen (21%) + Nitrogen (79%)

Enthalpy of combustion: Enthalpy of combustion of fuel is defined as the difference between the enthalpy of the products and enthalpy of reactants when complete combustion occurs at given temperature and pressure. It may be given as higher heating value or lower heating value. Higher heating value (HHV) of fuel is the enthalpy of combustion when all the water (H_2O) formed during combustion is in liquid phase. Lower heating value (LHV) of fuel refers to the enthalpy of combustion when all the water (H_2O) formed during combustion is in vapour form. The lower heating value will be less than higher heating value by the amount of heat required for evaporation of water.

HHV = LHV + (Heat required for evaporation of water)

It is also called calorific value of fuel and is defined as the number of heat units liberated when unit mass of fuel is burnt completely in a calorimeter under given conditions.

Enthalpy of formation: Enthalpy of formation of a compound is the energy released or absorbed when compound is formed from its elements at standard reference state. Thus enthalpy of formation shall equal heat transfer in a reaction during which compound is formed from its' elements at standard reference state. Enthalpy of formation will have positive (+ ve) value if formation is by an endothermic reaction and negative (– ve) value if formation is by an exothermic reaction.

Standard reference state: It refers to thermodynamic state at which the enthalpy datum can be set for study of reacting systems. At standard reference state, zero value is assigned arbitrarily to the enthalpy of stable elements. Generally, standard reference state is taken as 25°C and 1 atm,

i.e. $T_{ref} = 25°C = 298.15$ K, $p_{ref} = 1$ atm

Dissociation: It refers to the combustion products getting dissociated and thus absorbing some of energy. Such as, the case of carbon dioxide getting formed during combustion and subsequently getting dissociated can be explained as below,

Combustion: $C + O_2 \rightarrow CO_2 + \text{Heat}$

Dissociation: Heat + $CO_2 \rightarrow C + O_2$

Thus generally, dissociation has inherent requirement of high temperature and heat.

10.2 TYPES OF FUELS

'Fuel' refers to a combustible substance capable of releasing heat during its combustion. In general fuels have carbon, hydrogen and sulphur as the major combustible chemical elements. Sulphur is found to be relatively less contributor to the total heat released during combustion. Fuels may be classified as solid, liquid and gaseous fuel depending upon their state.

Solid fuel: Coal is the most common solid fuel. Coal is a dark brown/black sedimentary rock derived primarily from the unoxidized remains of carbon-bearing plant tissues. It can be further classified into different types based upon the composition. Composition can be estimated using either "proximate analysis" or by "ultimate analysis". Proximate analysis is the one in which the individual constituent element such as C, H_2, S, N_2 etc. are not determined rather only fraction of moisture, volatile matter, ash, carbon etc. are determined. Thus proximate analysis is not exact and gives only some idea about the fuel composition. Proximate analysis of coal gives, various constituents in following range, Moisture 3–30%, Volatile matter 3–50%, Ash 2–30% and Fixed carbon 16–92%.

In "ultimate analysis" the individual elements such as C, H_2, N_2, S and ash etc. present in the fuel are determined on mass basis. Thus, it gives relative amounts of chemical elements constituting fuel. In general the percentage by mass of different elements in coal lies in the following range:

Carbon	:	50 to 95%
Hydrogen	:	2 to 50%
Oxygen	:	2 to 40%
Nitrogen	:	0.5 to 3%
Sulphur	:	0.5 to 7%
Ash	:	2 to 30%

Different types of coal available are listed in the table hereunder. These are typical values for particular types of coal samples and may change for different types of coal.

Table 10.1 *Different types of coal*

Sl. No.	*Type*	*% by mass*		*Ultimate analysis, % by mass*					*Lower calorific value, kcal/kg*
		Moisture	*Volatile matter in dry coal*	*C*	H_2	O_2	N_2+S_2	*Ash*	
1.	Peat	20	65	43.70	6.42	44.36	1.52	4.00	3200
2.	Lignite	15	50	56.52	5.72	31.89	1.62	4.25	2450
3.	Bituminous coal	2	25	74.00	5.98	13.01	2.26	4.75	7300
4.	Anthracite coal	1	4	90.27	3.30	2.32	1.44	2.97	7950

Liquid fuels: Fuels in liquid form are called liquid fuels. Liquid fuels are generally obtained from petroleum and its by-products. These liquid fuels are complex mixture of different hydrocarbons, and obtained by refining the crude petroleum oil. Commonly used liquid fuels are petrol, kerosene diesel, aviation fuel, light fuel oil, heavy fuel oil etc.

Various liquid fuels of hydrocarbon family lie in Paraffin (C_nH_{2n+2} – chain structure), Olefins (C_nH_{2n} – chain structure), Napthalene (C_nH_{2n} – ring structure), Benzene (C_nH_{2n-6} – ring structure), Nepthalene (C_nH_{2n-12} – ring structure) category. Percentage by volume composition of some of liquid fuels is given below.

Table 10.2 *Composition of liquid fuels*

Sl.No.	*Fuel*		*% by volume*	
		Carbon	*Hydrogen*	*Sulphur*
1.	Petrol	85.5	14.4	0.1
2.	Kerosene	86.3	13.6	0.1
3.	Diesel	86.3	12.8	0.9
4.	Benzole	91.7	8.0	0.3
5.	Light fuel oil	86.2	12.4	1.4
6.	Heavy fuel oil	86.1	11.8	2.1

Liquid fuels offer following advantages over solid fuel.

(i) Better mixing of fuel and air is possible with liquid fuel.
(ii) Liquid fuels have no problem of ash formation.
(iii) Storage and handling of liquid fuels is easy compared to solid fuels.
(iv) Processing such as refining of liquid fuels is more convenient.

Gaseous fuels: These are the fuels in gaseous phase. Gaseous fuels are also generally hydrocarbon fuels derived from petroleum reserves available in nature. Most common gaseous fuel is natural gas. Gaseous fuels may also be produced artificially from burning solid fuel (coal) and water. Some of gaseous fuels produced artificially are coal gas, producer gas etc. Volumetric analysis of gaseous fuels is presented in Table 10.3. Gaseous fuels offer all advantages as there in liquid fuels except ease of storage.

Table 10.3 *Composition of gaseous fuels*

Sl.No.	*Fuel*		*% by volume*							
		H_2	O_2	N_2	CO	CH_4	C_2H_4	C_2H_6	C_4H_8	CO_2
1.	Natural gas	—	—	3	1	93	—	3	—	—
2.	Coal gas	53.6	0.4	6	9	25	—	—	3	3
3.	Producer gas	12	—	52	29	2.6	0.4	—	—	4

10.3 CALORIFIC VALUE OF FUEL

During combustion, the chemical energy of fuel gets transformed into molecular kinetic or molecular potential energy of products. This energy associated with combustion also called calorific value of fuel is very important to be known for thermodynamic design and calculations of combustion systems. "Bomb calorimeter" is one of the ways to get the heating value of solid and liquid fuels when burnt at constant volume. Different types of bomb calorimeters as given by At water, Davis, Emerson, Mahler, Parr, Peters and Williams are available. Bomb calorimeter as given by Emerson is discussed here. For getting the heating value of gaseous fuel the gas calorimeter is also discussed here.

10.4 BOMB CALORIMETER

Emerson's bomb calorimeter is shown in Fig. 10.1 here. Its major components are bomb, bucket, stirrer, crucible or fuel pan, jacket, thermometer etc. A known quantity of fuel under investigation is kept in the crucible. Crucible has a electric coil with d.c. supply in it. Bomb is charged with oxygen under pressure. Bomb is surrounded by a bucket containing water to absorb the heat released as fuel burns. Bomb also has an outer jacket with dead air space surrounding the bucket to minimise heat loss to surroundings. When electricity is flown into coil the fuel gets ignited. Bomb is actually a strong shell capable of withstanding about 100 atmosphere pressure. Inner bomb wall surface is lined with enamel and external wall surface is plated so as to prevent corrosion due to high temperature combustion products. Different operations while using it for calorific value measurement are as given ahead. First weigh the empty calorimeter bucket and fill it with a definite quantity of water at temperature about 2–3°C less than jacket water temperature. Charge bomb with oxygen at high pressure without disturbing the fuel kept in crucible. Ensure that there are no leaks in bomb and after ensuring it as leak proof place it in bomb jacket. Install thermometer and stirrer. Start stirrer for 3 to 4 minutes for temperature uniformity in bucket. Take temperature readings at definite interval, say after every five minutes.

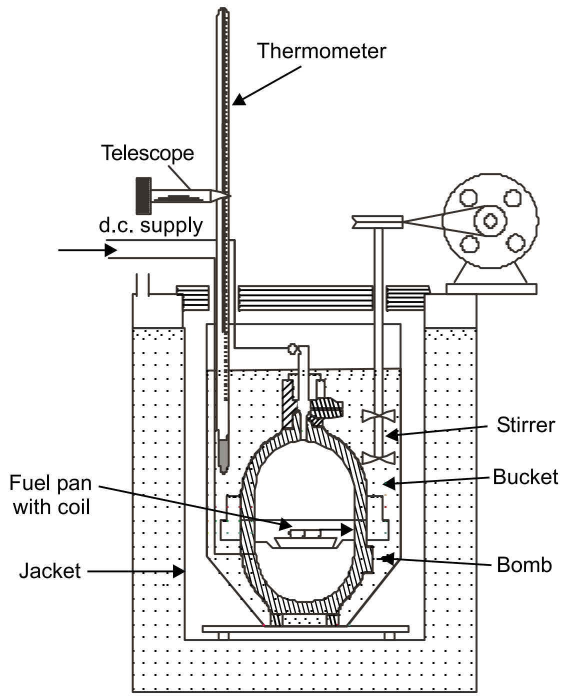


Fig. 10.1 *Bomb calorimeter*

Active the coil so as to fire fuel. Record temperature every 30 seconds until maximum temperature is reached and after attaining maximum temperature read temperature after every 5 minutes. These temperatures are required to account for the heat exchange with jacket water.

Later on remove bomb from calorimeter, release the gases and dismantle the bomb. Collect and weigh the iron fuse wire which remains. For getting accurate result the bomb should be washed with distilled water and washings titrated to obtain the amount of nitric acid and sulphuric acid formed. Corrections are made for small amount of heat transfer which occurs.

For making the calculations a curve of temperature is plotted with time. Determine the rate of temperature rise before firing.

Also, determine the rate of temperature drop after the maximum temperature is reached. Cooling correction may be added to the measured temperature rise. Heat balance may be applied as,

Heat released by fuel during combustion + Heat released by combustion of fuse wire
= Heat absorbed by water and calorimeter

For getting heat absorbed by calorimeter the water equivalent of calorimeter is used which can be determined by burning a fuel of known calorific value. Generally benzoic acid and napthalene having calorific value of 6325 kcal/kg and 9688 kcal/kg are being used for finding water equivalent of calorimeter.

Mathematically,

$$m_{\text{fuel}} \times \text{CV}_{\text{fuel}} + m_{fw} \times \text{CV}_{fw} = (m_w + m_c) \cdot R$$

$$\text{CV}_{\text{fuel}} = \left\{ \frac{(m_w + m_c) \cdot R - m_{fw} \cdot \text{CV}_{fw}}{m_{\text{fuel}}} \right\}$$

where m_{fuel} and m_{fw} are mass of fuel and mass of fuse wire, m_w and m_c are mass of water in calorimeter and water equivalent of calorimeter, R is correct temperature rise, CV_{fuel} is higher calorific value of fuel, CV_{fw} is calorific value of fuse wire. Calorific value of fuel estimated is higher calorific value as the water formed during combustion is condensed.

10.5 GAS CALORIMETER

Schematic of gas calorimeter is shown in Fig.10.2 It is used for estimating the heating value of gaseous fuels. It has burner with arrangement to regulate and measure flow rate and pressure of gaseous fuel. Combustion products pass through tubes which are surrounded by flowing water. Here the volume flow rate of gas flowing through calorimeter is measured. Water conditions are adjusted so as to cool the products of combustion to ambient air temperature. Rate of water flow through the calorimeter is measured and its' temperature rise is determined. If the heat exchange between the calorimeter and its' surroundings is neglected then the heat received by water shall be equal to the heating value of fuel. For precise estimation of heating value of gaseous fuels the procedure as specified by ASTM is to be followed.

10.6 COMBUSTION OF FUEL

Combustion of fuel refers to the chemical reaction that occurs between fuel and air to form combustion products with energy release i.e. oxidation of combustible fuel results into energy and products of combustion.

In other words, during chemical reaction the bonds within fuel molecules get broken and atoms and electrons rearrange themselves to yield products. During complete combustion carbon present in fuel transforms into carbon dioxide, hydrogen into water, sulphur into sulphur dioxide and nitrogen into nitrogen oxides. Thus it is obvious that the total mass before combustion and after combustion remains constant although the elements exist in form of different chemical compounds in reactants and products. Generic form of combustion equations shall be as follows;

(i) $$C + O_2 \rightarrow CO_2$$

On mass basis, $(12) + (32) \rightarrow (44)$

$$1 \text{ kg (C)} + \frac{8}{3} \text{ kg}(O_2) \rightarrow \frac{11}{3} \text{ kg}(CO_2)$$

$$1 \text{ mol (C)} + 1 \text{ mol } (O_2) \rightarrow 1 \text{ mol } (CO_2)$$

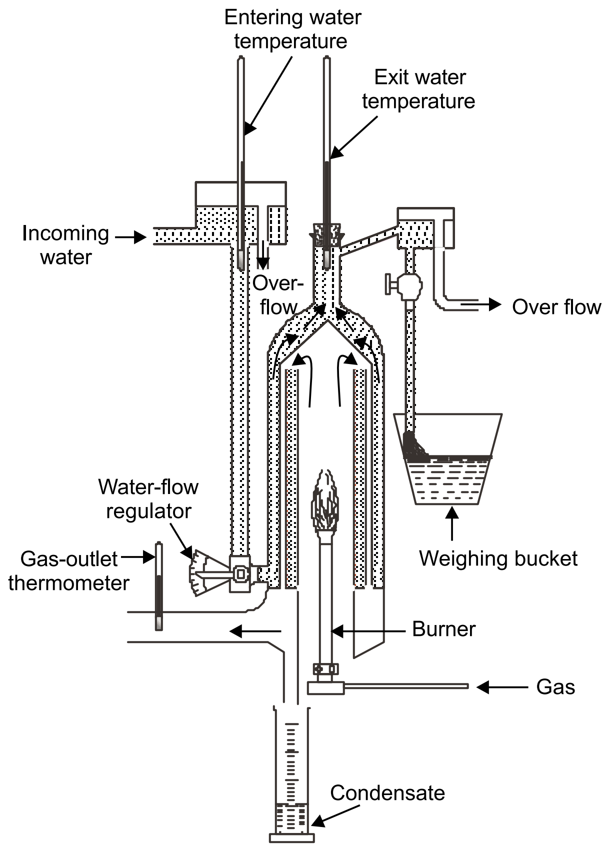


Fig. 10.2 *Gas calorimeter*

(ii) $$2C + O_2 \rightarrow 2CO$$

On mass basis, $(24) + (32) \rightarrow (56)$

$$1 \text{ kg (C)} + \frac{4}{3} \text{ kg } (O_2) \rightarrow \frac{7}{3} \text{ kg (CO)}$$

On mole basis, 2 mol (C) + 1 mol (O_2) $\rightarrow$ 2 mol (CO)

(iii) $$2CO + O_2 \rightarrow 2CO_2$$

On mass basis, $56 + 32 \rightarrow 88$

$$1 \text{ kg (CO)} + \frac{4}{7} \text{ kg } (O_2) \rightarrow \frac{11}{7} \text{ kg } (CO_2)$$

On mole basis, 2 mol (CO) + 1 mol (O_2) $\rightarrow$ 2 mol (CO_2)

(iv) $$2H_2 + O_2 \rightarrow 2H_2O$$

On mass basis, $4 + 32 \rightarrow 36$

1 kg (H_2) + 8 kg (O_2) $\rightarrow$ 9 kg (H_2O)

On mole basis, 2 mol (H_2) + 1 mol (O_2) $\rightarrow$ 2 mol (H_2O)

(v) $$S + O_2 \rightarrow SO_2$$

On mass basis, $32 + 32 \rightarrow 64$

$1 \text{ kg (S)} + 1 \text{ kg } (O_2) \rightarrow 2 \text{ kg } (SO_2)$

On mole basis, $1 \text{ mol (S)} + 1 \text{ mol } (O_2) \rightarrow 1 \text{ mol } (SO_2)$

(vi) $$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$$

On mass basis, $16 + 64 \rightarrow 44 + 36$

$$1 \text{ kg } (CH_4) + 4 \text{ kg } (O_2) \rightarrow \frac{11}{4} \text{ kg } (CO_2) + \frac{9}{4} \text{ kg } (H_2O)$$

On mole basis,

$1 \text{ mol } (CH_4) + 2 \text{ mol } (O_2) \rightarrow 1 \text{ mol } (CO_2) + 2 \text{ mol } (H_2O)$

(vii) $$2C_2H_6 + 7O_2 \rightarrow 4CO_2 + 6H_2O$$

On mass basis, $60 + 224 \rightarrow 176 + 108$

$$1 \text{ kg } (C_2H_6) + \frac{56}{15} \text{ kg } (O_2) \rightarrow \frac{44}{15} \text{ kg } (CO_2) + \frac{27}{15} \text{ kg } (H_2O)$$

On mole basis,

$2 \text{ mol } (C_2H_6) + 7 \text{ mol } (O_2) \rightarrow 4 \text{ mol } (CO_2) + 6 \text{ mol } (H_2O)$

(viii) In general, for any hydrocarbon's complete combustion,

$$a \cdot C_8H_{18} + b \cdot O_2 \rightarrow d \cdot CO_2 + e \cdot H_2O$$

Equating C, H and O on both sides of above equation,

$$a \times 8 = d \qquad \text{or,} \quad d = 8a$$

$$a \times 18 = e \times 2 \qquad \text{or,} \quad e = 9a$$

$$b \times 2 = (d \times 2) + e \qquad \text{or,} \quad 2b = 2d + e, \text{ or } 2b = 16a + 9a$$

Above yield, $b = 12.5a$, $d = 8a$, $e = 9a$

In order to avoid fraction let us round it off by multiplying by 2,

$$2a\, C_8H_{18} + 25a \cdot O_2 \rightarrow 16a \cdot CO_2 + 18a \cdot H_2O$$

Combustion equation shall now be,

$$2C_8H_{18} + 25O_2 \rightarrow 16CO_2 + 18H_2O$$

On mass basis, $228 + 800 \rightarrow 704 + 324$

$$1 \text{ kg } (C_8H_{18}) + \frac{200}{57} \text{ kg } (O_2) \rightarrow \frac{176}{57} \text{ kg } (CO_2) + \frac{81}{57} \text{ kg } (H_2O)$$

On mole basis,

$$2 \text{ mol } (C_8H_{18}) + 25 \text{ mol } (O_2) \rightarrow 16 \text{ mol } (CO_2) + 18 \text{ mol } (H_2O)$$

Theoretically, the calorific value of fuel can be determined based upon the fuel constituents and the heat evolved upon their oxidation.

Heat evolved during oxidation of some of the fuel constituents are as under,

Table 10.4

Fuel constituent	*Higher calorific value, kcal/kg*	*Lower calorific value, kcal/kg*	*Products of oxidation*
C	8100	—	CO_2
C	2420	—	CO
CO	2430	—	CO_2
H_2	34400	29000	H_2O
S	2200	—	SO_2

In any fuel containing carbon (C), hydrogen (H_2), oxygen (O) and sulphur (S) the higher calorific value of fuel can be estimated using respective calorific values for constituents. In the fuel oxygen is not present in free form but is associated with hydrogen.

$$\underset{(2)}{H_2} + \underset{(16)}{O} \rightarrow \underset{(18)}{H_2O}$$

On mass basis,

$$1 \text{ kg } (H_2) + 8 \text{ kg } (O) \rightarrow 9 \text{ kg } (H_2O)$$

$$\frac{1}{8} \text{ kg } (H_2) + 1 \text{ kg } (O) \rightarrow \frac{9}{8} \text{ kg } (H_2O)$$

Above chemical equation indicates that with every unit mass of oxygen, $\frac{1}{8}$ of oxygen mass shall be the mass of hydrogen associated with it. Thus, the free hydrogen available for oxidation (combustion) shall be only $\left(H - \frac{O}{8}\right)$ where H refers to hydrogen mass and O refers to oxygen mass.

The higher calorific value of fuel can thus be given as under using the mass fractions of constituent elements known. If percentage mass fractions of fuel constituents are given by C, H, O and S then.

$$\boxed{\text{H.C.V. of fuel} = \frac{1}{100}\left[8100 \cdot C + 34{,}400\left(H - \frac{O}{8}\right) + 2220 \cdot S\right] \text{ kcal/kg}}$$

Lower calorific value of fuel can be given by,

L.C.V. of fuel = (H.C.V. of fuel) – (Heat carried by water vapour formed per kg of fuel burnt)

The amount of latent heat carried depends upon the pressure at which evaporation takes place and quantity of water vapour formed. Generally, the evaporation is considered to take place at saturation temperature of 15°C and the latent heat of water vapour at this saturation temperature is 588.76 kcal/kg. During combustion of fuel the water shall be formed due to hydrogen present in fuel, therefore mass of water vapour can be given by the mass fraction of hydrogen. Thus,

$$\boxed{\text{L.C.V. of fuel} = \left\{(\text{H.C.V. of fuel}) - \left(\frac{9 \times H \times 588.76}{100}\right)\right\}, \text{ kcal/kg}}$$

10.7 COMBUSTION ANALYSIS

From earlier discussions it is seen how the combustion of a fuel can be written in the form of chemical reactions for the oxidation of different elements constituting fuel. Based on chemical reactions the mass of oxygen required per kg of element can be estimated. From these the oxygen requirement per kg of fuel constituents and subsequently mass of air required for the estimated oxygen requirement can be known. Let us see how air required for complete burning of a fuel having 85.5% carbon, 12.3% hydrogen and 2.2% ash is calculated;

Mass of oxygen required for constituents;

1 kg of carbon shall require $\frac{8}{3}$ kg of oxygen

1 kg of hydrogen shall require 8 kg of oxygen

Ash shall not undergo oxidation.

Table 10.5 *Air requirement for fuel (on mass basis)*

Mass of constituents per kg of fuel (A)	*Oxygen required per kg of constituent (B)*	*Oxygen required per kg of fuel (C) = (A) × (B)*	*Mass of air required, (theoretical, for air having 23% oxygen, 77% nitrogen)* $D = \frac{\Sigma(C) \times 100}{23}$
C = 0.855	8/3	2.28	
H_2 = 0.123	8	0.984	$= \frac{3.264 \times 100}{23} = 14.19$
Ash = 0.022	—	—	
		Σ(C) = 3.264	

Above calculations show that for the given fuel composition 14.19 kg of minimum air shall be theoretically required for complete combustion of one kg of fuel. Combustion analysis can be carried out either on mass basis or volume basis. Similar kind of analysis can be made on volume basis. Let us take heptane (C_7H_{16}) as the fuel and consider its' complete combustion.

Table 10.6 *Air required for fuel (on volume basis)*

Constituent	*Proportional mass, A*	*Proportional volume,* $B = \frac{A}{mol.wt.}$	*Percentage volume,* $C = \frac{B}{\Sigma B}$	*O_2 required per m^3 of D × 100*	*O_2 required per 100 m^3 of element, E = D × C element*	*Air required for 100 m^3 of fuel (theoretical for air having 21% O_2 and 79% N_2), F* $= \frac{\Sigma E}{21} \times 100$
C	12 × 7 = 84	7	46.67	1	46.67	$= \frac{73.335}{21} \times 100$
H_2	2 × 8 = 16	8	53.33	0.5	26.665	= 349.21
		ΣB = 15			ΣE = 73.335	

Hence volume of air required is 349.21 m^3 for 100 m^3 of fuel.

Volumetric analysis of combustion can be transformed into gravimetric analysis and vice versa. Such conversion is required because the volumetric analysis is available experimentally. Let us take an exhaust gas sample containing CO, CO_2, N_2 and O_2. Let their volume fractions be X_1, X_2, X_3 and X_4 respectively.

Table 10.7 Conversion from volume based analysis to mass based analysis

Constituent	*Volume of Constituent per m^3, V*	*Molecular weight, M*	*Proportional masses, X = M.V*	*Mass of constituent per kg of dry exhaust gases, $m = \frac{M.V}{\Sigma X}$*
CO	X_1	28	$28.X_1$	$\left(\frac{28 \cdot X_1}{\Sigma X}\right)$
CO_2	X_2	44	$44.X_2$	$\left(\frac{44 \cdot X_2}{\Sigma X}\right)$
N_2	X_3	28	$28.X_3$	$\left(\frac{28 \cdot X_3}{\Sigma X}\right)$
O_2	X_4	32	$32.X_4$	$\left(\frac{32 \cdot X_4}{\Sigma X}\right)$

Total of proportional masses $\Sigma X = 28X_1 + 44X_2 + 28X_3 + 32X_4$

10.8 DETERMINATION OF AIR REQUIREMENT

(a) From combustion equations of fuel constituents (percentage basis) such as C, H, O and S as described earlier the total oxygen required per kg of fuel for complete combustion can be given as;

$$= \frac{1}{100}\left[\frac{8}{3}\mathrm{C} + 8\left(\mathrm{H} - \frac{\mathrm{O}}{8}\right) + \mathrm{S}\right]$$

As air is taken to have 23% of oxygen by mass, therefore total air requirement per kg of fuel shall be;

$$= \frac{1}{100}\left[\frac{8}{3}\mathrm{C} + 8\left(\mathrm{H} - \frac{\mathrm{O}}{8}\right) + \mathrm{S}\right] \times \frac{100}{23}$$

$$\boxed{\text{Minimum air required per kg of fuel for complete combustion} = \frac{1}{23}\left[\frac{8}{3}\mathrm{C} + 8\left(\mathrm{H} - \frac{\mathrm{O}}{8}\right) + \mathrm{S}\right]}$$

(b) Air requirement per kg of fuel can also be determined if the volumetric analysis of dry flue gases and mass fraction of carbon per kg of fuel is known. Let us take the dry flue gases having mass fractions as defined earlier as,

$$1 \text{ kg of flue gas has } \left(\frac{28X_1}{\Sigma X}\right) \text{ kg of CO and } \left(\frac{44X_2}{\Sigma X}\right) \text{ kg of } CO_2.$$

Now the total mass of carbon present in CO and CO_2 present in 1 kg of flue gas can be estimated and equated with mass of carbon present per kg of fuel.

Mass of carbon per kg of flue gas = Mass of carbon in CO per kg flue gas
+ Mass of carbon in CO_2 per kg flue gas

$$\text{Mass of carbon in CO per kg flue gas} = \left(\frac{28X_1}{\Sigma X}\right) \times \frac{12}{28} = \left(\frac{12X_1}{\Sigma X}\right)$$

$$\text{Mass of carbon in } CO_2 \text{ per kg flue gas} = \left(\frac{44X_2}{\Sigma X}\right) \times \frac{12}{44} = \left(\frac{12X_2}{\Sigma X}\right)$$

$$\text{Total carbon per kg of flue gas} = \left\{\frac{12X_1}{\Sigma X} + \frac{12X_2}{\Sigma X}\right\}$$

$$= \frac{12}{\Sigma X} \; (X_1 + X_2) \text{kg per kg of flue gas.}$$

Now if we look at the fact that from where carbon is coming in flue gases then it is obvious that carbon is available only in fuel. Let us assume that carbon present in fuel completely goes into flue gases. Let us also assume that fuel does not contain nitrogen so what ever nitrogen is there in flue gas it will be because of nitrogen present in air. Let the mass of flue gases formed per kg of fuel after combustion be 'm_g'. The mass of carbon in per kg fuel say C can be equated to total mass of carbon calculated above.

Mass fraction of carbon in fuel = Total mass of carbon in m_g mass of flue gases.

$$C = \left\{\frac{12}{\Sigma X}(X_1 + X_2)\right\} \times m_g$$

$$\boxed{\text{Mass of flue gases, } m_g = \left\{\frac{C \cdot \Sigma X}{12(X_1 + X_2)}\right\}}$$

Mass of air supplied per kg of fuel can be known from the nitrogen fraction present in flue gases as only source of nitrogen is air. 0.77 kg of N_2 is available in 1 kg of air. From flue gas analysis $\left(\frac{28X_3}{\Sigma X}\right)$ is the mass of nitrogen available in per kg of flue gas so total mass of nitrogen present in flue gases (m_g) due to combustion of unit mass of fuel shall be $\left\{\left(\frac{28X_3}{\Sigma X}\right) \cdot m_g\right\}$.

From air composition 0.77 kg N_2 is present per kg of air so the mass of air supplied per kg of flue gas formed,

$$= \left(\frac{28X_3}{\Sigma X}\right) \times \frac{1}{0.77}, \text{ kg air per kg of flue gas}$$

$$\text{Mass of air supplied per kg of fuel} = \left(\frac{28X_3}{\Sigma X}\right) \times \frac{m_g}{0.77}$$

$$\text{Substituting for } m_g, \quad = \left(\frac{28X_3}{\Sigma X}\right) \times \frac{1}{0.77} \times \left\{\frac{C \cdot \Sigma X}{12(X_1 + X_2)}\right\}$$

$$\boxed{\text{Mass of air supplied per kg of fuel} = \frac{28X_3 \cdot C}{9.24(X_1 + X_2)}}$$

where X_1, X_2, X_3 are volume fractions of CO, CO_2 and N_2 present in unit volume of flue gas and C is mass fraction of carbon present in unit mass of fuel.

For complete combustion of fuel there shall be no CO and only CO_2 shall contain carbon i.e. $X_1 = 0$.

(c) Generally for ensuring complete combustion of fuel, excess air is supplied. In case of combustion if there is incomplete combustion of carbon resulting into formation of carbon monoxide, then additional oxygen shall be required for converting carbon monoxide (CO) into carbon dioxide (CO_2). From earlier discussions let us consider unit mass of flue gas containing $\left(\frac{28X_1}{\Sigma X}\right)$ kg of CO and $\left(\frac{32X_4}{\Sigma X}\right)$ kg of O_2. The oxygen present in fuel remains in association with hydrogen. For combustion of CO into CO_2 the mass of oxygen required is $\frac{4}{7}$ kg per kg of CO. Thus oxygen required for CO present in 1 kg flue gas

$$= \left(\frac{28X_1}{\Sigma X}\right) \times \frac{4}{7}$$

$$= \left(\frac{16X_1}{\Sigma X}\right) \text{ kg of oxygen}$$

Therefore, excess oxygen available per kg of flue gas formed,

$$= \left(\frac{32X_4}{\Sigma X}\right) - \left(\frac{16X_1}{\Sigma X}\right)$$

$$= \frac{16}{\Sigma X}\ (2X_4 - X_1)$$

$$\text{Excess oxygen supplied per kg of fuel burnt} = \left\{\frac{16}{\Sigma X}(2X_4 - X_1)m_g\right\}$$

$$= \frac{16}{\Sigma X}(2X_4 - X_1)\left\{\frac{C\Sigma X}{12(X_1 + X_2)}\right\}$$

$$= \frac{4}{3}\ \frac{(2X_4 - X_1)\cdot \text{C}}{(X_1 + X_2)}$$

Hence excess air supplied per kg of fuel

$$= \frac{4C(2X_4 - X_1)}{3(X_1 + X_2)} \times \frac{1}{0.23}$$

$$\text{Excess air supplied per kg of fuel burnt} = \frac{4C\cdot(2X_4 - X_1)}{0.69(X_1 + X_2)}$$

10.9 FLUE GAS ANALYSIS

Flue gas analysis refers to the determination of composition of exhaust gases. Flue gas analysis can be done theoretically and experimentally. Here experimental method of flue gas analysis is described. Various devices available for measuring the composition of products of combustion (flue gas) are Orsat Analyzer, Gas chromatograph. Infrared analyzer and Flame ionisation detector etc. Data from these devices can be used to determine the mole fraction of flue gases. Generally this analysis is done on dry basis which may also be termed as "dry product analysis" and it refers to describing mole fractions for all gaseous products except water vapour.

Orsat analyzer: It is also called as Orsat apparatus and is used for carrying out volumetric analysis of dry products of combustion. Schematic of apparatus is shown in Fig.10.3 It has three flasks containing different chemicals for absorption of CO_2, O_2 and CO respectively and a graduated eudiometer tube connected to an aspirator bottle filled with water.

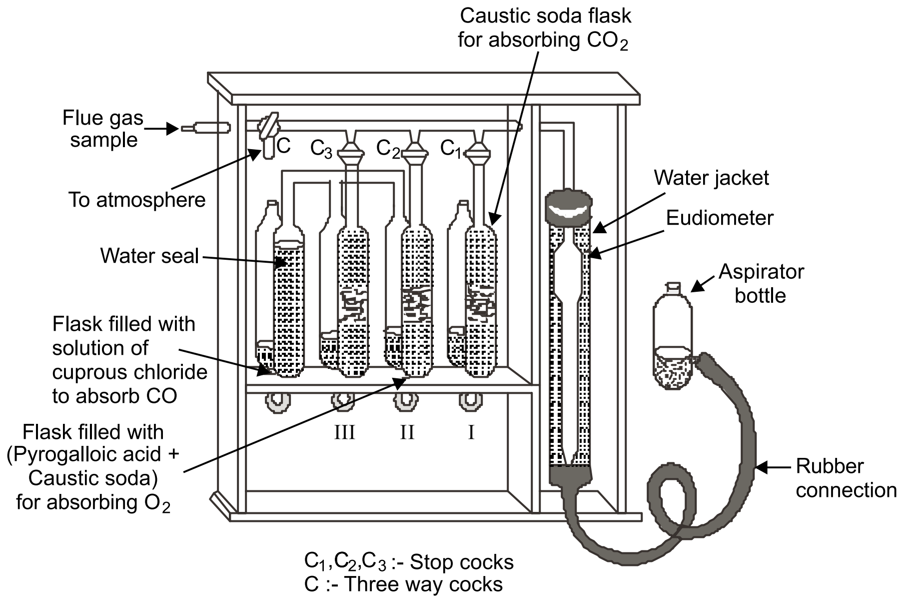


Fig. 10.3 *Orsat analyzer*

Flask I is filled with NaOH or KOH solution (about one part of KOH and 2 parts of water by mass). This 33% KOH solution shall be capable of absorbing about fifteen to twenty times its own volume of CO_2. Flask II is filled with alkaline solution of pyrogallic acid and above KOH solution. Here 5 gm of pyrogallic acid powder is dissolved in 100 cc of KOH solution as in Flask I. It is capable of absorbing twice its own volume of O_2. Flask III is filled with a solution of cuprous chloride which can absorb CO equal to its' volume. Cuprous chloride solution is obtained by mixing 5 mg of copper oxide in 100 cc of commercial HCl till it becomes colourless. Each flask has a valve over it and C_1, C_2, C_3 valves are put over flasks I, II and III. All the air or any other residual gas is removed from eudiometer by lifting the aspirator bottle and opening main value. The flue gas for analysis is taken by opening the main valve (three way valve) while valves C_1, C_2 and C_3 are closed. 100 cc of flue gas may be taken into eudiometer tube by lowering aspirator bottle until the level is zero and subsequently forced into flasks for absorbing different constituents. Aspirator bottle is lifted so as to inject flue gas into flask I with only valve C_1 in open state where CO_2 present shall be absorbed. Aspirator bottle is again lowered and reading of eudiometer tube taken. Difference in readings of eudiometer tube initially and after CO_2 absorption shall give percentage of CO_2 by volume. Similar steps may be repeated for getting O_2 and CO percentage by volume for which respective flask valve shall be opened and gas passed into flask. Thus Orsat analyzer directly gives percentage by volume of constituents. In case of other constituents to be estimated the additional flasks with suitable chemical may be used. The remaining volume in eudiometer after absorption of all constituents except N_2 shall give percentage volume of N_2 in flue gas.

As in combustion of hydrocarbon fuel the H_2O is present in flue gases but in orsat analysis dry flue gases are taken which means H_2O will be condensed and separated out. Therefore the percentage by volume of constituents estimated shall be on higher side as in actual product H_2O is there but in dry flue gas it is absent. Orsat analyzer does not give exact analysis.

10.10 FUEL CELLS

Fuel cell refers to a device having fuel and oxidizer in it which undergoes controlled chemical reaction to produce combustion products and provide electric current directly. In fuel cells the fuel and oxidizer react in stages on two separate electrodes i.e. anode (+ ive electrode) and cathode (– ive electrode). Two electrodes are separated by electrolyte in between. The chemical reaction is carried out to produce electric power without moving parts or the use of intermediate heat transfers as in power cycles. Thus fuel cell does not work on any cycle. Fuel cells based on hydrogen-oxygen fuel cells have been used to provide power to the spacecrafts. Fuel cells based on natural gas are also under process of development. Fuel cells are also being developed to power automobiles. Let us look at hydrogen-oxygen fuel cell in detail.

Figure 10.4 shows the schematic of hydrogen-oxygen fuel cell. Here H_2 supplied diffuses through the porous anode and reacts on anode surface with OH^-ions resulting into H_2O and free electrons, the reaction for it is given in figure.

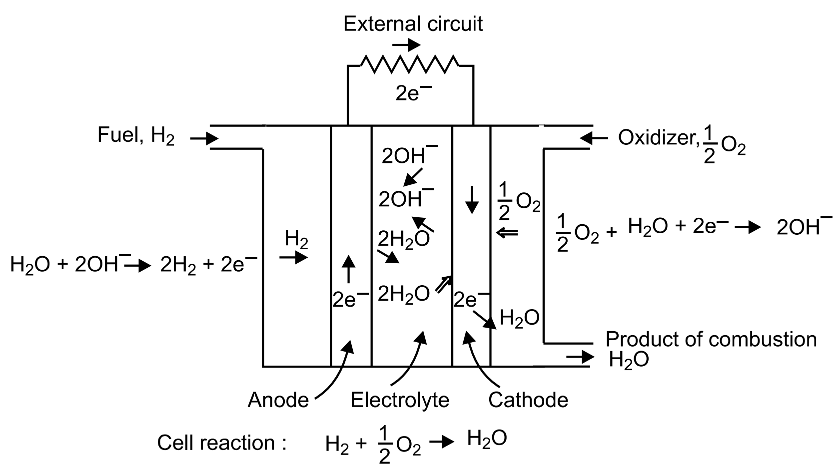


Fig. 10.4 *Hydrogen-Oxygen fuel cell*

Free electrons liberated enter the circuit while water goes into electrolyte. Oxygen supplied combines with water in electrolyte and electrons coming from electric circuit to produce OH^- ions and H_2O as per chemical reaction given in figure. OH^- ions are transported through the electrolyte. Overall fuel cell has chemical reaction as

$$H_2 + \frac{1}{2}O_2 \rightarrow H_2O$$

Thus in hydrogen-oxygen fuel cell electricity and water are produced.

EXAMPLES

1. *Coal having following composition by mass is burnt with theoretically correct amount of air.*

86% C, 6% H, 5% O, 2% N, 1% S

Determine the air-fuel ratio.

Solution:

Combustion equation for the coal (100 kg of coal) can be given as under;

$$\left(\frac{86}{12}\cdot C+\frac{6}{1}\cdot H+\frac{5}{16}\cdot O+\frac{2}{14}\cdot N+\frac{1}{32}\cdot S\right) + n(O_2 + 3.76\ N_2) \rightarrow$$
$$a\cdot CO_2 + b\cdot SO_2 + d\cdot N_2 + e\cdot H_2O$$

From above equation C, H, O, S, N can be equated on both the sides as under,

$$C;\ 7.16 = a$$
$$H;\ 6 = 2e$$
$$O;\ (0.3125 + 2n) = 2a + 2b + e$$
$$N;\ (0.1429 + 3.76\times 2n) = 2d$$
$$S;\ 0.03125 = b$$

Solving we get $a = 7.16$

$b = 0.03125$

$d = 32.163$

$e = 3$

$n = 8.535$

Amount of air required shall be [8.535 × (4.76)] kg mol per 100 kg of coal.

$$\text{Air-fuel ratio} = \frac{8.535\times 4.76\times 28.97}{100}$$
$$= 11.77 \text{ kg air per kg of fuel}$$

$\boxed{\text{Air-fuel ratio} = 11.77}$ ***Ans.***

2. *One kg C_8H_{18} fuel is supplied to an engine with 13 kg of air. Determine the percentage by volume of CO_2 in dry exhaust gas considering exhaust gas to consist of CO_2, CO and N_2.*

Solution:

Combustion equation in mol. basis for one kg of fuel supplied shall be as under.

$$\frac{1}{114}(C_8H_{18}) + n(0.21\ O_2 + 0.79\ N_2) \rightarrow a\cdot CO_2 + b\cdot CO + d\cdot N_2 + e\cdot H_2O$$

Equating the coefficients on both sides,

$$C;\ \frac{8}{114} = 0.0702 = a + b$$
$$O_2;\ 0.21\ n = a + \frac{b}{2} + \frac{e}{2}$$
$$H;\ \frac{18}{114} = 0.1579 = 2e$$

N_2; 0.79 $n = d$. Also it is given that 13 kg of air per kg of fuel is supplied, therefore

$$n = \frac{13}{28.97} = 0.4487$$

Solving above following are available,

$a = 0.0393$

$b = 0.0309$

$d = 0.3545$
$e = 0.07895$
$n = 0.4487$

Constituents of dry exhaust gas shall be CO_2, CO and N_2 as indicated,

$$\text{Therefore dry exhaust gas} = (a + b + d)$$
$$= (0.0393 + 0.0309 + 0.3545) = 0.4247$$

$$\text{Percentage by volume of } CO_2 \text{ in dry exhaust gas} = \frac{a}{(a+b+d)} \times 100$$
$$= \frac{0.0393 \times 100}{0.4247}$$
$$= 9.25\%$$

$\boxed{\text{\% by volume of } CO_2 = 9.25\%}$ ***Ans.***

3. *In a boiler the coal having 88% C, 3.8% H_2, 2.2% O_2 and remaining ash is burnt in the furnace. It is found that CO_2 going with flue gases constitute to be 12% and temperature of flue gases is 260°C. The flue gas sample is analyzed using Orsat apparatus at room temperature. Determine the percentage of CO_2 that would be there for complete combustion of fuel.*

Solution:

Actual percentage of CO_2 in flue gases = 12%

Constituent mass per kg of fuel	*Combustion product (x)*		*Molecular weight (y)*		*Volume fraction z = (x)/(y)*	
	CO_2	H_2O	CO_2	H_2O	CO_2	H_2O
C = 0.88	$\left(0.88 \times \frac{44}{12}\right)$		44		0.0733	
H_2 = 0.038		$\left(0.038 \times \frac{18}{2}\right)$		18		0.019

Exhaust gases shall comprise of CO_2, H_2O, O_2 and N_2

$$\text{Actual percentage of } CO_2 = \frac{CO_2 \times 100}{(CO_2 + H_2O + O_2 + N_2)}$$

$$12 = \frac{0.0733 \times 100}{(0.0733 + 0.019 + (O_2 + N_2))}$$

$$(O_2 + N_2) = 0.5185$$

Orsat apparatus analyzes only dry flue gas so the percentage of CO_2 can be obtained as,

$$\% \, CO_2 = \frac{CO_2 \times 100}{(CO_2 + O_2 + N_2)} = \frac{0.0733 \times 100}{(0.0733 + 0.5185)} = 12.38\%$$

$\boxed{\% \, CO_2 = 12.38\%}$ ***Ans.***

4. *Determine the percentage analysis of combustion products by mass and by volume when gravimetric analysis of a hydrocarbon fuel indicates 86% C and 14% H_2. Excess air supplied is 50% for combustion. Consider air to have 23.2% of oxygen by mass and remaining as nitrogen, molecular weights of carbon, oxygen, nitrogen and hydrogen are 12, 16, 14 and 1 respectively. Also estimate the change in internal energy of per kg of products when cooled from 2100°C to 900°C when the internal energies of combustion products are as under,*

T°C	Internal energy, kJ/kg			
	CO_2	H_2O	N_2	O_2
2100	2040	3929	1823	1693
900	677	1354	694	635

Solution:

Theoretical mass of oxygen required per kg of fuel depending upon its' constituents

$$C + O_2 \rightarrow CO_2$$
$$12 \text{ kg} + 32 \text{ kg} \rightarrow 44 \text{ kg}$$

or, $$1 \text{ kg} + \frac{8}{3} \text{ kg} \rightarrow \frac{11}{3} \text{ kg}$$

or, $$0.86 \text{ kg} + 2.29 \text{ kg} \rightarrow 3.15 \text{ kg}$$

For hydrogen,

$$2H_2 + O_2 \rightarrow 2H_2O$$
$$4 \text{ kg} + 32 \text{ kg} \rightarrow 36 \text{ kg}$$
$$1 \text{ kg} + 8 \text{ kg} \rightarrow 9 \text{ kg}$$
$$0.14 \text{ kg} + 1.12 \text{ kg} \rightarrow 1.26 \text{ kg}$$

Hence, minimum mass of oxygen required kg per kg of fuel $= \left(0.86 \times \frac{8}{3} + 0.14 \times 8\right) = 3.41$ kg

As it is given that 50% excess air is supplied so the excess oxygen $= 3.41 \times 0.5 = 1.705$ kg

Total oxygen supplied $= 3.41 + 1.705 = 5.115$ kg

Constituents of exhaust gases per kg of fuel, $CO_2 = \frac{11}{3} \times 0.86 = 3.15$ kg

$$H_2O = 9 \times 0.14 = 1.26 \text{ kg}$$

$$N_2 = \left(\frac{5.115 \times 76.8}{23.2}\right) = 16.932 \text{ kg}$$

Excess $O_2 = 1.705$ kg

For one kg of fuel the composition of combustion products is as under,

Exaust gas constituents (x)	*Gravimetric analysis %* $g = \frac{(x \times 100)}{\Sigma(x)}$	*Molecular weight, y*	*Proportional volume,* $z = \frac{g}{y}$	*Volumetric analysis %,* $v = \frac{(z \times 100)}{\Sigma(z)}$
CO_2 = 3.15 kg	13.67	44	0.311	8.963
H_2O = 1.26 kg	5.47	18	0.304	8.761
N_2 = 16.932 kg	73.47	28	2.624	75.619
O_2 = 1.705 kg	7.39	32	0.231	6.657
Total = 23.047	100%		3.47	100%

Change in internal energy for given temperature variation shall be estimated as under, on gravimetric basis

Mass fraction of constituents of exhaust gases (g)	*Change in internal energy per kg of constituents, (b) kJ/kg*	*Change in internal energy (c) = (g × b), kJ*
CO_2 = 0.1367	2040 – 677 = 1363	186.32
H_2O = 0.0547	3929 – 1354 = 2575	140.85
N_2 = 0.7347	1823 – 694 = 1129	829.48
O_2 = 0.0739	1693 – 635 = 1058	78.19

Total change in internal energy = 1234.84 kJ/kg of exhaust gases

$$\boxed{\text{Change in internal energy} = 1234.84 \text{ kJ/kg of exhaust gases}}$$ ***Ans.***

5. *Determine the percentage excess air supplied to boiler for burning the coal having following composition on mass basis,*

C: 0.82
H_2: 0.05
O_2: 0.08
N_2: 0.03
S: 0.005
moisture: 0.015

Volumetric analysis of dry flue gases shows the following composition,

CO_2 = 10%
CO = 1%
N_2 = 82%
O_2 = 7%

Solution:

For constituent components combustion reactions are as under. Out of given constituents only C, H_2, and S shall require oxygen for burning reaction.

$$C + O_2 \rightarrow CO_2$$
$$12 \text{ kg} + 32 \text{ kg} \rightarrow 44 \text{ kg}$$
$$1 \text{ kg} + \frac{8}{3} \text{ kg} \rightarrow \frac{11}{3} \text{ kg}$$
$$2H_2 + O_2 \rightarrow 2H_2O$$
$$4 \text{ kg} + 32 \text{ kg} \rightarrow 36 \text{ kg}$$
$$1 \text{ kg} + 8 \text{ kg} \rightarrow 9 \text{ kg}$$
$$S + O_2 \rightarrow SO_2$$
$$32 \text{ kg} + 32 \text{ kg} \rightarrow 64 \text{ kg}$$
$$1 \text{ kg} + 1 \text{ kg} \rightarrow 2 \text{ kg}$$

Net mass of oxygen required per kg of coal

$$= \{(0.82 \times \frac{8}{3}) + (0.05 \times 8) + (0.005 \times 1) - (0.08)\}$$
$$= 2.512 \text{ kg}$$

Mass of air required per kg of coal, considering oxygen to be 23% in the air by mass,

$$= \frac{100}{23} \times 2.512$$

$= 10.92$ kg air per kg of coal

Volume of constituent per mol. of dry flue gas % (a)	*Molecular wt. (b)*	*Proportional mass of constituents $c = (a \times b)$*	*Mass per kg of flue gas $d = \frac{c}{\Sigma(c)}$*	*Mass of carbon per kg of dry flue gas*
$CO_2 = 10$	44	440	0.147	$\frac{0.147 \times 12}{44} = 0.04009$
$CO = 1$	28	28	0.0094	$\frac{0.0094 \times 12}{28} = 0.00403$
$N_2 = 82$	28	2296	0.768	
$O_2 = 7$	32	224	0.075	Total carbon per kg of dry flue gas = 0.04412
		$\Sigma(c) = 2988$		

Carbon is given to be 0.82 kg in per unit mass of coal and the mass of carbon per unit mass of dry flue gas is 0.04412 kg so the mass of dry flue gases per kg of coal shall be

$$= \frac{\text{Carbon mass per kg of dry flue gases}}{\text{Carbon mass per kg of coal}}$$

$$= \frac{0.82}{0.04412}$$

$= 18.59$ kg

Therefore, the mass of CO per kg of coal $= 18.59 \times 0.0094$

$= 0.1747$ kg

The mass of excess O_2 per kg of coal (i.e. unutilized O_2) $= 18.59 \times 0.075$

$= 1.394$ kg

The CO produced in combustion products per kg of coal shall further require O_2 for its' complete burning to CO_2.

$$2CO + O_2 \rightarrow 2CO_2$$

$$56 \text{ kg} + 32 \text{ kg} \rightarrow 88 \text{ kg}$$

$$1 \text{ kg} + \frac{4}{7} \text{ kg} \rightarrow \frac{11}{7} \text{ kg}$$

The mass of O_2 required for complete burning of 0.1747 kg CO per kg of coal shall be $= 0.1747 \times \frac{4}{7} = 0.998$ kg.

Out of excess O_2 coming out with dry flue gases 0.0998 kg of O_2 shall be utilized for complete burning of CO. Thus, the net excess O_2 per kg of coal $= 1.394 - 0.0998 = 1.2942$ kg O_2

Hence, excess air required for 1.2942 kg $O_2 = \frac{1.2942 \times 100}{23} = 5.627$ kg air

$$\% \text{ excess air} = \frac{5.627}{10.92} \times 100 = 51.53\%$$

$\boxed{\% \text{ excess air} = 51.53\%}$ ***Ans.***

6. *C_2H_6 burns completely with air when the air-fuel ratio is 18 on mass basis. Determine the percent excess or percent deficiency of air as appropriate and the dew point temperature of combustion products when cooled at 1 atm.*

Solution:

Given air-fuel ratio on mass basis is first transformed into the air-fuel ratio on molar basis.

$$\text{A/F ratio on molar basis} = \frac{\text{No. of moles of air}}{\text{No. of moles of fuel}} = \frac{\left(\frac{\text{Mass of air}}{\text{Mol. wt. of air}}\right)}{\left(\frac{\text{Mass of fuel}}{\text{Mol. wt. of fuel}}\right)}$$

$$= \frac{\text{Mass of air}}{\text{Mass of fuel}} \times \frac{\text{Mol. wt. of fuel}}{\text{Mol. wt. of air}}$$

$$= (\text{A/F ratio on mass basis}) \times \frac{\text{Mol. wt. of } C_2H_6}{\text{Mol. wt. of air}}$$

$$= 18 \times \frac{30}{29}$$

A/F ratio on molar basis = 18.62

Chemical reaction for complete combustion may be given as,

$$C_2H_6 + n(O_2 + 3.76\,N_2) \rightarrow aCO_2 + bN_2 + dO_2 + eH_2O$$

Also, for complete combustion there will be $\left(\frac{18.62}{4.76}\right)$ moles of air for each mole of fuel,

So, $n = 3.912$

Equating coefficient

C: $2 = a$	Solving we get,	$a = 2$
H: $6 = 2e$		$b = 14.71$
O: $2n = 2a + 2d + e$		$d = 0.412$
N: $(3.76 \times 2n) = 2b$		$e = 3$

Complete combustion reaction may be given as,

$$C_2H_6 + 3.912\,(O_2 + 3.76\,N_2) \rightarrow 2CO_2 + 14.71N_2 + 0.412O_2 + 3H_2O$$

or, $C_2H_6 + 3.5(O_2 + 3.76\,N_2) \rightarrow 2CO_2 + 13.16\,N_2 + 3H_2O$

Hence air-fuel ratio (theoretical) on mol. basis

$$(\text{A/F}) = \frac{3.5 \times 4.76}{1} = 16.66$$

Since theoretical air-fuel ratio is less than actual so it means excess air is supplied.

$$\text{Percentage of excess air} = \frac{(\text{A/F})_{\text{actual}} - (\text{A/F})_{\text{theoretical}}}{(\text{A/F})_{\text{theoretical}}} \times 100$$

$$= \left(\frac{18.62 - 16.66}{16.66}\right) \times 100$$

$= 11.76\%$

$\boxed{\text{Percentage of excess air} = 11.76\%}$ ***Ans.***

Total amount of mixture $= (a + b + d + e)$

$= (2 + 14.71 + 0.412 + 3)$

$= 20.122$ kg/mol. of fuel

$$\text{Partial pressure of water vapour } P = \left(\frac{e}{a+b+d+e}\right) \times 1.013$$

$= 0.151$ bar

From steam table saturation temperature corresponding to 0.151 bar is seen to be 54°C.

$\boxed{\text{Dew point temperature} = 54°\text{C}}$ ***Ans.***

7. *Obtain the volumetric composition of combustion products obtained after combustion of C_7H_{16} (heptane) being burnt with 50% excess air. Also obtain the average molecular weight and specific volume of combustion products at S.T.P. Consider the volume per kg mol. at S.T.P. to be 22.4 m^3 and air to have 21% O_2 and 79% N_2 by volume.*

Solution:

Here excess air supplied shall result in unutilized O_2 with combustion products.

*Excess O_2 supplied $= 0.5 \times 73.33 = 36.665\ m^3$

Total O_2 supplied $= 73.33 + 36.665 = 109.995 m^3$

**Total N_2 supplied $= (109.995) \times \frac{79}{21} = 413.79 m^3$

Total volume of combustion products $= \{46.67 + 53.33 + 36.665 + 413.79\}$

$= 550.455\ m^3$

Average molecular weight of combustion products can be estimated by summation of mass of constituents per mol. of combustion products.

Constituents of combustion products	*Fraction of constituents per unit products* $b = \frac{a}{\Sigma(a)}$	*Molecular weight (c)*	*Mass of constituents per mol. of combustion* $(d = b \times c)$	*Average molecular weight of combustion products* $c = \Sigma(d)$
CO_2(= 46.67)	$\frac{46.67}{550.455} = 0.0848$	44	3.7312	
H_2O(= 53.33)	$\frac{53.33}{550.455} = 0.0969$	18	1.7442	28.65
O_2(= 36.665)	$\frac{36.665}{550.455} = 0.0666$	32	2.1312	
N_2(= 413.79)	$\frac{413.79}{550.455} = 0.7517$	28	21.0476	

Constituents of C_7H_{16}	*Proportional mass*	*Proportional volume*	*% Volume*	O_2 *required per* m^3 *of fuel*	O_2 *required for* 100 m^3 *of fuel*	*% Combustion product per*100 m^3 *of fuel*			
	(a)	$(b) = \frac{a}{mol.wt}$	$(c) = \frac{b \times 100}{\Sigma(b)}$	(d)	$(e) = c \times d$	CO_2	H_2O	O_2	N_2
C (mol. wt. = 12)	$12 \times 7 = 84$	7	46.67	1 ($C + O_2 \rightarrow CO_2$) (1 mol 1 mol $\rightarrow$ 1 mol.)	46.67	46.67			
H_2 (mol. wt. = 2)	$2 \times 8 = 16$	8	53.33	0.5 ($2H_2 + O_2 \rightarrow 2H_2O$) 2 mol 1 mol 2 mol	26.66		53.33		
		Σ(b) = 15			73.33	46.67	53.33	36.665*	413.79

Specific volume = $\frac{22.4}{28.65}$ = 0.7818 m^3/kg

$$\boxed{\begin{array}{l}\text{Average mol. wt.} = 28.65\\ \text{Specific volume} = 0.7818\ m^3/kg\end{array}}$$ ***Ans.***

Mass of fuel having 0.864 kg of carbon = $\frac{0.864}{\left(\begin{array}{c}\text{Mass of carbon}\\ \text{per kg of fuel}\end{array}\right)} = \frac{0.864}{0.847}$ = 1.02kg

It shows that one mol of dry flue gases shall require burning of 1.02 kg of fuel and 29.89 kg of air.

So the air-fuel ratio = $\frac{29.89}{1.02}$ = 29.3

% Excess air = $\frac{(29.3 - 15) \times 100}{15}$ = 95.33%

$$\boxed{\begin{array}{l}\text{Air fuel ratio} = 29.3\\ \text{\% excess air} = 95.33\%\end{array}}$$ ***Ans.***

Here the calculations above have been based on balancing the carbon present. These can also be done based on oxygen-hydrogen balance as under;

Alternate approach (Not very accurate)

Mass of oxygen present in one mol of dry flue gas = Mass of oxygen in CO_2 + Mass of O_2 appearing with dry flue gas

$$= \left(\frac{3.168 \times 32}{44}\right) + (0.108 \times 32)$$

$$= 5.76 \text{ kg}$$

Mass of oxygen present with nitrogen in air = $\left(\frac{22.96 \times 23.2}{76.8}\right)$ = 6.94 kg

Here, the mass of oxygen supplied in air (6.94 kg) is more than oxygen present in dry flue gases (5.76 kg) which indicates that some oxygen gets consumed in formation of water. Orsat analysis analyzes dry flue gas.

Mass of oxygen forming water (H_2O) = 6.94 – 5.76 = 1.18 kg

$$2H_2 + O_2 \rightarrow 2H_2O$$
$$4 \text{ kg} + 32 \text{ kg} \rightarrow 36 \text{ kg}$$
$$1 \text{ kg} + 8 \text{ kg} \rightarrow 9 \text{ kg}$$

Mass of hydrogen in fuel = $\left(\frac{1.18}{8}\right)$ = 0.1475 kg

Mass of fuel = $\frac{0.1475}{0.153}$ = 0.964 kg

Thus one mol of dry flue gases are generated when 0.964 kg of fuel is burnt with 29.89 kg of air.

8. *During Orsat analysis of the combustion products of an engine running on diesel ($C_{12}H_{26}$) the CO_2 and O_2 are found to be 7.2% and 10.8% respectively and rest is N_2 by volume. Determine the air-fuel ratio and percentage excess air considering air to have O_2 and N_2 in proportion of 23.2% and 76.8% respectively by mass.*

Solution:

There are two approaches for getting the air-fuel ratio and subsequently excess air. One is by carbon-balancing and other by balancing of oxygen-hydrogen.

Minimum air required for complete combustion of 1 kg of fuel can be obtained considering the composition of fuel i.e. $C_{12}H_{26}$.

Mass of constituent per kg of fuel, $C_{12}H_{26}$ (a)	*Oxygen required per kg of constituents (b)*	*Oxygen required per kg of fuel (c) = a × b*	*Minimum air required per kg of fuel for complete combustion*
$C = \dfrac{12 \times 12}{(12 \times 12 + 26 \times 1)} = 0.847$	$\dfrac{8}{3}$	$\dfrac{8}{3} \times 0.847 = 2.259$	$\dfrac{100}{23.2} \times 3.483$
$H_2 = \dfrac{26 \times 1}{(12 \times 12 + 26 \times 1)} = 0.153$	8	$8 \times 0.153 = 1.224$	= 15 kg
		Total = 3.483	

Dry flue gases contain 7.2% of CO_2, 10.8% O_2 and 82% N_2 by volume. This volumetric composition is to be converted to gravimetric (mass basis) i.e. mol fractions may be estimated as mol is unit of volume.

Mass of constituent in one mol of dry flue gases = Volume fraction × Mol. wt. of constituent

Mass of CO_2 in one mol of dry flue gases = 0.072 × 44 = 3.168 kg

Mass of N_2 in one mol of dry flue gases = 0.82 × 28 = 22.96 kg

Corresponding to N_2 the mass of air per mol of dry flue gases can be obtained as,

$$= \frac{22.96 \times 100}{76.8} = 29.89 \text{ kg}$$

Carbon present in one mol of dry flue gases = $\left(3.168 \times \dfrac{12}{44}\right)$ = 0.864 kg

Therefore, from the carbon present in one mol of dry flue gases the amount of fuel containing this much of carbon can be calculated.

So, air fuel ratio = $\dfrac{29.89}{0.964}$ = 31.0

% excess air = $\dfrac{(31 - 15)}{15} \times 100$ = 106.67%

Air-fuel ratio and % excess air calculated differ from those estimated using balancing of carbon. Generally, carbon balancing is used for getting accurate results.

9. *Fuel having 88% C and 12% H_2 is burnt using 98% of air compared to theoretical air requirement for combustion. Determine the following considering that H_2 is completely burnt and carbon burns to CO and CO_2 leaving no free carbon.*

(*a*) *% analysis of dry flue gases by volume*

(*b*) *% heat loss due to incomplete combustion. Consider gross calorific values in kJ/kg as, C to CO_2 = 34694 kJ/kg, C to CO = 10324 kJ/kg, H_2 = 143792 kJ/kg*

Solution:

Mass of constituent per kg of fuel (*a*)	*Oxygen required per kg of constituent* (*b*)	*Oxygen required per kg of fuel* $c = (a \times b)$	*Minimum air required per kg of fuel for complete combustion* $\frac{100}{23.2} \times \Sigma(c)$
C = 0.88	$\frac{8}{3}$	2.35	$\frac{3.30 \times 100}{23.2} = 14.27$ kg
$H_2 = 0.12$	8	0.95	
		Total 3.30	

Actual air supplied for burning = 0.98 × 14.27 = 13.98 kg

Less amount of air supplied = 14.27 – 13.98 = 0.29 kg. Due to this amount of air complete carbon could not get burnt into CO_2 and CO is there.

Let us write burning of carbon to yield CO and CO_2

$$C + O_2 \rightarrow CO_2 \qquad\qquad 2C + O_2 \rightarrow 2CO$$

$$12\text{ kg} + 32\text{ kg} \rightarrow 44\text{ kg} \qquad 24\text{ kg} + 32\text{ kg} \rightarrow 56\text{ kg}$$

$$1\text{ kg} + 2.67\text{ kg} \rightarrow 3.67\text{ kg} \qquad 1\text{kg} + 1.33\text{ kg} \rightarrow 2.33\text{ kg}$$

Thus the amount of air saved by burning C to CO instead of CO_2, per kg of carbon

$$= (2.67 - 1.33) \times \frac{100}{23.2}$$

$$= 5.78 \text{ kg}$$

Thus the amount of carbon burnt to CO per kg of fuel shall be

$$= \frac{0.29 \times 0.88}{5.78} = 0.044 \text{ kg}$$

Out of 0.88 kg C per kg of fuel only 0.044 kg C gives CO while remaining (0.88 – 0.044 = 0.836) 0.836 kg C shall yield CO_2.

Volumetric analysis of combustion products is present as under.

Constituents of dry flue gas	*Mass of constituent per kg of fuel* (*a*)	*Proportional Volume in mol* $(b) = \left(\frac{a}{Mol.wt}\right)$	*Percentage volume* $(c) = \frac{b}{\Sigma(b)} \times 100$
CO_2	$0.836 \times \frac{44}{12} = 3.06$	$\frac{3.06}{44} = 0.069$	15.11%
CO	$0.044 \times \frac{28}{12} = 0.103$	$\frac{0.103}{28} = 0.0037$	0.81%
N_2	$0.768 \times 13.98 = 10.74$	$\frac{10.74}{28} = 0.384$	84.08%
		$\Sigma(b) = 0.4567$	100%

$$\boxed{\begin{array}{l}\text{Volumetric analysis of dry flue gas}\\ \qquad\qquad = 15.11\%\ CO_2,\ 0.81\%\ CO,\ 84.08\ \%\ N_2\end{array}}$$ ***Ans.***

Calorific value of fuel in actual case due to incomplete burning of carbon.

$$= (0.836 \times 34694) + (0.044 \times 10324)$$
$$= 29458.44 \text{ kJ}$$

Theoretical calorific value of fuel with complete burning of carbon

$$= (0.88 \times 34694) = 30530.7 \text{ kJ}$$

Theoretical calorific value of fuel $= \{(0.88 \times 34694) + (0.12 \times 143792)\}$

$$= 47785.76 \text{ kJ}$$

Percentage heat loss due to incomplete combustion

$$= \frac{(30530.72 - 29458.44)}{47785.76} \times 100$$
$$= 2.24\%$$

$$\boxed{\%\ \text{Heat loss} = 2.24\%}$$ ***Ans.***

10. *During production of gas the air and steam are passed through an incandescent coal bed. The coal is seen to have 95% of carbon and remaining as incombustible. The gas produced has hydrogen, nitrogen and carbon monoxide. Determine, the steam required per kg of coal and total air required per kg of coal when the heat of formation for steam is 147972 kJ/kg of hydrogen and for carbon monoxide it is 10324 kJ/kg of carbon. Also obtain volumetric analysis of gas. Take temperature of water as 20°C at 1 atm pressure. Take air to have 23.2%* O_2 *and 76.8%* N_2 *by mass.*

Solution:

Combustion equation can be written, separately for reaction with air and with steam as both are passed through.

Reaction with air

$$2C + O_2 \rightarrow 2CO$$
$$24 \text{ kg} + 32 \text{ kg} \rightarrow 56 \text{ kg}$$
$$1 \text{ kg of C} + \frac{4}{3} \text{ kg of } O_2 \rightarrow \frac{7}{3} \text{ kg of CO} + 10324 \text{ kJ} \quad ...(i)$$

Reaction with steam

$$C + H_2O \rightarrow CO + H_2$$
$$12 \text{ kg} + 18 \text{ kg} \rightarrow 28 \text{ kg} + 2 \text{ kg}$$
$$1 \text{ kg C} + \frac{3}{2} \text{ kg } H_2O \rightarrow \frac{7}{3} \text{ kg CO} + \frac{1}{6} \text{ kg } H_2 + \{10324 - \frac{1}{6}$$
$$\times 147972 - \frac{3}{2} \underbrace{((100-20) \times 4.18 + 2253)\}}_{\text{Sensible heat + Latent heat}}$$
$$1 \text{ kg C} + \frac{3}{2} \text{ kg } H_2O \rightarrow \frac{7}{3} \text{ kg CO} + \frac{1}{6} \text{ kg } H_2 - 182191 \text{ kJ} \quad ...(ii)$$

Two equations for burning with air and reaction with steam can be balanced in terms of heat released provided equation (ii) is multiplied by $\left(\frac{10324}{182191}\right)$ throughout, so equation (ii) gets modified into,

$$0.057 \text{ kg C} + 0.085 \text{ kg } H_2O \text{ (steam)} \rightarrow 0.1322 \text{ kg CO} + 0.0094 \text{ kg } H_2 - 10324 \text{ kJ} \quad ...(iii)$$

Thus, total carbon used from reaction (i) and (iii) with no heat release can be given as,

$$= (1 + 0.057) \text{ kg C} = 1.057 \text{ kg carbon.}$$

$$\text{Steam required per kg of coal} = \frac{0.085 \times 0.95}{1.057} = 0.0764 \text{ kg}$$

$$\text{Total air required per kg of coal} = \left(\frac{4}{3} \times \frac{0.95}{1.057}\right) \times \frac{100}{23.2} = 5.16 \text{ kg}$$

$$\boxed{\begin{array}{r} \text{Steam required per kg of coal} = 0.0764 \text{ kg} \\ \text{Air required per kg of coal} = 5.16 \text{ kg} \end{array}}$$ ***Ans.***

$$\text{Total carbon monoxide} = (2.333 + 0.132)$$

$$= 2.465 \text{ kg}$$

$$\text{Total hydrogen} = 0.0094 \text{ kg}$$

$$\text{Total nitrogen} = \frac{4}{3} \times \frac{76.8}{23.2} = 4.414 \text{ kg}$$

Volumetric analysis

Constituents (a)	*Mol. wt.* (b)	*Proportional volume in mol.* (c) = $\frac{a}{b}$	*% volume of gas* (d) = $\frac{c}{\Sigma(c)} \times 100$
CO = 2.465 kg H_2 = 0.0094 kg N_2 = 4.414 kg	28 2 28	0.088 0.0047 0.1576	35.16%, CO 1.88%, H_2 62.96%, N_2
		Σ(c) = 0.2503	100%

$$\boxed{\text{\% by volume} = 35.16\% \text{ CO}, 1.88\% \ H_2, 62.96\% \ N_2}$$ ***Ans.***

11. *Determine the higher and lower calorific values of coal for which following observations are made in bomb calorimeter.*

Mass of coal sample = 1 gm
Mass of water in bomb calorimeter = 2.5 kg
Initial temperature of water = 20°C
Maximum recorded temperature of water = 22.6°C
Water equivalent of apparatus = 750 gm
Cooling correction = + 0.018°C

Consider coal to have 5% H_2 in it.

Solution:

Using the cooling correction the corrected rise in temperature of water can be obtained as,

$$= (22.6 - 20) + 0.018 = 2.618°C$$

Heat supplied to calorimeter and water $= (2500 + 750) \times 2.618$

$= 8508.5$ Cal/gm

Higher calorific value of coal $= 8508.5$ Cal/gm

In order to get lower calorific value the heat carried by the steam formation is to be excluded from higher calorific value.

$$2H_2 + O_2 \rightarrow 2H_2O$$

$$1 \text{ kg } H_2 + 8 \text{ kg } O_2 \rightarrow 9 \text{ kg } H_2O$$

(Mass of steam formed per kg of coal) $= 0.05 \times 9$

$= 0.45$ gm steam per gm of coal.

From steam table h_{fg} at 20°C $= 586$ kcal/kg

$$\text{Lower heating value of coal} = 8508.5 - \left(0.45 \times \frac{586 \times 10^3}{10^3}\right)$$

$= 8244.8$ cal/gm

Higher heating value = 8508.5 kcal/kg
Lower heating value = 8244.8 kcal/kg

Ans.

12. *Composition of a fuel by volume is as follows:*

H_2 – 52%
CH_4 – 20%
CO – 16%
CO_2 – 3%
O_2 – 2%
N_2 – 7%

Higher calorific value of fuel constituents H_2, CO and CH_4 may be taken as 28424 kJ/m^3, 27463 kJ/m^3, 87780 kJ/m^3 at 1 bar and 0°C respectively. The latent heat of one kg of water for transformation to steam at 1 bar, 0°C may be taken as 2445 kJ. Air may be considered to have 21% O_2 by volume. Determine

(*a*) *the volume of air required for complete combustion of 1 m^3 of fuel.*
(*b*) *the volumetric analysis of dry flue gases considering 20% excess air.*
(*c*) *the lower calorific value of fuel.*

Solution:

Combustion reactions for constituents are,

$2H_2 + O_2 \rightarrow 2H_2O$,
1 mol 0.5 mol 1 mol

$CH_4 + 2O_2 \rightarrow 2H_2O + CO_2$,
1 mol 2 mol 2 mol 1 mol

$2CO + O_2 \rightarrow 2CO_2$,
1 mol 0.5 mol 1 mol

$$\text{Minimum air required} = 0.72 \times \frac{100}{21} = 3.43 \text{ m}^3$$

$$\boxed{\text{Volume of air required for complete combustion} = 3.43 \text{ m}^3}$$ ***Ans.***

Volumetric analysis

Constituents per m^3 of fuel (a)	O_2 *required per m^3 of constituents of fuel (b)*	O_2 *required per m^3 of fuel (c) = a × b*	*Combustion products per m^3 of fuel*			
			CO_2	H_2O	O_2	N_2
H_2 = 0.52	0.5	0.26	—	0.52	—	—
CH_4 = 0.20	2	0.40	0.20	0.40	—	—
CO = 0.16	0.5	0.08	0.16	—	—	—
CO_2 = 0.03	—	—	0.03	—	—	—
O_2 = 0.02	—	(– 0.02)	—	—	0.144	—
N_2 = 0.07	—	—	—	—	—	3.322
			ΣCO_2 = 0.39	ΣH_2O = 0.92	ΣO_2 = 0.144	ΣN_2 = 3.322

Total combustion product per m^3 of fuel = 4.776 m^3

Total dry flue gases per m^3 of fuel = 3.856 m^3

Excess air supplied is 20% so total volume of air supplied = Minimum air required + excess air supplied.

Excess air supplied = $0.2 \times 3.43 = 0.686$ m^3

Total air supplied = $3.43 + 0.686 = 4.116$ m^3 air per m^3 of fuel

**Oxygen present in excess air = $0.72 \times 0.2 = 0.144$ m^3, This will be with combustion products

$$\text{Nitrogen present in total air} = \frac{4.116 \times 79}{21} = 3.252 \text{ m}^3$$

**Since 0.07 m^3 N_2 per m^3 of fuel is already present in fuel so total nitrogen available in combustion products = $3.252 + 0.07 = 3.322$ m^3 per m^3 of fuel.

$$\% \text{ by volume of } CO_2 = \frac{(0.20 + 0.16 + 0.03) \times 100}{3.856} = 10.12\%$$

$$\% \text{ by volume of } O_2 = \frac{0.144 \times 100}{3.856} = 3.73\%$$

$$\% \text{ by volume of } N_2 = \frac{3.322 \times 100}{3.856} = 86.15\%$$

Volumetric composition of dry flue gases = 10.12% CO_2, 3.73% O_2, 86.15% N_2

Ans.

Higher calorific value may be obtained from the values given for constituents H_2, CH_4 and CO.

$= \{(0.52 \times 28424) + (0.20 \times 27463) + (0.16 \times 87780)\}$

$= 34317.88$ kJ/m^3

Steam produced during combustion of 1 m^3 of fuel.

$$m_{\text{steam}} = \frac{0.92}{22.4} = 0.0411 \text{ kg}$$

Lower calorific value = Higher calorific value – ($m_{\text{steam}} \times$ latent heat)

$= \{34317.88 - (0.0411 \times 2445)\}$

$= 34217.39$ kJ/m^3

Lower calorific value = 34217.39 kJ/m^3

Ans.

13. *Determine the volume of air supplied for complete combustion of one m^3 of fuel gas. If 40% excess air is supplied then find the percentage contraction in volume after the products of combustion have been cooled. The composition of fuel gas by volume is, H_2 = 20%, CH_4 = 3%, CO = 22%, CO_2 = 8%, N_2 = 47%.Consider air to have 21% O_2 and 79% N_2 by volume.*

Solution:

Combustion reactions for constituents are

$$2H_2 + O_2 \rightarrow 2H_2O$$
1 mol 0.5 mol 1 mol

$$CH_4 + 2O_2 \rightarrow 2H_2O + CO_2$$
1 mol 2 mol 2 mol 1 mol

$$2CO + O_2 \rightarrow 2CO_2$$
1 mol 0.5 mol 1 mol

Volumetric analysis (without excess air)

Constituents per m^3 of fuel gas	*O_2 required per m^3 of fuel constituents*	*O_2 required per m^3 of fuel*	*Combustion products per m^3 of fuel*			
(a)	*(b)*	*(c) = a × b*	*CO_2*	*H_2O*	*O_2*	*N_2*
H_2 = 0.20	0.5	0.1	—	0.2	—	—
CH_4 = 0.03	2	0.06	0.03	0.06	—	—
CO = 0.22	0.5	0.11	0.22	—	—	—
CO_2 = 0.08	—	—	0.08	—	—	
N_2 = 0.47	—	—	—	—	—	0.47
		$\Sigma O_2 = 0.27$	$\Sigma CO_2 = 0.33$	$\Sigma H_2O = 0.26$	—	$\Sigma N_2 = 0.47$

Minimum volume of air required = $\dfrac{0.27 \times 100}{21}$ = 1.286 m^3 air per m^3 of fuel.

$\boxed{\text{Minimum air required} = 1.286\ m^3/m^3 \text{ of fuel}}$ ***Ans.***

With excess air there shall be additional amount of O_2 and N_2 present in combustion products.

With 40% excess air the dry flue gas shall comprise of following:

(*i*) CO_2 = 0.33 m^3 per m^3 of fuel.

(*ii*) N_2 = 0.47 + nitrogen from total air (minimum air + excess air)

$$= 0.47 + \left(1.286 \times 1.4 \times \frac{79}{100}\right)$$
$$= 1.89\ m^3 \text{ per } m^3 \text{ of fuel}$$

(*iii*) O_2 = oxygen from excess air

$$= \left(\frac{1.286 \times 0.4 \times 21}{100}\right)$$
$$= 0.108\ m^3 \text{ per } m^3 \text{ of fuel}$$

Total volume of dry flue gas formed = (0.33 + 1.89 + 0.108) m^3 per m^3 of fuel

= 2.328 m^3 per m^3 of fuel.

Volume before combustion = Volume of fuel gas + Volume of air supplied

= (1 + 1.286 ´ 1.5)m^3 = 2.929 m^3

% Contraction in volume = $\left(\dfrac{2.929 - 2.328}{2.929}\right) \times 100 = 20.52\%$

$\boxed{\%\ \text{contraction in volume} = 20.52\%}$ ***Ans.***

14. *A hydrocarbon fuel when burned with air gave the following Orsat analysis, CO_2: 11.94%, O_2: 2.26%, CO: 0.41%, N_2: 83.39%*

Determine,

(i) the air-fuel ratio on mass basis

(ii) the percent of carbon and hydrogen in the fuel on mass basis, and

(iii) percentage of theoretical air supplied.

Assume air to have 21% oxygen.

Solution:

Orsat analysis gives, CO_2 = 11.94%, O_2 = 2.26%, CO = 0.41%, N_2 = 83.39%

$$H_2O = 100 - (11.94 + 2.26 + 0.41 + 83.39)$$
$$H_2O = 2\%$$

Let hydrogen fuel be 'C_aH_b'.

$$C_aH_b + d \cdot O_2 + \left(\frac{79}{21} \cdot d\right) N_2 \rightarrow e \cdot CO + f \cdot CO_2 + \frac{b}{2}\ H_2O + g \cdot O_2 + h \cdot N_2$$

On mass basis,

$$\rightarrow 28e + 44f + 9b + 32g + 28h$$

Mass of constituents of exhaust gas, $CO_2 = 44f$, $CO = 28e$, $H_2O = 9b$, $O_2 = 32g$, $N_2 = 28h$

Exhaust gas mass = $(28e + 44f + 9b + 32g + 28h)$

As per analysis,

$$44f = 0.1194\ (28e + 44f + 9b + 32g + 28h)$$
$$28e = 0.0041\ (28e + 44f + 9b + 32g + 28h)$$
$$9b = 0.02\ (28e + 44f + 9b + 32g + 28h)$$
$$32g = 0.0226\ (28e + 44f + 9b + 32g + 28h)$$
$$28b = 0.8339\ (28e + 44f + 9b + 32g + 28h)$$

Also,

$$12 \cdot a = 12e + 12f, \text{ or, } a = e + f$$

$$\frac{79}{21} \cdot d \cdot 28 = 28h$$
$$h = 3.76d$$

Solving above we get

$$a = 19.53e$$
$$b = 15.18e$$
$$d = 54.09e$$
$$f = 18.53e$$
$$g = 4.82e$$
$$h = 203.37e$$

$$\text{Air-fuel ratio} = \frac{\left(32 \times d + \frac{79}{21} \times d \times 28\right)}{(12 \times a + b)} = 29.77$$

$$\boxed{\text{Air-fuel ratio} = 29.77}$$ ***Ans.***

$$\text{Carbon fraction} = \frac{12a}{(12a + b)} = 0.9392$$

$$\text{Hydrogen fraction} = \frac{b}{(12a + b)} = 0.0608$$

$$\boxed{\%\ \text{Carbon} = 93.92\%}$$ ***Ans.***

% Hydrogen = 6.08%

$$\text{Percentage theoretical air supplied} = \frac{\left(32d + \frac{79}{21} \times d \times 28\right) \times 100}{\left(32d + \frac{79}{21} d \times 28 + 12a + b\right)}$$

$$= 96.75$$

$\boxed{\text{\% Theoretical air supplied} = 96.75\%}$ ***Ans.***

15. *Obtain the stoichiometric reaction equation for CH_4 being burnt with*

(i) 100% theoretical air

(ii) 200% excess air

(iii) 20% less than theoretical air requirement

Consider air to have 21% oxygen and 79% nitrogen by mass.

Solution:

Let number of air molecules required for one molecule of CH_4 be 'n'. Combustion equation for CH_4,

$$CH_4 + n\left(O_2 + \frac{79}{21} \cdot N_2\right) \to a \cdot CO_2 + b \cdot N_2 + d \cdot H_2O$$

From above equation the C, H_2 and O_2 can be equated as under;

$$C,\ 1 = a, \text{ or } a = 1$$

$$O_2,\ n = a + \frac{d}{2}, \text{ or } 2n = 2a + d$$

$$H_2,\ 2 = d, \text{ or } d = 2$$

$$N_2,\ n \times \frac{79}{21} = b, \text{ or } b = 3.76\,n$$

It gives

$$n = 2,\ a = 1$$

$$b = 7.52,\ d = 2$$

Stoichiometric combustion equation shall be,

For 100% theoretical air
$CH_4 + 2(O_2 + 3.76\ N_2) \to CO_2 + 7.52N_2 + 2H_2O$

Ans.

When 200% of excess air is added then total air supplied will be 300% i.e. (200% excess + 100% theoretical). Due to this excess O_2 shall be there in combustion products.

Thus modified form of combustion equation shall be,

$$CH_4 + 3 \times 2\ (O_2 + 3.76\ N_2) \to a \cdot CO_2 + b \cdot N_2 + d \cdot H_2O + e \cdot O_2$$

Equating C, H_2, O_2 and N_2 we get

$$C;\ a = 1$$

$$O_2;\ a + \frac{d}{2} + e = 6$$

$$H_2;\ d = 2$$

$$N_2;\ b = 6 \times 3.76$$

It yields,

$a = 1$
$b = 22.56$
$d = 2$
$e = 4$

Now the combustion equation with 200% excess air shall be;

With 200% excess air;
$CH_4 + 6(O_2 + 3.76N_2) \rightarrow CO_2 + 22.56N_2 + 2H_2O + 4O_2$

Ans.

When there is 20% less air then actual air supplied will be 20% less than 100% theoretical air. Due to less than theoretical air supplied there will be incomplete combustion and CO will be present with combustion products.

$$CH_4 + 0.8 \times 2\,(O_2 + 3.76N_2) \rightarrow aCO_2 + bCO + dN_2 + eH_2O$$

Equating C, H_2, O_2 and N_2 we get

C; $a + b = 1$

O_2; $a + \frac{b}{2} + \frac{e}{2} = 1.6$

H_2; $e = 2$

N_2; $d = 1.2 \times 3.76 = 4.512$

It yields,

$a = 0.2$
$b = 0.8$
$d = 4.512$
$e = 2$

Hence for 20% less air the combustion equation shall be as under

For 20% less air.
$CH_4 + 1.6\,(O_2 + 3.76N_2) \rightarrow 0.2CO_2 + 0.8CO + 4.512N_2 + 2H_2O$

Ans.

16. *Determine the adiabatic flame temperature for the combustion of carbon monoxide (CO) with 150 percent theoretical amount of oxygen to form CO_2. The reactants enter the steady flow reactor at 25°C, 1 atm and the products are CO_2 and excess O_2. Enthalpy of O_2, CO and CO_2 are 0 kJ/kg mole, – 110418 kJ/kg and – 393,137 kJ/kg mole respectively. The constant pressure specific heats of CO_2 and O_2 at 1 atm, may be assumed to be 56.43 and 36.5 kJ/kg mole K, respectively.[U.P.S.C. 1993]*

Solution:

Combustion equation of carbon monoxide yields,

$$2CO + \frac{3}{2}O_2 \rightarrow 2CO_2 + \frac{1}{2}O_2$$

Since the process is adiabatic so the total enthalpy of reactants and products shall remain same,

$$\text{Enthalpy of reactants} = 2\,(-\,110{,}418) + \frac{1}{2}(0)$$

$$= -\,220{,}836 \text{ kJ/kg} \cdot \text{mole}$$

STEAM AND ITS PROPERTIES

- Steam is the gaseous phase of water. It utilizes heat during the process and carries large quantities of heat later. Hence, it could be used as a working substance for heat engines.
- Steam is generated in boilers at constant pressure. Generally, steam may be obtained starting from ice or straight away from the water by adding heat to it.

FORMATION OF STEAM

- temperature-Enthalpy Diagram t-h diagram)
- The graphical representation of transformation of 1 kg of ice into 1 kg of superheated steam at constant pressure (temperature vs. enthalpy) is known as t-h diagram. shows the various stages of formation of steam starting from ice shows the corresponding t-h diagram.

FORMATION OF STEAM

- Consider 1 kg of ice in a pistion -cylinder arrangement as shown. it is under an Absolute Pressure say P bar and at temperature –t 0 C (below the freezing point). Keeping the pressure constant, the gradual heating of the ice leads to note the following changes in it, These are represented on a t-h diagram on heating, the temperature of the ice will gradually rises from p to Q i.e. from –t C till reaches the freezing temperature 0.

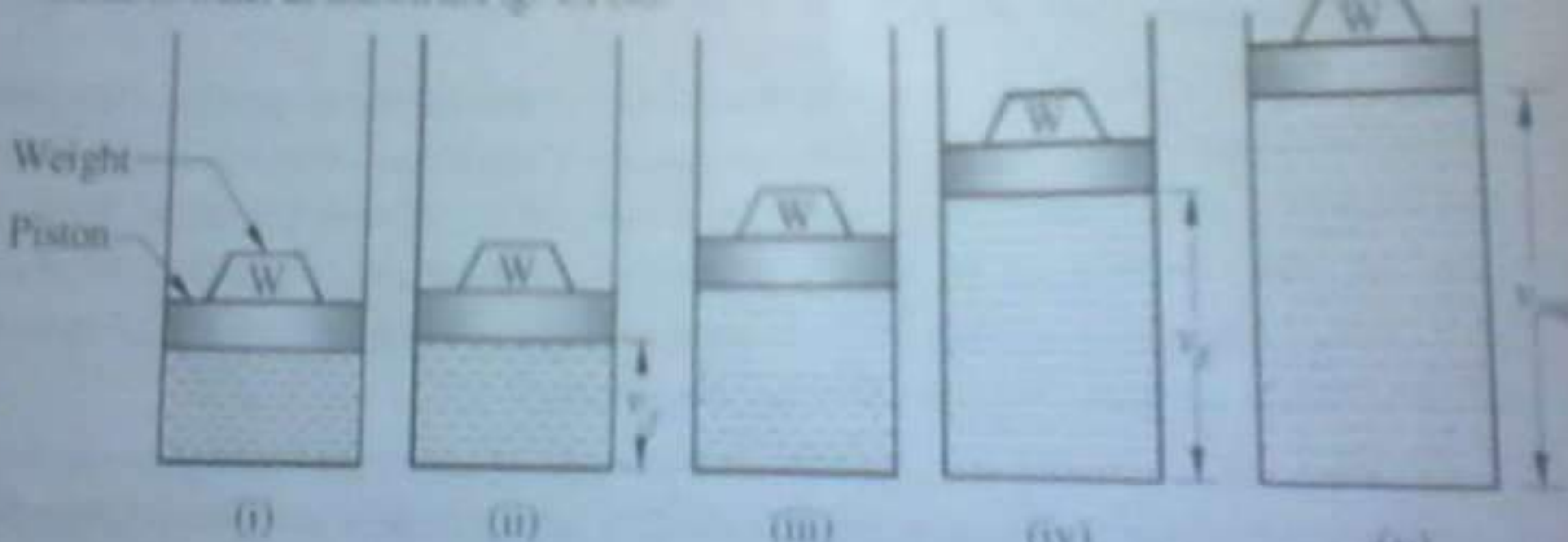


Fig. 4.1 Formation of Steam.

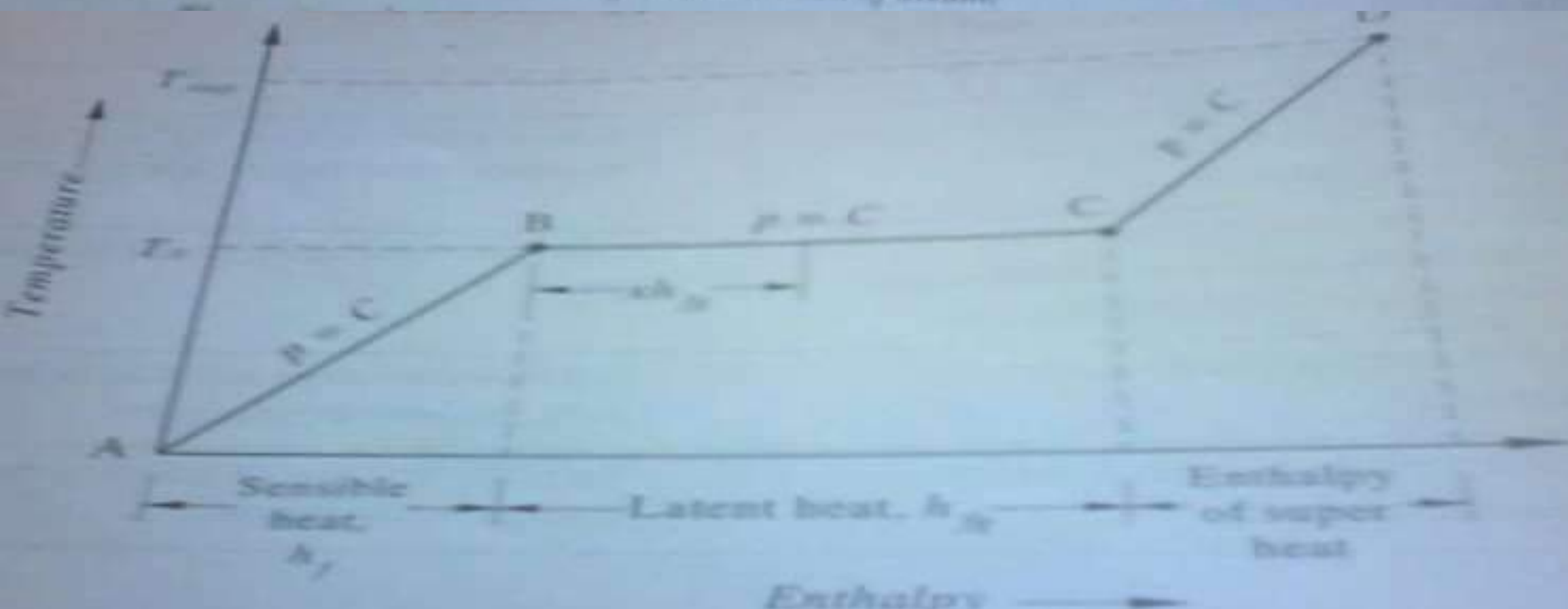


Fig. 4.2 Temperature - Enthalpy diagram

FORMATION OF STEAM

- Adding more heat, the ice starts melting without changing in the temperature till the entire ice is converted into water from Q to R. The amount of heat during this period from Q to R is called Latent heat of fusion of ice or simply Latent heat of ice.
- Continuous heating raises the temperature to its boiling point t C known as Saturation Temperature. The corresponding pressure is called saturation pressure. it is the stage of vaporization at 1.01325 bar atmospheric pressure (760mm . As pressure increases, the value of saturation temperature also increases. The amount of heat added during R to S is called Sensible Heat or Enthalpy of Saturated Water or Total Heat of Water (h, or h "').

FORMATION OF STEAM

- During the process, a slight increase in volume of water (saturated water) may be noted. The resulting volume is known as Specific volume of Saturated Water (Vf or vW).
- (d) On further heating beyond S, the water will gradually starts evaporate and starts convert it to steam, but the temperature remains constant. As long as the steam is in contact with water, it is called Wet Steam or saturated steam

FORMATION OF STEAM

- On further heating the temperature remains constant, but the entire water converts to steam. But still it will be wet steam. The total heat supplied from OOC is called Enthalpy of Wet Steam (h wet). The resulting volume is known as Specific Volume of Wet Steam (v wet)
- On further heating the wet steam, the water particles, which are in suspension, will start evaporating gradually and at a particular moment the final particles just evaporates. The steam at that moment corresponding to point T is called Dry Steam or Dry Saturated Steam. The resulting volume is known as Specific Volume of Dry Steam (vg). This steam not obeys the gas laws. The amount of heat added during S to T is called Latent Heat of Vaporization of Steam or Latent Heat of Steam (hfg). During the process, the saturation temperature remains constant. The total heat supplied from O'C is called Enthalpy of Dry Steam (hg).

FORMATION OF STEAM

- On further heating beyond point T to U the temperature starts from ts to tu, the point of interest. This
- process is called Super heating. The steam so obtained is called Super Heated Steam. It obeys gas laws.

TYPES STEAM

The steam during the steam generation process can exist in three types:

1. Wet steam (saturated steam)
2. Dry steam (dry saturated steam)
3. Superheated steam

1. WET STEAM :

- Both the water molecules and steam coexist to form a two phase mixture, called wet steam.
- Which will be in thermal equilibrium because both of them will be at the same saturation temperature.

2. DRY SATURATED STEAM:

- A steam at the saturation temperature corresponding to a given pressure and having no water molecules in it is known as dry saturated steam or dry steam.

- Since the dry saturated steam does not contain any water molecules in it, its dryness fraction will be unity.

3. SUPERHEATED STEAM:

- When a dry saturated steam is heated further at the given constant pressure, its temperature rise beyond its saturation temperature. The steam in this state is said to be superheated.

ENTHALPY OF STEAM

- Enthalpy Of liquid:

  $$hf=Cpw(tf-0)$$

- Enthalpy of Dry saturated steam:

  $$hg=hf + hfg$$

- Enthalpy of Wet steam:

  $$h=hf + xhfg$$

- Enthalpy of Superheted steam:

  $$hsup= hg + Cps(Tsup - Tsat)$$

SPECIFIC VOLUME OF STEAM

- Specific volume of saturated water: v_f
- Specific volume of dry saturated steam: v_g
- Specific volume of wet steam:

  $$v = xv_g + (1-x)v_f$$

- Specific volume of superheated steam:

  $$V_g/T_s = V_{sup}/T_{sup}$$

INTERNAL ENERGY OF STEAM

- It is defined as the difference between the enthalpy of steam and external work of evaporation.

Internal energy of dry steam : $u_g = h_g – pv_g$ kJ/kg

Internal energy of wet steam : $u = h_f + xh_{fg} – pxv_g$ kJ/kg

Internal energy of superheated steam : $u_{sup} = h_{sup} – pv_{sup}$ kJ/kg

- Internal Latent heat:

It is algebraic difference between the enthalpy of evaporation at given pressure and work of evaporation.

Internal Latent heat : $h_{fg} - pv$

THROTTLING PROCESS

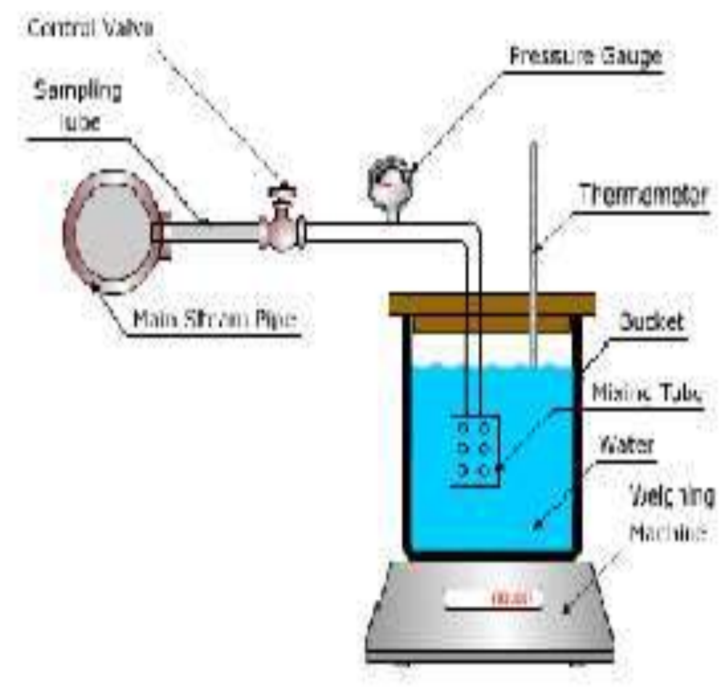
- The temperature change of a gas or liquid when it is forced through a valve or porous plug while kept insulated so that no heat is exchanged with the environment. This procedure is called a Throttling process.

MEASUREMENT OF DRYNESS FRACTION

- The dryness fraction of steam can be measured experimentally.
- Calorimeters are used for measurement of dryness fraction of steam.
- There are for methods of determining the dryness fraction of steam.

1. Bucket or barrel calorimeter
2. Throttling calorimeter
3. Separating calorimeter
4. Combined separating & throttling calorimeter

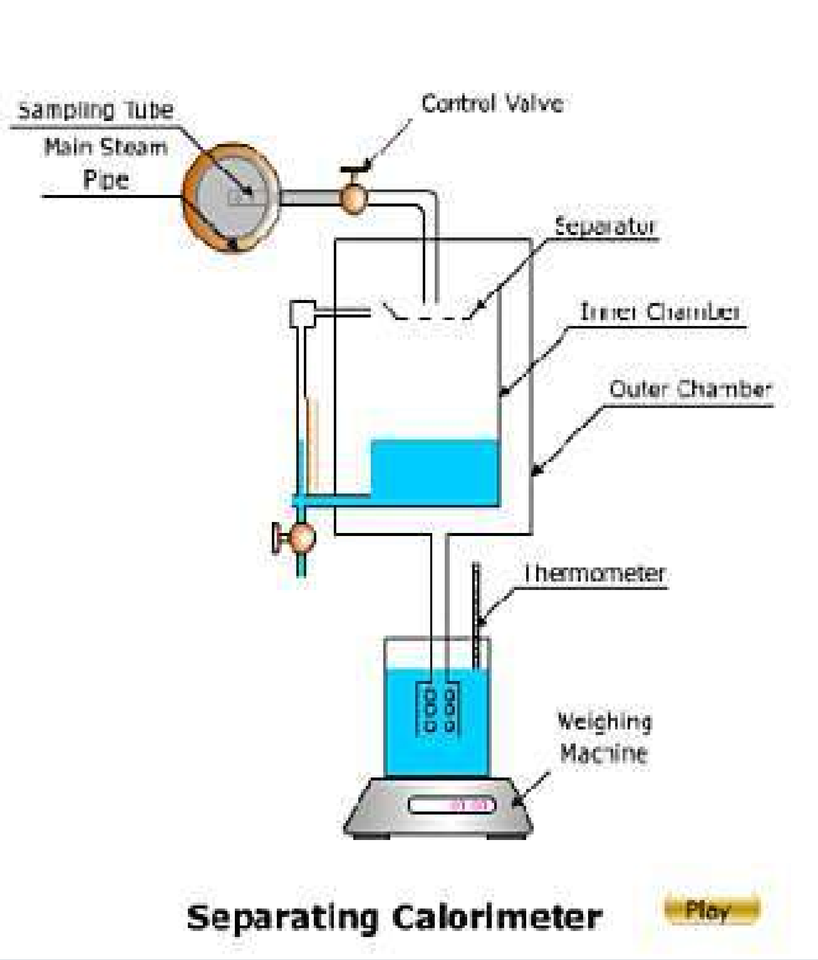
BUCKET OR BARREL CALORIMETER



Bucket or Barrel Calorimeter

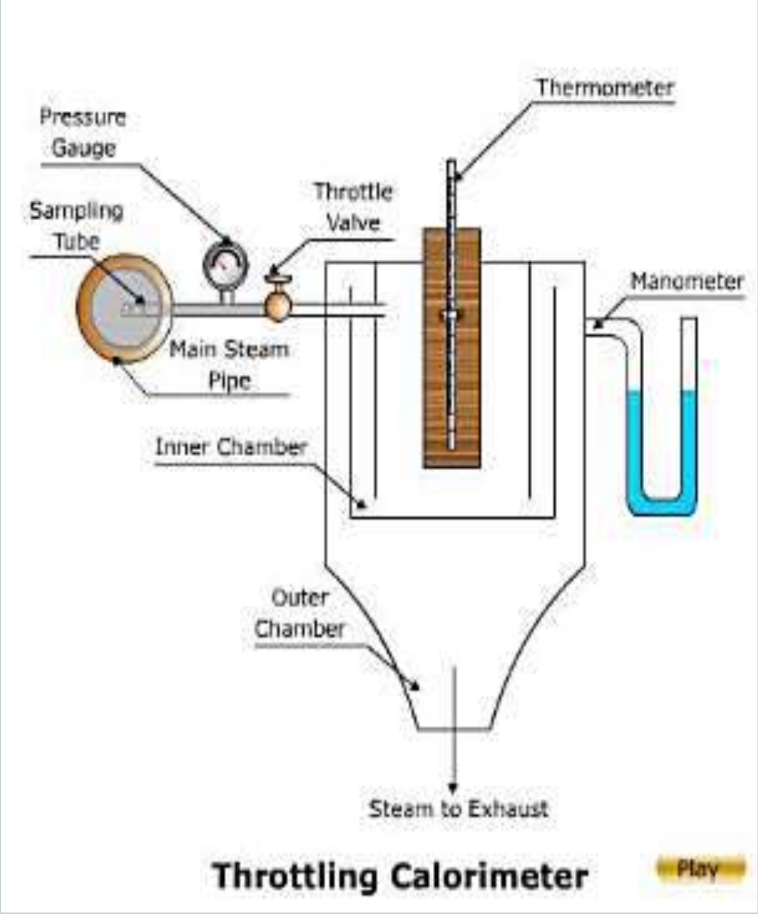
- In this calorimeter a known mass of water and then heat loss by steam is equated to heat gained by water. The steam is passed through a sample tube into bucket calorimeter contains known weight of water.
- The weight of calorimeter with water before mixing steam & after mixing the steam is measured by thermometer.

SEPARATING CALORIMETER



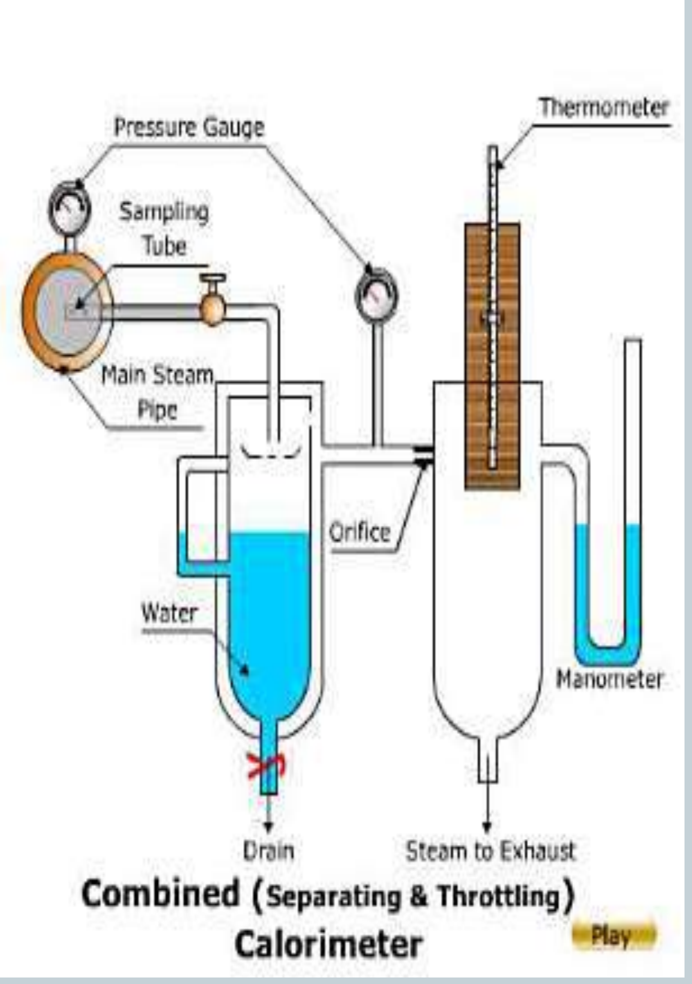
- This calorimeter is used to measure dryness fraction of very wet steam. The steam is passed through sampling tube. The moisture is separated mechanically from steam passing through the separator .
- The water partials are separated due to inertia of water partials as steam is passed through the perforated trays.
- The out going steams is then condensed in the bucket calorimeter.

THROTTLING CALORIMETER



- This types of calorimeter is used to measure dryness fraction of steam whose dryness fraction is considerably high.
- The steam sample is passed through a throttle valve & is allowed to throttle down to pressure unit until it comes out in dry saturated or super heated condition.
- The pressure & temperature of steam coming out of throttling calorimeter is measured with water manometer & thermometer respectively.

COMBINED SEPARATING & THROTTLING CALORIMETER



COMBINED SEPARATING & THROTTLING CALORIMETER

- The combined separating & throttling calorimeter gives the dryness fraction of wide quality steam very accurately.
- In this calorimeter, the stream from sampling tube is first passed through the separating calorimeter where most of the moisture is removed & steam partly dried.
- This steam is further passed to throttling calorimeter where it comes out as dry saturated or in superheated form.
- The steam coming out from throttling calorimeter is condensed in condenser coming out of condenser is recorded.
- The weight of water separated In separating calorimeter & the pressure & temperature of steam coming out from throttling valve are also recorded.

Properties of Pure Substances

$-5^\circ C$ Ice $\longrightarrow 250^\circ C$ Vapour

(i) $-5^\circ C$ Ice $\longrightarrow 0^\circ C$ Ice

$$ds = c \ln\left(\frac{T_f}{T_{in}}\right) = 2.093 \ln\left(\frac{273}{268}\right) = 0.038 \ \text{kJ/kg K}$$

(ii) $0^\circ C$ Ice $\longrightarrow 0^\circ C$ water

$$S_3 - S_2 = \frac{dQ}{T} = \frac{L}{T} = \frac{334.91}{273} = 1.23 \ \text{kJ/kg K}$$

(iii) $0^\circ C$ water $\longrightarrow 100^\circ C$ water

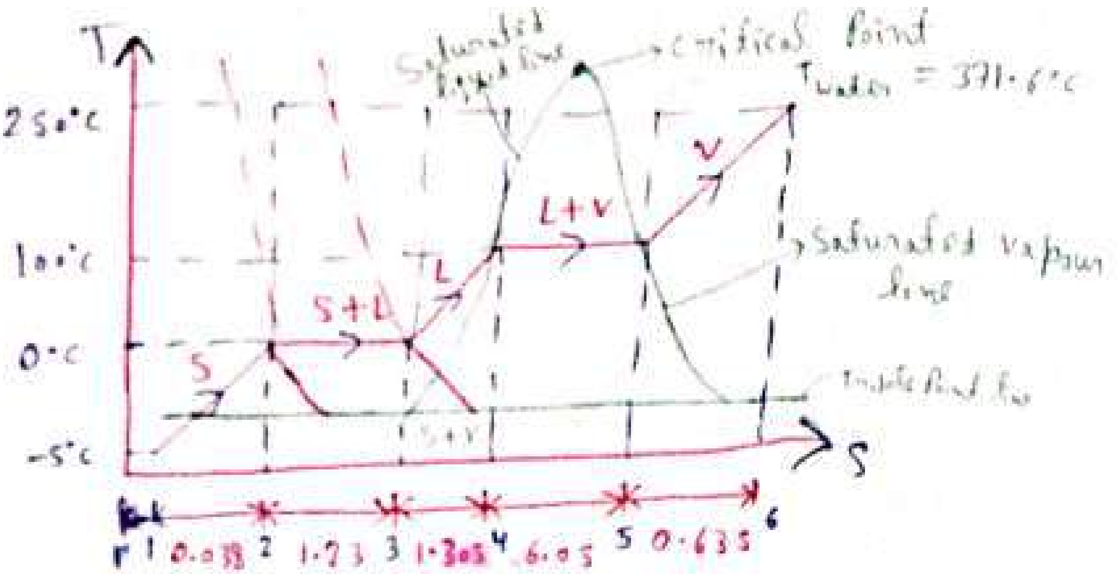
$$S_4 - S_3 = c \ln\left(\frac{T_4}{T_4}\right) = 4.18 \ln\left(\frac{373}{273}\right) = 1.305 \ \text{kJ/kg K}$$

(iv) $100^\circ C$ water $\longrightarrow 100^\circ C$ vapour

$$S_5 - S_4 = \frac{dQ}{T} = \frac{2257}{373} = 6.05 \ \text{kJ/kg K}$$

(v) $100^\circ C$ Vapour $\longrightarrow 250^\circ C$ vapour

$$S_6 - S_5 = 1.860 \ln\left(\frac{250+273}{100+273}\right) = 0.635 \ \text{kJ/kg K}$$



Critical Point :- It is the point at which saturated liquid line and saturated vapour curve meet.

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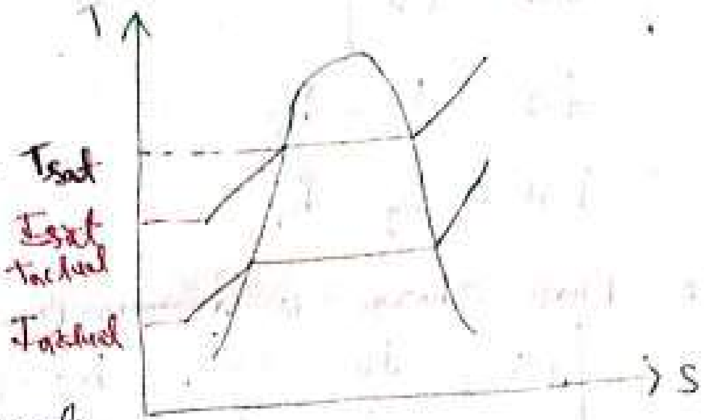
The temp of water at critical point is 371.6°C.

Subcooled or Undercooled Region ⇒

It is a region in which the actual temp is less than saturation temp corresponding to that Pressure.

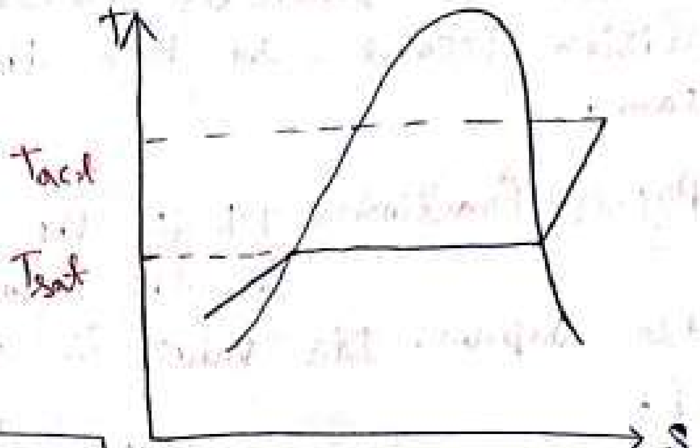


Degree of undercooling / subcooling

$$\boxed{DOS = T_{sat} - T_{actual}}$$

Super heated Region ⇒

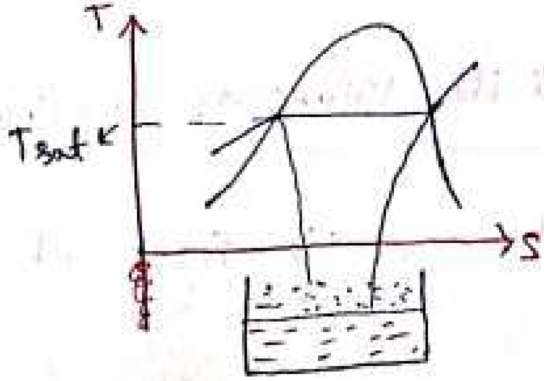
It is a region in which the actual temp is more than the saturation temp corresponding to that Pressure.



$$\boxed{\text{Degree of superheated} = T_{act} - T_{sat}}$$

Wet Region ⇒ It is the region in which both liquid & vapour exist together.

Note: No ideal gas apply in wet region.



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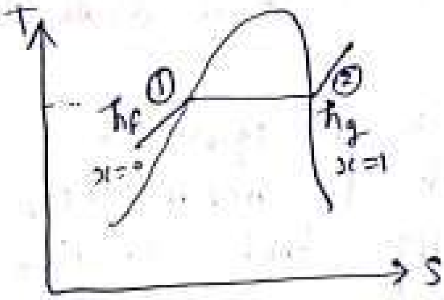
Latent heat ⇒ The heat transfer during phase change is called latent Heat.

$$\boxed{đQ = dH} = h_2 - h_1$$

$$\boxed{đQ = LH}$$

$$đQ = h_g - h_f$$

$$\therefore LH = h_g - h_f$$



Since Phase change is a rev. Process

$$\therefore \boxed{đS = \frac{đQ}{T} = \frac{LH}{T} = \frac{h_g - h_f}{T}}$$

* With Rise in Pressure latent heat of Vapourization decreases & at critical Point latent heat is zero, directly liquid is converted into steam. Due to increase of Pressure collision b/w particles increases & at critical pt this collision became so high that it ~~converts~~ flashes into steam.

Dryness fraction ⇒ It is the ratio of mass of water vapour to the total mass of liquid including water vapour. Its value is not defined at the critical Point.

$$\boxed{x = \frac{m_V}{m_{\ell} + m_V}}$$

All constant dryness fraction line convergs at critical Point. Its value lies b/w 0 to 1.

Specific volume of Mixture ⇒ Since volume is the extensive Property therefore it can be written in the form of summation.

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$$V = V_l + V_v$$

$$v = \frac{V}{m} \Rightarrow V = vm$$

$$m = m_v + m_l$$

$$\therefore V = m_v v_v + m_l v_l$$

$$\therefore mv = m_l v_l + m_v v_v$$

$$\text{or} \quad v = \frac{m_l v_l + m_v v_v}{m}$$

$$= \frac{m_l v_l}{m} + \frac{m_v v_v}{m}$$

$$v = \frac{m_l v_l}{m_l + m_v} + \frac{m_v v_v}{m_l + m_v}$$

$$\boxed{\text{VHSU}}$$

$$v = x v_v + (1-x) v_l$$

$$v = x v_g + (1-x) v_f$$

$$\boxed{v = v_f + x(v_g - v_f)} \longrightarrow ①$$

$$h = h_f + x(h_g - h_f) \longrightarrow ②$$

$$s = s_f + x(s_g - s_f) \longrightarrow ③$$

$$u = u_f + x(u_g - u_f) \longrightarrow ④$$

These eq^n are valid only for wet regions.

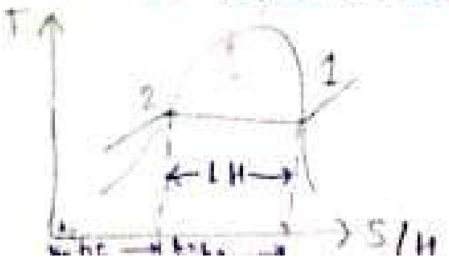
Enthalpy calculations at various Points :-

Case-I :- When the point is on saturated vapour curve.

$$h_1 = h_2 + LH$$

$$h_g = h_f + LH$$

$$\therefore \boxed{LH = h_g - h_f}$$



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Case-II :→ When the point is in the wet Region.

$$\boxed{h_1 = h_f + x(LH)}$$

Case-III :→ when the Point is in superheated region.

$$h_1 = h_g + \Delta h$$

$$\boxed{h_1 = h_g + C_{p\,vapour}(T_{sup} - T_{sat})}$$

$\Downarrow$

1.88 kJ/kgK

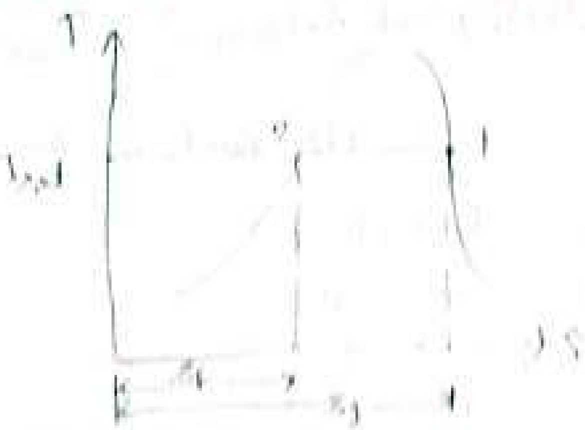
Note :→ In superheated region vapour can be treated as ideal gas if no information is given.

Entropy of various Points :→

Case-I :→ When the Point is on the saturated vapour curve.

$$ds = \frac{dQ}{T_{sat}} = \frac{LH}{T_{sat}}$$

$$\boxed{s_g - s_f = \frac{h_g - h_f}{T_{sat}}}$$



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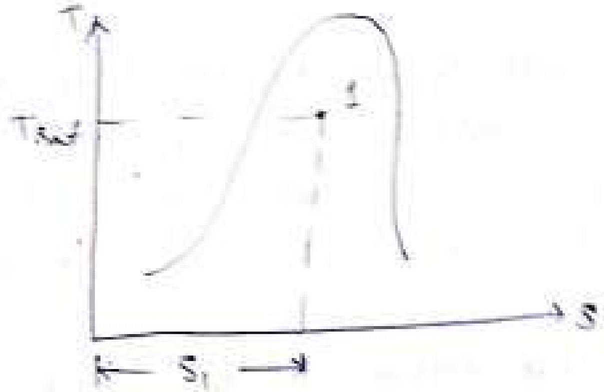
Case-II ⇒ When the Point is in the wet Region.

$$\boxed{S_1 = S_f + x\frac{(LH)}{T_{sat}}}$$



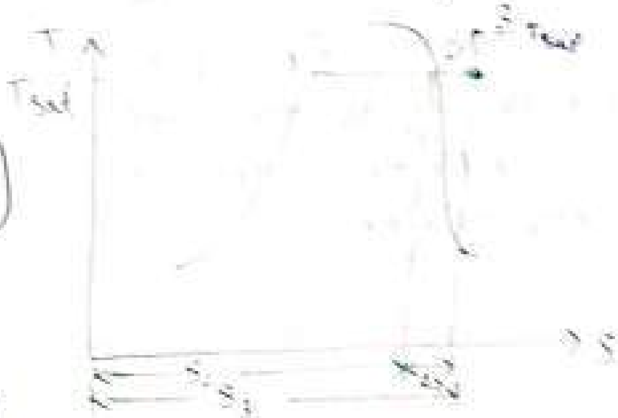
Case-III ⇒ When the Point is in the superheated region.

$$S_3 = S_2 + \Delta S$$

$$\Delta S = C_{p_{vapour}} \ln\left(\frac{T_{sup}}{T_{sat}}\right) - \cancel{R \ln\left(\frac{P_{sat}}{P_{sat}}\right)}$$

$$\Delta S = C_{p_{vapour}} \ln\left(\frac{T_{sup}}{T_{sat}}\right)$$

$$\boxed{S_3 = S_2 + C_{p_{vapour}} \ln\left(\frac{T_{sup}}{T_{sat}}\right)}$$



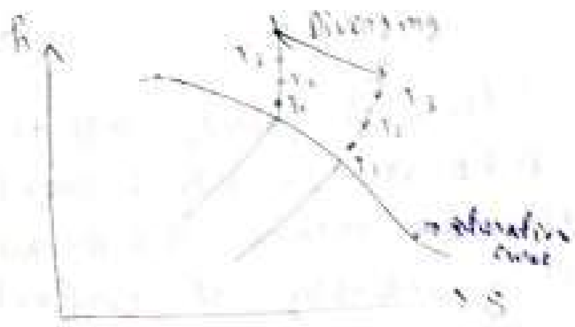
Mollier Diagram ⇒ (h - s diagram)

$$T\,dS = dh - v\,dP$$

$P =$ constant

$dP = 0$

$$\left(\frac{dh}{dS}\right)_{P=const} = T$$



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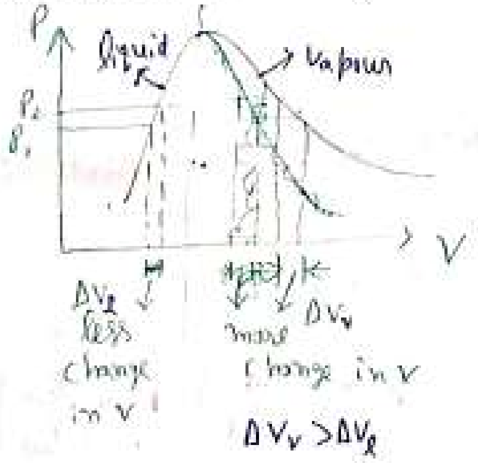
* The slope of Isobaric curve on Mollier diagram is equal to T :-

In wet region as the temp is constant, the slope is also constant. In superheated region as the temp increases the slope also increases & hence constant pressure line diverges in the superheated region.

* For the same increasement in the Pressure $(P_2 - P_1)$ there is more change in the vapours & less change in the volume for liquid Because liquids are almost incompressible & vapours are compressible.



Note :-

Solid → Liquid (Melting)

Liquid → Vapour (Vaporization)

Solid → Vapour (Sublimation)

Liquid → Solid (Freezing)

Vapour → liquid (condensation)

Vapour → Solid (Ablimation)

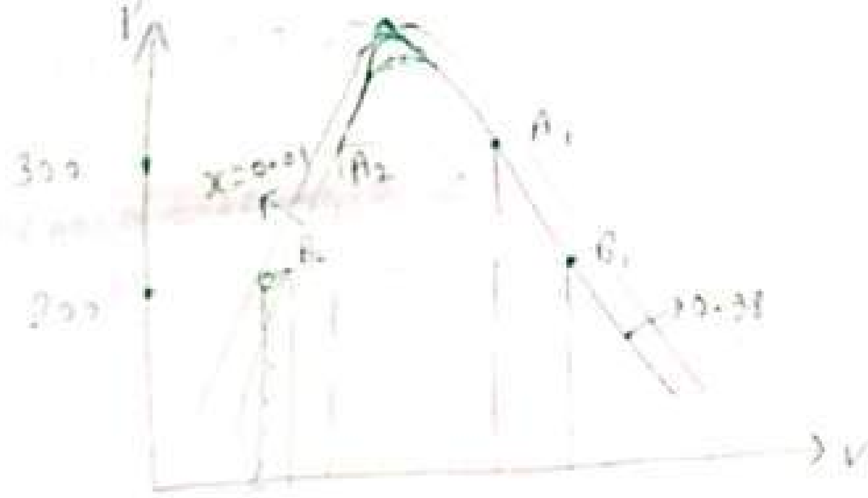
Q - Steam of quality 0.98 is present in two separate containers, A & B at 300 kPa & 200 kPa respectively. Specific volume of steam in container A & B are initialy V_{A_1} & V_{B_1} respectively. steam condensing at constant pressure in such a way that final quality of steam in both the container is 0.01 & specific volume of steam in container A & B are V_{A_2} & V_{B_2}. which of following statement is true.

(a) $v_{A_1} > v_{B_1}$ & $v_{A_2} > v_{B_2}$

(b) $v_{A_1} < v_{B_1}$ & $v_{A_2} < v_{B_2}$

(c) $v_{A_1} > v_{B_1}$ & $v_{A_2} < v_{B_2}$

(d) $v_{A_1} < v_{B_1}$ & $v_{A_2} > v_{B_2}$ ✓



$v_{A_2} > v_{B_2}$ $\quad$ $v_{A_1} < v_{B_1}$

Imp

Q. A Pressure cooker contains saturated water vapour mixture at 100°C with volume of vapour being 8 times the volume of liquid. The specific volume of saturated liquid & sat. vapour at 100°C are $v_f = 0.001044\ m^3/kg$ & $v_g = 1.6729\ m^3/kg$ respectively. Calculate the quality of mixture.

Vol. of vapour = 8 × Vol. of liquid

$$v_f = v_l = 0.001044\ m^3/kg$$

$$v_g = v_v = 1.6729\ m^3/kg$$

$$x = \frac{m_v}{m_l + m_v}$$

$$= \frac{1}{\left[\frac{m_l}{m_v} + 1\right]}$$

$$= \frac{1}{\left[\frac{(V/v)_l}{\quad} + 1\right]}$$

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$$= \frac{1}{\left[\frac{V_l}{v_l} \times \frac{v_v}{V_v} + 1\right]}$$

$$\cancel{= \frac{1}{8 \times \frac{0.001044}{1.6729} + 1}} = \frac{1}{\frac{1}{8} \times \frac{1.6729}{0.001044} + 1}$$

$$= 0.00496 = 0.005 \; (\checkmark)$$

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TABLE A-4

Saturated water—Temperature table

		Specific volume, m³/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg · K		
Temp., T °C	Sat. press., P_{sat} kPa	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
0.01	0.6117	0.001000	206.00	0.000	2374.9	2374.9	0.001	2500.9	2500.9	0.0000	9.1556	9.1556
5	0.8725	0.001000	147.03	21.019	2360.8	2381.8	21.020	2489.1	2510.1	0.0763	8.9487	9.0249
10	1.2281	0.001000	106.32	42.020	2346.6	2388.7	42.022	2477.2	2519.2	0.1511	8.7488	8.8999
15	1.7057	0.001001	77.885	62.980	2332.5	2395.5	62.982	2465.4	2528.3	0.2245	8.5559	8.7803
20	2.3392	0.001002	57.762	83.913	2318.4	2402.3	83.915	2453.5	2537.4	0.2965	8.3696	8.6661
25	3.1698	0.001003	43.340	104.83	2304.3	2409.1	104.83	2441.7	2546.5	0.3672	8.1895	8.5567
30	4.2469	0.001004	32.879	125.73	2290.2	2415.9	125.74	2429.8	2555.6	0.4368	8.0152	8.4520
35	5.6291	0.001006	25.205	146.63	2276.0	2422.7	146.64	2417.9	2564.6	0.5051	7.8466	8.3517
40	7.3851	0.001008	19.515	167.53	2261.9	2429.4	167.53	2406.0	2573.5	0.5724	7.6832	8.2556
45	9.5953	0.001010	15.251	188.43	2247.7	2436.1	188.44	2394.0	2582.4	0.6386	7.5247	8.1633
50	12.352	0.001012	12.026	209.33	2233.4	2442.7	209.34	2382.0	2591.3	0.7038	7.3710	8.0748
55	15.763	0.001015	9.5639	230.24	2219.1	2449.3	230.26	2369.8	2600.1	0.7680	7.2218	7.9898
60	19.947	0.001017	7.6670	251.16	2204.7	2455.9	251.18	2357.7	2608.8	0.8313	7.0769	7.9082
65	25.043	0.001020	6.1935	272.09	2190.3	2462.4	272.12	2345.4	2617.5	0.8937	6.9360	7.8296
70	31.202	0.001023	5.0396	293.04	2175.8	2468.9	293.07	2333.0	2626.1	0.9551	6.7989	7.7540
75	38.597	0.001026	4.1291	313.99	2161.3	2475.3	314.03	2320.6	2634.6	1.0158	6.6655	7.6812
80	47.416	0.001029	3.4053	334.97	2146.6	2481.6	335.02	2308.0	2643.0	1.0756	6.5355	7.6111
85	57.868	0.001032	2.8261	355.96	2131.9	2487.8	356.02	2295.3	2651.4	1.1346	6.4089	7.5435
90	70.183	0.001036	2.3593	376.97	2117.0	2494.0	377.04	2282.5	2659.6	1.1929	6.2853	7.4782
95	84.609	0.001040	1.9808	398.00	2102.0	2500.1	398.09	2269.6	2667.6	1.2504	6.1647	7.4151
100	101.42	0.001043	1.6720	419.06	2087.0	2506.0	419.17	2256.4	2675.6	1.3072	6.0470	7.3542
105	120.90	0.001047	1.4186	440.15	2071.8	2511.9	440.28	2243.1	2683.4	1.3634	5.9319	7.2952
110	143.38	0.001052	1.2094	461.27	2056.4	2517.7	461.42	2229.7	2691.1	1.4188	5.8193	7.2382
115	169.18	0.001056	1.0360	482.42	2040.9	2523.3	482.59	2216.0	2698.6	1.4737	5.7092	7.1829
120	198.67	0.001060	0.89133	503.60	2025.3	2528.9	503.81	2202.1	2706.0	1.5279	5.6013	7.1292
125	232.23	0.001065	0.77012	524.83	2009.5	2534.3	525.07	2188.1	2713.1	1.5816	5.4956	7.0771
130	270.28	0.001070	0.66808	546.10	1993.4	2539.5	546.38	2173.7	2720.1	1.6346	5.3919	7.0265
135	313.22	0.001075	0.58179	567.41	1977.3	2544.7	567.75	2159.1	2726.9	1.6872	5.2901	6.9773
140	361.53	0.001080	0.50850	588.77	1960.9	2549.6	589.16	2144.3	2733.5	1.7392	5.1901	6.9294
145	415.68	0.001085	0.44600	610.19	1944.2	2554.4	610.64	2129.2	2739.8	1.7908	5.0919	6.8827
150	476.16	0.001091	0.39248	631.66	1927.4	2559.1	632.18	2113.8	2745.9	1.8418	4.9953	6.8371
155	543.49	0.001096	0.34648	653.19	1910.3	2563.5	653.79	2098.0	2751.8	1.8924	4.9002	6.7927
160	618.23	0.001102	0.30680	674.79	1893.0	2567.8	675.47	2082.0	2757.5	1.9426	4.8066	6.7492
165	700.93	0.001108	0.27244	696.46	1875.4	2571.9	697.24	2065.6	2762.8	1.9923	4.7143	6.7067
170	792.18	0.001114	0.24260	718.20	1857.5	2575.7	719.08	2048.8	2767.9	2.0417	4.6233	6.6650
175	892.60	0.001121	0.21659	740.02	1839.4	2579.4	741.02	2031.7	2772.7	2.0906	4.5335	6.6242
180	1002.8	0.001127	0.19384	761.92	1820.9	2582.8	763.05	2014.2	2777.2	2.1392	4.4448	6.5841
185	1123.5	0.001134	0.17390	783.91	1802.1	2586.0	785.19	1996.2	2781.4	2.1875	4.3572	6.5447
190	1255.2	0.001141	0.15636	806.00	1783.0	2589.0	807.43	1977.9	2785.3	2.2355	4.2705	6.5059
195	1398.8	0.001149	0.14089	828.18	1763.6	2591.7	829.78	1959.0	2788.8	2.2831	4.1847	6.4678
200	1554.9	0.001157	0.12721	850.46	1743.7	2594.2	852.26	1939.8	2792.0	2.3305	4.0997	6.4302

TABLE A-4

Saturated water—Temperature table (*Continued*)

		Specific volume, m³/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg · K		
Temp., T °C	Sat. press., P_{sat} kPa	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
205	1724.3	0.001164	0.11508	872.86	1723.5	2596.4	874.87	1920.0	2794.8	2.3776	4.0154	6.3930
210	1907.7	0.001173	0.10429	895.38	1702.9	2598.3	897.61	1899.7	2797.3	2.4245	3.9318	6.3563
215	2105.9	0.001181	0.094680	918.02	1681.9	2599.9	920.50	1878.8	2799.3	2.4712	3.8489	6.3200
220	2319.6	0.001190	0.086094	940.79	1660.5	2601.3	943.55	1857.4	2801.0	2.5176	3.7664	6.2840
225	2549.7	0.001199	0.078405	963.70	1638.6	2602.3	966.76	1835.4	2802.2	2.5639	3.6844	6.2483
230	2797.1	0.001209	0.071505	986.76	1616.1	2602.9	990.14	1812.8	2802.9	2.6100	3.6028	6.2128
235	3062.6	0.001219	0.065300	1010.0	1593.2	2603.2	1013.7	1789.5	2803.2	2.6560	3.5215	6.1775
240	3347.0	0.001229	0.059707	1033.4	1569.8	2603.1	1037.5	1765.5	2803.0	2.7018	3.4405	6.1424
245	3651.2	0.001240	0.054656	1056.9	1545.7	2602.7	1061.5	1740.8	2802.2	2.7476	3.3596	6.1072
250	3976.2	0.001252	0.050085	1080.7	1521.1	2601.8	1085.7	1715.3	2801.0	2.7933	3.2788	6.0721
255	4322.9	0.001263	0.045941	1104.7	1495.8	2600.5	1110.1	1689.0	2799.1	2.8390	3.1979	6.0369
260	4692.3	0.001276	0.042175	1128.8	1469.9	2598.7	1134.8	1661.8	2796.6	2.8847	3.1169	6.0017
265	5085.3	0.001289	0.038748	1153.3	1443.2	2596.5	1159.8	1633.7	2793.5	2.9304	3.0358	5.9662
270	5503.0	0.001303	0.035622	1177.9	1415.7	2593.7	1185.1	1604.6	2789.7	2.9762	2.9542	5.9305
275	5946.4	0.001317	0.032767	1202.9	1387.4	2590.3	1210.7	1574.5	2785.2	3.0221	2.8723	5.8944
280	6416.6	0.001333	0.030153	1228.2	1358.2	2586.4	1236.7	1543.2	2779.9	3.0681	2.7898	5.8579
285	6914.6	0.001349	0.027756	1253.7	1328.1	2581.8	1263.1	1510.7	2773.7	3.1144	2.7066	5.8210
290	7441.8	0.001366	0.025554	1279.7	1296.9	2576.5	1289.8	1476.9	2766.7	3.1608	2.6225	5.7834
295	7999.0	0.001384	0.023528	1306.0	1264.5	2570.5	1317.1	1441.6	2758.7	3.2076	2.5374	5.7450
300	8587.9	0.001404	0.021659	1332.7	1230.9	2563.6	1344.8	1404.8	2749.6	3.2548	2.4511	5.7059
305	9209.4	0.001425	0.019932	1360.0	1195.9	2555.8	1373.1	1366.3	2739.4	3.3024	2.3633	5.6657
310	9865.0	0.001447	0.018333	1387.7	1159.3	2547.1	1402.0	1325.9	2727.9	3.3506	2.2737	5.6243
315	10,556	0.001472	0.016849	1416.1	1121.1	2537.2	1431.6	1283.4	2715.0	3.3994	2.1821	5.5816
320	11,284	0.001499	0.015470	1445.1	1080.9	2526.0	1462.0	1238.5	2700.6	3.4491	2.0881	5.5372
325	12,051	0.001528	0.014183	1475.0	1038.5	2513.4	1493.4	1191.0	2684.3	3.4998	1.9911	5.4908
330	12,858	0.001560	0.012979	1505.7	993.5	2499.2	1525.8	1140.3	2666.0	3.5516	1.8906	5.4422
335	13,707	0.001597	0.011848	1537.5	945.5	2483.0	1559.4	1086.0	2645.4	3.6050	1.7857	5.3907
340	14,601	0.001638	0.010783	1570.7	893.8	2464.5	1594.6	1027.4	2622.0	3.6602	1.6756	5.3358
345	15,541	0.001685	0.009772	1605.5	837.7	2443.2	1631.7	963.4	2595.1	3.7179	1.5585	5.2765
350	16,529	0.001741	0.008806	1642.4	775.9	2418.3	1671.2	892.7	2563.9	3.7788	1.4326	5.2114
355	17,570	0.001808	0.007872	1682.2	706.4	2388.6	1714.0	812.9	2526.9	3.8442	1.2942	5.1384
360	18,666	0.001895	0.006950	1726.2	625.7	2351.9	1761.5	720.1	2481.6	3.9165	1.1373	5.0537
365	19,822	0.002015	0.006009	1777.2	526.4	2303.6	1817.2	605.5	2422.7	4.0004	0.9489	4.9493
370	21,044	0.002217	0.004953	1844.5	385.6	2230.1	1891.2	443.1	2334.3	4.1119	0.6890	4.8009
373.95	22,064	0.003106	0.003106	2015.7	0	2015.7	2084.3	0	2084.3	4.4070	0	4.4070

Source: Tables A–4 through A–8 are generated using the Engineering Equation Solver (EES) software developed by S. A. Klein and F. L. Alvarado. The routine used in calculations is the highly accurate Steam_IAPWS, which incorporates the 1995 Formulation for the Thermodynamic Properties of Ordinary Water Substance for General and Scientific Use, issued by The International Association for the Properties of Water and Steam (IAPWS). This formulation replaces the 1984 formulation of Haar, Gallagher, and Kell (NBS/NRC Steam Tables, Hemisphere Publishing Co., 1984), which is also available in EES as the routine STEAM. The new formulation is based on the correlations of Saul and Wagner (J. Phys. Chem. Ref. Data, 16, 893, 1987) with modifications to adjust to the International Temperature Scale of 1990. The modifications are described by Wagner and Pruss (J. Phys. Chem. Ref. Data, 22, 783, 1993). The properties of ice are based on Hyland and Wexler, "Formulations for the Thermodynamic Properties of the Saturated Phases of H_2O from 173.15 K to 473.15 K," *ASHRAE Trans.*, Part 2A, Paper 2793, 1983.

TABLE A–5

Saturated water—Pressure table

		Specific volume, m^3/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg · K		
Press., P kPa	Sat. temp., T_{sat} °C	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
1.0	6.97	0.001000	129.19	29.302	2355.2	2384.5	29.303	2484.4	2513.7	0.1059	8.8690	8.9749
1.5	13.02	0.001001	87.964	54.686	2338.1	2392.8	54.688	2470.1	2524.7	0.1956	8.6314	8.8270
2.0	17.50	0.001001	66.990	73.431	2325.5	2398.9	73.433	2459.5	2532.9	0.2606	8.4621	8.7227
2.5	21.08	0.001002	54.242	88.422	2315.4	2403.8	88.424	2451.0	2539.4	0.3118	8.3302	8.6421
3.0	24.08	0.001003	45.654	100.98	2306.9	2407.9	100.98	2443.9	2544.8	0.3543	8.2222	8.5765
4.0	28.96	0.001004	34.791	121.39	2293.1	2414.5	121.39	2432.3	2553.7	0.4224	8.0510	8.4734
5.0	32.87	0.001005	28.185	137.75	2282.1	2419.8	137.75	2423.0	2560.7	0.4762	7.9176	8.3938
7.5	40.29	0.001008	19.233	168.74	2261.1	2429.8	168.75	2405.3	2574.0	0.5763	7.6738	8.2501
10	45.81	0.001010	14.670	191.79	2245.4	2437.2	191.81	2392.1	2583.9	0.6492	7.4996	8.1488
15	53.97	0.001014	10.020	225.93	2222.1	2448.0	225.94	2372.3	2598.3	0.7549	7.2522	8.0071
20	60.06	0.001017	7.6481	251.40	2204.6	2456.0	251.42	2357.5	2608.9	0.8320	7.0752	7.9073
25	64.96	0.001020	6.2034	271.93	2190.4	2462.4	271.96	2345.5	2617.5	0.8932	6.9370	7.8302
30	69.09	0.001022	5.2287	289.24	2178.5	2467.7	289.27	2335.3	2624.6	0.9441	6.8234	7.7675
40	75.86	0.001026	3.9933	317.58	2158.8	2476.3	317.62	2318.4	2636.1	1.0261	6.6430	7.6691
50	81.32	0.001030	3.2403	340.49	2142.7	2483.2	340.54	2304.7	2645.2	1.0912	6.5019	7.5931
75	91.76	0.001037	2.2172	384.36	2111.8	2496.1	384.44	2278.0	2662.4	1.2132	6.2426	7.4558
100	99.61	0.001043	1.6941	417.40	2088.2	2505.6	417.51	2257.5	2675.0	1.3028	6.0562	7.3589
101.325	99.97	0.001043	1.6734	418.95	2087.0	2506.0	419.06	2256.5	2675.6	1.3069	6.0476	7.3545
125	105.97	0.001048	1.3750	444.23	2068.8	2513.0	444.36	2240.6	2684.9	1.3741	5.9100	7.2841
150	111.35	0.001053	1.1594	466.97	2052.3	2519.2	467.13	2226.0	2693.1	1.4337	5.7894	7.2231
175	116.04	0.001057	1.0037	486.82	2037.7	2524.5	487.01	2213.1	2700.2	1.4850	5.6865	7.1716
200	120.21	0.001061	0.88578	504.50	2024.6	2529.1	504.71	2201.6	2706.3	1.5302	5.5968	7.1270
225	123.97	0.001064	0.79329	520.47	2012.7	2533.2	520.71	2191.0	2711.7	1.5706	5.5171	7.0877
250	127.41	0.001067	0.71873	535.08	2001.8	2536.8	535.35	2181.2	2716.5	1.6072	5.4453	7.0525
275	130.58	0.001070	0.65732	548.57	1991.6	2540.1	548.86	2172.0	2720.9	1.6408	5.3800	7.0207
300	133.52	0.001073	0.60582	561.11	1982.1	2543.2	561.43	2163.5	2724.9	1.6717	5.3200	6.9917
325	136.27	0.001076	0.56199	572.84	1973.1	2545.9	573.19	2155.4	2728.6	1.7005	5.2645	6.9650
350	138.86	0.001079	0.52422	583.89	1964.6	2548.5	584.26	2147.7	2732.0	1.7274	5.2128	6.9402
375	141.30	0.001081	0.49133	594.32	1956.6	2550.9	594.73	2140.4	2735.1	1.7526	5.1645	6.9171
400	143.61	0.001084	0.46242	604.22	1948.9	2553.1	604.66	2133.4	2738.1	1.7765	5.1191	6.8955
450	147.90	0.001088	0.41392	622.65	1934.5	2557.1	623.14	2120.3	2743.4	1.8205	5.0356	6.8561
500	151.83	0.001093	0.37483	639.54	1921.2	2560.7	640.09	2108.0	2748.1	1.8604	4.9603	6.8207
550	155.46	0.001097	0.34261	655.16	1908.8	2563.9	655.77	2096.6	2752.4	1.8970	4.8916	6.7886
600	158.83	0.001101	0.31560	669.72	1897.1	2566.8	670.38	2085.8	2756.2	1.9308	4.8285	6.7593
650	161.98	0.001104	0.29260	683.37	1886.1	2569.4	684.08	2075.5	2759.6	1.9623	4.7699	6.7322
700	164.95	0.001108	0.27278	696.23	1875.6	2571.8	697.00	2065.8	2762.8	1.9918	4.7153	6.7071
750	167.75	0.001111	0.25552	708.40	1865.6	2574.0	709.24	2056.4	2765.7	2.0195	4.6642	6.6837

TABLE A–5

Saturated water—Pressure table (*Continued*)

		Specific volume, m^3/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg · K		
Press., P kPa	Sat. temp., T_{sat} °C	Sat. liquid, v_f	Sat. vapor, v_g	Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g	Sat. liquid, s_f	Evap., s_{fg}	Sat. vapor, s_g
800	170.41	0.001115	0.24035	719.97	1856.1	2576.0	720.87	2047.5	2768.3	2.0457	4.6160	6.6616
850	172.94	0.001118	0.22690	731.00	1846.9	2577.9	731.95	2038.8	2770.8	2.0705	4.5705	6.6409
900	175.35	0.001121	0.21489	741.55	1838.1	2579.6	742.56	2030.5	2773.0	2.0941	4.5273	6.6213
950	177.66	0.001124	0.20411	751.67	1829.6	2581.3	752.74	2022.4	2775.2	2.1166	4.4862	6.6027
1000	179.88	0.001127	0.19436	761.39	1821.4	2582.8	762.51	2014.6	2777.1	2.1381	4.4470	6.5850
1100	184.06	0.001133	0.17745	779.78	1805.7	2585.5	781.03	1999.6	2780.7	2.1785	4.3735	6.5520
1200	187.96	0.001138	0.16326	796.96	1790.9	2587.8	798.33	1985.4	2783.8	2.2159	4.3058	6.5217
1300	191.60	0.001144	0.15119	813.10	1776.8	2589.9	814.59	1971.9	2786.5	2.2508	4.2428	6.4936
1400	195.04	0.001149	0.14078	828.35	1763.4	2591.8	829.96	1958.9	2788.9	2.2835	4.1840	6.4675
1500	198.29	0.001154	0.13171	842.82	1750.6	2593.4	844.55	1946.4	2791.0	2.3143	4.1287	6.4430
1750	205.72	0.001166	0.11344	876.12	1720.6	2596.7	878.16	1917.1	2795.2	2.3844	4.0033	6.3877
2000	212.38	0.001177	0.099587	906.12	1693.0	2599.1	908.47	1889.8	2798.3	2.4467	3.8923	6.3390
2250	218.41	0.001187	0.088717	933.54	1667.3	2600.9	936.21	1864.3	2800.5	2.5029	3.7926	6.2954
2500	223.95	0.001197	0.079952	958.87	1643.2	2602.1	961.87	1840.1	2801.9	2.5542	3.7016	6.2558
3000	233.85	0.001217	0.066667	1004.6	1598.5	2603.2	1008.3	1794.9	2803.2	2.6454	3.5402	6.1856
3500	242.56	0.001235	0.057061	1045.4	1557.6	2603.0	1049.7	1753.0	2802.7	2.7253	3.3991	6.1244
4000	250.35	0.001252	0.049779	1082.4	1519.3	2601.7	1087.4	1713.5	2800.8	2.7966	3.2731	6.0696
5000	263.94	0.001286	0.039448	1148.1	1448.9	2597.0	1154.5	1639.7	2794.2	2.9207	3.0530	5.9737
6000	275.59	0.001319	0.032449	1205.8	1384.1	2589.9	1213.8	1570.9	2784.6	3.0275	2.8627	5.8902
7000	285.83	0.001352	0.027378	1258.0	1323.0	2581.0	1267.5	1505.2	2772.6	3.1220	2.6927	5.8148
8000	295.01	0.001384	0.023525	1306.0	1264.5	2570.5	1317.1	1441.6	2758.7	3.2077	2.5373	5.7450
9000	303.35	0.001418	0.020489	1350.9	1207.6	2558.5	1363.7	1379.3	2742.9	3.2866	2.3925	5.6791
10,000	311.00	0.001452	0.018028	1393.3	1151.8	2545.2	1407.8	1317.6	2725.5	3.3603	2.2556	5.6159
11,000	318.08	0.001488	0.015988	1433.9	1096.6	2530.4	1450.2	1256.1	2706.3	3.4299	2.1245	5.5544
12,000	324.68	0.001526	0.014264	1473.0	1041.3	2514.3	1491.3	1194.1	2685.4	3.4964	1.9975	5.4939
13,000	330.85	0.001566	0.012781	1511.0	985.5	2496.6	1531.4	1131.3	2662.7	3.5606	1.8730	5.4336
14,000	336.67	0.001610	0.011487	1548.4	928.7	2477.1	1571.0	1067.0	2637.9	3.6232	1.7497	5.3728
15,000	342.16	0.001657	0.010341	1585.5	870.3	2455.7	1610.3	1000.5	2610.8	3.6848	1.6261	5.3108
16,000	347.36	0.001710	0.009312	1622.6	809.4	2432.0	1649.9	931.1	2581.0	3.7461	1.5005	5.2466
17,000	352.29	0.001770	0.008374	1660.2	745.1	2405.4	1690.3	857.4	2547.7	3.8082	1.3709	5.1791
18,000	356.99	0.001840	0.007504	1699.1	675.9	2375.0	1732.2	777.8	2510.0	3.8720	1.2343	5.1064
19,000	361.47	0.001926	0.006677	1740.3	598.9	2339.2	1776.8	689.2	2466.0	3.9396	1.0860	5.0256
20,000	365.75	0.002038	0.005862	1785.8	509.0	2294.8	1826.6	585.5	2412.1	4.0146	0.9164	4.9310
21,000	369.83	0.002207	0.004994	1841.6	391.9	2233.5	1888.0	450.4	2338.4	4.1071	0.7005	4.8076
22,000	373.71	0.002703	0.003644	1951.7	140.8	2092.4	2011.1	161.5	2172.6	4.2942	0.2496	4.5439
22,064	373.95	0.003106	0.003106	2015.7	0	2015.7	2084.3	0	2084.3	4.4070	0	4.4070

TABLE A–6

Superheated water

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	P = 0.01 MPa (45.81°C)*				P = 0.05 MPa (81.32°C)				P = 0.10 MPa (99.61°C)			
Sat.†	14.670	2437.2	2583.9	8.1488	3.2403	2483.2	2645.2	7.5931	1.6941	2505.6	2675.0	7.3589
50	14.867	2443.3	2592.0	8.1741								
100	17.196	2515.5	2687.5	8.4489	3.4187	2511.5	2682.4	7.6953	1.6959	2506.2	2675.8	7.3611
150	19.513	2587.9	2783.0	8.6893	3.8897	2585.7	2780.2	7.9413	1.9367	2582.9	2776.6	7.6148
200	21.826	2661.4	2879.6	8.9049	4.3562	2660.0	2877.8	8.1592	2.1724	2658.2	2875.5	7.8356
250	24.136	2736.1	2977.5	9.1015	4.8206	2735.1	2976.2	8.3568	2.4062	2733.9	2974.5	8.0346
300	26.446	2812.3	3076.7	9.2827	5.2841	2811.6	3075.8	8.5387	2.6389	2810.7	3074.5	8.2172
400	31.063	2969.3	3280.0	9.6094	6.2094	2968.9	3279.3	8.8659	3.1027	2968.3	3278.6	8.5452
500	35.680	3132.9	3489.7	9.8998	7.1338	3132.6	3489.3	9.1566	3.5655	3132.2	3488.7	8.8362
600	40.296	3303.3	3706.3	10.1631	8.0577	3303.1	3706.0	9.4201	4.0279	3302.8	3705.6	9.0999
700	44.911	3480.8	3929.9	10.4056	8.9813	3480.6	3929.7	9.6626	4.4900	3480.4	3929.4	9.3424
800	49.527	3665.4	4160.6	10.6312	9.9047	3665.2	4160.4	9.8883	4.9519	3665.0	4160.2	9.5682
900	54.143	3856.9	4398.3	10.8429	10.8280	3856.8	4398.2	10.1000	5.4137	3856.7	4398.0	9.7800
1000	58.758	4055.3	4642.8	11.0429	11.7513	4055.2	4642.7	10.3000	5.8755	4055.0	4642.6	9.9800
1100	63.373	4260.0	4893.8	11.2326	12.6745	4259.9	4893.7	10.4897	6.3372	4259.8	4893.6	10.1698
1200	67.989	4470.9	5150.8	11.4132	13.5977	4470.8	5150.7	10.6704	6.7988	4470.7	5150.6	10.3504
1300	72.604	4687.4	5413.4	11.5857	14.5209	4687.3	5413.3	10.8429	7.2605	4687.2	5413.3	10.5229

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	P = 0.20 MPa (120.21°C)				P = 0.30 MPa (133.52°C)				P = 0.40 MPa (143.61°C)			
Sat.	0.88578	2529.1	2706.3	7.1270	0.60582	2543.2	2724.9	6.9917	0.46242	2553.1	2738.1	6.8955
150	0.95986	2577.1	2769.1	7.2810	0.63402	2571.0	2761.2	7.0792	0.47088	2564.4	2752.8	6.9306
200	1.08049	2654.6	2870.7	7.5081	0.71643	2651.0	2865.9	7.3132	0.53434	2647.2	2860.9	7.1723
250	1.19890	2731.4	2971.2	7.7100	0.79645	2728.9	2967.9	7.5180	0.59520	2726.4	2964.5	7.3804
300	1.31623	2808.8	3072.1	7.8941	0.87535	2807.0	3069.6	7.7037	0.65489	2805.1	3067.1	7.5677
400	1.54934	2967.2	3277.0	8.2236	1.03155	2966.0	3275.5	8.0347	0.77265	2964.9	3273.9	7.9003
500	1.78142	3131.4	3487.7	8.5153	1.18672	3130.6	3486.6	8.3271	0.88936	3129.8	3485.5	8.1933
600	2.01302	3302.2	3704.8	8.7793	1.34139	3301.6	3704.0	8.5915	1.00558	3301.0	3703.3	8.4580
700	2.24434	3479.9	3928.8	9.0221	1.49580	3479.5	3928.2	8.8345	1.12152	3479.0	3927.6	8.7012
800	2.47550	3664.7	4159.8	9.2479	1.65004	3664.3	4159.3	9.0605	1.23730	3663.9	4158.9	8.9274
900	2.70656	3856.3	4397.7	9.4598	1.80417	3856.0	4397.3	9.2725	1.35298	3855.7	4396.9	9.1394
1000	2.93755	4054.8	4642.3	9.6599	1.95824	4054.5	4642.0	9.4726	1.46859	4054.3	4641.7	9.3396
1100	3.16848	4259.6	4893.3	9.8497	2.11226	4259.4	4893.1	9.6624	1.58414	4259.2	4892.9	9.5295
1200	3.39938	4470.5	5150.4	10.0304	2.26624	4470.3	5150.2	9.8431	1.69966	4470.2	5150.0	9.7102
1300	3.63026	4687.1	5413.1	10.2029	2.42019	4686.9	5413.0	10.0157	1.81516	4686.7	5412.8	9.8828

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	P = 0.50 MPa (151.83°C)				P = 0.60 MPa (158.83°C)				P = 0.80 MPa (170.41°C)			
Sat.	0.37483	2560.7	2748.1	6.8207	0.31560	2566.8	2756.2	6.7593	0.24035	2576.0	2768.3	6.6616
200	0.42503	2643.3	2855.8	7.0610	0.35212	2639.4	2850.6	6.9683	0.26088	2631.1	2839.8	6.8177
250	0.47443	2723.8	2961.0	7.2725	0.39390	2721.2	2957.6	7.1833	0.29321	2715.9	2950.4	7.0402
300	0.52261	2803.3	3064.6	7.4614	0.43442	2801.4	3062.0	7.3740	0.32416	2797.5	3056.9	7.2345
350	0.57015	2883.0	3168.1	7.6346	0.47428	2881.6	3166.1	7.5481	0.35442	2878.6	3162.2	7.4107
400	0.61731	2963.7	3272.4	7.7956	0.51374	2962.5	3270.8	7.7097	0.38429	2960.2	3267.7	7.5735
500	0.71095	3129.0	3484.5	8.0893	0.59200	3128.2	3483.4	8.0041	0.44332	3126.6	3481.3	7.8692
600	0.80409	3300.4	3702.5	8.3544	0.66976	3299.8	3701.7	8.2695	0.50186	3298.7	3700.1	8.1354
700	0.89696	3478.6	3927.0	8.5978	0.74725	3478.1	3926.4	8.5132	0.56011	3477.2	3925.3	8.3794
800	0.98966	3663.6	4158.4	8.8240	0.82457	3663.2	4157.9	8.7395	0.61820	3662.5	4157.0	8.6061
900	1.08227	3855.4	4396.6	9.0362	0.90179	3855.1	4396.2	8.9518	0.67619	3854.5	4395.5	8.8185
1000	1.17480	4054.0	4641.4	9.2364	0.97893	4053.8	4641.1	9.1521	0.73411	4053.3	4640.5	9.0189
1100	1.26728	4259.0	4892.6	9.4263	1.05603	4258.8	4892.4	9.3420	0.79197	4258.3	4891.9	9.2090
1200	1.35972	4470.0	5149.8	9.6071	1.13309	4469.8	5149.6	9.5229	0.84980	4469.4	5149.3	9.3898
1300	1.45214	4686.6	5412.6	9.7797	1.21012	4686.4	5412.5	9.6955	0.90761	4686.1	5412.2	9.5625

*The temperature in parentheses is the saturation temperature at the specified pressure.

† Properties of saturated vapor at the specified pressure.

TABLE A–6

Superheated water (*Continued*)

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	P = 1.00 MPa (179.88°C)				P = 1.20 MPa (187.96°C)				P = 1.40 MPa (195.04°C)			
Sat.	0.19437	2582.8	2777.1	6.5850	0.16326	2587.8	2783.8	6.5217	0.14078	2591.8	2788.9	6.4675
200	0.20602	2622.3	2828.3	6.6956	0.16934	2612.9	2816.1	6.5909	0.14303	2602.7	2803.0	6.4975
250	0.23275	2710.4	2943.1	6.9265	0.19241	2704.7	2935.6	6.8313	0.16356	2698.9	2927.9	6.7488
300	0.25799	2793.7	3051.6	7.1246	0.21386	2789.7	3046.3	7.0335	0.18233	2785.7	3040.9	6.9553
350	0.28250	2875.7	3158.2	7.3029	0.23455	2872.7	3154.2	7.2139	0.20029	2869.7	3150.1	7.1379
400	0.30661	2957.9	3264.5	7.4670	0.25482	2955.5	3261.3	7.3793	0.21782	2953.1	3258.1	7.3046
500	0.35411	3125.0	3479.1	7.7642	0.29464	3123.4	3477.0	7.6779	0.25216	3121.8	3474.8	7.6047
600	0.40111	3297.5	3698.6	8.0311	0.33395	3296.3	3697.0	7.9456	0.28597	3295.1	3695.5	7.8730
700	0.44783	3476.3	3924.1	8.2755	0.37297	3475.3	3922.9	8.1904	0.31951	3474.4	3921.7	8.1183
800	0.49438	3661.7	4156.1	8.5024	0.41184	3661.0	4155.2	8.4176	0.35288	3660.3	4154.3	8.3458
900	0.54083	3853.9	4394.8	8.7150	0.45059	3853.3	4394.0	8.6303	0.38614	3852.7	4393.3	8.5587
1000	0.58721	4052.7	4640.0	8.9155	0.48928	4052.2	4639.4	8.8310	0.41933	4051.7	4638.8	8.7595
1100	0.63354	4257.9	4891.4	9.1057	0.52792	4257.5	4891.0	9.0212	0.45247	4257.0	4890.5	8.9497
1200	0.67983	4469.0	5148.9	9.2866	0.56652	4468.7	5148.5	9.2022	0.48558	4468.3	5148.1	9.1308
1300	0.72610	4685.8	5411.9	9.4593	0.60509	4685.5	5411.6	9.3750	0.51866	4685.1	5411.3	9.3036

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	P = 1.60 MPa (201.37°C)				P = 1.80 MPa (207.11°C)				P = 2.00 MPa (212.38°C)			
Sat.	0.12374	2594.8	2792.8	6.4200	0.11037	2597.3	2795.9	6.3775	0.09959	2599.1	2798.3	6.3390
225	0.13293	2645.1	2857.8	6.5537	0.11678	2637.0	2847.2	6.4825	0.10381	2628.5	2836.1	6.4160
250	0.14190	2692.9	2919.9	6.6753	0.12502	2686.7	2911.7	6.6088	0.11150	2680.3	2903.3	6.5475
300	0.15866	2781.6	3035.4	6.8864	0.14025	2777.4	3029.9	6.8246	0.12551	2773.2	3024.2	6.7684
350	0.17459	2866.6	3146.0	7.0713	0.15460	2863.6	3141.9	7.0120	0.13860	2860.5	3137.7	6.9583
400	0.19007	2950.8	3254.9	7.2394	0.16849	2948.3	3251.6	7.1814	0.15122	2945.9	3248.4	7.1292
500	0.22029	3120.1	3472.6	7.5410	0.19551	3118.5	3470.4	7.4845	0.17568	3116.9	3468.3	7.4337
600	0.24999	3293.9	3693.9	7.8101	0.22200	3292.7	3692.3	7.7543	0.19962	3291.5	3690.7	7.7043
700	0.27941	3473.5	3920.5	8.0558	0.24822	3472.6	3919.4	8.0005	0.22326	3471.7	3918.2	7.9509
800	0.30865	3659.5	4153.4	8.2834	0.27426	3658.8	4152.4	8.2284	0.24674	3658.0	4151.5	8.1791
900	0.33780	3852.1	4392.6	8.4965	0.30020	3851.5	4391.9	8.4417	0.27012	3850.9	4391.1	8.3925
1000	0.36687	4051.2	4638.2	8.6974	0.32606	4050.7	4637.6	8.6427	0.29342	4050.2	4637.1	8.5936
1100	0.39589	4256.6	4890.0	8.8878	0.35188	4256.2	4889.6	8.8331	0.31667	4255.7	4889.1	8.7842
1200	0.42488	4467.9	5147.7	9.0689	0.37766	4467.6	5147.3	9.0143	0.33989	4467.2	5147.0	8.9654
1300	0.45383	4684.8	5410.9	9.2418	0.40341	4684.5	5410.6	9.1872	0.36308	4684.2	5410.3	9.1384

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	P = 2.50 MPa (223.95°C)				P = 3.00 MPa (233.85°C)				P = 3.50 MPa (242.56°C)			
Sat.	0.07995	2602.1	2801.9	6.2558	0.06667	2603.2	2803.2	6.1856	0.05706	2603.0	2802.7	6.1244
225	0.08026	2604.8	2805.5	6.2629								
250	0.08705	2663.3	2880.9	6.4107	0.07063	2644.7	2856.5	6.2893	0.05876	2624.0	2829.7	6.1764
300	0.09894	2762.2	3009.6	6.6459	0.08118	2750.8	2994.3	6.5412	0.06845	2738.8	2978.4	6.4484
350	0.10979	2852.5	3127.0	6.8424	0.09056	2844.4	3116.1	6.7450	0.07680	2836.0	3104.9	6.6601
400	0.12012	2939.8	3240.1	7.0170	0.09938	2933.6	3231.7	6.9235	0.08456	2927.2	3223.2	6.8428
450	0.13015	3026.2	3351.6	7.1768	0.10789	3021.2	3344.9	7.0856	0.09198	3016.1	3338.1	7.0074
500	0.13999	3112.8	3462.8	7.3254	0.11620	3108.6	3457.2	7.2359	0.09919	3104.5	3451.7	7.1593
600	0.15931	3288.5	3686.8	7.5979	0.13245	3285.5	3682.8	7.5103	0.11325	3282.5	3678.9	7.4357
700	0.17835	3469.3	3915.2	7.8455	0.14841	3467.0	3912.2	7.7590	0.12702	3464.7	3909.3	7.6855
800	0.19722	3656.2	4149.2	8.0744	0.16420	3654.3	4146.9	7.9885	0.14061	3652.5	4144.6	7.9156
900	0.21597	3849.4	4389.3	8.2882	0.17988	3847.9	4387.5	8.2028	0.15410	3846.4	4385.7	8.1304
1000	0.23466	4049.0	4635.6	8.4897	0.19549	4047.7	4634.2	8.4045	0.16751	4046.4	4632.7	8.3324
1100	0.25330	4254.7	4887.9	8.6804	0.21105	4253.6	4886.7	8.5955	0.18087	4252.5	4885.6	8.5236
1200	0.27190	4466.3	5146.0	8.8618	0.22658	4465.3	5145.1	8.7771	0.19420	4464.4	5144.1	8.7053
1300	0.29048	4683.4	5409.5	9.0349	0.24207	4682.6	5408.8	8.9502	0.20750	4681.8	5408.0	8.8786

TABLE A–6

Superheated water (*Continued*)

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	P = 4.0 MPa (250.35°C)				P = 4.5 MPa (257.44°C)				P = 5.0 MPa (263.94°C)			
Sat.	0.04978	2601.7	2800.8	6.0696	0.04406	2599.7	2798.0	6.0198	0.03945	2597.0	2794.2	5.9737
275	0.05461	2668.9	2887.3	6.2312	0.04733	2651.4	2864.4	6.1429	0.04144	2632.3	2839.5	6.0571
300	0.05887	2726.2	2961.7	6.3639	0.05138	2713.0	2944.2	6.2854	0.04535	2699.0	2925.7	6.2111
350	0.06647	2827.4	3093.3	6.5843	0.05842	2818.6	3081.5	6.5153	0.05197	2809.5	3069.3	6.4516
400	0.07343	2920.8	3214.5	6.7714	0.06477	2914.2	3205.7	6.7071	0.05784	2907.5	3196.7	6.6483
450	0.08004	3011.0	3331.2	6.9386	0.07076	3005.8	3324.2	6.8770	0.06332	3000.6	3317.2	6.8210
500	0.08644	3100.3	3446.0	7.0922	0.07652	3096.0	3440.4	7.0323	0.06858	3091.8	3434.7	6.9781
600	0.09886	3279.4	3674.9	7.3706	0.08766	3276.4	3670.9	7.3127	0.07870	3273.3	3666.9	7.2605
700	0.11098	3462.4	3906.3	7.6214	0.09850	3460.0	3903.3	7.5647	0.08852	3457.7	3900.3	7.5136
800	0.12292	3650.6	4142.3	7.8523	0.10916	3648.8	4140.0	7.7962	0.09816	3646.9	4137.7	7.7458
900	0.13476	3844.8	4383.9	8.0675	0.11972	3843.3	4382.1	8.0118	0.10769	3841.8	4380.2	7.9619
1000	0.14653	4045.1	4631.2	8.2698	0.13020	4043.9	4629.8	8.2144	0.11715	4042.6	4628.3	8.1648
1100	0.15824	4251.4	4884.4	8.4612	0.14064	4250.4	4883.2	8.4060	0.12655	4249.3	4882.1	8.3566
1200	0.16992	4463.5	5143.2	8.6430	0.15103	4462.6	5142.2	8.5880	0.13592	4461.6	5141.3	8.5388
1300	0.18157	4680.9	5407.2	8.8164	0.16140	4680.1	5406.5	8.7616	0.14527	4679.3	5405.7	8.7124

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	P = 6.0 MPa (275.59°C)				P = 7.0 MPa (285.83°C)				P = 8.0 MPa (295.01°C)			
Sat.	0.03245	2589.9	2784.6	5.8902	0.027378	2581.0	2772.6	5.8148	0.023525	2570.5	2758.7	5.7450
300	0.03619	2668.4	2885.6	6.0703	0.029492	2633.5	2839.9	5.9337	0.024279	2592.3	2786.5	5.7937
350	0.04225	2790.4	3043.9	6.3357	0.035262	2770.1	3016.9	6.2305	0.029975	2748.3	2988.1	6.1321
400	0.04742	2893.7	3178.3	6.5432	0.039958	2879.5	3159.2	6.4502	0.034344	2864.6	3139.4	6.3658
450	0.05217	2989.9	3302.9	6.7219	0.044187	2979.0	3288.3	6.6353	0.038194	2967.8	3273.3	6.5579
500	0.05667	3083.1	3423.1	6.8826	0.048157	3074.3	3411.4	6.8000	0.041767	3065.4	3399.5	6.7266
550	0.06102	3175.2	3541.3	7.0308	0.051966	3167.9	3531.6	6.9507	0.045172	3160.5	3521.8	6.8800
600	0.06527	3267.2	3658.8	7.1693	0.055665	3261.0	3650.6	7.0910	0.048463	3254.7	3642.4	7.0221
700	0.07355	3453.0	3894.3	7.4247	0.062850	3448.3	3888.3	7.3487	0.054829	3443.6	3882.2	7.2822
800	0.08165	3643.2	4133.1	7.6582	0.069856	3639.5	4128.5	7.5836	0.061011	3635.7	4123.8	7.5185
900	0.08964	3838.8	4376.6	7.8751	0.076750	3835.7	4373.0	7.8014	0.067082	3832.7	4369.3	7.7372
1000	0.09756	4040.1	4625.4	8.0786	0.083571	4037.5	4622.5	8.0055	0.073079	4035.0	4619.6	7.9419
1100	0.10543	4247.1	4879.7	8.2709	0.090341	4245.0	4877.4	8.1982	0.079025	4242.8	4875.0	8.1350
1200	0.11326	4459.8	5139.4	8.4534	0.097075	4457.9	5137.4	8.3810	0.084934	4456.1	5135.5	8.3181
1300	0.12107	4677.7	5404.1	8.6273	0.103781	4676.1	5402.6	8.5551	0.090817	4674.5	5401.0	8.4925

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	P = 9.0 MPa (303.35°C)				P = 10.0 MPa (311.00°C)				P = 12.5 MPa (327.81°C)			
Sat.	0.020489	2558.5	2742.9	5.6791	0.018028	2545.2	2725.5	5.6159	0.013496	2505.6	2674.3	5.4638
325	0.023284	2647.6	2857.1	5.8738	0.019877	2611.6	2810.3	5.7596				
350	0.025816	2725.0	2957.3	6.0380	0.022440	2699.6	2924.0	5.9460	0.016138	2624.9	2826.6	5.7130
400	0.029960	2849.2	3118.8	6.2876	0.026436	2833.1	3097.5	6.2141	0.020030	2789.6	3040.0	6.0433
450	0.033524	2956.3	3258.0	6.4872	0.029782	2944.5	3242.4	6.4219	0.023019	2913.7	3201.5	6.2749
500	0.036793	3056.3	3387.4	6.6603	0.032811	3047.0	3375.1	6.5995	0.025630	3023.2	3343.6	6.4651
550	0.039885	3153.0	3512.0	6.8164	0.035655	3145.4	3502.0	6.7585	0.028033	3126.1	3476.5	6.6317
600	0.042861	3248.4	3634.1	6.9605	0.038378	3242.0	3625.8	6.9045	0.030306	3225.8	3604.6	6.7828
650	0.045755	3343.4	3755.2	7.0954	0.041018	3338.0	3748.1	7.0408	0.032491	3324.1	3730.2	6.9227
700	0.048589	3438.8	3876.1	7.2229	0.043597	3434.0	3870.0	7.1693	0.034612	3422.0	3854.6	7.0540
800	0.054132	3632.0	4119.2	7.4606	0.048629	3628.2	4114.5	7.4085	0.038724	3618.8	4102.8	7.2967
900	0.059562	3829.6	4365.7	7.6802	0.053547	3826.5	4362.0	7.6290	0.042720	3818.9	4352.9	7.5195
1000	0.064919	4032.4	4616.7	7.8855	0.058391	4029.9	4613.8	7.8349	0.046641	4023.5	4606.5	7.7269
1100	0.070224	4240.7	4872.7	8.0791	0.063183	4238.5	4870.3	8.0289	0.050610	4233.1	4864.5	7.9220
1200	0.075492	4454.2	5133.6	8.2625	0.067938	4452.4	5131.7	8.2126	0.054342	4447.7	5127.0	8.1065
1300	0.080733	4672.9	5399.5	8.4371	0.072657	4671.3	5398.0	8.3874	0.058147	4667.3	5394.1	8.2819

TABLE A–6

Superheated water (*Concluded*)

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	P = 15.0 MPa (342.16°C)				P = 17.5 MPa (354.67°C)				P = 20.0 MPa (365.75°C)			
Sat.	0.010341	2455.7	2610.8	5.3108	0.007932	2390.7	2529.5	5.1435	0.005862	2294.8	2412.1	4.9310
350	0.011481	2520.9	2693.1	5.4438								
400	0.015671	2740.6	2975.7	5.8819	0.012463	2684.3	2902.4	5.7211	0.009950	2617.9	2816.9	5.5526
450	0.018477	2880.8	3157.9	6.1434	0.015204	2845.4	3111.4	6.0212	0.012721	2807.3	3061.7	5.9043
500	0.020828	2998.4	3310.8	6.3480	0.017385	2972.4	3276.7	6.2424	0.014793	2945.3	3241.2	6.1446
550	0.022945	3106.2	3450.4	6.5230	0.019305	3085.8	3423.6	6.4266	0.016571	3064.7	3396.2	6.3390
600	0.024921	3209.3	3583.1	6.6796	0.021073	3192.5	3561.3	6.5890	0.018185	3175.3	3539.0	6.5075
650	0.026804	3310.1	3712.1	6.8233	0.022742	3295.8	3693.8	6.7366	0.019695	3281.4	3675.3	6.6593
700	0.028621	3409.8	3839.1	6.9573	0.024342	3397.5	3823.5	6.8735	0.021134	3385.1	3807.8	6.7991
800	0.032121	3609.3	4091.1	7.2037	0.027405	3599.7	4079.3	7.1237	0.023870	3590.1	4067.5	7.0531
900	0.035503	3811.2	4343.7	7.4288	0.030348	3803.5	4334.6	7.3511	0.026484	3795.7	4325.4	7.2829
1000	0.038808	4017.1	4599.2	7.6378	0.033215	4010.7	4592.0	7.5616	0.029020	4004.3	4584.7	7.4950
1100	0.042062	4227.7	4858.6	7.8339	0.036029	4222.3	4852.8	7.7588	0.031504	4216.9	4847.0	7.6933
1200	0.045279	4443.1	5122.3	8.0192	0.038806	4438.5	5117.6	7.9449	0.033952	4433.8	5112.9	7.8802
1300	0.048469	4663.3	5390.3	8.1952	0.041556	4659.2	5386.5	8.1215	0.036371	4655.2	5382.7	8.0574

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	P = 25.0 MPa				P = 30.0 MPa				P = 35.0 MPa			
375	0.001978	1799.9	1849.4	4.0345	0.001792	1738.1	1791.9	3.9313	0.001701	1702.8	1762.4	3.8724
400	0.006005	2428.5	2578.7	5.1400	0.002798	2068.9	2152.8	4.4758	0.002105	1914.9	1988.6	4.2144
425	0.007886	2607.8	2805.0	5.4708	0.005299	2452.9	2611.8	5.1473	0.003434	2253.3	2373.5	4.7751
450	0.009176	2721.2	2950.6	5.6759	0.006737	2618.9	2821.0	5.4422	0.004957	2497.5	2671.0	5.1946
500	0.011143	2887.3	3165.9	5.9643	0.008691	2824.0	3084.8	5.7956	0.006933	2755.3	2997.9	5.6331
550	0.012736	3020.8	3339.2	6.1816	0.010175	2974.5	3279.7	6.0403	0.008348	2925.8	3218.0	5.9093
600	0.014140	3140.0	3493.5	6.3637	0.011445	3103.4	3446.8	6.2373	0.009523	3065.6	3399.0	6.1229
650	0.015430	3251.9	3637.7	6.5243	0.012590	3221.7	3599.4	6.4074	0.010565	3190.9	3560.7	6.3030
700	0.016643	3359.9	3776.0	6.6702	0.013654	3334.3	3743.9	6.5599	0.011523	3308.3	3711.6	6.4623
800	0.018922	3570.7	4043.8	6.9322	0.015628	3551.2	4020.0	6.8301	0.013278	3531.6	3996.3	6.7409
900	0.021075	3780.2	4307.1	7.1668	0.017473	3764.6	4288.8	7.0695	0.014904	3749.0	4270.6	6.9853
1000	0.023150	3991.5	4570.2	7.3821	0.019240	3978.6	4555.8	7.2880	0.016450	3965.8	4541.5	7.2069
1100	0.025172	4206.1	4835.4	7.5825	0.020954	4195.2	4823.9	7.4906	0.017942	4184.4	4812.4	7.4118
1200	0.027157	4424.6	5103.5	7.7710	0.022630	4415.3	5094.2	7.6807	0.019398	4406.1	5085.0	7.6034
1300	0.029115	4647.2	5375.1	7.9494	0.024279	4639.2	5367.6	7.8602	0.020827	4631.2	5360.2	7.7841

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	P = 40.0 MPa				P = 50.0 MPa				P = 60.0 MPa			
375	0.001641	1677.0	1742.6	3.8290	0.001560	1638.6	1716.6	3.7642	0.001503	1609.7	1699.9	3.7149
400	0.001911	1855.0	1931.4	4.1145	0.001731	1787.8	1874.4	4.0029	0.001633	1745.2	1843.2	3.9317
425	0.002538	2097.5	2199.0	4.5044	0.002009	1960.3	2060.7	4.2746	0.001816	1892.9	2001.8	4.1630
450	0.003692	2364.2	2511.8	4.9449	0.002487	2160.3	2284.7	4.5896	0.002086	2055.1	2180.2	4.4140
500	0.005623	2681.6	2906.5	5.4744	0.003890	2528.1	2722.6	5.1762	0.002952	2393.2	2570.3	4.9356
550	0.006985	2875.1	3154.4	5.7857	0.005118	2769.5	3025.4	5.5563	0.003955	2664.6	2901.9	5.3517
600	0.008089	3026.8	3350.4	6.0170	0.006108	2947.1	3252.6	5.8245	0.004833	2866.8	3156.8	5.6527
650	0.009053	3159.5	3521.6	6.2078	0.006957	3095.6	3443.5	6.0373	0.005591	3031.3	3366.8	5.8867
700	0.009930	3282.0	3679.2	6.3740	0.007717	3228.7	3614.6	6.2179	0.006265	3175.4	3551.3	6.0814
800	0.011521	3511.8	3972.6	6.6613	0.009073	3472.2	3925.8	6.5225	0.007456	3432.6	3880.0	6.4033
900	0.012980	3733.3	4252.5	6.9107	0.010296	3702.0	4216.8	6.7819	0.008519	3670.9	4182.1	6.6725
1000	0.014360	3952.9	4527.3	7.1355	0.011441	3927.4	4499.4	7.0131	0.009504	3902.0	4472.2	6.9099
1100	0.015686	4173.7	4801.1	7.3425	0.012534	4152.2	4778.9	7.2244	0.010439	4130.9	4757.3	7.1255
1200	0.016976	4396.9	5075.9	7.5357	0.013590	4378.6	5058.1	7.4207	0.011339	4360.5	5040.8	7.3248
1300	0.018239	4623.3	5352.8	7.7175	0.014620	4607.5	5338.5	7.6048	0.012213	4591.8	5324.5	7.5111

TABLE A–7

Compressed liquid water

T °C	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K	v m³/kg	u kJ/kg	h kJ/kg	s kJ/kg · K
	P = 5 MPa (263.94°C)				P = 10 MPa (311.00°C)				P = 15 MPa (342.16°C)			
Sat.	0.0012862	1148.1	1154.5	2.9207	0.0014522	1393.3	1407.9	3.3603	0.0016572	1585.5	1610.3	3.6848
0	0.0009977	0.04	5.03	0.0001	0.0009952	0.12	10.07	0.0003	0.0009928	0.18	15.07	0.0004
20	0.0009996	83.61	88.61	0.2954	0.0009973	83.31	93.28	0.2943	0.0009951	83.01	97.93	0.2932
40	0.0010057	166.92	171.95	0.5705	0.0010035	166.33	176.37	0.5685	0.0010013	165.75	180.77	0.5666
60	0.0010149	250.29	255.36	0.8287	0.0010127	249.43	259.55	0.8260	0.0010105	248.58	263.74	0.8234
80	0.0010267	333.82	338.96	1.0723	0.0010244	332.69	342.94	1.0691	0.0010221	331.59	346.92	1.0659
100	0.0010410	417.65	422.85	1.3034	0.0010385	416.23	426.62	1.2996	0.0010361	414.85	430.39	1.2958
120	0.0010576	501.91	507.19	1.5236	0.0010549	500.18	510.73	1.5191	0.0010522	498.50	514.28	1.5148
140	0.0010769	586.80	592.18	1.7344	0.0010738	584.72	595.45	1.7293	0.0010708	582.69	598.75	1.7243
160	0.0010988	672.55	678.04	1.9374	0.0010954	670.06	681.01	1.9316	0.0010920	667.63	684.01	1.9259
180	0.0011240	759.47	765.09	2.1338	0.0011200	756.48	767.68	2.1271	0.0011160	753.58	770.32	2.1206
200	0.0011531	847.92	853.68	2.3251	0.0011482	844.32	855.80	2.3174	0.0011435	840.84	858.00	2.3100
220	0.0011868	938.39	944.32	2.5127	0.0011809	934.01	945.82	2.5037	0.0011752	929.81	947.43	2.4951
240	0.0012268	1031.6	1037.7	2.6983	0.0012192	1026.2	1038.3	2.6876	0.0012121	1021.0	1039.2	2.6774
260	0.0012755	1128.5	1134.9	2.8841	0.0012653	1121.6	1134.3	2.8710	0.0012560	1115.1	1134.0	2.8586
280					0.0013226	1221.8	1235.0	3.0565	0.0013096	1213.4	1233.0	3.0410
300					0.0013980	1329.4	1343.3	3.2488	0.0013783	1317.6	1338.3	3.2279
320									0.0014733	1431.9	1454.0	3.4263
340									0.0016311	1567.9	1592.4	3.6555
	P = 20 MPa (365.75°C)				P = 30 MPa				P = 50 MPa			
Sat.	0.0020378	1785.8	1826.6	4.0146								
0	0.0009904	0.23	20.03	0.0005	0.0009857	0.29	29.86	0.0003	0.0009767	0.29	49.13	−0.0010
20	0.0009929	82.71	102.57	0.2921	0.0009886	82.11	111.77	0.2897	0.0009805	80.93	129.95	0.2845
40	0.0009992	165.17	185.16	0.5646	0.0009951	164.05	193.90	0.5607	0.0009872	161.90	211.25	0.5528
60	0.0010084	247.75	267.92	0.8208	0.0010042	246.14	276.26	0.8156	0.0009962	243.08	292.88	0.8055
80	0.0010199	330.50	350.90	1.0627	0.0010155	328.40	358.86	1.0564	0.0010072	324.42	374.78	1.0442
100	0.0010337	413.50	434.17	1.2920	0.0010290	410.87	441.74	1.2847	0.0010201	405.94	456.94	1.2705
120	0.0010496	496.85	517.84	1.5105	0.0010445	493.66	525.00	1.5020	0.0010349	487.69	539.43	1.4859
140	0.0010679	580.71	602.07	1.7194	0.0010623	576.90	608.76	1.7098	0.0010517	569.77	622.36	1.6916
160	0.0010886	665.28	687.05	1.9203	0.0010823	660.74	693.21	1.9094	0.0010704	652.33	705.85	1.8889
180	0.0011122	750.78	773.02	2.1143	0.0011049	745.40	778.55	2.1020	0.0010914	735.49	790.06	2.0790
200	0.0011390	837.49	860.27	2.3027	0.0011304	831.11	865.02	2.2888	0.0011149	819.45	875.19	2.2628
220	0.0011697	925.77	949.16	2.4867	0.0011595	918.15	952.93	2.4707	0.0011412	904.39	961.45	2.4414
240	0.0012053	1016.1	1040.2	2.6676	0.0011927	1006.9	1042.7	2.6491	0.0011708	990.55	1049.1	2.6156
260	0.0012472	1109.0	1134.0	2.8469	0.0012314	1097.8	1134.7	2.8250	0.0012044	1078.2	1138.4	2.7864
280	0.0012978	1205.6	1231.5	3.0265	0.0012770	1191.5	1229.8	3.0001	0.0012430	1167.7	1229.9	2.9547
300	0.0013611	1307.2	1334.4	3.2091	0.0013322	1288.9	1328.9	3.1761	0.0012879	1259.6	1324.0	3.1218
320	0.0014450	1416.6	1445.5	3.3996	0.0014014	1391.7	1433.7	3.3558	0.0013409	1354.3	1421.4	3.2888
340	0.0015693	1540.2	1571.6	3.6086	0.0014932	1502.4	1547.1	3.5438	0.0014049	1452.9	1523.1	3.4575
360	0.0018248	1703.6	1740.1	3.8787	0.0016276	1626.8	1675.6	3.7499	0.0014848	1556.5	1630.7	3.6301
380					0.0018729	1782.0	1838.2	4.0026	0.0015884	1667.1	1746.5	3.8102

TABLE A–8

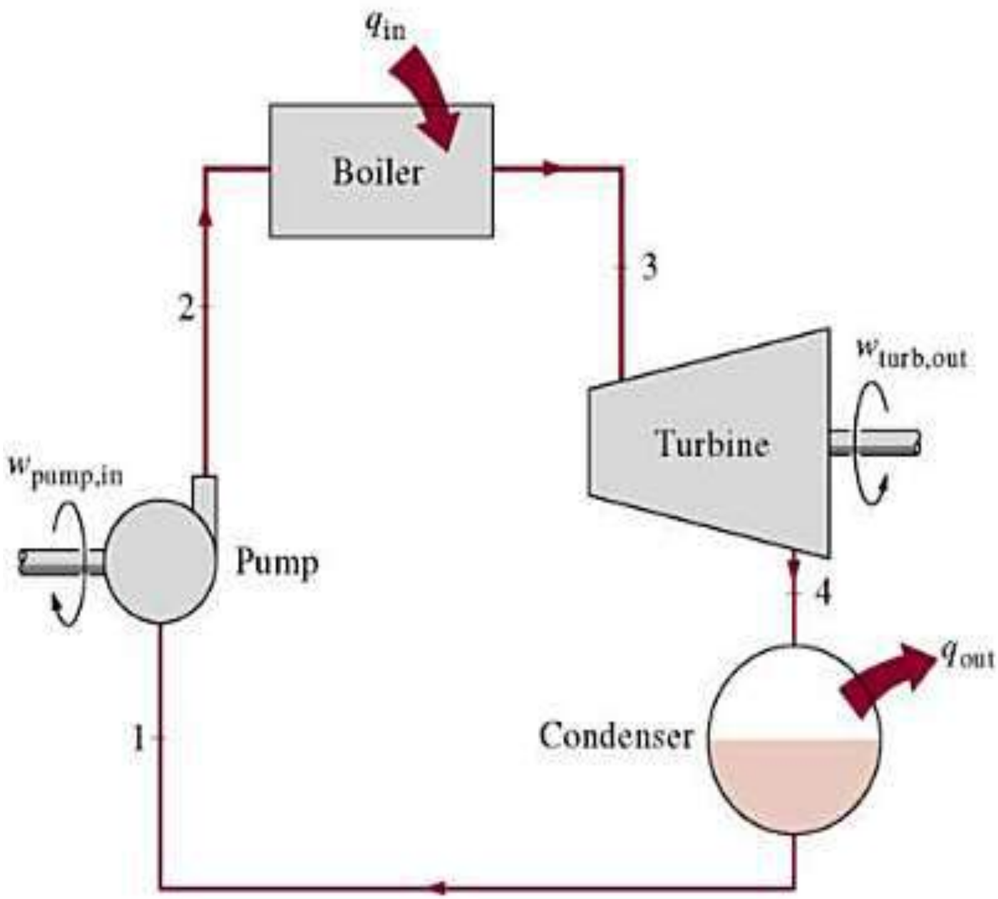
Saturated ice–water vapor

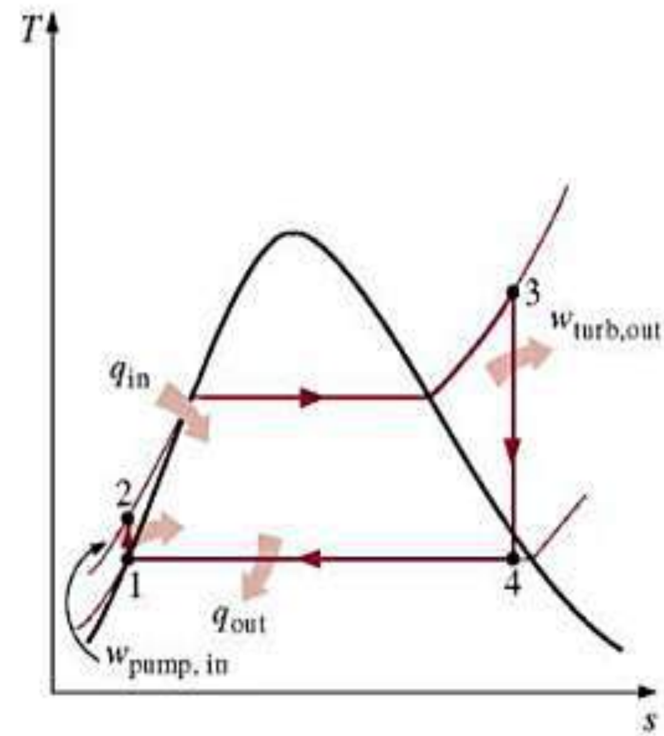
		Specific volume, m³/kg		Internal energy, kJ/kg			Enthalpy, kJ/kg			Entropy, kJ/kg · K		
Temp., T °C	Sat. press., P_{sat} kPa	Sat. ice, v_i	Sat. vapor, v_g	Sat. ice, u_i	Subl., u_{ig}	Sat. vapor, u_g	Sat. ice, h_i	Subl., h_{ig}	Sat. vapor, h_g	Sat. ice, s_i	Subl., s_{ig}	Sat. vapor, s_g
0.01	0.61169	0.001091	205.99	−333.40	2707.9	2374.5	−333.40	2833.9	2500.5	−1.2202	10.374	9.154
0	0.61115	0.001091	206.17	−333.43	2707.9	2374.5	−333.43	2833.9	2500.5	−1.2204	10.375	9.154
−2	0.51772	0.001091	241.62	−337.63	2709.4	2371.8	−337.63	2834.5	2496.8	−1.2358	10.453	9.218
−4	0.43748	0.001090	283.84	−341.80	2710.8	2369.0	−341.80	2835.0	2493.2	−1.2513	10.533	9.282
−6	0.36873	0.001090	334.27	−345.94	2712.2	2366.2	−345.93	2835.4	2489.5	−1.2667	10.613	9.347
−8	0.30998	0.001090	394.66	−350.04	2713.5	2363.5	−350.04	2835.8	2485.8	−1.2821	10.695	9.413
−10	0.25990	0.001089	467.17	−354.12	2714.8	2360.7	−354.12	2836.2	2482.1	−1.2976	10.778	9.480
−12	0.21732	0.001089	554.47	−358.17	2716.1	2357.9	−358.17	2836.6	2478.4	−1.3130	10.862	9.549
−14	0.18121	0.001088	659.88	−362.18	2717.3	2355.2	−362.18	2836.9	2474.7	−1.3284	10.947	9.618
−16	0.15068	0.001088	787.51	−366.17	2718.6	2352.4	−366.17	2837.2	2471.0	−1.3439	11.033	9.689
−18	0.12492	0.001088	942.51	−370.13	2719.7	2349.6	−370.13	2837.5	2467.3	−1.3593	11.121	9.761
−20	0.10326	0.001087	1131.3	−374.06	2720.9	2346.8	−374.06	2837.7	2463.6	−1.3748	11.209	9.835
−22	0.08510	0.001087	1362.0	−377.95	2722.0	2344.1	−377.95	2837.9	2459.9	−1.3903	11.300	9.909
−24	0.06991	0.001087	1644.7	−381.82	2723.1	2341.3	−381.82	2838.1	2456.2	−1.4057	11.391	9.985
−26	0.05725	0.001087	1992.2	−385.66	2724.2	2338.5	−385.66	2838.2	2452.5	−1.4212	11.484	10.063
−28	0.04673	0.001086	2421.0	−389.47	2725.2	2335.7	−389.47	2838.3	2448.8	−1.4367	11.578	10.141
−30	0.03802	0.001086	2951.7	−393.25	2726.2	2332.9	−393.25	2838.4	2445.1	−1.4521	11.673	10.221
−32	0.03082	0.001086	3610.9	−397.00	2727.2	2330.2	−397.00	2838.4	2441.4	−1.4676	11.770	10.303
−34	0.02490	0.001085	4432.4	−400.72	2728.1	2327.4	−400.72	2838.5	2437.7	−1.4831	11.869	10.386
−36	0.02004	0.001085	5460.1	−404.40	2729.0	2324.6	−404.40	2838.4	2434.0	−1.4986	11.969	10.470
−38	0.01608	0.001085	6750.5	−408.07	2729.9	2321.8	−408.07	2838.4	2430.3	−1.5141	12.071	10.557
−40	0.01285	0.001084	8376.7	−411.70	2730.7	2319.0	−411.70	2838.3	2426.6	−1.5296	12.174	10.644

Rankine Cycle: The Ideal Cycle for Vapor Power Cycles

- Operating principles
- Vapor power plants
- The ideal Rankine vapor power cycle
- Efficiency
  - Improved efficiency - superheat

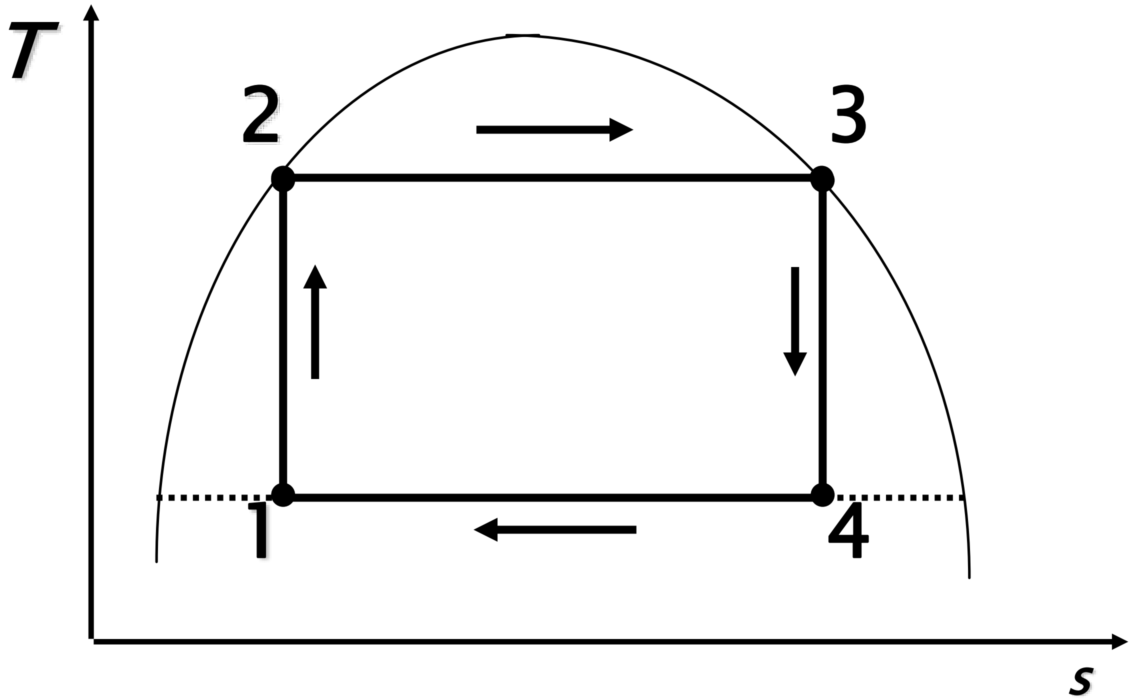
The simple ideal Rankine cycle.



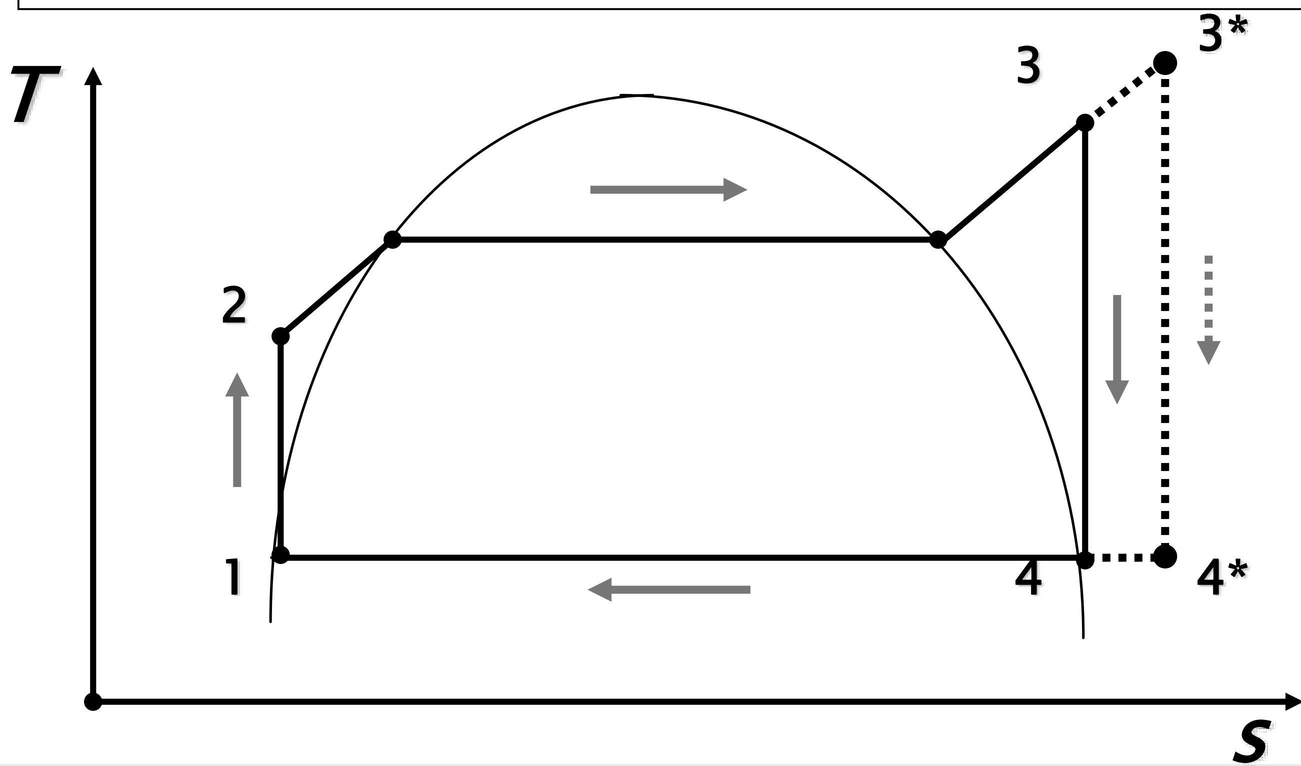


A hypothetical vapor power cycle

- Assume a Carnot cycle operating between
- two fixed temperatures as shown.

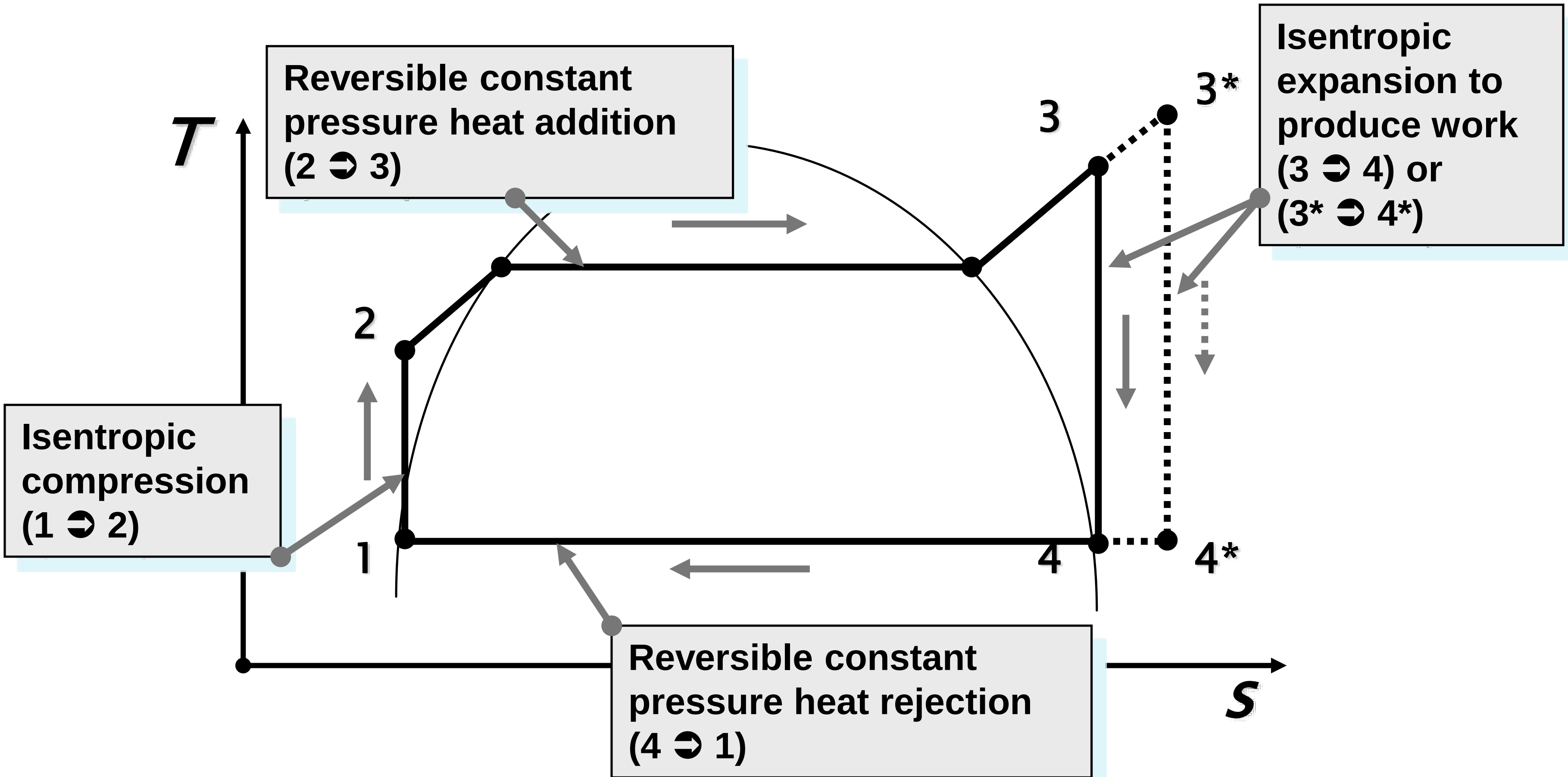


The ideal Rankine cycle



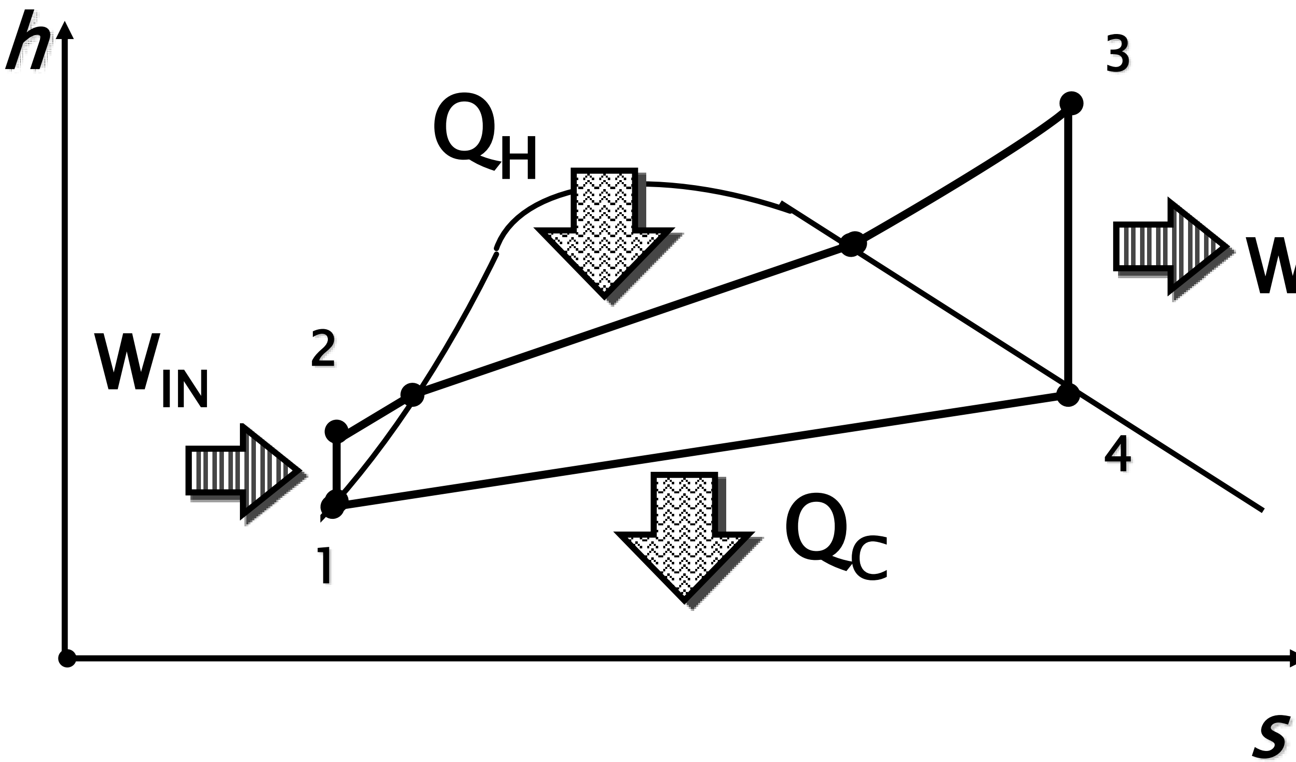
All processes are internally reversible.

The ideal Rankine cycle

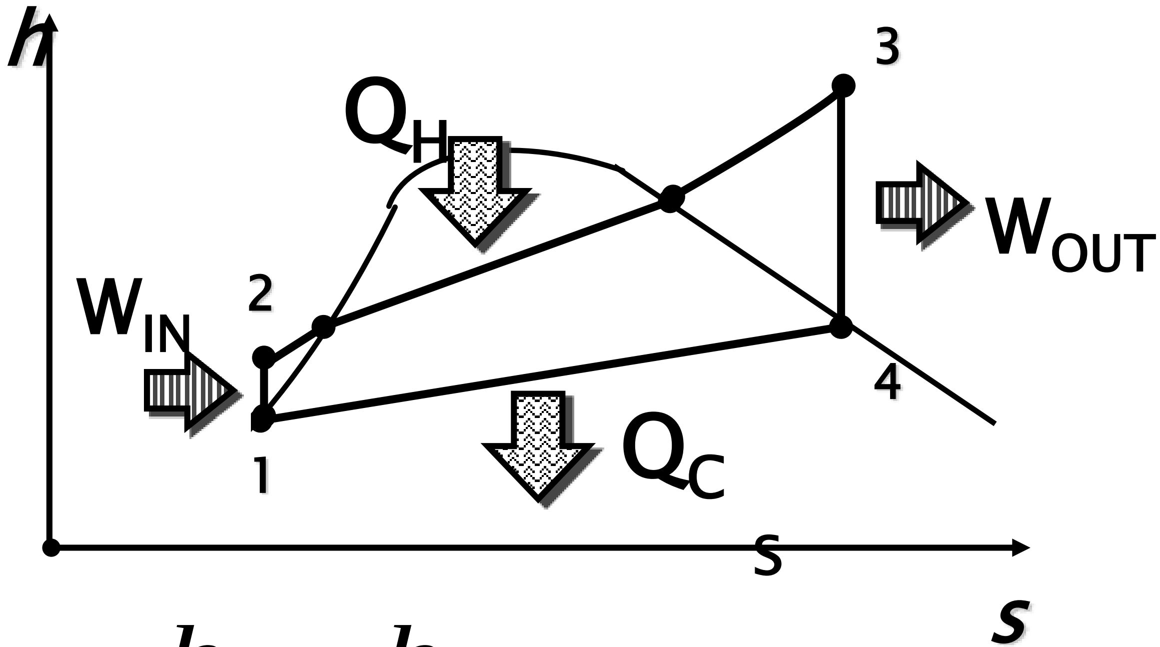


All processes are internally reversible.

The ideal Rankine cycle (h-s diagram)



Rankine cycle efficiency



$$W_{OUT} = h_3 - h_4$$

$$W_{IN} = h_2 - h_1$$

$$Q_H = h_3 - h_4$$

$$\eta = \frac{W_{NET}}{Q_H} \qquad (3)$$

FIGURE 10–6

The effect of lowering the condenser pressure on the ideal Rankine cycle.

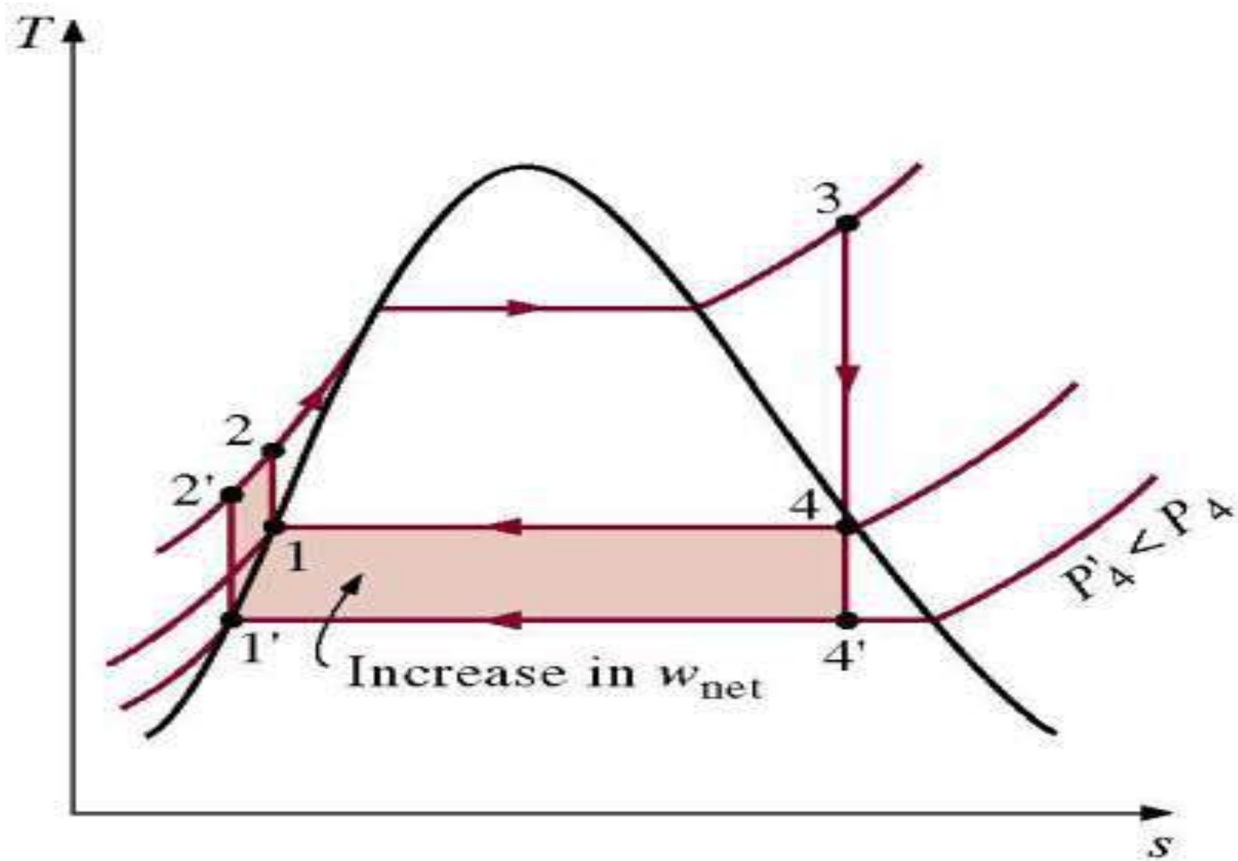


FIGURE 10-7

The effect of superheating the steam to higher temperatures on the ideal Rankine cycle.

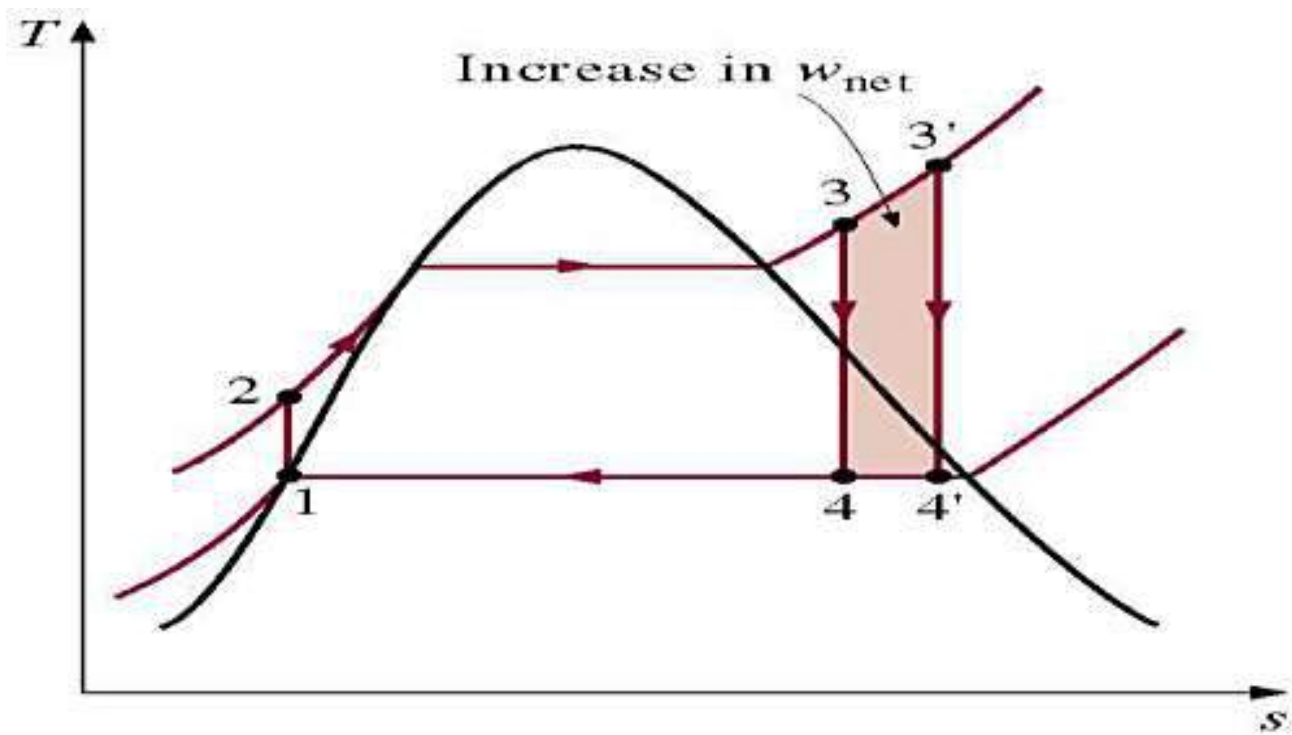


FIGURE 10-8
The effect of increasing the boiler pressure on the ideal Rankine cycle.

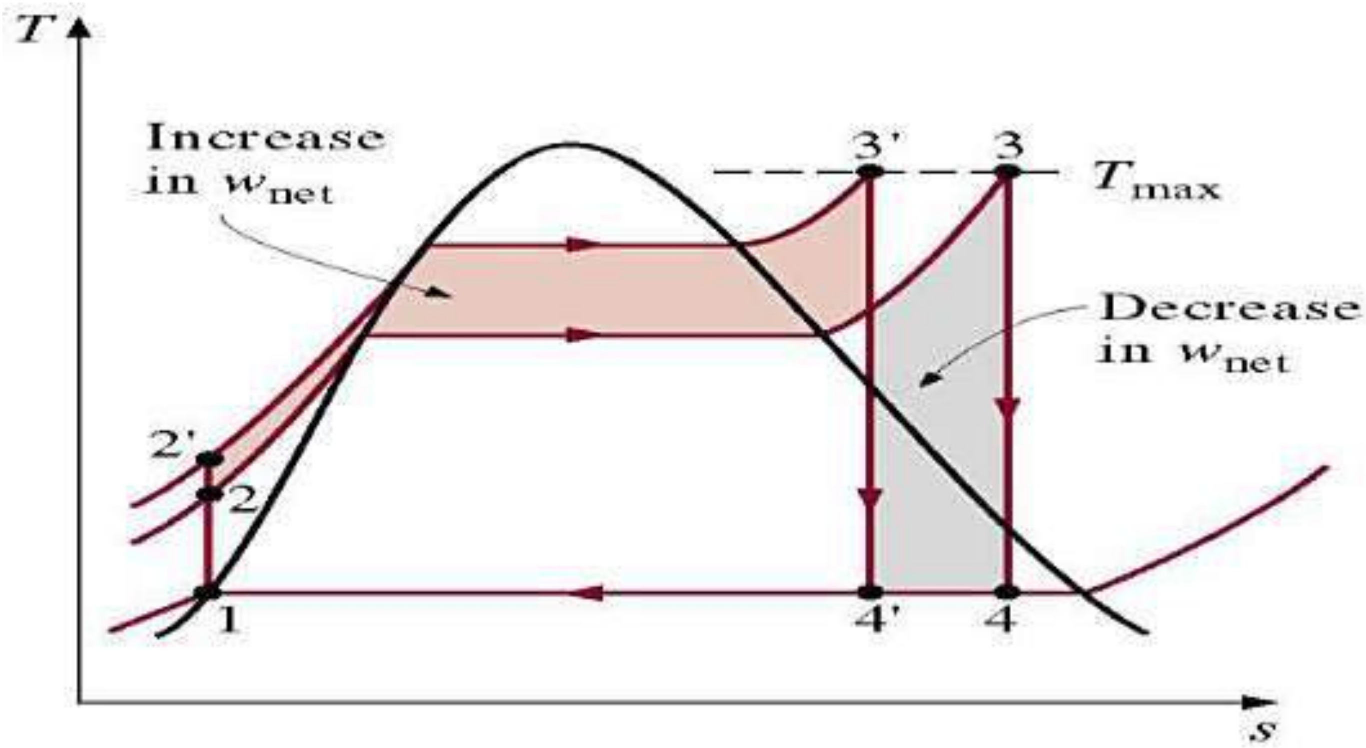
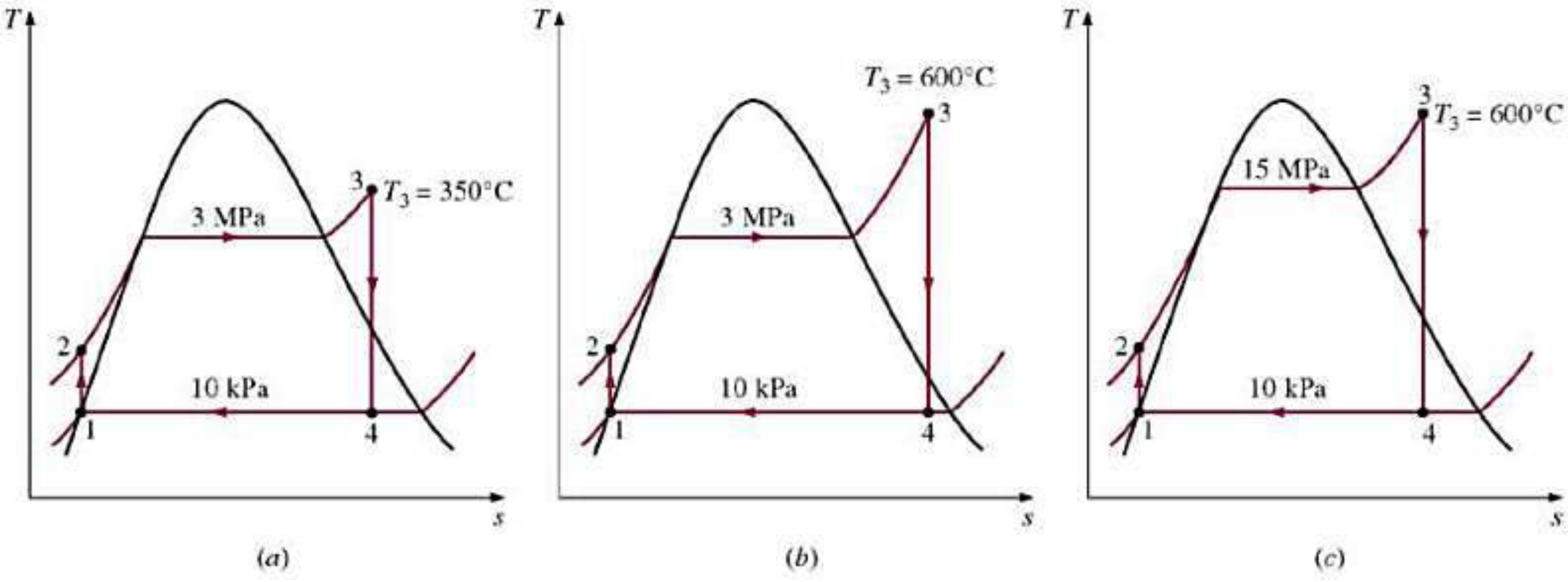
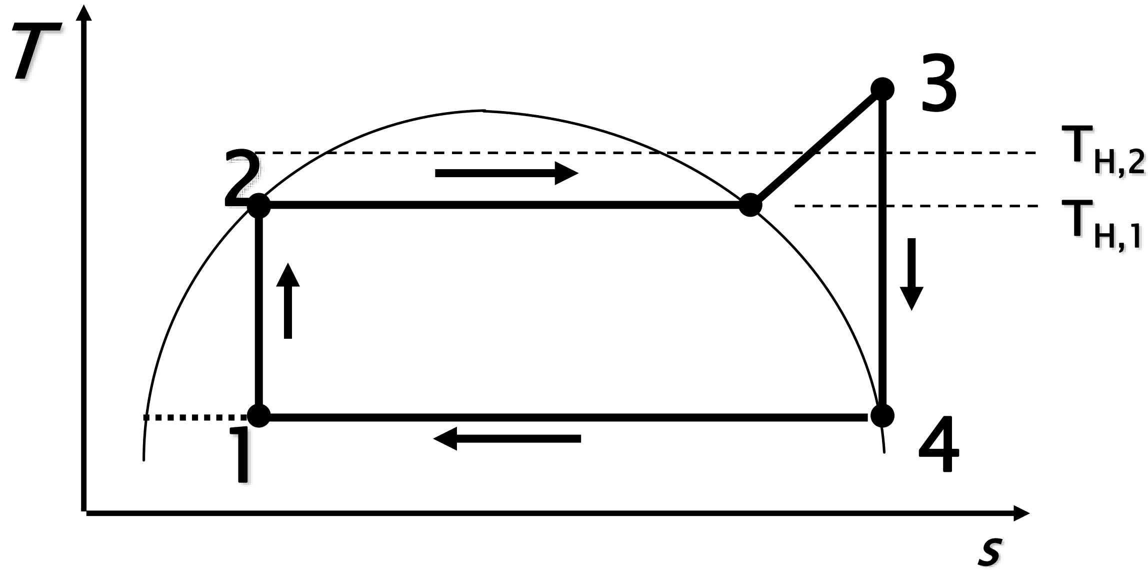


FIGURE 10-10
T-s **diagrams of the three cycles discussed in Example 9–3.**

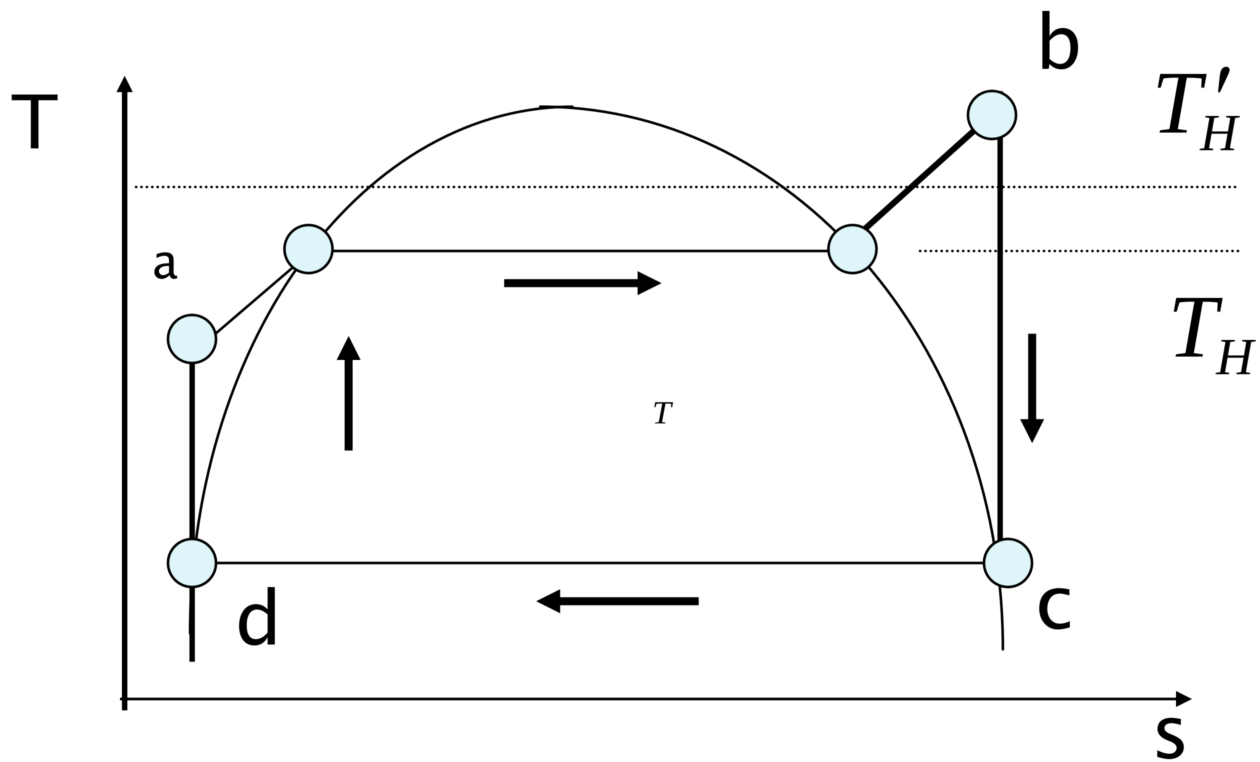


A hypothetical vapor power cycle with superheat



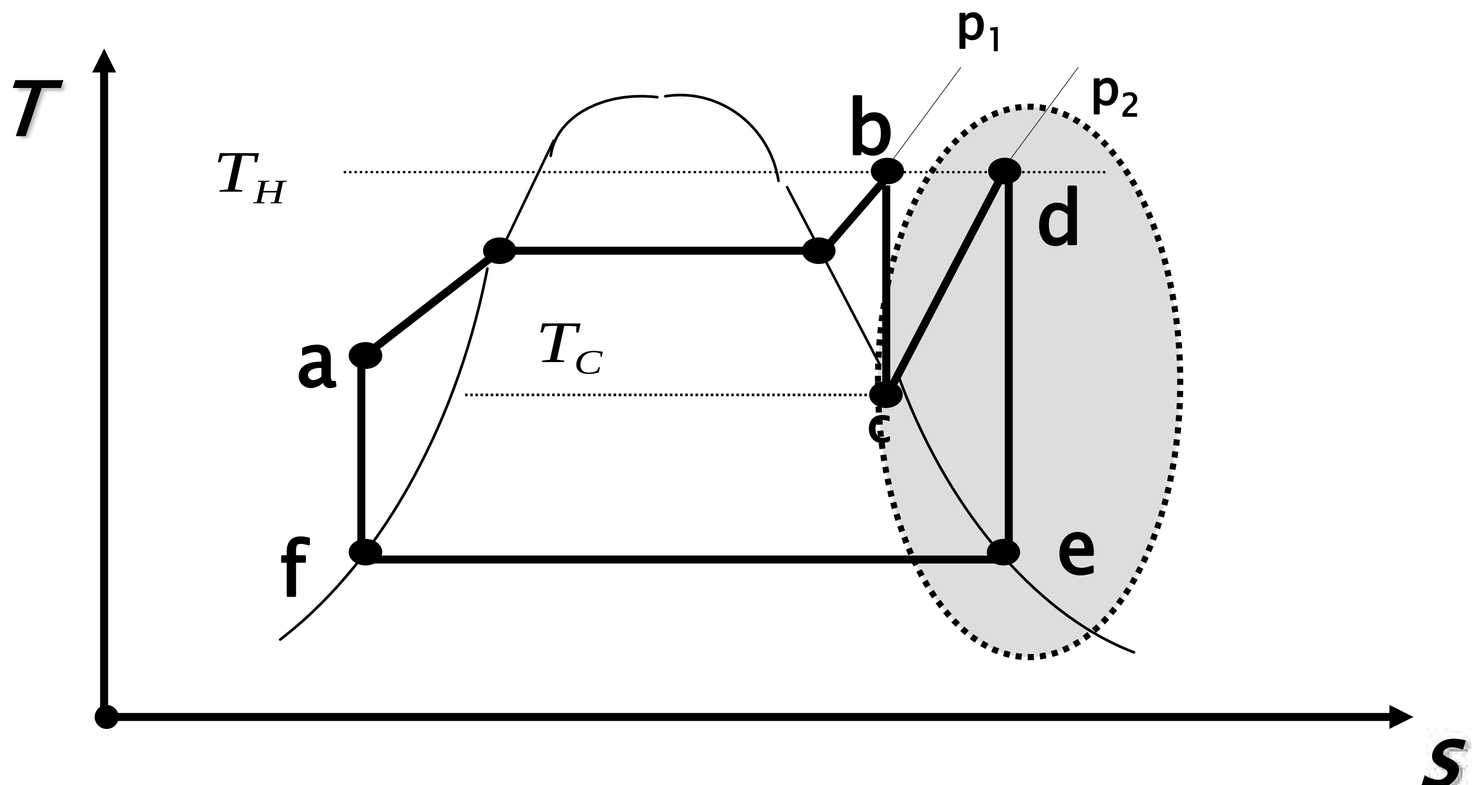
Superheating the working fluid raises the average temperature of heat addition.

A hypothetical vapor power cycle: A Rankine cycle with superheat



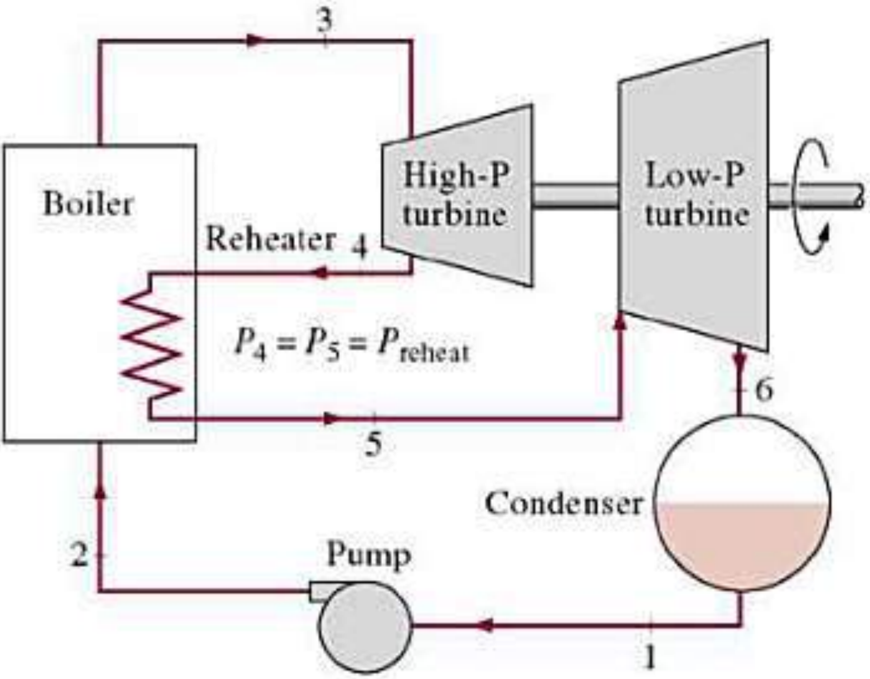
Superheating the working fluid raises the average temperature with a reservoir at a higher temperature.

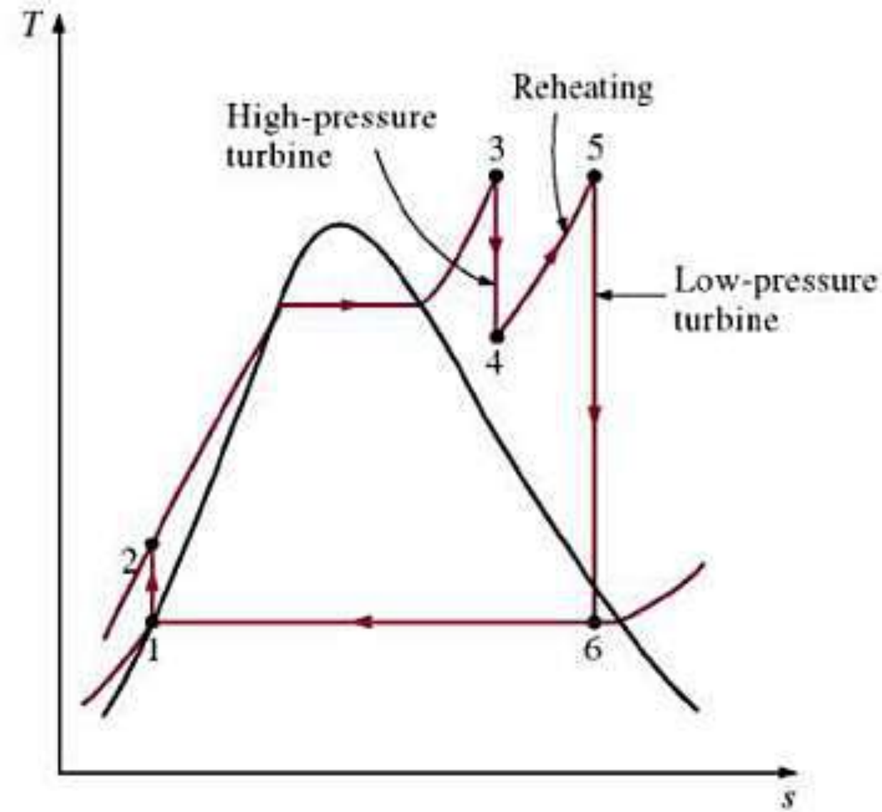
The Rankine cycle with reheat



The extra expansion via reheating to state “d” allows a greater enthalpy to be released between states “c” to “e”.

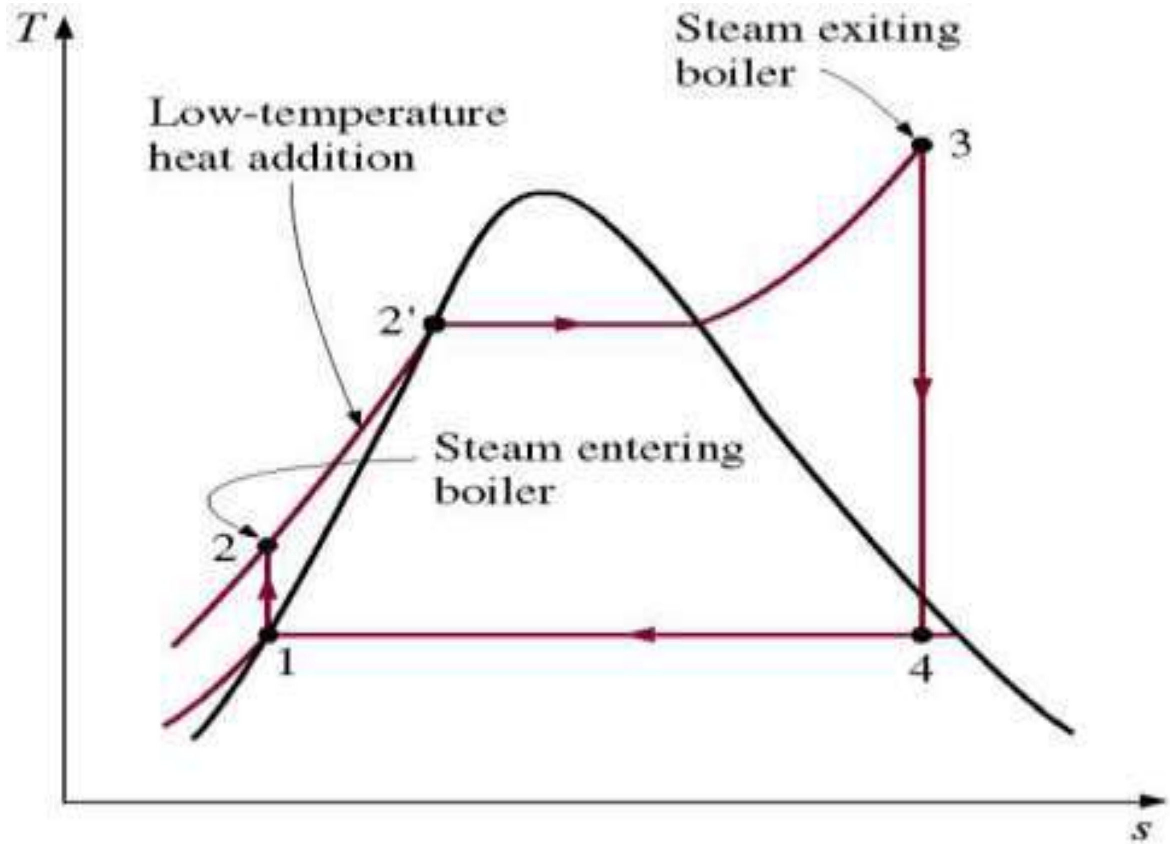
FIGURE 10–11
The ideal reheat Rankine cycle.



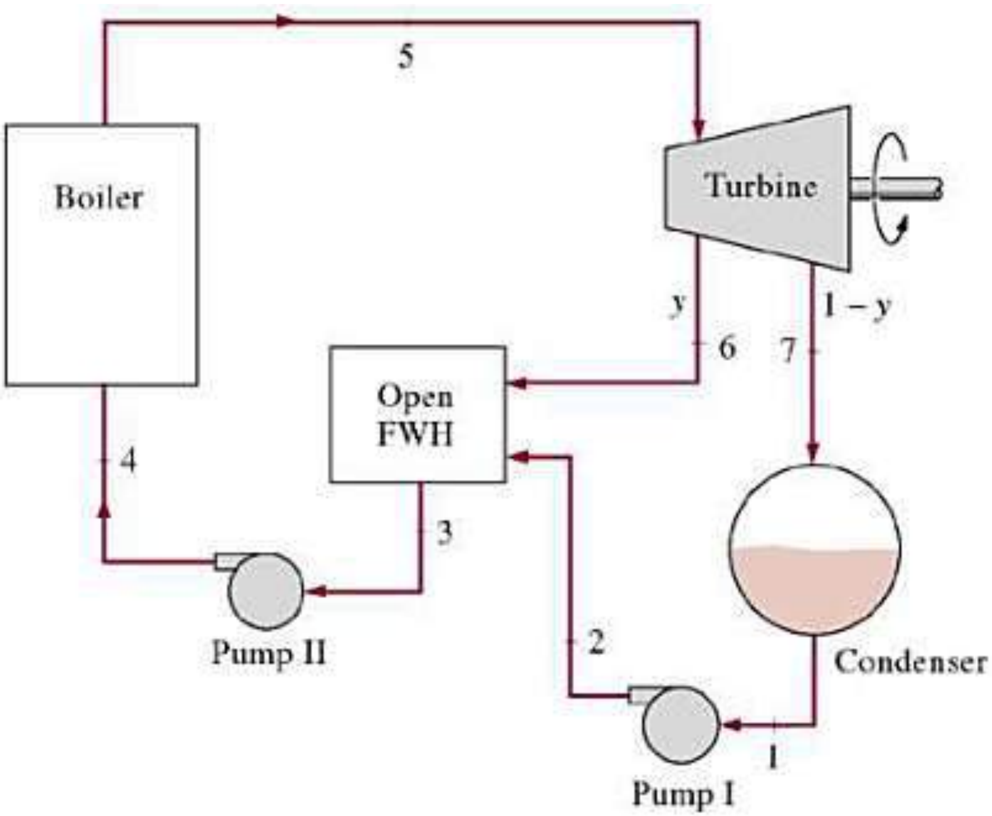


The Rankine cycle with regeneration

The first part of the heat-addition process in the boiler takes place at relatively low temperatures.



The ideal regenerative Rankine cycle with an open feedwater heater.



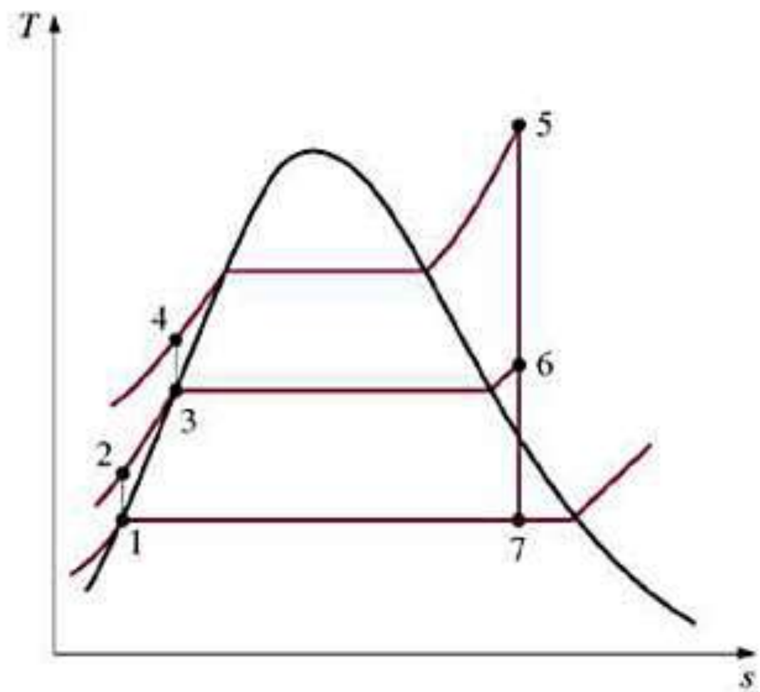
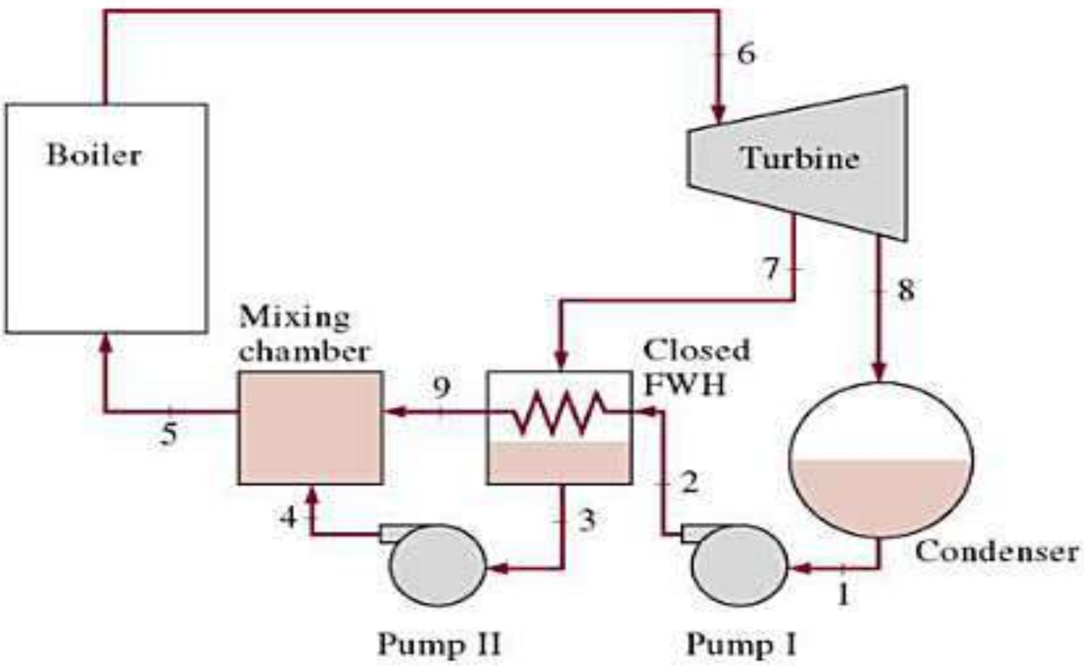


FIGURE 10-16
The ideal regenerative Rankine cycle with a closed feedwater heater.



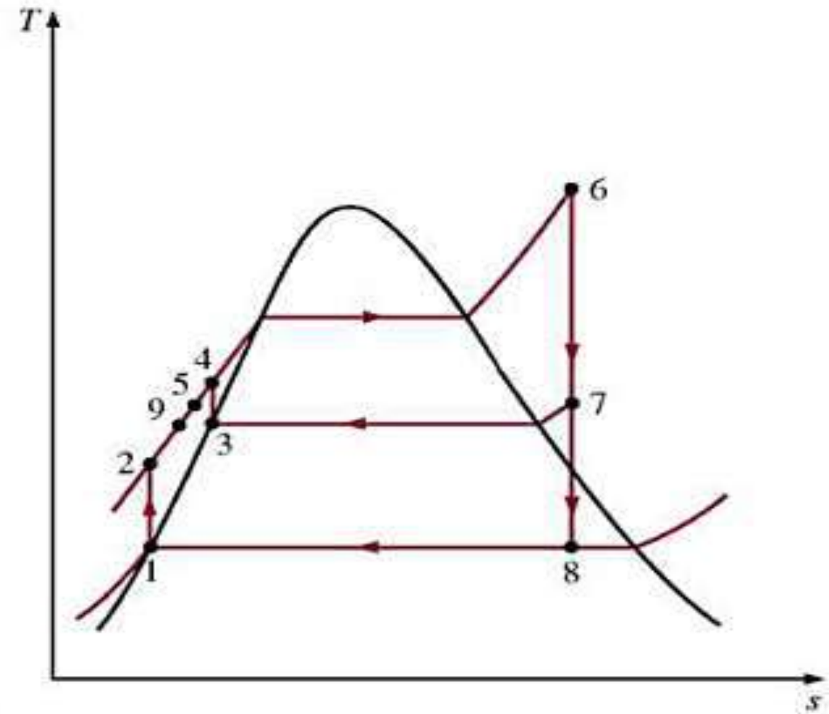
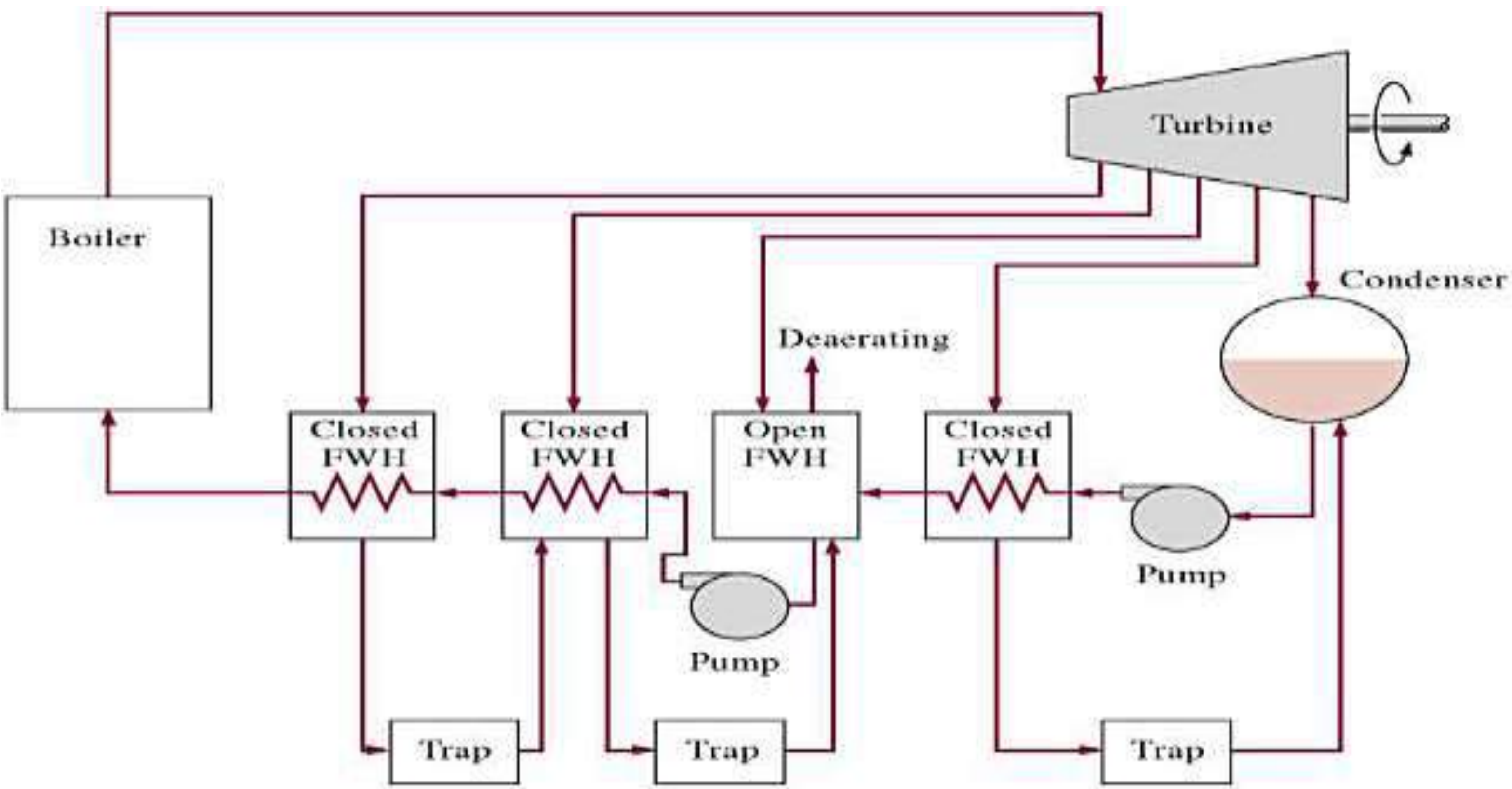


FIGURE 10-17

A steam power plant with one open and three closed feedwater heaters.



Commercial steam-power plants

- Reheat (multiple stages)
- Regeneration (multiple extractions)
- Nearly ideal heat addition
  - Constant temperature boiling for water

Commercial steam-power plants

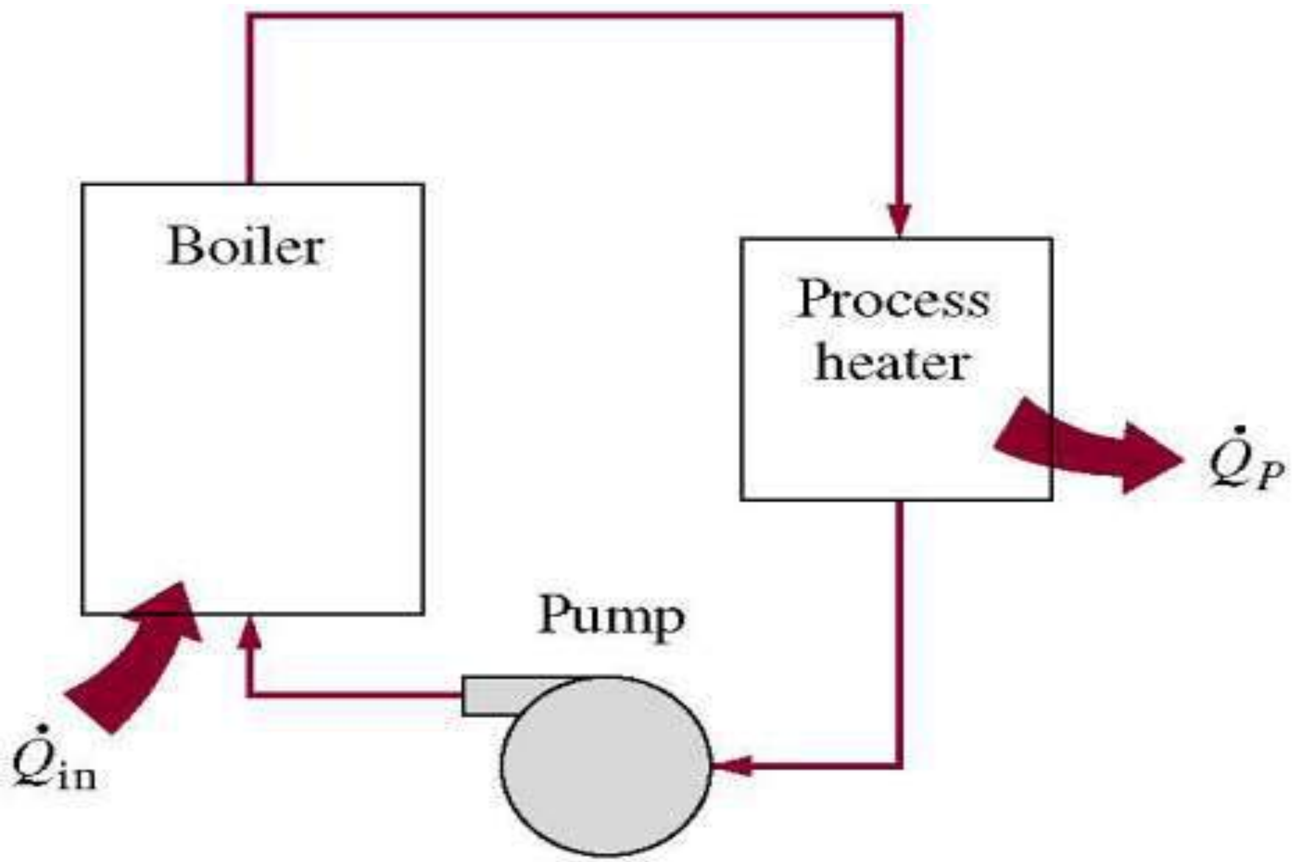
- Heat transfer characteristics of steam and water permit external combustion systems
- Compression of condensed liquid produces a favorable work ratio.

Commercial steam-power plants

- The Rankine cycle with reheat and regeneration is advantageous for large plants.
- Small plants do not have economies of scale
  - Internal combustion for heat addition.
  - A different thermodynamic cycle

Cogeneration

A simple process-heating plant.



An ideal cogeneration plant.

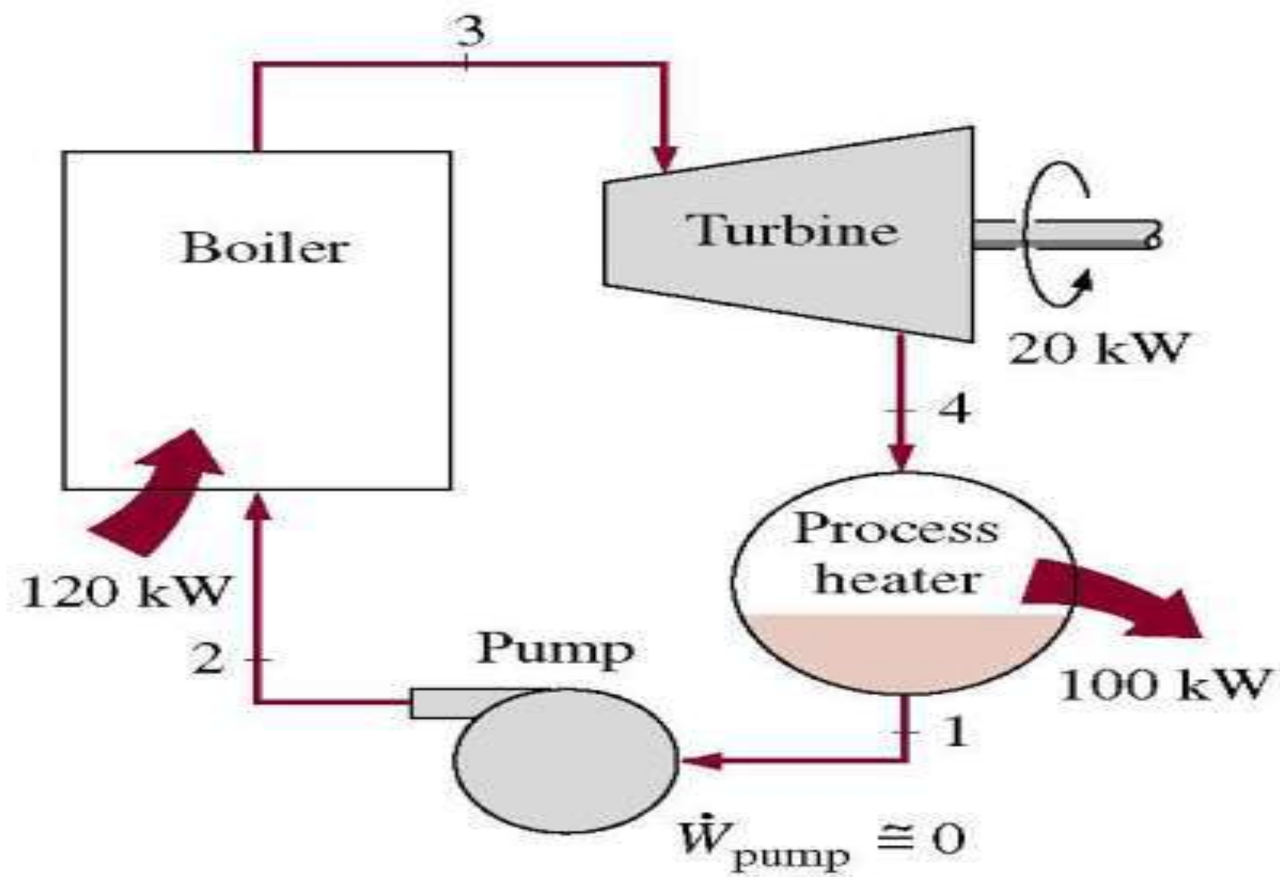
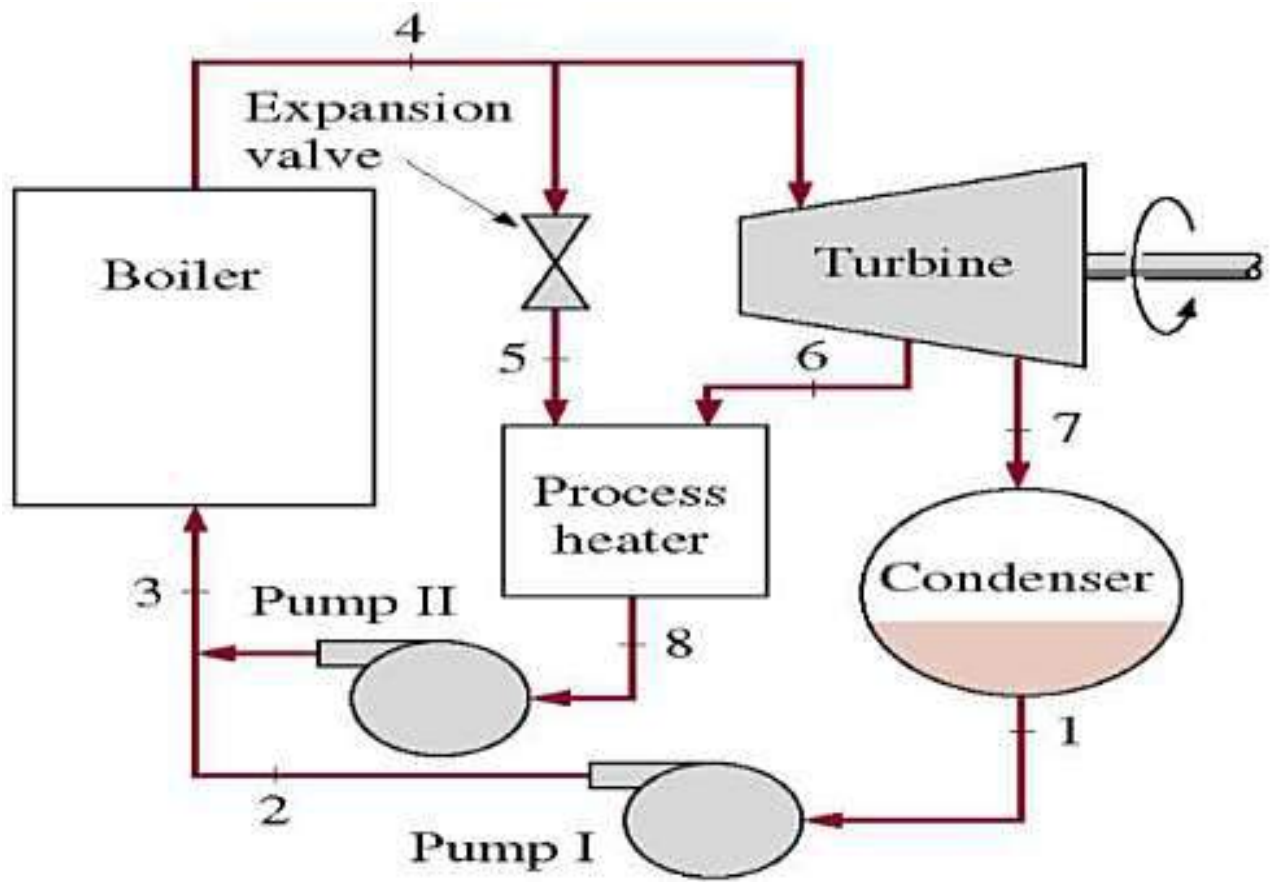
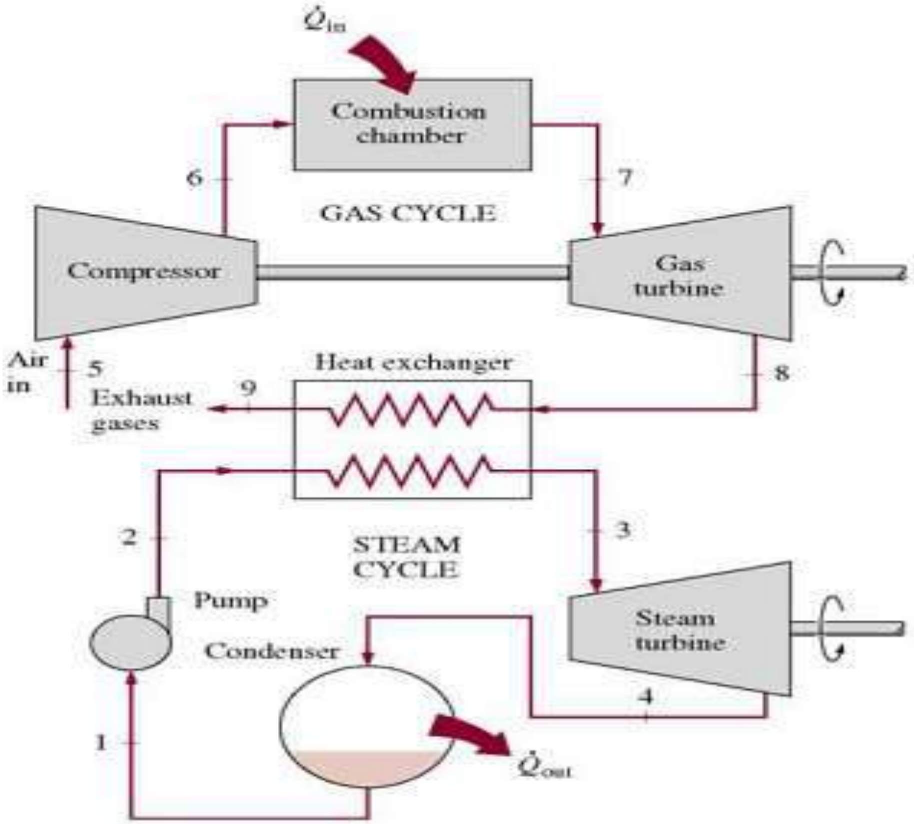


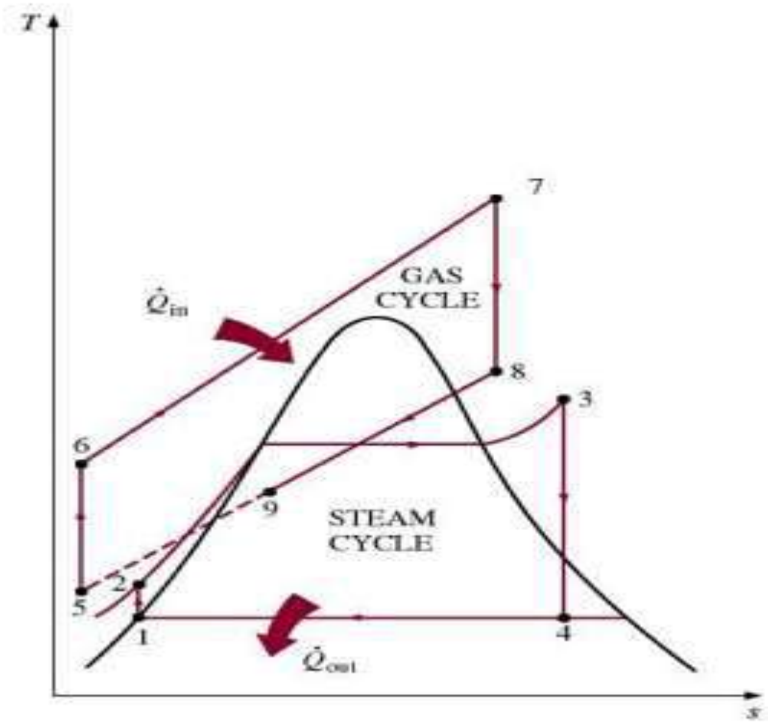
FIGURE 10-22
A cogeneration plant with adjustable loads.



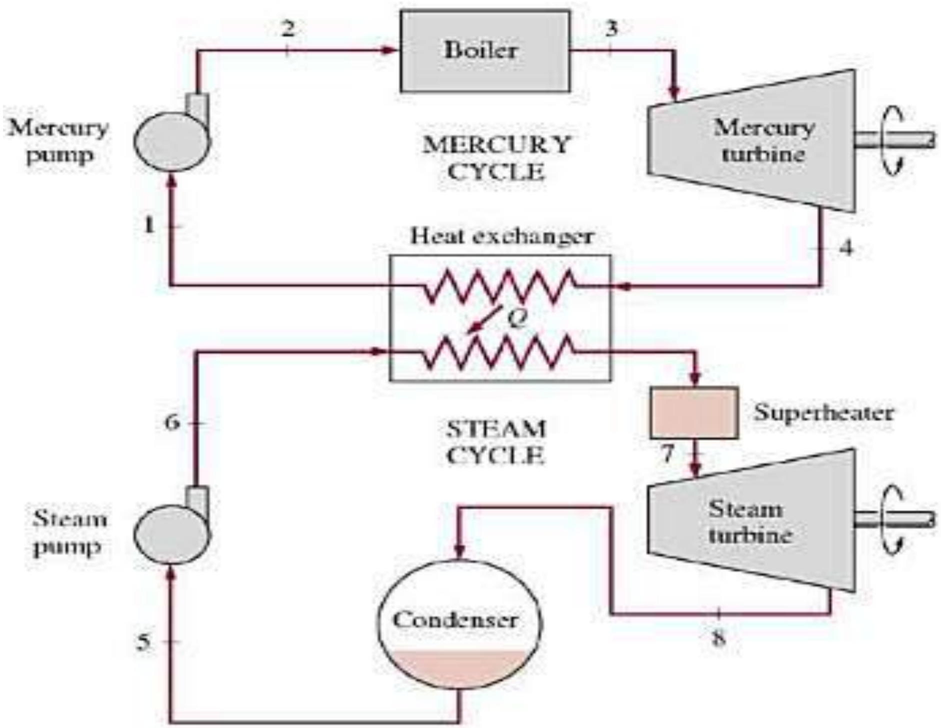
10-9 Combined /gas-Vapor Power Cycles

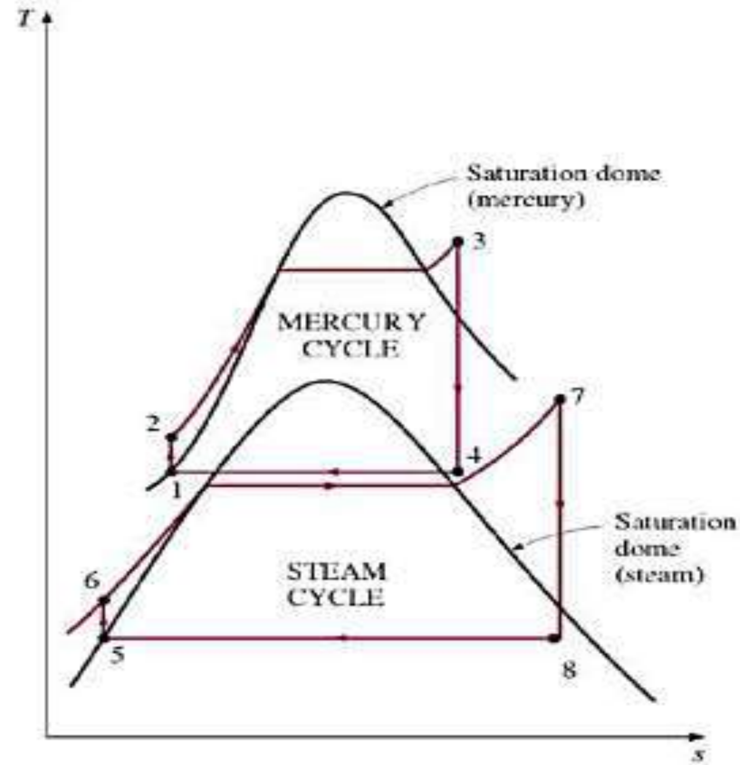
Combined gas–steam power plant.





Mercury–water binary vapor cycle.

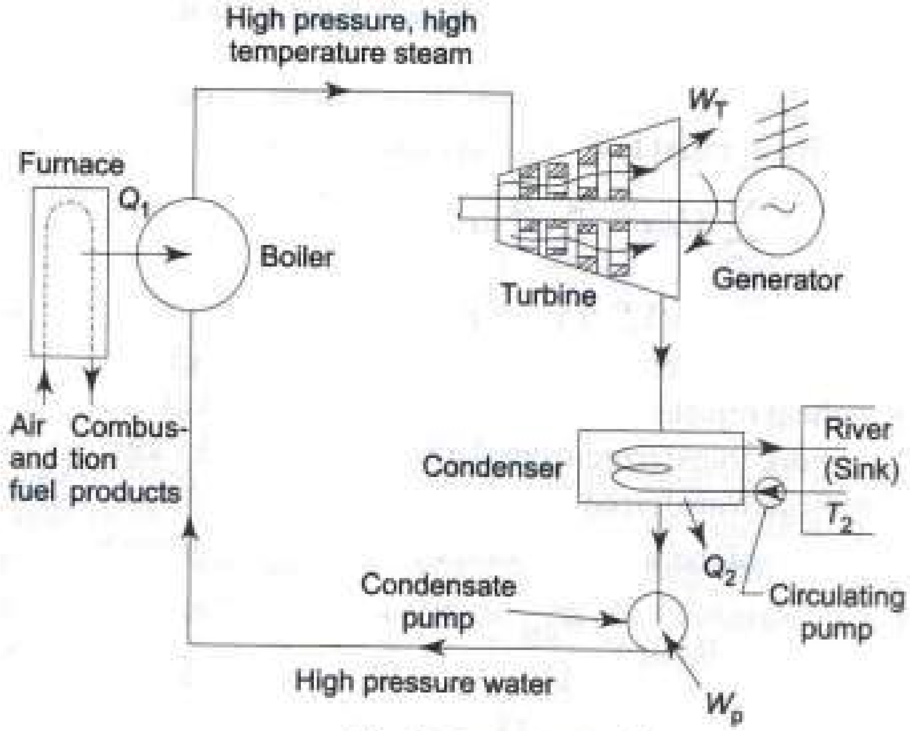




UNIT – 4
VAPOUR POWER CYCLES

Carnot vapour power cycle, drawbacks as a reference cycle, simple Rankine cycle; description, T-s diagram, analysis for performance. Comparison of Carnot and Rankine cycles. Effects of pressure and temperature on Rankine cycle performance. Actual vapour power cycles. Ideal and practical regenerative Rankine cycles, open and closed feed water heaters. Reheat Rankine cycle.

Vapour power cycles are used in steam power plants. In a power cycle heat energy (released by the burning of fuel) is converted into work (shaft work), in which a working fluid repeatedly performs a succession of processes. In a vapour power cycle, the working fluid is water, which undergoes a change of phase.



Simple steam power plant

Figure shows a simple steam power plant working on the vapour power cycle. Heat is transferred to the water in the boiler (Q_H) from an external source. (Furnace, where fuel is continuously burnt) to raise steam, the high pressure high temperature steam leaving the boiler expands in the turbine to produce shaft work (W_T), the steam leaving the turbine condenses into water in the condenser (where cooling water circulates), rejecting heat (Q_L), and then the water is pumped back (W_P) to the boiler.

Since the fluid is undergoing a cyclic process, the net energy transferred as heat during the cycle must equal the net energy transfer as work from the fluid.

By the 1st law of Thermodynamics,

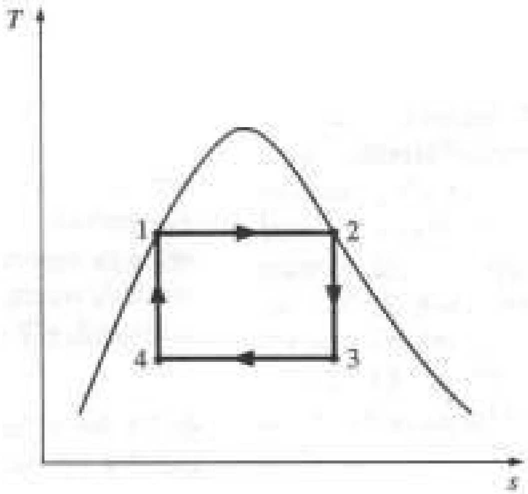
$$\sum_{cycle} Q_{net} = \sum_{cycle} W_{net}$$

Or $$Q_H - Q_L = W_T - W_P$$

Where Q_H = heat transferred to the working fluid (kJ/kg)
Q_L = heat rejected from the working fluid (kJ/kg)
W_T = work transferred from the working fluid (kJ/kg)
W_P = work transferred into the working fluid (kJ/kg)

$$\therefore \quad \eta_{cycle} = \frac{W_{net}}{Q_H} = \frac{W_T - W_P}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

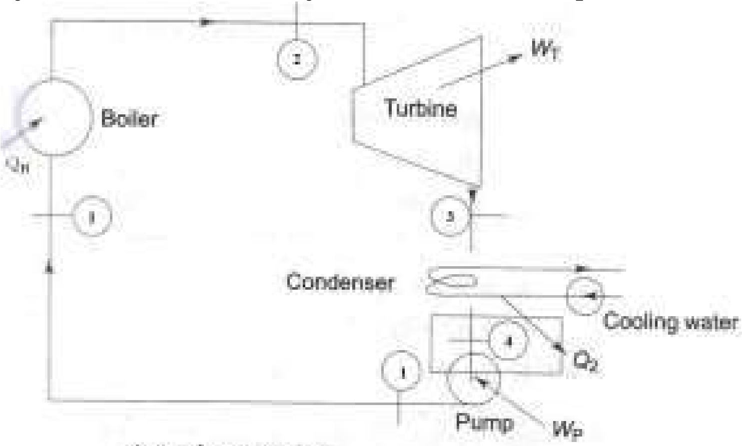
Idealized steam power cycles:



We know that the efficiency of a Carnot engine is maximum and it does not depend on the working fluid. It is, therefore, natural to examine of a steam power plant can be operated on the Carnot cycle.

Figure shows the Carnot cycle on the T-S diagram. Heat addition at constant pressure P_2, can be achieved isothermally in the process 1-2 in a boiler. The decrease in pressure from P_2 to P_3 in the process 2-3 can also be attained through the performance of work in a steam turbine. But in order to bring back the saturated liquid water to the boiler at the state 1, the condensation process 3-4 in the condenser must be terminated at the state 4, where the working fluid is a mixture of liquid water and vapour. But it is practically impossible to attain a condensation of this kind. Difficulty is also experienced in compressing isentropically the binary mixture from state 4 to the initial state 1, where the working fluid is entirely in the liquid state. Due to these inherent practical difficulties, Carnot cycle remains an ideal one.

Rankine Cycle: The simplest way of overcoming the inherent practical difficulties of the Carnot cycle without deviating too much from it is to keep the processes 1-2 and 2-3 of the latter unchanged and to continue the process 3-4 in the condenser until all the vapour has been converted into liquid water. Water is then pumped into the boiler upto the pressure corresponding to the state 1 and the cycle is completed. Such a cycle is known as the Rankine cycle. This theoretical cycle is free of all the practical limitations of the Carnot cycle.

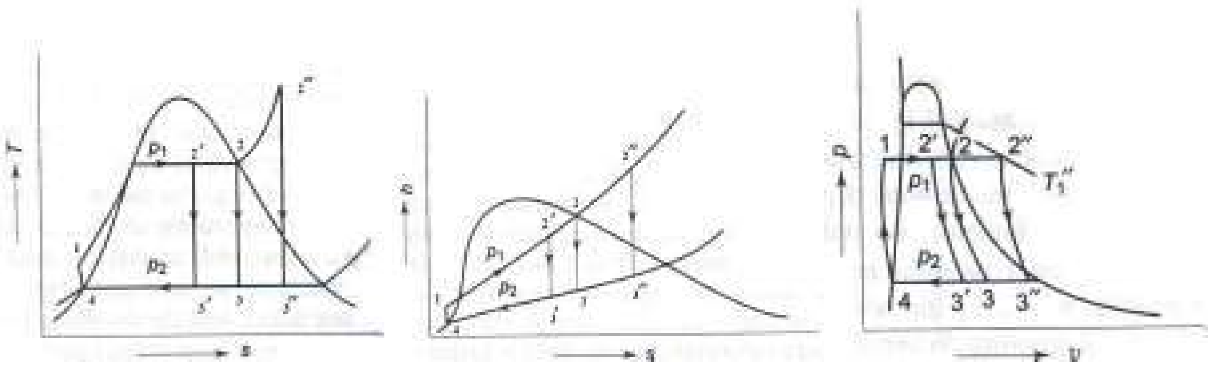


A simple steam plant

Figure (a) shows the schematic diagram for a simple steam power cycle which works on the principle of a Rankine cycle. Figure (b) represents the T-S diagram of the cycle.

The Rankine cycle comprises the following processes.

Process 1-2: Constant pressure heat transfer process in the boiler
Process 2-3: Reversible adiabatic expansion process in the steam turbine
Process 3-4: Constant pressure heat transfer process in the condenser and
Process 4-1: Reversible adiabatic compression process in the pump.



The numbers on the plots correspond to the numbers on the schematic diagram. For any given pressure, the steam approaching the turbine may be dry saturated (state 2), wet (state 2^1) or superheated (state 2^{11}), but the fluid approaching the pump is, in each case, saturated liquid (state 4). Steam expands reversibly and adiabatically in the turbine from state 2 to state 3 (or 2^1 to 3^1 or 2^{11} to 3^{11}), the steam leaving the turbine condenses to water in the condenser reversibly at constant pressure from state 3 (or 3^1, or 3^{11}) to state 4. Also, the water is heated in the boiler to form steam reversibly at constant pressure from state 1 to state 2 (or 2^1 or 2^{11})

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Applying SFEE to each of the processes on the basis of unit mass of fluid and neglecting changes in KE & PE, the work and heat quantities can be evaluated.

For 1kg of fluid, the SFEE for the boiler as the CV, gives,
$h_1 + Q_H = h_2$ i.e., $Q_H = h_2 - h_1$ --- (1)
SFEE to turbine, $h_2 = W_T + h_3$ i.e., $W_T = h_2 - h_3$ --- (2)
SFEE to condenser, h_3 $Q_L + h_4$ i.e., $Q_L = h_3 - h_4$ --- (3)
SFEE to pump, $h_4 + W_P = h_1$ i.e., $W_P = h_1 - h_4$ --- (4)

The efficiency of Rankine cycle is $\eta = \dfrac{W_{net}}{Q_H} = \dfrac{W_T - W_P}{Q_H}$

i.e., $\eta = \dfrac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$ or $\eta = \dfrac{(h_2 - h_1) - (h_3 - h_4)}{(h_2 - h_1)}$

The pump handles liquid water which is incompressible i.e., its density or specific volume undergoes little change with an increase in pressure.

For reversible adiabatic compression, we have Tds = dh – vdp; since ds = 0
We have, dh = vdp

Since change in specific volume is negligible, $\Delta h = v \, \Delta P$

Or $(h_1 - h_4) = v_4 (P_2 - P_3)$

Usually the pump work is quite small compared to the turbine work and is sometimes neglected. In that case, $h_1 = h_4$

$$\eta_{rankine} \cong \frac{(h_2 - h_3)}{(h_2 - h_1)} \cong \frac{(h_2 - h_3)}{(h_2 - h_4)}$$

The efficiency of the Rankine cycle is presented graphically in the T-S diagram

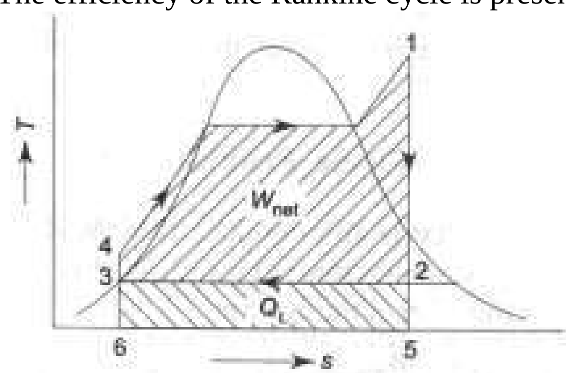


Fig. *Q_1, W_{net} and Q_L are proportional to areas*

$Q_H \propto$ area 2-5-6-1,
$Q_L \propto$ area 3-5-6-4

$W_{net} = (Q_H - Q_L)$ = area 1-2-3-4 enclosed by the cycle.

The capacity of the steam plant is expressed in terms of steam rate defined as the rate of steam flow (kg/h) required to produce unit shaft output (1kW)

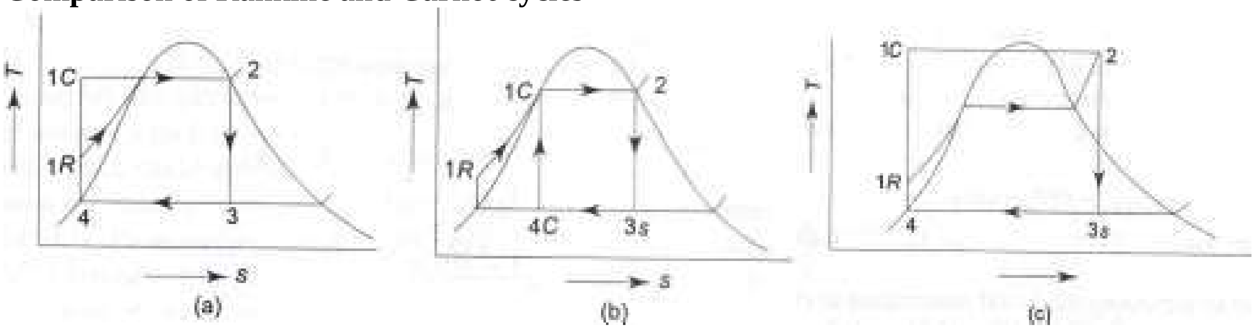
$$\therefore \quad \text{Steam rate} = \frac{1}{W_T - W_P} \; \frac{kg}{kJ} \; \frac{1kJ/s}{1kW} = \text{Specific Steam Consumption}(SSC)$$
$$= \frac{1}{W_T - W_P} \; \frac{kg}{kWs} = \frac{3600}{W_T - W_P} \; \frac{kg}{kWh}$$

The cycle efficiency also expressed alternatively as heat rate which is the rate of heat input (Q_H) required to produce unit work output (1kW)

$$\text{Heat rate} = \frac{3600\, Q_H}{W_T - W_P} = \frac{3600}{\eta_{cycle}} \; \frac{kJ}{kWh}$$

$$\text{Lastly, work ratio } r_w = \frac{\oint \delta W}{\text{positive work}} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_3)}$$

Comparison of Rankine and Carnot cycles



Carnot cycle has the maximum possible efficiency for the given limits of temperature. But it is not suitable in steam power plants. Figure shows the Rankine and Carnot cycles on the T-S diagram.
The reversible adiabatic expansion in the turbine, the constant temperature heat rejection in the condenser, and the Reversible adiabatic compression in the pump, are similar characteristic features of both the Rankine and Carnot cycles. But whereas the heat addition process in the Rankine cycle is reversible and at constant pressure, in the carnot cycle it is reversible and isothermal.

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In Figures (a) and (c), Q_L is the same in both the cycles, but since Q_H is more, $\eta_C > \eta_R$. The two carnot cycles in Figure (a) and (b) have the same thermal efficiency. $\therefore$ in Figure (b) also $\eta_C > \eta_R$.

But the Carnot cycle cannot be realized in practice because the pump work is very large. Whereas in (a) and (c) it is impossible to add heat at infinite pressures and at constant temperature from state 1C to state 2, in (b), it is difficult to control the quality at 4C, so that isentropic compression leads to a saturated liquid state.

Mean temperature of Heat addition

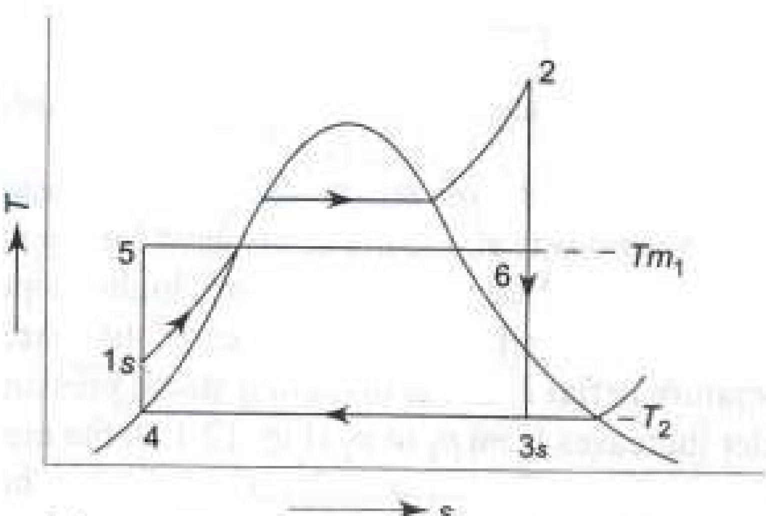

Fig. *Mean temperature of heat addition*

In the Rankine cycle, heat is added reversibly at a constant pressure, but at infinite temperatures. Let T_{m1}, is the mean temperature of heat addition, so that area under 1s and 2 is equal to the area under 5-6.

Heat added, $Q_H = h_2 - h_{1S} = T_{m1}(S_2 - S_{1S})$

$\therefore T_{m1}$ = Mean temperature of heat addition $= \dfrac{h_2 - h_{1S}}{S_2 - S_{1S}}$

Q_L = heat rejected $= h_{3S} - h_4$

$= T_3(S_2 - S_{1S})$

$$\therefore \quad \eta_R = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_3(S_2 - S_{1S})}{T_{m1}(S_2 - S_{1S})}$$

$$\eta_R = 1 - \frac{T_3}{T_{m1}}$$ where T_3 = temperature of heat rejection.

As T_3 is lowered for a given T_{m1}, the $\eta_R \uparrow$. But the lowest practical temperature of heat rejection is the ambient temperature T_0

i.e., $\eta_R = f(T_{m1})$ only.

Or higher the mean temperature of heat addition, the higher will be the cycle efficiency.

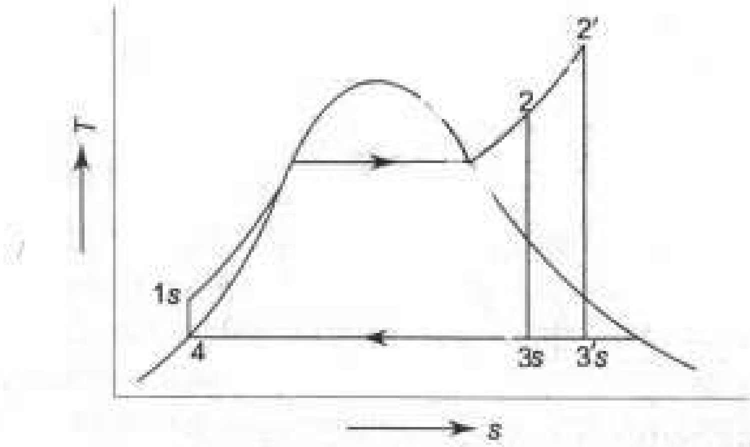

Fig. *Effect of superheat on mean temperature of heat addition*

The effect of increasing the initial temperature at constant pressure on cycle efficiency is shown in Figure. When the initial state changes from 2 to 2^1, T_{m1}, between 2 and 2^1 is higher than T_{m1} between 1s and 2. So an increase in the superheat at constant pressure increases the mean temperature of heat addition and hence the cycle η.

But the maximum temperature of steam that can be used is fixed from metallurgical considerations (i.e., materials used for the manufacture of the components which are subjected to high pressure, high temperature steam such as super heaters, valves, pipelines, inlet stages of turbines etc).

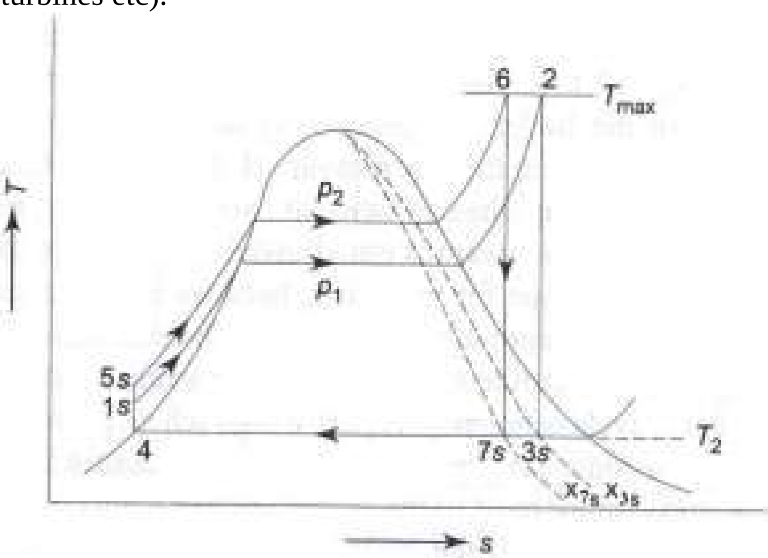

Fig. *Effect of increase of pressure on Rankine cycle*

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When the maximum temperature is fixed, as the operating steam pressure at which heat is added in the boiler increases from P_1 to P_2, the mean temperature of heat addition increases (since T_{m1} between 5_S and 6 higher than between 1_S and 2). But when the turbine inlet pressure increases from P_1 to P_2, the ideal expansion line shifts to the left and the moisture content at the exhaust increases $(\because x_{7S} < x_{3S})$

If the moisture content of steam in the turbine is higher the entrained water particles along with the vapour coming out from the nozzles with high velocity strike the blades and erode their surfaces, as a result of which the longevity of the blades decreases. From this consideration, moisture content at the turbine exhaust is not allowed to exceed 15% or x < 0.85.

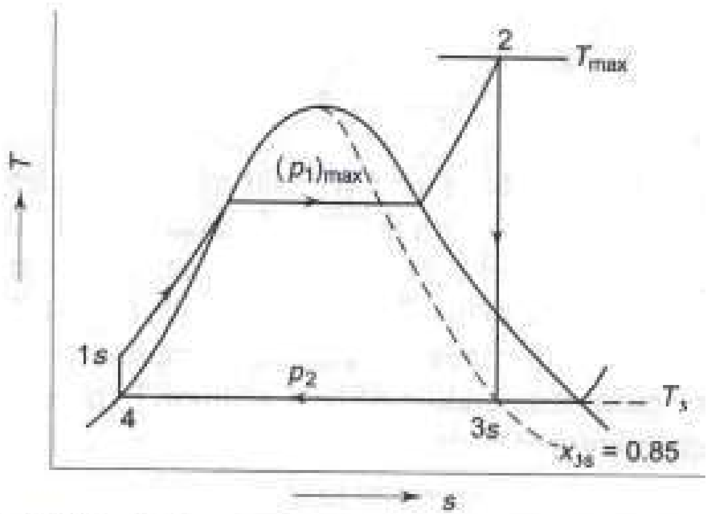


Fig. *Fixing of exhaust quality, maximum temperature and maximum pressure in Rankine cycle*

∴ With the maximum steam temperature at the turbine inlet, the minimum temperature of heat rejection and the minimum quality of steam at the turbine exhaust fixed, the maximum steam pressure at the turbine inlet also gets fixed. The vertical line drawn from 3S, fixed by T_3 and x_{3S}, intersects the T_{max} line, fixed by material, at 2, which gives maximum steam pressure at the turbine inlet.

Effect of Boiler Pressure (Using Molliar Diagram i.e., h-s diagram)

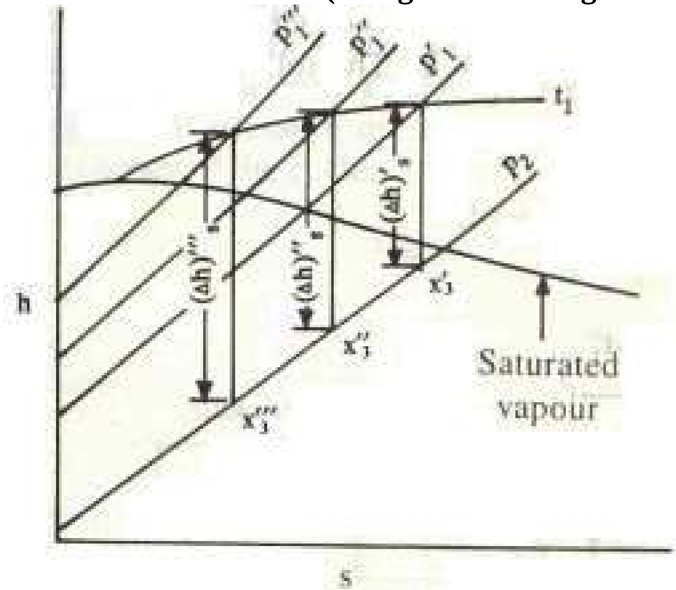


Fig. 13.5 Effect of boiler pressure

We have,

$$\eta_{th} = \frac{(h_2 - h_3) - (h_1 - h_4)}{h_2 - h_1} \text{ but } W_P << W_T$$

$$\therefore \eta_{th} = \frac{h_2 - h_3}{h_2 - h_1} = \frac{(\Delta h)_S}{(h_2 - h_1)}$$

i.e., Rankine cycle η depends on h_2, h_1 and Δh_S. From figure as $P_1''' > P_1'' > P_1'$ for the fixed maximum temperature of the steam t_1 and condenser pressure P_2, Isentropic heat drops increases with boiler pressure i.e., from the figure therefore it is evident that as boiler pressure increases, the isentropic heat drop $(\Delta h)_S$ increases, but the enthalpy of the steam entering the turbine decreases, with the result that the Rankine η increases. But quality of the steam at the exit of the turbine suffers i.e., $x_3''' < x_3'' < x_3'$, which leads to serious wear of the turbine blades.

Effect of Super Heating (Using Molliar Diagram i.e., h-s diagram)

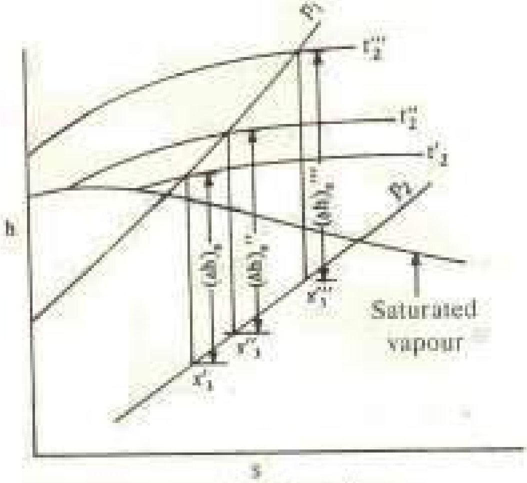
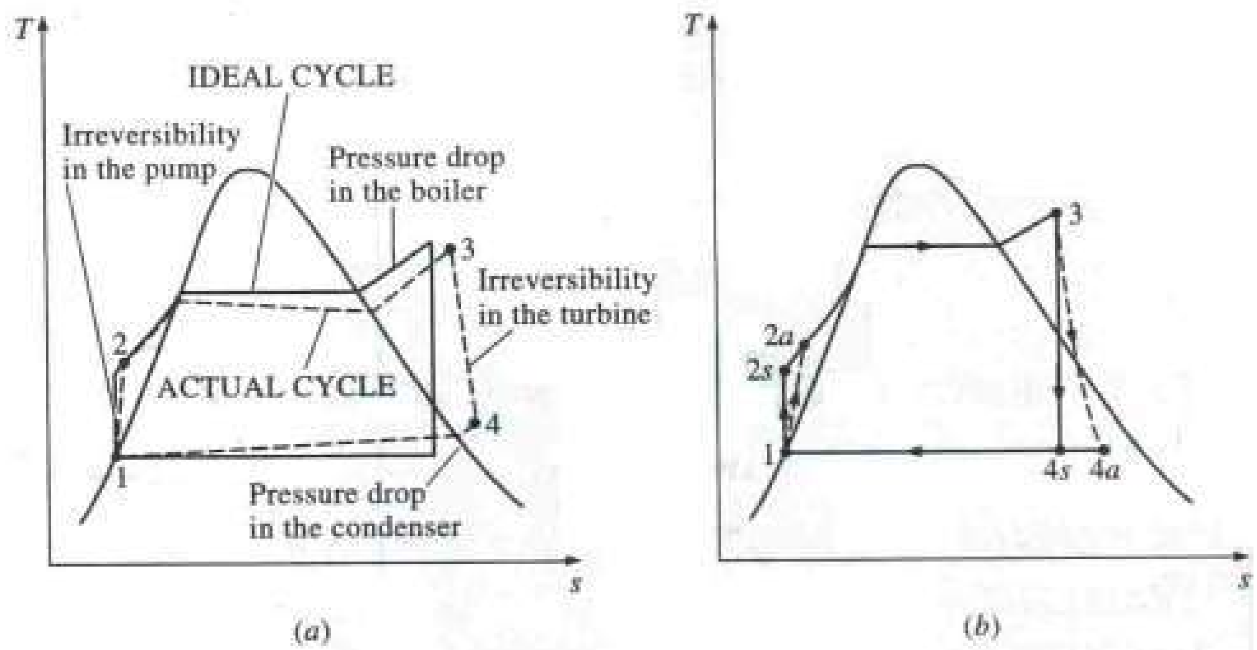


Fig. Effect of superheating

The moisture in the steam at the end of the expansion may be reduced by increasing the super heated temperature of steam t_2. This can be seen in figure where $t_2''' > t_2'' > t_2'$, but $x_3' < x_3'' < x_3'''$. It is, therefore, natural that to avoid erosion of the turbine blades, an increase in the boiler pressure must be accompanied by super heating at a higher temperature and since this raises the mean average temperature at which heat is transferred to the steam, the Rankine η increases.

Deviation of Actual Vapour Power cycles from Ideal cycle



The actual Vapour power cycle differs from the ideal Rankine cycle, as shown in figure, as a result of irreversibilities in various components mainly because of fluid friction and heat loss to the surroundings.

Fluid friction causes pressure drops in the boiler, the condenser, and the piping between various components. As a result, steam leaves the boiler at a lower pressure. Also the pressure at the turbine inlet is lower than that at the boiler exit due to pressure drop in the connecting pipes. The pressure drop in the condenser is usually very small. To compensate these pressure drops, the water must be pumped to sufficiently higher pressure which requires the larger pump and larger work input to the pump.

The other major source of irreversibility is the heat loss from the steam to the surroundings as the steam flows through various components. To maintain the same level of net work output, more heat needs to be transferred to the steam in the boiler to compensate for these undesired heat losses. As a result, cycle efficiency decreases.

As a result of irreversibilities, a pump requires a greater work input, and a turbine produces a smaller work output. Under the ideal conditions, the flow through these devices are isentropic. The deviation of actual pumps and turbines from the isentropic ones can be accounted for by utilizing isentropic efficiencies, defined as

$$\eta_P = \frac{W_S}{W_a} = \frac{h_{1S} - h_4}{h_1 - h_4}$$

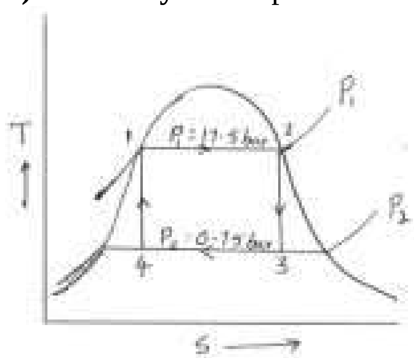
And $\eta_t = \dfrac{W_a}{W_S} = \dfrac{h_2 - h_3}{h_2 - h_{3S}}$

Problems:

1. **Dry saturated steam at 17.5 bar enters the turbine of a steam power plant and expands to the condenser pressure of 0.75 bar. Determine the Carnot and Rankine cycle efficiencies. Also find the work ratio of the Rankine cycle.**

Solution: P_1 = 17.5 bar $\quad$ P_2 = 0.75 bar $\quad \eta_{Carnot}$ = ? $\eta_{Rankine}$ = ?

a) Carnot cycle: At pressure 17.5 bar from steam tables,



P	t_S	h_f	h_{fg}	h_g	S_f	S_{fg}	S_g
17	204.3	871.8	1921.6	2793.4	2.3712	4.0246	6.3958
18	207.11	884.5	1910.3	2794.8	2.3976	3.9776	6.3751

For P = 17.5 bar, using linear interpolation

For t_S, $204.3 + \frac{207.11 - 204.3}{1} x 0.5 = 205.71^0 C$

$= 478.71$ K

Similarly, $h_f = 878.15$ kJ/kg $h_{fg} = 1915.95$ kJ/kg $h_g = 2794.1$ kJ/kg
$S_f = 2.3844$ kJ/kg^{0}K $S_{fg} = 4.0011$ kJ/kg^{0}K $S_g = 6.3855$ kJ/kg K

Also at pressure 0.75 bar from steam tables

P	t_S	h_f	h_{fg}	h_g	S_f	S_{fg}	S_g
0.8	93.51	391.7	2274.0	2665.8	1.233	6.2022	7.4352
0.7	89.96	376.8	2283.3	2660.1	1.1921	6.2883	7.4804

$\therefore$ For 0.75 bar, using linear interpolation,
$t_S = 91.74^0$C $h_f = 384.25$ $h_{fg} = 2278.65$ $h_g = 2662.95$
$S_f = 1.2126$ $S_{fg} = 6.2453$ $S_g = 7.4578$

The Carnot cycle η, $\eta_C = \frac{T_1 - T_2}{T_1} = \frac{478.71 - 364.74}{478.71} = 0.2381$

Steam rate or SSC = $\frac{1}{\oint \delta W} = \frac{1}{W_T - W_P}$

Since the expansion work is isentropic, $S_2 = S_3$

But $S_2 = S_g = 6.3855$ and $S_3 = S_{f3} + x_3 S_{fg3}$

i.e., $6.3855 = 1.2126 + x_3 (6.2453)$ $\therefore x_3 = 0.828$

$\therefore$ Enthalpy at state 3, $h_3 = h_{f3} + x_3 h_{fg3}$
$= 384.25 + 0.828 (2278.65) = 2271.63$ kJ/kg

$\therefore$ Turbine work or expansion work or positive work $= h_2 - h_3$
$= 2794.1 - 2271.63 = 522.47$ kJ/kg

Again since the compression process is isentropic
i.e., $S_4 = S_1 = S_{f1} = 2.3844$

Hence $2.3844 = S_{f4} + x_4 S_{fg4}$
$= 1.2126 + x_4 (6.2453)$ $\therefore x_4 = 0.188$

$\therefore$ Enthalpy at state 4 is $h_4 = h_{f4} + x_4\, h_{fg4}$

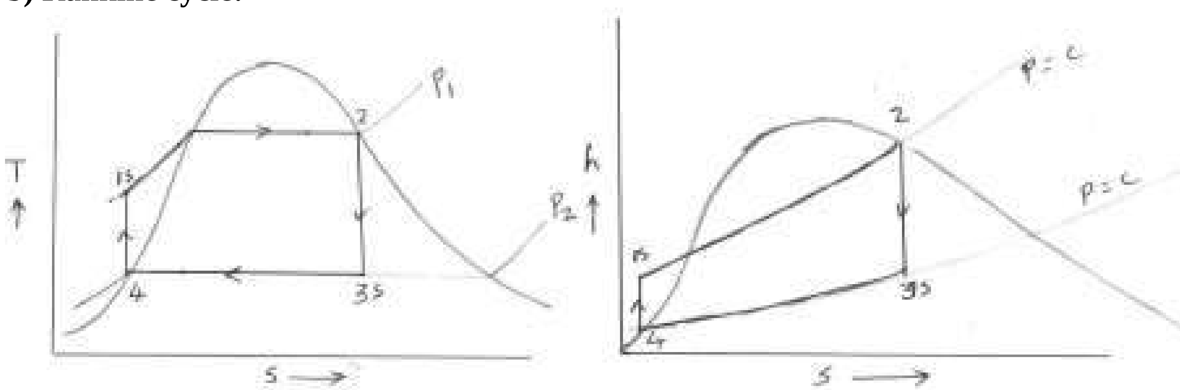
$= 384.25 + 0.188\ (2278.65)$
$= 811.79$ kJ/kg

$\therefore$ Compression work, $= h_1 - h_4 = 878.15 - 811.79$
$W_P = 66.36$ kJ/kg

$$\therefore SSC = \frac{1}{522.47 - 66.36} = 2.192\ x\,10^{-3}\,kg/kJ$$

$$\text{work ratio} = r_w = \frac{\oint \delta w}{+ve\,work} = \frac{W_T - W_P}{W_T} = \frac{456.11}{522.47} = 0.873$$

b) Rankine cycle:



$$\eta_R = \frac{W_T - W_P}{Q_H} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$

Since the change in volume of the saturated liquid water during compression from state 4 to state 1 is very small, v_4 may be taken as constant. In a steady flow process, work W = $-v\int dp$

$\therefore W_P = h_{1S} - h_4 = v_{fP2}\,(P_1 - P_2)$
$= 0.001037\ (17.5 - 0.75)\ x\ 10^5\ x\ (1/1000)$
$= 1.737$ kJ/kg

$\therefore h_{1S} = 1.737 + 384.25 = 385.99$ kJ/kg

Hence, turbine work $= W_T = h_2 - h_3 = 522.47$kJ/kg
Heat supplied $= Q_H = h_2 - h_{1S} = 2.794.1 - 385.99 = 2408.11$ kJ/kg

$$\therefore \eta_R = \frac{522.47 - 1.737}{2408.11} = 0.2162$$

$$\therefore SSC = \frac{1}{522.47 - 1.737} = 19204\ x\,10^{-3}\,kg/kJ$$

Work ratio, $r_w = \dfrac{522.47 - 1737}{522.47} = 0.9967$

2. If in problem (1), the turbine and the pump have each 85% efficiency, find the % reduction in the net work and cycle efficiency for Rankine cycle.

Solution: If $\eta_P = 0.85$, $\eta_T = 0.85$

$$W_P = \frac{W_P}{0.85} = \frac{1.737}{0.85} = 2.0435 kJ / kg$$

$W_T = \eta_T W_T = 0.85\ (522.47) = 444.09$ kJ/kg

$\therefore W_{net} = W_T - W_P = 442.06$ kJ/kg

$\therefore$% reduction in work output = $\dfrac{520.73 - 442.06}{520.73} = 15.11\%$

$W_P = h_{1S} - h_4$ $\therefore h_{1S} = 2.0435 + 384.25 = 386.29$ kJ/kg

$\therefore Q_H - h_2 - h_{1S} = 2794.1 - 386.29 = 2407.81$ kJ/kg

$\therefore \eta_{cycle} = \dfrac{442.06}{2407.81} = 0.1836$

$\therefore$% reduction in cycle efficiency $= \dfrac{0.2162 - 0.1836}{0.2162} = 15.08\%$

Note: Alternative method for problem 1 using h-s diagram (Mollier diagram) though the result may not be as accurate as the analytical solution. The method is as follows

Since steam is dry saturated at state 2, locate this state at the pressure P_2 = 17.5 bar on the saturation line and read the enthalpy at this state. This will give the value of h_2.

As the expansion process 2-3 is isentropic, draw a vertical line through the state 2 to meet the pressure line, P = 0.75 bar. The intersection of the vertical line with the pressure line will fix state 3. From the chart, find the value of h_3.

The value of h_4 can be found from the steam tables at pressure, P = 0.75 bar, as $h_4 = h_{f4}$. After finding the values of h_2, h_3 and h_4, apply the equation used in the analytical solution for determining the Rankine cycle η and SSC.

Effect of Boiler Pressure (Using Molliar Diagram i.e., h-s diagram)

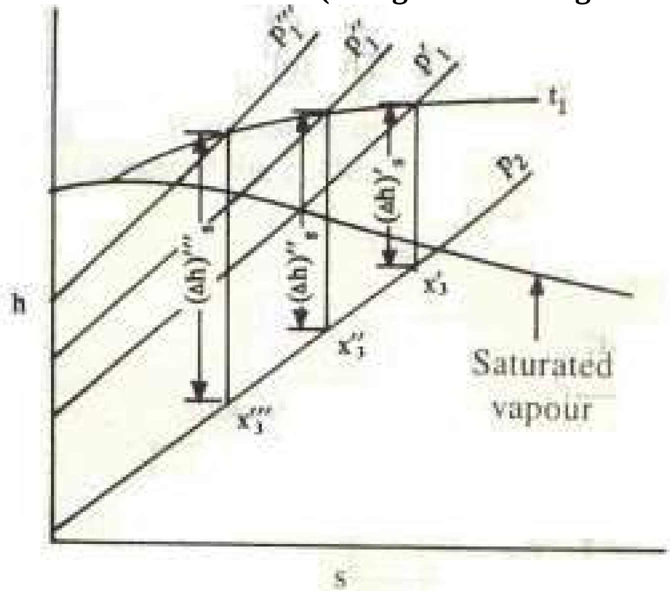


Fig. 13.5 Effect of boiler pressure

We have,

$$\eta_{th} = \frac{(h_2 - h_3) - (h_1 - h_4)}{h_2 - h_1} \text{ but } W_P << W_T$$

$$\therefore \eta_{th} = \frac{h_2 - h_3}{h_2 - h_1} = \frac{(\Delta h)_S}{(h_2 - h_1)}$$

i.e., Rankine cycle η depends on h_2, h_1 and Δh_S. From figure as $P_1''' > P_1'' > P_1'$ for the fixed maximum temperature of the steam t_1 and condenser pressure P_2, Isentropic heat drops increases with boiler pressure i.e., from the figure therefore it is evident that as boiler pressure increases, the isentropic heat drop $(\Delta h)_S$ increases, but the enthalpy of the steam entering the turbine decreases, with the result that the Rankine η increases. But quality of the steam at the exit of the turbine suffers i.e., $x_3''' < x_3'' < x_3'$, which leads to serious wear of the turbine blades.

Effect of Super Heating (Using Molliar Diagram i.e., h-s diagram)

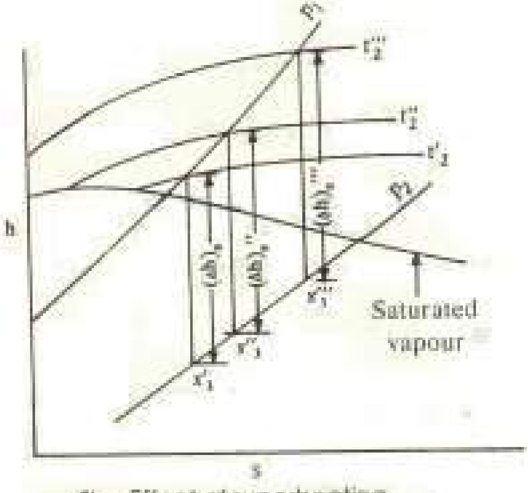
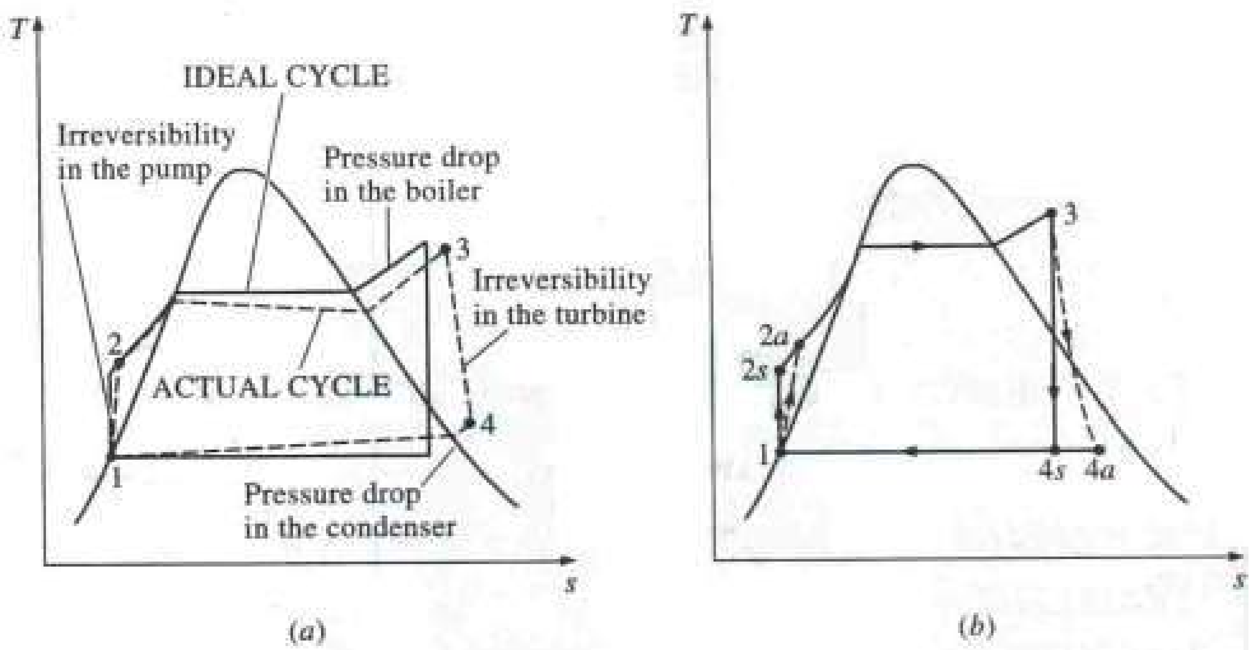


Fig. Effect of superheating

The moisture in the steam at the end of the expansion may be reduced by increasing the super heated temperature of steam t_1. This can be seen in figure where $t_1''' > t_1'' > t_1'$, but $x_3' < x_3'' < x_3'''$. It is, therefore, natural that to avoid erosion of the turbine blades, an increase in the boiler pressure must be accompanied by super heating at a higher temperature and since this raises the mean average temperature at which heat is transferred to the steam, the Rankine η increases.

Deviation of Actual Vapour Power cycles from Ideal cycle



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Fluid friction causes pressure drops in the boiler, the condenser, and the piping between various components. As a result, steam leaves the boiler at a lower pressure. Also the pressure at the turbine inlet is lower than that at the boiler exit due to pressure drop in the connecting pipes. The pressure drop in the condenser is usually very small. To compensate these pressure drops, the water must be pumped to sufficiently higher pressure which requires the larger pump and larger work input to the pump.

The other major source of irreversibility is the heat loss from the steam to the surroundings as the steam flows through various components. To maintain the same level of net work output, more heat needs to be transferred to the steam in the boiler to compensate for these undesired heat losses. As a result, cycle efficiency decreases.

As a result of irreversibilities, a pump requires a greater work input, and a turbine produces a smaller work output. Under the ideal conditions, the flow through these devices are isentropic. The deviation of actual pumps and turbines from the isentropic ones can be accounted for by utilizing isentropic efficiencies, defined as

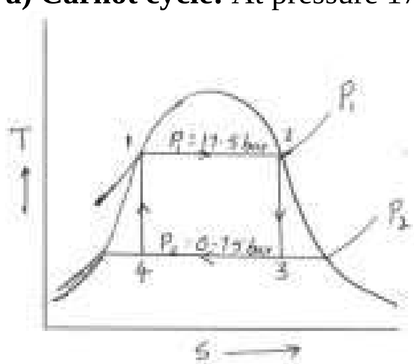
$$\eta_P = \frac{W_S}{W_a} = \frac{h_{1S} - h_4}{h_1 - h_4}$$

$$\text{And } \eta_t = \frac{W_a}{W_S} = \frac{h_2 - h_3}{h_2 - h_{3S}}$$

Numerical Problems:

1. **Dry saturated steam at 17.5 bar enters the turbine of a steam power plant and expands to the condenser pressure of 0.75 bar. Determine the Carnot and Rankine cycle efficiencies. Also find the work ratio of the Rankine cycle.**

Solution: P_1 = 17.5 bar P_2 = 0.75 bar η_{Carnot} = ? $\eta_{Rankine}$ = ?
a) Carnot cycle: At pressure 17.5 bar from steam tables,



P	t_S	h_f	h_{fg}	h_g	S_f	S_{fg}	S_g
17	204.3	871.8	1921.6	2793.4	2.3712	4.0246	6.3958
18	207.11	884.5	1910.3	2794.8	2.3976	3.9776	6.3751

For P = 17.5 bar, using linear interpolation

For t_S, $204.3 + \frac{207.11 - 204.3}{1} x 0.5 = 205.71^0 C$

$= 478.71$ K

Similarly, $h_f = 878.15$ kJ/kg $\quad h_{fg} = 1915.95$ kJ/kg $\quad h_g = 2794.1$ kJ/kg

$S_f = 2.3844$ kJ/kg^{0}K $\quad S_{fg} = 4.0011$ kJ/kg^{0}K $\quad S_g = 6.3855$ kJ/kg K

Also at pressure 0.75 bar from steam tables

P	t_S	h_f	h_{fg}	h_g	S_f	S_{fg}	S_g
0.8	93.51	391.7	2274.0	2665.8	1.233	6.2022	7.4352
0.7	89.96	376.8	2283.3	2660.1	1.1921	6.2883	7.4804

$\therefore$ For 0.75 bar, using linear interpolation,

$t_S = 91.74^0$C $\quad h_f = 384.25 \quad h_{fg} = 2278.65 \quad h_g = 2662.95$

$S_f = 1.2126 \quad S_{fg} = 6.2453 \quad S_g = 7.4578$

The Carnot cycle η, $\eta_C = \frac{T_1 - T_2}{T_1} = \frac{478.71 - 364.74}{478.71} = 0.2381$

Steam rate or SSC = $\frac{1}{\oint \delta W} = \frac{1}{W_T - W_P}$

Since the expansion work is isentropic, $S_2 = S_3$

But $S_2 = S_g = 6.3855$ and $S_3 = S_{f3} + x_3 S_{fg3}$

i.e., $6.3855 = 1.2126 + x_3 (6.2453) \quad \therefore x_3 = 0.828$

$\therefore$ Enthalpy at state 3, $\quad h_3 = h_{f3} + x_3 h_{fg3}$

$= 384.25 + 0.828 (2278.65) = 2271.63$ kJ/kg

$\therefore$ Turbine work or expansion work or positive work $= h_2 - h_3$

$= 2794.1 - 2271.63 = 522.47$ kJ/kg

Again since the compression process is isentropic

i.e., $S_4 = S_1 = S_{f1} = 2.3844$

Hence $2.3844 = S_{f4} + x_4 S_{fg4}$

$= 1.2126 + x_4 (6.2453) \quad \therefore x_4 = 0.188$

$\therefore$ Enthalpy at state 4 is $h_4 = h_{f4} + x_4 h_{fg4}$

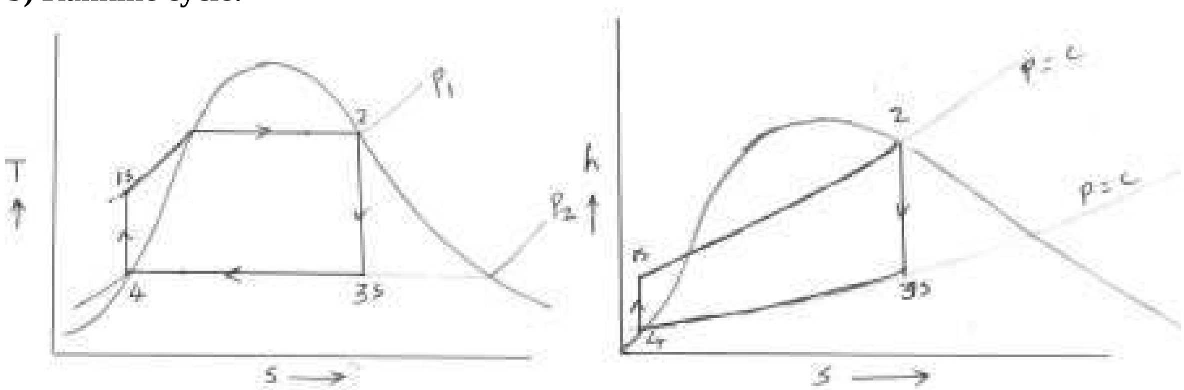
$= 384.25 + 0.188 (2278.65)$
$= 811.79$ kJ/kg

$\therefore$ Compression work, $= h_1 - h_4 = 878.15 - 811.79$
$W_P = 66.36$ kJ/kg

$$\therefore SSC = \frac{1}{522.47 - 66.36} = 2.192\ x\,10^{-3}\,kg/kJ$$

$$\text{work ratio} = r_w = \frac{\oint \delta w}{+ve\,work} = \frac{W_T - W_P}{W_T} = \frac{456.11}{522.47} = 0.873$$

b) Rankine cycle:



$$\eta_R = \frac{W_T - W_P}{Q_H} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$

Since the change in volume of the saturated liquid water during compression from state 4 to state 1 is very small, v_4 may be taken as constant. In a steady flow process, work W = $-v\int dp$

$\therefore W_P = h_{1S} - h_4 = v_{fP2} (P_1 - P_2)$
$= 0.001037 (17.5 - 0.75) \times 10^5 \times (1/1000)$
$= 1.737$ kJ/kg
$\therefore h_{1S} = 1.737 + 384.25 = 385.99$ kJ/kg

Hence, turbine work = $W_T = h_2 - h_3 = 522.47$kJ/kg
Heat supplied = $Q_H = h_2 - h_{1S} = 2.794.1 - 385.99 = 2408.11$ kJ/kg

$$\therefore \eta_R = \frac{522.47 - 1.737}{2408.11} = 0.2162$$

$$\therefore SSC = \frac{1}{522.47 - 1.737} = 19204\ x\,10^{-3}\,kg/kJ$$

Work ratio, $r_w = \frac{522.47 - 1737}{522.47} = 0.9967$

2. If in problem (1), the turbine and the pump have each 85% efficiency, find the % reduction in the net work and cycle efficiency for Rankine cycle.

Solution: If $\eta_P = 0.85$, $\eta_T = 0.85$

$$W_P = \frac{W_P}{0.85} = \frac{1.737}{0.85} = 2.0435 kJ/kg$$

$W_T = \eta_T W_T = 0.85 (522.47) = 444.09$ kJ/kg

$\therefore W_{net} = W_T - W_P = 442.06$ kJ/kg

$\therefore$% reduction in work output = $\frac{520.73 - 442.06}{520.73} = 15.11\%$

$W_P = h_{1S} - h_4$ $\therefore h_{1S} = 2.0435 + 384.25 = 386.29$ kJ/kg

$\therefore Q_H - h_2 - h_{1S} = 2794.1 - 386.29 = 2407.81$ kJ/kg

$$\therefore \eta_{cycle} = \frac{442.06}{2407.81} = 0.1836$$

$\therefore$% reduction in cycle efficiency $= \frac{0.2162 - 0.1836}{0.2162} = 15.08\%$

Note: Alternative method for problem 1 using h-s diagram (Mollier diagram) though the result may not be as accurate as the analytical solution. The method is as follows

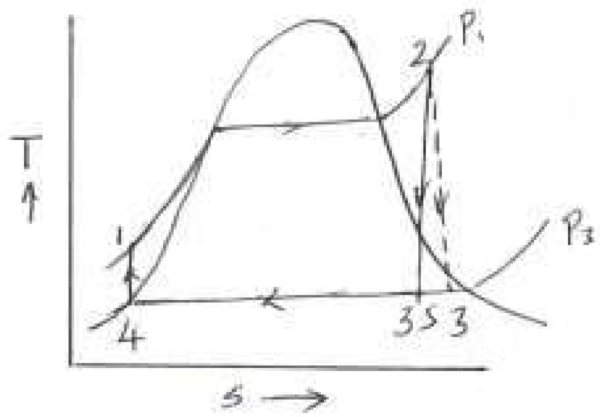
Since steam is dry saturated at state 2, locate this state at the pressure P_2 = 17.5 bar on the saturation line and read the enthalpy at this state. This will give the value of h_2.

As the expansion process 2-3 is isentropic, draw a vertical line through the state 2 to meet the pressure line, P = 0.75 bar. The intersection of the vertical line with the pressure line will fix state 3. From the chart, find the value of h_3.

The value of h_4 can be found from the steam tables at pressure, P = 0.75 bar, as $h_4 = h_{f4}$. After finding the values of h_2, h_3 and h_4, apply the equation used in the analytical solution for determining the Rankine cycle η and SSC.

3. Steam enters the turbine of a steam power plant, operating on Rankine cycle, at 10 bar, 300^0C. The condenser pressure is 0.1 bar. Steam leaving the turbine is 90% dry. Calculate the adiabatic efficiency of the turbine and also the cycle η, neglecting pump work.

Solution:



P_1 = 10 bar $t_2 = 300^0$C P_3 = 0.1 bar

$x_3 = 0.9$ η_t = ? η_{cycle} = ? Neglect W_P

From superheated steam tables,

For P_2 = 10 bar and $t_2 = 300^0$C, h_2 = 3052.1 kJ/kg, s_2 = 7.1251 kJ/kg

From table A – 1, For P_3 = 0.1 bar

$t_S = 45.83^0$C h_f = 191.8 h_{fg} = 2392.9

S_f = 0.6493 S_{fg} = 7.5018

Since $x_3 = 0.9$, $h_3 = h_{f4} + x_3\, h_{fg3}$

$= 191.8 + 0.9\,(2392.9)$

$= 2345.4$ kJ/kg

Also, since process 2-3s is isentropic, $S_2 = S_{3S}$

i.e., $7.1251 = S_{fg4} + x_{3S}\, S_{fg3}$

$= 0.6493 + x_{3S}\,(7.5018)$

$\therefore x_{3S} = 0.863$

$\therefore h_{3S} = 191.8 + 0.863\,(2392.9) = 2257.43$ kJ/kg

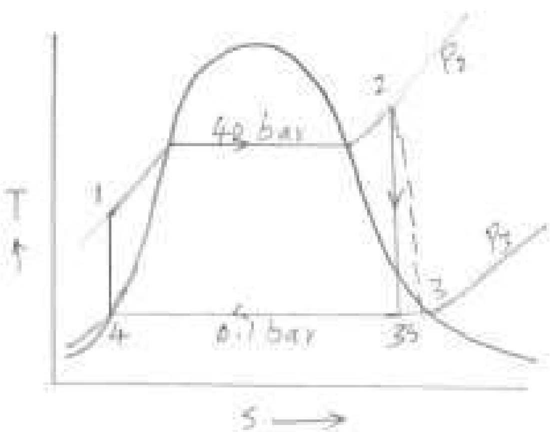
$$\therefore Turbine\ efficiency, \eta_t = \frac{h_2 - h_3}{h_2 - h_{3S}} = \frac{3052.1 - 2345.4}{3052.1 - 2257.43} = 0.89$$

$$\eta_{cycle} = \frac{W_T}{Q_H} = \frac{h_2 - h_3}{h_2 - h_1} \quad but \quad h_1 = 191.8 \text{ kJ/kg}$$

$$= \frac{3052.1 - 2345.4}{3052.1 - 191.8} = 0.25 \quad i.e., \quad 25\%$$

4. **A 40 mW steam plant working on Rankine cycle operates between boiler pressure of 4 MPa and condenser pressure of 10 KPa. The steam leaves the boiler and enters the steam turbine at 400^{0}C. The isentropic η of the steam turbine is 85%. Determine (i) the cycle η (ii) the quality of steam from the turbine and (iii) the steam flow rate in kg per hour. Consider pump work.**

Solution:



P_2 = 4 MPa = 40 bar P_3 = 10 KPa = 0.1 bar

P = 40000kW $t_2 = 400^0$C $\eta_t = 0.85$ η_{cycle} = ? x_3 = ?

$\dot{m} = ?$

$h_2 = h\big|_{40\,bar,\,400^0C} = 3215.7 kJ/kg$ and s_2 = 6.7733 kJ/kg-K

$h_4 = h_f\big|_{0.1bar} = 191.8 kJ/kg$

Process 2-3s is isentropic i.e., $S_2 = S_{3S}$

$6.7733 = 0.6493 + x_{3S}\,(7.5018)$

$\therefore x_{3S} = 0.816$

$\therefore h_{3S} = h_{f3} + x_{3S}\, h_{fg3} = 191.8 + 0.816\ (2392.9)$

$= 2145.2$ kJ/kg

$$But \quad \eta_t = \frac{h_2 - h_3}{h_2 - h_{3S}} \quad i.e., \quad 0.85 = \frac{3215.7 - h_3}{3215.7 - 2145.2}$$

$\therefore h_3 = 2305.8$ kJ/kg

$\therefore W_T = h_2 - h_3 = 3215.7 - 2305.8 = 909.9$ kJ/kg

$W_P = v\int dP \quad = 0.0010102\ (40 - 0.1)\ 10^5/10^2$

$= 4.031$ kJ/kg

$= h_1 - h_4 \qquad \therefore h_1 = 195.8$ kJ/kg

$$(i)\ \eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{909.9 - 4.031}{(3215.7 - 195.8)} = 29.9\%$$

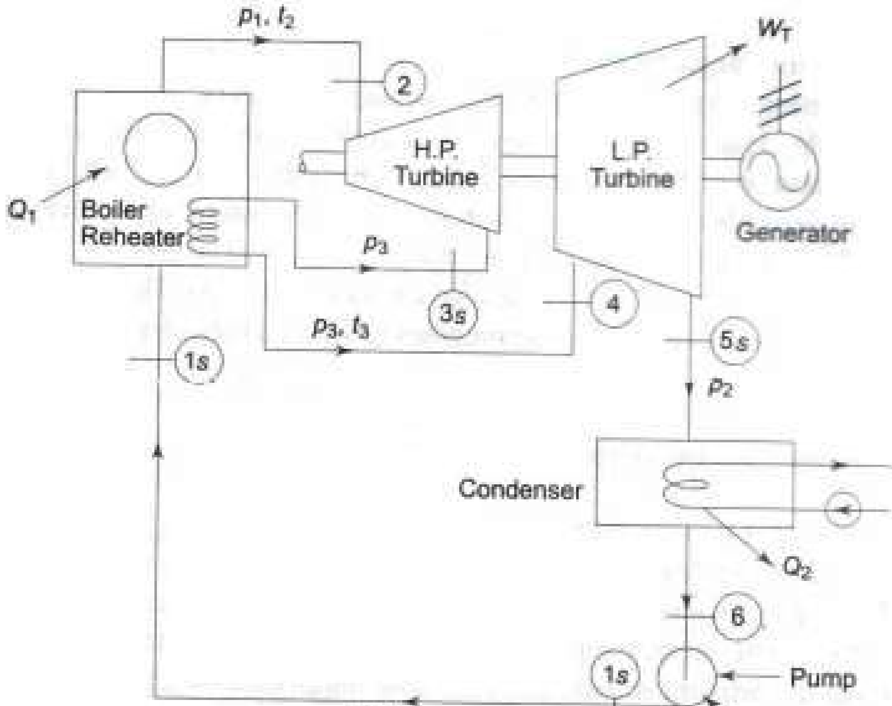
(ii) $x_3 = ?$ we have $2305.8 = 191.8 + x_3\ (2392.9)$ $\therefore x_3 = 0.88$

(iii) $P = \dot{m}\, W_{net}$ i.e., $40000 = \dot{m}\ (905.87)$

$\therefore \dot{m} = 44.2$ kg/s

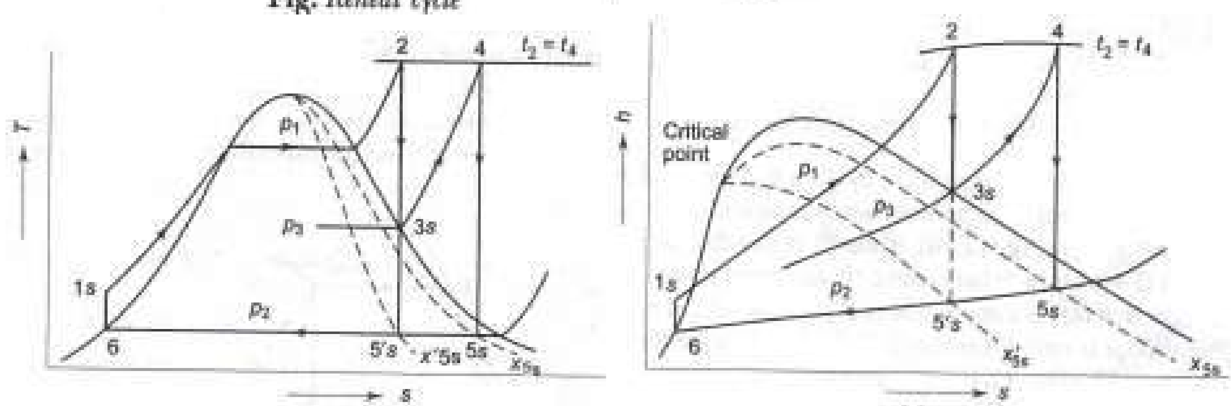
$= 159120$ kg/hr

Ideal Reheat cycle: We know that, the efficiency of the Rankine cycle could be increased by increasing steam pressure in the boiler and superheating the steam. But this increases the moisture content of the steam in the lower pressure stages in the turbine, which may lead to erosion of the turbine blade. $\therefore$The reheat cycle has been developed to take advantage of the increased pressure of the boiler, avoiding the excessive moisture of the steam in the low pressure stages. In the reheat cycle, steam after partial expansion in the turbine is brought back to the boiler, reheated by combustion gases and then fed back to the turbine for further expansion.



(a)

Fig. *Reheat cycle*



(b) (c)

In the reheat cycle the expansion of steam from the initial state (2) to the condenser pressure is carried out in two or more steps, depending upon the number of reheats used.

In the first step, steam expands in HP turbine from state 2 to approximate the saturated vapour line (process 2-3s). The steam is then reheated (or resuperheated) at constant pressure in the boiler (or in a reheater) process 3s-4 and the remaining expansion process 4s-5 is carried out in the LP turbine.

Note: 1) To protect the reheater tubes, steam is not allowed to expand deep into the two-phase region before it is taken for reheating, because in that case the moisture particles in steam while evaporating would leave behind solid deposits in the form of scale which is difficult to remove. Also a low reheat pressure may bring down T_{m1} and hence cycle η. Again a high reheat pressure increases the moisture content at turbine exhaust. Thus reheat pressure is optimized. Optimum reheat pressure is about 0.2 to 0.25 of initial pressure.

We have for 1 kg of steam

$Q_H = (h_2 - h_{1S}) + (h_4 - h_{3S})$; $Q_L = h_{5S} - h_6$
$W_T = (h_2 - h_{3S}) + (h_4 - h_{5S})$; $W_P = h_{1S} - h_6$

$$\therefore \quad \eta_R = \frac{W_T - W_P}{Q_H} \;;$$

Steam rate $= \dfrac{3600}{(W_T - W_P)} kg/kWh$

Since higher reheat pressure is used, W_P work is appreciable.

2) In practice, the use of reheat gives a marginal increase in cycle η, but it increases the net work output by making possible the use of higher pressures, keeping the quality of steam at turbine exhaust within a permissible limit. The quality improves from $x_{5^1 S}$ to x_{5S} by the use of reheat.

Ideal Regenerative cycle: The mean temperature of heat addition can also be increased by decreasing the amount of heat added at low temperatures. In a simple Rankine cycle (saturated steam entering the turbine), a considerable part of the total heat supplied is in the liquid phase when heating up water from 1 to 1^1, at a temperature lower than T_2, the maximum temperature of the cycle. For maximum η, all heat should be supplied at T_2, and feed water should enter the boiler at 1^1. This may be accomplished in what is known as an ideal regenerative cycle as shown in figures (a) and (b).

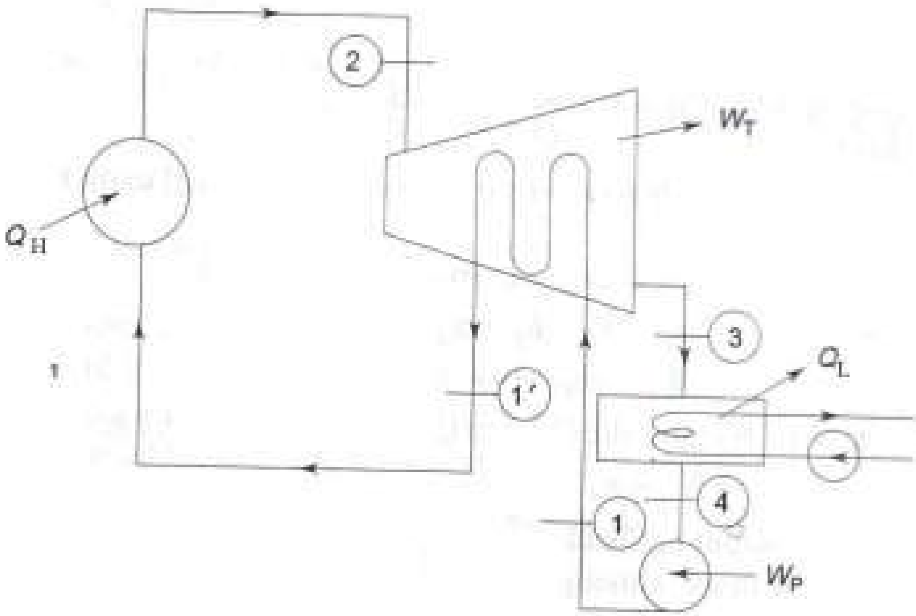


Fig. *Ideal regenerative cycle*

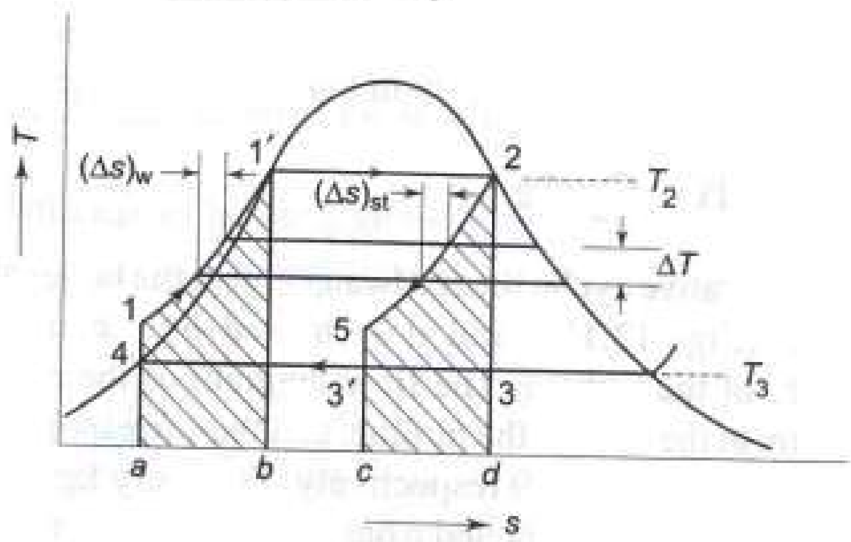


Fig. *Ideal regenerative cycle on T-s plot*

$$\Delta T(\text{water}) = -\Delta T\ (\text{steam})$$

The unique feature of the ideal regenerative cycle is that the condensate, after leaving the pump circulates around the turbine casing, counter-flow to the direction of vapour flow in the turbine. Thus it is possible to transfer heat from the vapour as it flows through the turbine to the liquid flowing around the turbine.

Let us assume that this is a reversible heat transfer i.e., at each point, the temperature of the vapour is only infinitesimally higher than the temperature of the liquid. $\therefore$The process $2\text{-}3^1$ represents reversible expansion of steam in the turbine with reversible heat rejection. i.e., for any small step in the process of heating the water $\Delta T_{(water)} = -\Delta T_{(steam)}$ and $(\Delta S)_{water} = (\Delta S)_{steam}$. Then the slopes of lines $2\text{-}3^1$ and $1^1\text{-}4$ will be identical at every temperature and the lines will be identical in contour. Areas $1\text{-}1^1\text{-b-a-}1$ and $3^1\text{-}2\text{-d-c-}3^1$ are not only equal but congruous. $\therefore$, all heat added from external source (Q_H) is at constant temperature T_2 and all heat rejected (Q_L) is at constant temperature T_3, both being reversible.

Then $Q_H = h_2 - h_1^1 = T_2 (S_2 - S_1^1)$

$Q_L = h_3^1 - h_4 = T_3 (S_3^1 - S_4)$

Since $S_1^1 - S_4 = S_2 - S_3^1$ or $S_2 - S_1^1 = S_3^1 - S_4$

$\therefore \quad \eta_{Reg} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_3}{T_2}$ i.e., the η of ideal regenerative cycle is thus equal to the Carnot cycle η.

Writing SFEE to turbine,

$h_2 + h_1 = W_T + h_1^1 + h_3^1$

i.e., $W_T = (h_2 - h_3^1) - (h_1^1 - h_1)$

or $W_T = (h_2 - h_3^1) - (h_1^1 - h_1)$ --- (1)

and the W_P is same as simple rankine cycle i.e., $W_P = (h_1 - h_4)$

$\therefore$The net work output of the ideal regenerative cycle is less and hence its steam rate will be more. Although it is more efficient when compared to rankine cycle, this cycle is not practicable for the following reasons.

1) Reversible heat transfer cannot be obtained in finite time.
2) Heat exchanger in the turbine is mechanically impracticable.
3) The moisture content of the steam in the turbine is high.

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Regenerative cycle:

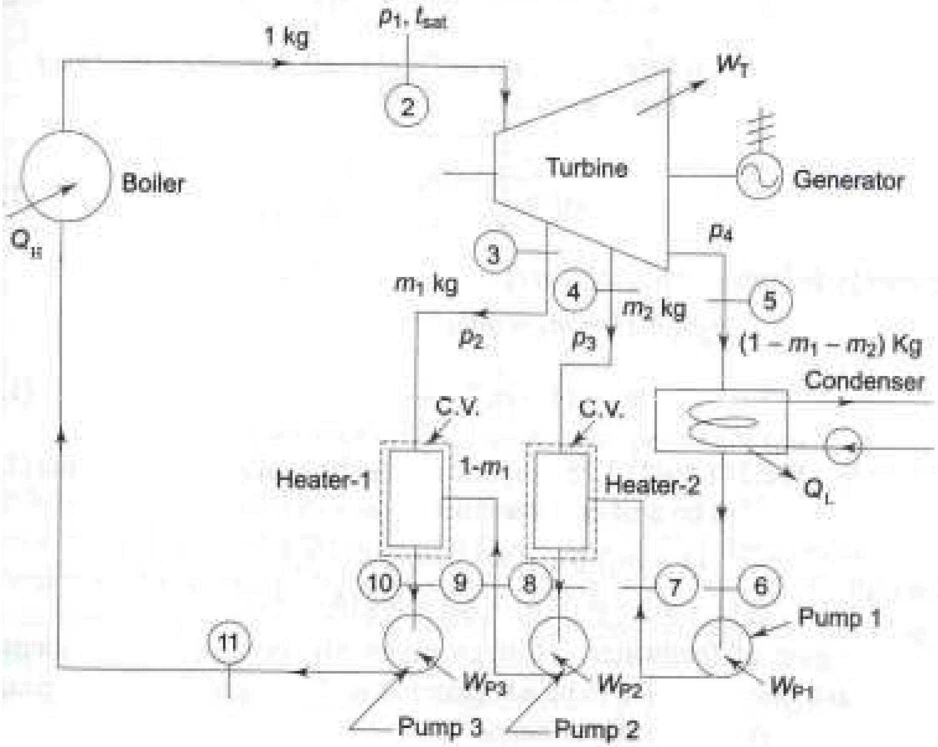
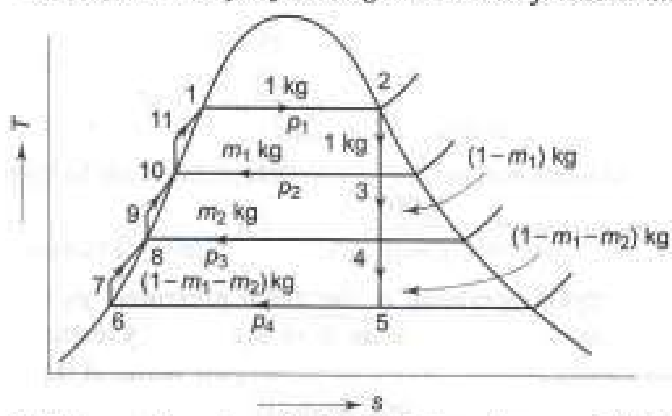
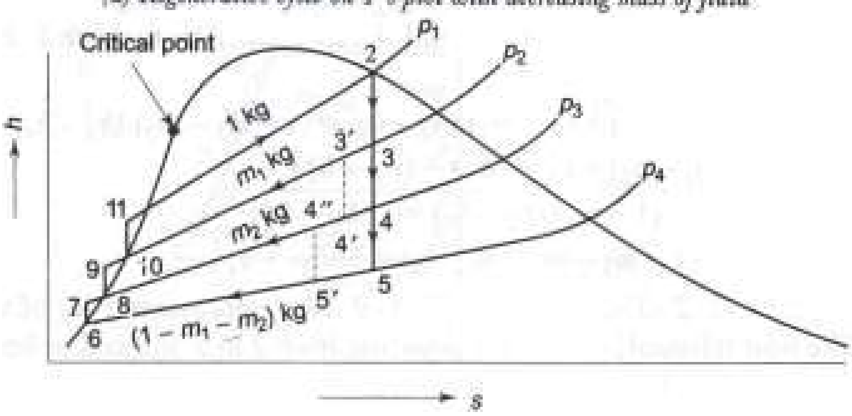


Fig. *Regenerative cycle flow diagram with two feedwater heaters*



(a) Regenerative cycle on T-s plot with decreasing mass of fluid



In a practical regenerative cycle, the feed water enters the boiler at a temperature between 1 and 1^1 (previous article figure), and it is heated by steam extracted from intermediate stages of the turbine. The flow diagram of the regenerative cycle with saturated steam at the inlet to the turbine and the corresponding T-S diagram are shown in figure.

For every kg of steam entering the turbine, let m_1 kg steam be extracted from an intermediate stage of the turbine where the pressure is P_2, and it is used to heat up feed water [$(1 - m_1)$ kg at state 9] by mixing in heater (1). The remaining $(1-m_1)$ kg of steam then expands in the turbine from pressure P_2 (state 3) to pressure P_3 (state 4) when m_2 kg of steam is extracted for heating feed water in heater (2). So $(1 - m_1 - m_2)$kg of steam then expands in the remaining stages of the turbine to pressure P_4, gets condensed into water in the condenser, and then pumped to heater (2), where it mixes with m_2 kg of steam extracted at pressure P_3. Then $(1-m_1)$ kg of water is pumped to heater (1) where it mixes with m_1 kg of steam extracted at pressure P_2. The resulting 1kg of steam is then pumped to the boiler where heat from an external source is supplied. Heaters 1 and 2 thus operate at pressure P_2 and P_3 respectively. The amounts of steam m_1 and m_2 extracted from the turbine are such that at the exit from each of the heaters, the state is saturated liquid at the respective pressures.

$\therefore$ Turbine work, $W_T = 1(h_2 - h_3) + (1 - m_1)(h_3 - h_4) + (1 - m_1 - m_2)(h_4 - h_5)$

Pump work, $W_P = W_{P1} + W_{P2} + W_{P3}$

$= (1 - m_1 - m_2)(h_7 - h_6) + (1 - m_1)(h_9 - h_8) + 1(h_{11} - h_{10})$

$Q_H = (h_2 - h_{11})$; $\qquad Q_L = (1 - m_1 - m_2)(h_5 - h_6)$

$$\therefore \text{Cycle efficiency, } \eta = \frac{Q_H - Q_L}{Q_H} = \frac{W_T - W_P}{Q_H}$$

$$SSC = \frac{3600}{W_T - W_P} kg/kWh$$

In the Rankine cycle operating at the given pressure P_1 and P_4, the heat addition would have been from state 7 to state 2. By using two stages of regenerative feed water heating., feed water enters the boiler at state 11, instead of state 7, and heat addition is, therefore from state 11 to state 2.

Therefore $(T_{m1})_{with\ regeneration} = \dfrac{h_2 - h_{11}}{S_2 - S_{11}}$

And $(T_{m1})_{without\ regeneration} = \dfrac{h_2 - h_7}{S_2 - S_7}$

Since $(T_{m1})_{\text{with regenerative}} > (T_{m1})_{\text{without regenerative}}$, the η of the regenerative cycle will be higher than that of the Rankine cycle.

The energy balance for heater 1,

$m_1 h_3 + (1 - m_1) h_9 = 1\, h_{10}$

$$\therefore m_1 = \frac{h_{10} - h_9}{h_3 - h_9} \qquad \text{--- (1)}$$

The energy balance for heater 2,

$$m_2 h_4 + (1 - m_1 - m_2) h_7 = (1 - m_1) h_8$$

$$\text{Or } m_2 = (1 - m_1)\frac{(h_8 - h_7)}{(h_4 - h_7)} \quad \text{--- (2)}$$

Above equations (1) and (2) can also be written alternatively as

$$(1 - m_1)(h_{10} - h_9) = m_1 (h_3 - h_{10})$$

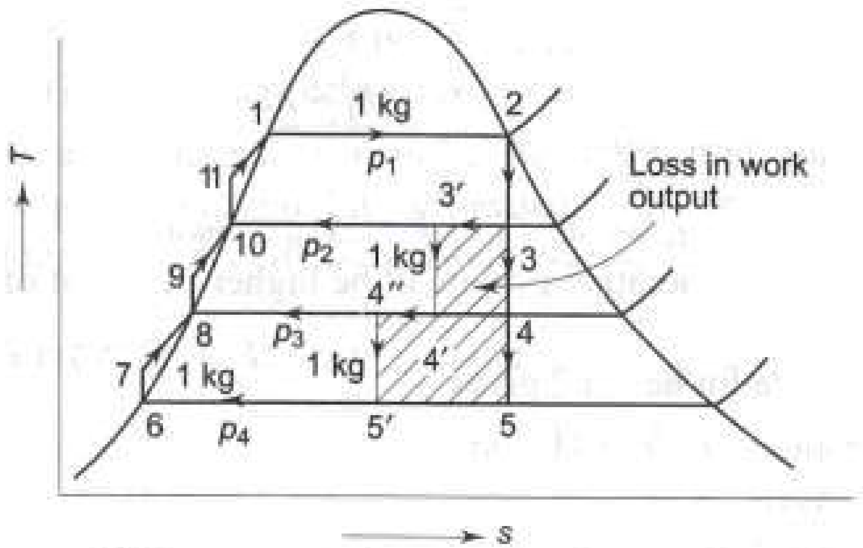
$$\text{and } (1 - m_1 - m_2)(h_8 - h_7) = m_2 (h_4 - h_8)$$

Energy gain of feed water = energy given off by vapour in condensation.

Heaters have been assumed to be adequately insulated and there is no heat gain from, or heat loss to, the surroundings.

In figure (a) path 2-3-4-5 represents the states of a decreasing mass of fluid.

For 1kg of steam, the states would be represented by the path $2\text{-}3^1\text{-}4^{11}\text{-}5^1$. [Figure (b)].



(b) Regenerative cycle on T-s plot for unit mass of fluid

$$\begin{aligned} \text{We have } W_T &= (h_2 - h_3) + (1 - m_1)(h_3 - h_4) + (1 - m_1 - m_2)(h_4 - h_5) \\ &= (h_2 - h_3) + (h_3^1 - h_4^1) + (h_4^{11} - h_5^1) \text{ [From Figure b]} \quad \text{--- (3)} \end{aligned}$$

The cycle $2 - 3 - 3^1 - 4^1 - 4^{11} - 5^1 - 6 - 7 - 8 - 9 - 10 - 11 - 2$ represents 1kg of working fluid. The heat released by steam condensing from 3 to 3^1 is utilized in heating up the water from 9 to 10.

$$\therefore \quad 1 (h_3 - h_3^1) = 1 (h_{10} - h_9) \quad \text{--- (4)}$$

$$\text{Similarly, } 1 (h_4^1 - h_4^{11}) = 1 (h_8 - h_7) \quad \text{--- (5)}$$

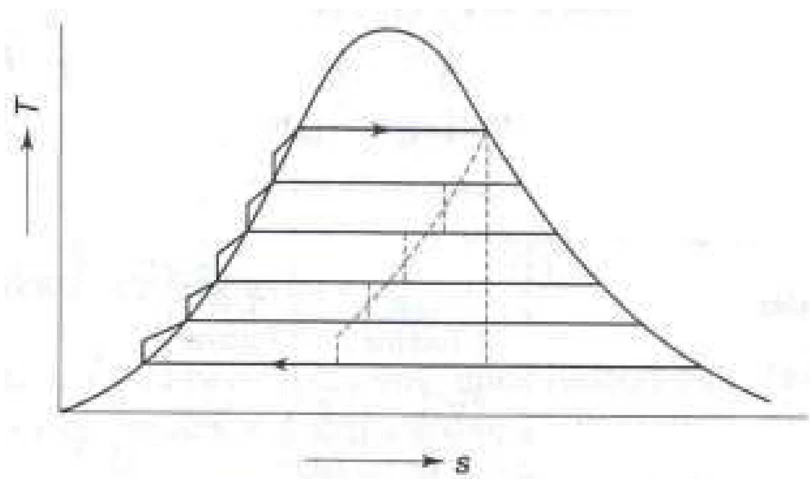
From equation (3), (4) and (5),

$$W_T = (h_2 - h_5^1) - (h_3 - h_3^1) - (h_4^1 - h_4^{11})$$
$$= (h_2 - h_5^1) - (h_{10} - h_9) - (h_8 - h_7) \qquad \text{--- (6)}$$

Also from Ideal regenerative cycle, [Previous article]

$$W_T = (h_2 - h_3^1) - (h_1^1 - h_1) \qquad \text{--- (1)}$$

The similarity of equations (6) and equation (1) from previous article is notices. It is seen that the stepped cycle $2 - 3^1 - 4^1 - 4^{11} - 5^1 - 6 - 7 - 8 - 9 - 10 - 11$ approximates the ideal regenerative cycle in Figure (1) [previous article] and that a greater no. of stages would give a closer approximation. Thus the heating of feed water by steam 'bled' from the turbine, known as regeneration, "Carnotizes" the Rankine cycle.



Regenerative cycle with many stages of feedwater heating

The heat rejected Q_L in the cycle decreases from $(h_5 - h_6)$ to $(h_5^1 - h_6)$. There is also loss in work output by the amount (area under $3 - 3^1$ + area under $4^1 - 4^{11}$ – area under $5 - 5^1$) as shown by the hatched area in Figure (b). So the steam rate increases by regeneration i.e., more steam has to circulate per hour to produce unit shaft output.

Reheat – Regenerative cycle:

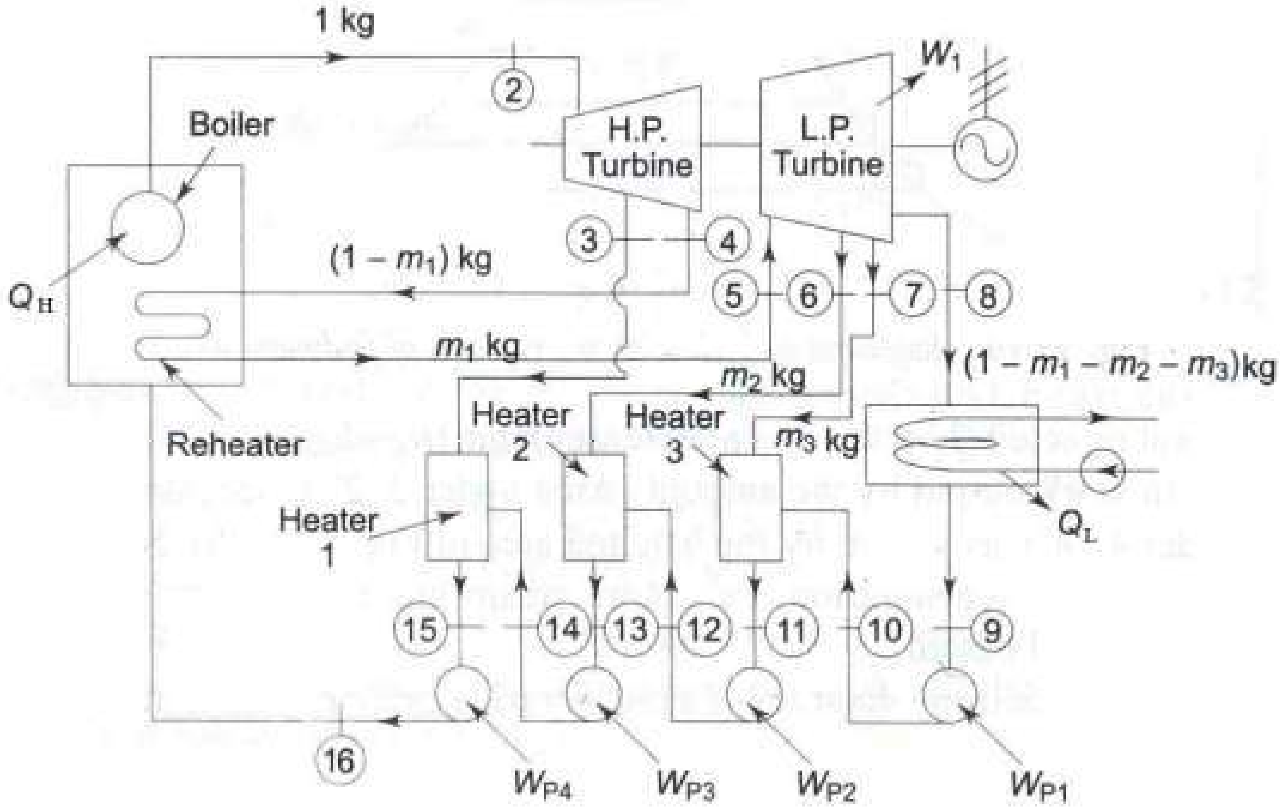


Fig. *Reheat-regenerative cycle flow diagram*

Reheat – regenerative cycle flow diagram (Three-stages of feed water heating)

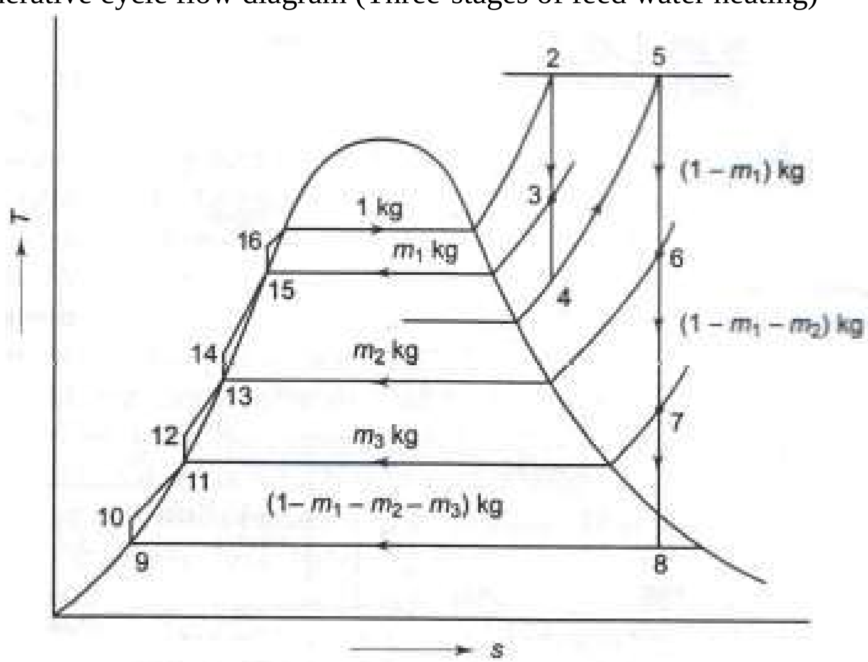


Fig. *T–s diagram of reheat-regenerative cycle*

The reheating of steam is employed when the vapourization pressure is high reheat alone on the thermal η is very small. ∴Regeneration or the heating up of feed water by steam extracted from the turbine will effect in more increasing in the η_{th}.

Turbine work, $W_T = (h_1 - h_2) + (1 - m_1)(h_2 - h_3) + (1 - m_1)(h_4 - h_5) + (1 - m_1 - m_2)(h_5 - h_6)$
$+ (1 - m_1 - m_2 - m_3)(h_6 - h_7)$ kJ/kg
Pump work, $W_P = (1 - m_1 - m_2 - m_3)(h_9 - h_8) + (1 - m_1 - m_2)(h_{11} - h_{10})$
$+ (1 - m_1)(h_{13} - h_{12}) + 1\,(h_{15} - h_{14})$ kJ/kg

Heat added, $Q_H = (h_1 - h_{15}) + (1 - m_1)(h_4 - h_3)$ kJ/kg
Heat rejected, $Q_L = (1 - m_1 - m_2 - m_3)(h_7 - h_8)$ kJ/kg

The energy balance of heaters 1, 2 and 3 gives

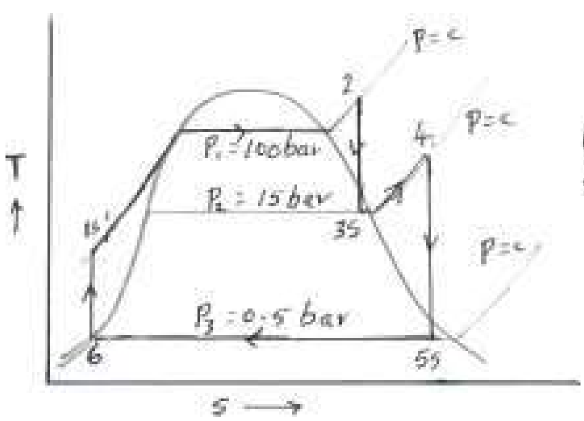
$$m_1 h_2 + (1 - m_1) h_{13} = 1 \times h_{14}$$
$$m_2 h_5 + (1 - m_1 - m_2) h_{11} = (1 - m_1) h_{12}$$
$$m_3 h_6 + (1 - m_1 - m_2 - m_3) h_9 = (1 - m_1 - m_2) h_{10}$$

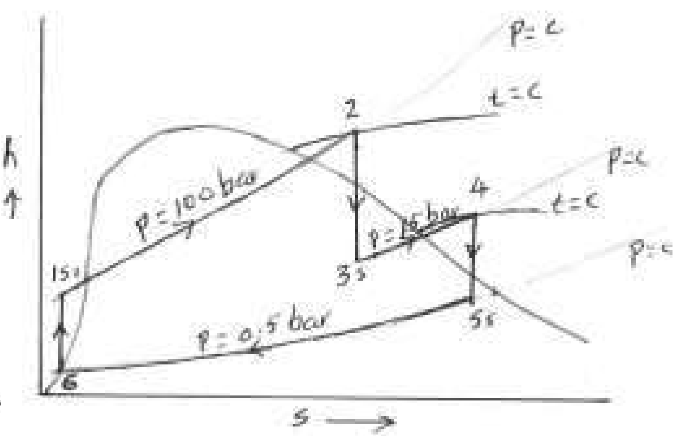
From which m_1, m_2 and m_3 can be evaluated

Numerical Problems:

1. **An ideal reheat cycle utilizes steam as the working fluid. Steam at 100 bar, 400^{0}C is expanded in the HP turbine to 15 bar. After this, it is reheated to 350^{0}C at 15 bar and is then expanded in the LP turbine to the condenser pressure of 0.5 bar. Determine the thermal η and steam rate.**

Solution:





From steam tables

P = 100 bar t = 400^{0}C = v = 0.026408m^3/kg
h = 3099.9 kJ/kg
S = 6.2182 kJ/kg-K

P = 15 bar t_S = 192.28^{0}C, v_f = 0.0011538m^3/kg, v_g = 0.13167m^3/kg
h_f = 844.6 kJ/kg, h_{fg} = 1845.3 kJ/kg, h_g = 2789.9 kJ/kg
s_f = 2.3144kJ/kg-K, s_{fg} = 4.1262 kJ/kg-K, s_g = 6.4406 kJ/kg-K

P = 0.5 bar $t_S = 81.35^0C$, $v_f = 0.0010301\ m^3/kg$, $v_g = 3.2401\ m^3/kg$
$h_f = 340.6$ kJ/kg, $h_{fg} = 2305.4$ kJ/kg $h_g = 2646.0$ kJ/kg
$s_f = 1.0912$ kJ/kg-K $s_{fg} = 6.5035$ kJ/kg-K, $s_g = 7.5947$ kJ/kg-K

$h_2 = 3099.9$ kJ/kg,
Process 2-3s is isentropic, i.e., $S_2 = S_{3S}$
$6.2182 = 2.3144 + x_{3S}(4.1262)$
$\therefore x_{3S} = 0.946$
$\therefore h_{3S} = 844.6 + x_{3S}(1845.3)$
$= 2590.44$ kJ/kg

$\therefore$ Expansion of steam in the HP turbine $= h_2 - h_{3S}$
$= 3099.9 - 2590.44$
$= 509.46$ kJ/kg

P = 15 bar, t = 350^0C = v = 0.18653
h = 3148.7
s = 7.1044

Expansion of steam in the LP cylinder = $h_4 - h_{5s}$
$h_4 = 3148.7$ kJ/kg
To find h_{5s}:
We have $S_4 = S_{5s}$
$7.1044 = S_{f5} + x_{5S} S_{fg5}$
$= 1.0912 + x_{5S}(6.5035)$
$\therefore x_{5S} = 0.925$
$\therefore h_{5s} = 340.6 + 0.925(2305.4) = 2473.09$ kJ/kg
$\therefore$ Expansion of steam in the LP turbine $= 3148.7 - 2473.09$
$= 675.61$ kJ/kg
$h_6 = h_f$ for P_3 =0.5 bar i.e., $h_6 = 340.6$ kJ/kg
Pump work, $W_P = h_{1s} - h_6 = v_{f5}(P_3 - P_1) = 0.0010301\ (100 - 0.501 \times 10^5)$
$= 10.249$ kJ/kg
$\therefore h_{1s} = 350.85$ kJ/kg
$\therefore$ Heat supplied, $Q_H = (h_2 - h_{1S}) + (h_4 - h_{3S})$
$= (3099.9 - 350.85) + (3148.7 - 2590.44)$
$= 2749.05$ kJ/kg + 558.26
$= 3307.31$ kJ/kg

$$\therefore \quad \eta_{th} = \frac{W_{net}}{Q_H} = \frac{(W)_{HP} + (W)_{LP} - W_P}{Q_H}$$

$$= \frac{509.46 + 675.61 - 10.25}{3307.31} = 0.355$$

Steam rate, SSC $= \dfrac{3600}{W_{net}} = 3.064 kg / kWh$

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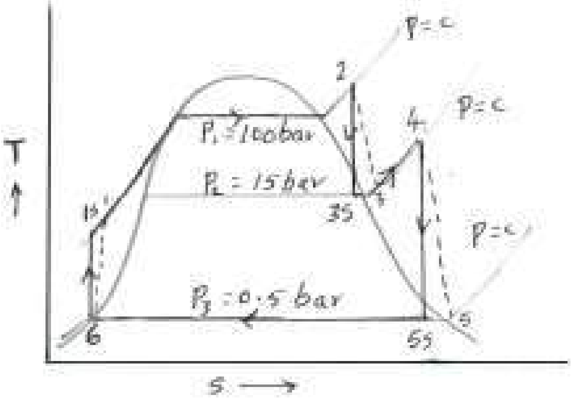
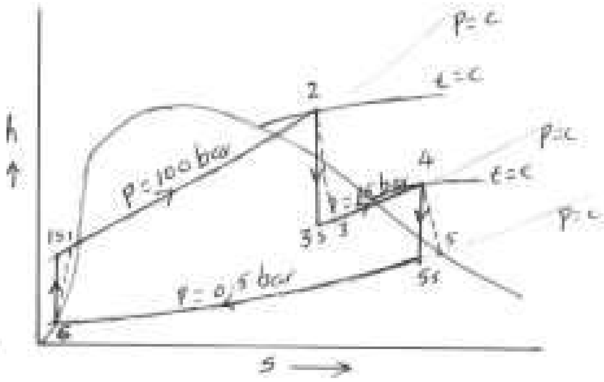
1. b) When η of the HP turbine, LP turbine and feed pump are 80%, 85% and 90% respectively.





$$\eta_{tHP} = \frac{h_2 - h_3}{h_2 - h_{3s}} = 0.8 = \frac{3099.9 - h_3}{3099.9 - 2590.44}$$

$\therefore h_3 = 2692.33$ kJ/kg

$$\eta_{tLP} = \frac{h_4 - h_5}{h_4 - h_{5s}} = 0.85 = \frac{3148.7 - h_5}{3148.7 - 2473.09}$$

$\therefore h_5 = 2574.43$ kJ/kg

$$\eta_P = \frac{h_{15} - h_6}{h_1 - h_6} = 0.9 = \frac{350.85 - 340.6}{h_1 - 340.6}$$

$\therefore h_1 = 351.99$ kJ/kg

$$\therefore \eta_{th} = \frac{(h_2 - h_3) + (h_4 - h_5) - (h_1 - h_6)}{(h_2 - h_1) + (h_4 - h_3)}$$

$$= \frac{(3099.9 - 2692.33) + (3148.7 - 2574.43) - (351.99 - 340.6)}{(3099.9 - 351.99) + (3148.7 - 2692.33)}$$

= 0.303 or 30.3%

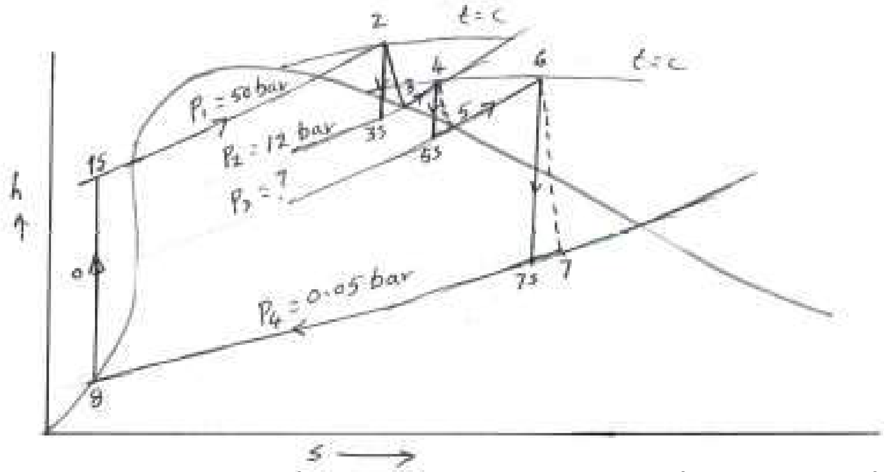
$\therefore$ SSC = 3.71 kg/kWh

Using Mollier-chart: $h_2 = 3095$ kJ/kg, $h_{3s} = 2680$ kJ/kg, $h_4 = 3145$ kJ/kg
$h_5 = 2475$ kJ/kg, $h_6 = 340.6$ kJ/kg (from steam tables)
$W_P = 10.249$ kJ/kg

2. **Steam at 50 bar, 350^{0}C expands to 12 bar in a HP stage, and is dry saturated at the stage exit. This is now reheated to 280^{0}C without any pressure drop. The reheat steam expands in an intermediate stage and again emerges dry and saturated at a low pressure, to be reheated a second time to 280^{0}C. Finally, the steam expands in a LP stage to 0.05 bar. Assuming the work output is the same for the high and intermediate**

stages, and the efficiencies of the high and low pressure stages are equal, find: (a) η of the HP stage (b) Pressure of steam at the exit of the intermediate stage, (c) Total power output from the three stages for a flow of 1kg/s of steam, (d) Condition of steam at exit of LP stage and (e) Then η of the reheat cycle. Also calculate the thermodynamic mean temperature of energy addition for the cycle.

Solution:



$P_1 = 50$ bar $t_2 = 350^0$C $P_2 = 12$ bar $t_4 = 280^0$C, $t_6 = 280^0$C

$P_3 = ?$ $P_4 = 0.05$ bar

From Mollier diagram

$h_2 = 3070$kJ/kg $h_{3s} = 2755$ kJ/kg $h_3 = 2780$ kJ/kg $h_4 = 3008$ kJ/kg

(a) η_t for HP stage $= \dfrac{h_2 - h_3}{h_2 - h_{3s}} = \dfrac{3070 - 2780}{3070 - 2755}$

$= 0.921$

(b) Since the power output in the intermediate stage equals that of the HP stage, we have

$h_2 - h_3 = h_4 - h_5$

i.e., $3070 - 2780 = 3008 - h_5$

$\therefore h_5 = 2718$ kJ/kg

Since state 5 is on the saturation line, we find from Mollier chart, $P_3 = 2.6$ bar,

Also from Mollier chart, $h_{5s} = 2708$ kJ/kg, $h_6 = 3038$ kJ/kg, $h_{7s} = 2368$ kJ/kg

Since η_t is same for HP and LP stages,

$$\eta_t = \frac{h_6 - h_7}{h_6 - h_{7s}} = 0.921 = \frac{3038 - h_7}{3038 - 2368} \quad \therefore h_7 = 2420.93\text{kJ/kg}$$

$\therefore$ At a pressure 0.05 bar, $h_7 = h_{f7} + x_7\, h_{fg7}$

$2420.93 = 137.8 + x_7\,(2423.8)$

$\therefore x_7 = 0.941$

Total power output $= (h_2 - h_3) + (h_4 - h_5) + (h_6 - h_7)$

$= (3070 - 2780) + (3008 - 2718) + (3038 - 2420.93)$

$= 1197.07$ kJ/kg

$\therefore$ Total power output /kg of steam = 1197.07 kW

For P_4 = 0.05 bar from steam tables, h_8 = 137.8 kJ/kg;

$W_P = 0.0010052\,(50 - 0.05)\,10^2 = 5.021$ kJ/kg

$= h_8 - h_{1s}$

$\therefore h_{1s} = 142.82$ kJ/kg

Heat supplied, Q_H $= (h_2 - h_{1s}) + (h_4 - h_3) + (h_6 - h_5)$

$= (3070 - 142.82) + (3008 - 2780) + (3038 - 2718)$

$= 3475.18$ kJ/kg

$W_{net} = W_T - W_P = 1197.07 - 5.021 = 1192.05$ kJ/kg

$$\therefore \eta_{th} = \frac{W_{net}}{Q_H} = \frac{1192.05}{3475.18} = 0.343$$

$$\eta_{th} = 1 - \frac{T_o}{T_m} = 1 - \frac{(273 + 32.9)}{T_m} = 0.343,$$

$$0.657 = \frac{305.9}{T_m}$$

$\therefore T_m = 465.6$ K

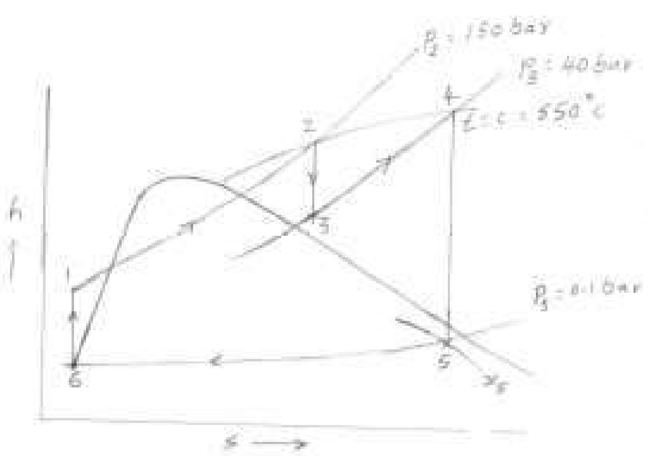
Or

$$T_m = \frac{h_2 - h_{1s}}{S_2 - S_{1s}} = \frac{3070 - 142.82}{6.425 - 0.4763} = 492K$$

$$SSC = \frac{3600}{1192.05} = 3.02 kg/kWh$$

3. A steam power station uses the following cycle: Steam at boiler outlet – 150 bar; reheat at 40 bar, 550^{0}C; condenser at 0.1 bar. Using Mollier chart and assuming that all processes are ideal, find (i) quality at turbine exhaust (ii) cycle η (iii) steam rate.

Solution:



P_2 = 150 bar $t_2 = 550^0$C P_3 = 40 bar $t_3 = 550^0$C

P_5 = 0.1 bar

From Mollier diagram i.e., h-s diagram

$$h_2 = h\Big|_{150\,bar,550^0C} = 3450 kJ/kg$$

$$h_4 = h\Big|_{40\,bar,550^0C} = 3562 kJ/kg$$

h_3 = 3050 kJ/kg

h_5 = 2290 kJ/kg

x_5 = 0.876 kJ/kg

h_6 can not determined from h-s diagram, hence steam tables are used.

$$h_6 = h_f\Big|_{0.1\,bar} = 191.8 kJ/kg$$

Process 6-1 is isentropic pump work i.e., $W_{P1} = v\int dP$

= 0.0010102 (40 – 01) $10^5/10^3$ = 4.031 kJ/kg

= (h_1 – h_6)

$\therefore h_1$ = 195.8 kJ/kg

(i) Quality of steam at turbine exhaust = x_5 = 0.876

$$(ii)\ \eta_{cycle} = \frac{W_T - W_P}{Q_H}$$

Turbine work, W_T = W_{T1} + W_{T2}

= (h_2 – h_3) + (h_4 – h_5)

= (3450 – 3050) + (3562 – 2290)

= 1672 kJ/kg

$Q_H = Q_1 + Q_R$ = (h_2- h_1) + (h_4 – h_3)

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$= (3450 - 195.8) + (3562 - 3050)$

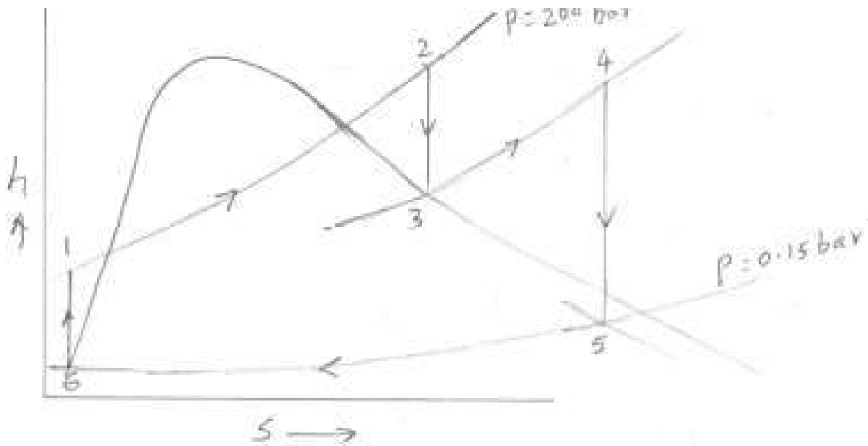
$= 3766.2$ kJ/kg

$$\therefore \eta_{cycle} = \frac{1672 - 4.031}{3766.2} = \frac{1667.97}{3766.2} = 0.443$$

$$(iii) \quad Steam\ rate = \frac{3600}{1667.97} = 2.16 kg / kWh$$

4. **An ideal Rankine cycle with reheat is designed to operate according to the following specification. Pressure of steam at high pressure turbine = 20 MPa, Temperature of steam at high pressure turbine inlet = 550^{0}C, Temperature of steam at the end of reheat = 550^{0}C, Pressure of steam at the turbine exhaust = 15 KPa. Quality of steam at turbine exhaust = 90%. Determine (i) the pressure of steam in the reheater (ii) ratio of pump work to turbine work, (iii) ratio of heat rejection to heat addition, (iv) cycle η.**

Solution:



$P_2 = 200$ bar $\quad t_2 = 550^0$C $\quad t_4 = 550^0$C $\quad P_5 = 0.15$ bar $\quad x_5 = 0.9$

From Mollier diagram,

$h_2 = 3370$ kJ/kg

$h_3 = 2800$ kJ/kg

$h_4 = 3580$ kJ/kg

$h_5 = 2410$ kJ/kg

$x_5 = 0.915$

$P_3 = P_4 = 28$ bar

But given in the data $x_5 = 0.9$

From steam tables $h_6 = 226$ kJ/kg

Pump work $W_P = v\int dP$

$= 0.001014\,(200 - 0.15)\,10^5/10^3$

$= 20.26$ kJ/kg

But $W_P = h_1 - h_6$ $\quad\therefore h_1 = 246.26$ kJ/kg

(i) Pressure of steam in the reheater = 28 bar

(ii) Turbine work $W_T = (h_2 - h_3) + (h_4 - h_5)$

$= (3370 - 2800) + (3580 - 2410)$

$= 1740$ kJ/kg

$$\therefore \text{Ratio of } \frac{W_P}{W_T} = 0.0116 \quad i.e., \quad 1.2\%$$

(iii) $Q_L = (h_5 - h_6) = (2410 - 226) = 2184$ kJ/kg

$Q_H = (h_2 - h_1) + (h_4 - h_3)$

$= (3370 - 226) + (3580 - 2800)$

$= 3924$ kJ/kg

$$\therefore \frac{Q_L}{Q_H} = 0.5565 \quad i.e., \quad 55.65\%$$

$$(iv)\ \eta_{cycle} = \frac{W_{net}}{Q_{Total}} = \frac{(1740 - 20.26)}{3924} = 0.4383 \quad i.e., \quad 43.8\%$$

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Feedwater Heaters (FWH)

A practical Regeneration process in steam power plants is accomplished by extracting or bleeding, steam from the turbine at various points. This steam, which could have produced more work by expanding further in the turbine, is used to heat the feed water instead. The device where the feedwater heated by regeneration is called a Regenerator or a Feedwater Heater (FWH).

A feedwater heater is basically a heat exchanger where heat is transferred from the steam to the feedwater either by mixing the two streams (open feedwater heaters) or without mixing them (closed feedwater heaters).

Open Feedwater Heaters

An open (or direct-contact) feedwater heater is basically a mixing chamber, where the steam extracted from the turbine mixes with the feedwater exiting the pump. Ideally, the mixture leaves the heater as a saturated liquid at the heater pressure.

The advantages of open heater are simplicity, lower cost, and high heat transfer capacity. The disadvantage is the necessity of a pump at each heater to handle the large feedwater stream.

Closed Feedwater Heaters

In closed feedwater heater, the heat is transferred from the extracted steam to the feedwater without mixing taking place. The feedwater flows through the tubes in the heater and extracted steam condenses on the outside of the tubes in the shell. The heat released from the condensation is transferred to the feedwater through the walls of the tubes. The condensate (saturated water at the steam extraction pressure), some times called the heater-drip, then passes through a trap into the next lower pressure heater. This, to some extent, reduces the steam required by that heater. The trap passes only liquid and no vapour. The drip from the lowest pressure heater could similarly be trapped to the condenser, but this would be throwing away energy to the condenser cooling water. The avoid this waste, the drip pump feed the drip directly into the feedwater stream.

A closed heaters system requires only a single pump for the main feedwater stream regardless of the number of heaters. The drip pump, if used is relatively small. Closed heaters are costly and may not give as high a feedwater temperature as do open heaters.

In most steam power plants, closed heaters are favoured, but atleast one open heater is used, primarily for the purpose of feedwater deaeration. The open heater in such a system is called deaerator.

Note: The higher the number of heater used, the higher will be the cycle efficiency. The number of heater is fixed up by the energy balance of the whole plant when it is found that the cost of adding another does not justify the saving in Q_H or the marginal increase in cycle efficiency. An

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increase in feedwater temperature may, in some cases, cause a reduction in boiler efficiency. So the number of heaters get optimized. Five feedwater heaters are often used in practice.

Characteristics of an Ideal working fluid

The maximum temperature that can be used in steam cycles consistent with the best available material is about 600^0C, while the critical temperature of steam is 375^0C, which necessitates large superheating and permits the addition of only an infinitesimal amount of heat at the highest temperature.

The desirable characteristics of the working fluid in a vapour power cycle to obtain best thermal η are as follows:

a) The fluid should have a high critical temperature so that the saturation pressure at the maximum permissible temperature (metallurgical limit) is relatively low. It should have a large enthalpy of evaporation at that pressure.
b) The saturation pressure at the temperature of heat rejection should be above atmosphere pressure so as to avoid the necessity of maintaining vacuum in the condenser.
c) The specific heat of liquid should be small so that little heat transfer is required to raise the liquid to the boiling point.
d) The saturation vapour line of the T-S diagram should be steep, very close to the turbine expansion process so that excessive moisture does not appear during expansion.
e) The freezing point of the fluid should be below room temperature, so that it does not get solidified while flowing through the pipe lines.
f) The fluid should be chemically stable and should not contaminate the materials of construction at any temperature.
g) The fluid should be nontoxic, non corrosive, not excessively viscous, and low in cost.

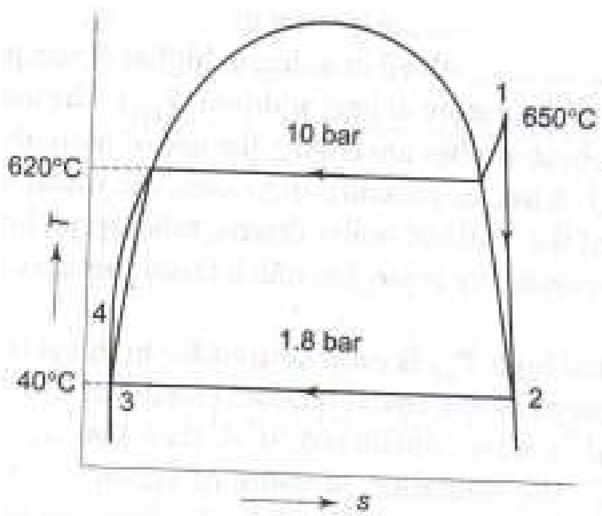
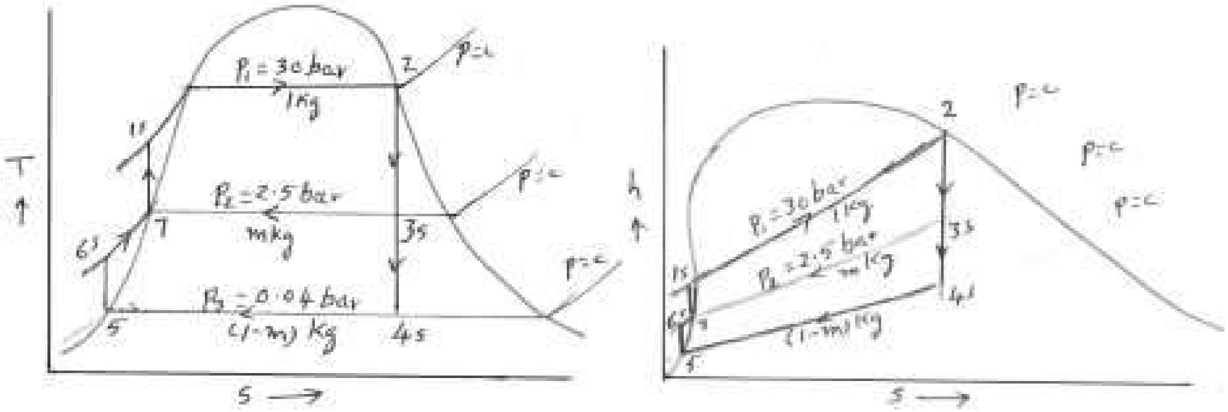


Fig. T-S diagram for an Ideal Working Fluid for a Vapour Power Cycle

Numerical Problems:

1. **An ideal regenerative cycle operates with dry saturated steam, the maximum and minimum pressures being 30 bar and 0.04 bar respectively. The plant is installed with a single mixing type feed water heater. The bled steam pressure is 2.5 bar. Determine (a) the mass of the bled steam, (b) the thermal η of the cycle, and (c) SSC in kg/kWh.**

Solution:



$P_1 = 30$ bar $\quad P_2 = 2.5$ bar $\quad P_3 = 0.04$ bar

From steam tables, For $P_1 = 30$ bar, $\quad h_2 = 2802.3$ kJ/kg, $\quad S_2 = 6.1838$ kJ/kg-^{0}k

But $S_2 = S_{3s}$ i.e., $6.1838 = 1.6072 + x_3 (5.4448)$

$\therefore x_3 = 0.841$

$\therefore h_3 = 535.4 + 0.841 (2281.0)$
$= 2452.68$ kJ/kg

Also $S_2 = S_{4s}$ i.e., $6.1838 = 0.4225 + x_4 (8.053)$

$\therefore x_4 = 0.715$

$\therefore h_4 = 121.4 + 0.715 (2433.1)$
$= 1862.1$ kJ/kg

At $P_3 = 0.04$ bar, $\quad h_5 = 121.4$ kJ/kg, $\quad v_5 = 0.001004$ m^3/kg

$\therefore$Condensate pump work $= (h_6 – h_5) = v_5 (P_2 – P_3)$
$= 0.001004 (2.5 – 0.04) (10^5/10^3)$
$= 0.247$ kJ/kg

$\therefore h_6 = 0.247 + 121.4 = 121.65$ kJ/kg

Similarly, $h_1 = h_7 + v_7 (P_1 – P_2) (10^5/10^3)$
$= 535.4 + 0.0010676 (30 – 2.5) 10^2$
$= 538.34$ kJ/kg

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a) **Mass of the bled steam:**

Applying the energy balance to the feed water heater

$mh_3 + (1 - m)\, h_6 = 1\, (h_7)$

$$\therefore m = \frac{(h_7 - h_6)}{(h_3 - h_6)} = \frac{(535.4 - 121.65)}{(2452.68 - 121.65)} = 0.177 kg/kg \text{ of steam}$$

b) **Thermal η:**

Turbine work, W_T $= 1\,(h_2 - h_{3s}) + (1 - m)\,(h_3 - h_{4s})$
$= 1\,(2802.3 - 2452.65) + (1 - 0.177)\,(2452.68 - 1862.1)$
$= 835.67$ kJ/kg

Pump work, W_P $= (1 - m)\,(h_{6s} - h_5) + 1\,(h_{1s} - h_7)$
$= (1 - 0.177)\,(121.65 - 121.4) + 1\,(538.34 - 535.4)$
$= 3.146$ kJ/kg

$\therefore W_{net} = W_T - W_P = 832.52$ kJ/kg

Heat supplied, Q_H $= 1\,(h_2 - h_{1s})$
$= 1\,(2802.3 - 538.34)$
$= 2263.96$ kJ/kg

$$\therefore \eta_{th} = \frac{W_{net}}{Q_H} = \frac{832.52}{2263.96} = 0.368 \text{ or } \mathbf{36.8\%}$$

c) **SSC:**

$$SSC = \frac{3600}{W_{net}} = 4.324 kg/kWh$$

2. In problem (3), also calculate the increase in mean temperature of heat addition, efficiency and steam rate as compared to the Rankine cycle (without regeneration)

Solution: Tm_1 (with regeneration) $= \dfrac{h_2 - h_1}{S_2 - S_1} = \dfrac{2263.96}{(6.1838 - 1.6072)} = 494.68k$

Tm_1 (without regeneration) $= \dfrac{h_2 - h_6}{S_2 - S_6} = \dfrac{2802.3 - 121.65}{(6.1838 - 0.4225)} = 465.29k$

$\therefore$ Increase in Tm_1 due to regeneration $= 494.68 - 465.29 =$ **29.39^{0}K**

W_T (without regeneration) $= h_2 - h_4 = 2802.3 - 1862.1 = 940.2$ kJ/kg

W_P (without regeneration) $= (h_1 - h_5) = v_5\,(30 - 0.04)\,10^2$
$= 0.001004\,(29.96)\,10^2 = 3.01$ kJ/kg

$\therefore h_1 = 3.01 + 121.4 = 124.41$ kJ/kg

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$$\therefore \eta_{th} \text{ (without regeneration) } = \frac{W_{net}}{Q_H} = \frac{(940.2 - 3.01)}{2802.3 - 124.41} = 0.349$$

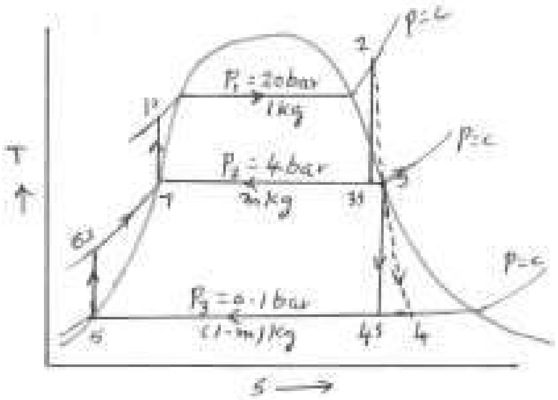
$\therefore$ Increase in η_{th} due to regeneration = 0.368 – 0.349 = 0.018 i.e., 1.8%

Steam rate (without regeneration) = 3.84 kg/kWh

$\therefore$ Increase in steam rate due to regeneration = 4.324 – 3.84
= 0.484 kg/kWh

3. **Steam at 20 bar and 300^{0}C is supplied to a turbine in a cycle and is bled at 4 bar. The bled-steam just comes out saturated. This steam heats water in an open heater to its saturation state. The rest of the steam in the turbine expands to a condenser pressure of 0.1 bar. Assuming the turbine efficiency to be the same before and after bleeding, find: a) the turbine η and the steam quality at the exit of the last stage; b) the mass flow rate of bled steam 1kg of steam flow at the turbine inlet; c) power output / (kg/s) of steam flow; and d) overall cycle η.**

Solution:



P_1 = 20 bar t_1 = 300^{0}C P_2 = 4 bar P_3 = 0.1 bar

From steam tables,

For P_1 = 20 bar and t_1 = 300^{0}C

v_2 = 0.12550 h_2 = 3025.0 S_2 = 6.7696

For P_2 = 4 bar, h_3 = 2737.6, t_s = 143.63

h_f = 604.7, h_{fg} = 2132.9, S_f = 1.7764, S_{fg} = 5.1179, S_g = 6.8943

For P_2 = 0.1 bar, 45.83, 191.8, 2392.9, 2584.8, 0.6493, 7.5018, 8.1511

We have, $S_2 = S_{3s}$ i.e., 6.7696 = 1.7764 + x_3 (5.1179)

$\therefore x_3 = 0.976$

$\therefore h_{3s}$ = 604.7 + 0.976 (2132.9) = 2685.63 kJ/kg

$$\therefore \eta_t = \frac{h_2 - h_3}{h_2 - h_{3s}} = \frac{3025 - 2737.6}{3025 - 2685.63} = 0.847$$

$S_3 = S_{4s}$ i.e., 6.8943 = 0.6493 + x_4 (7.5018)

$\therefore x_{4s} = 0.832$

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$\therefore h_{4s} = 191.8 + 0.832\ (2392.9) = 2183.81$kJ/kg

But η_t is same before and after bleeding i.e., $\eta_t = \dfrac{h_3 - h_4}{h_3 - h_{4s}}$

$$\text{i.e., } 0.847 = \frac{2737.6 - h_4}{2737.6 - 2183.81}$$

$$\therefore h_4 = 2268.54 \text{ kJ/kg}$$

$\therefore h_4 = h_{f4} + x_4\ h_{fg4}$ $\quad\quad \therefore x_4 = 0.868$

b) Applying energy balance to open heater, $mh_3 + (1 - m)\ h_{6s} = 1\ (h_7)$

$$\therefore m = \frac{h_7 - h_6}{h_3 - h_6}$$

Condensate pump work, $(h_{6s} - h_5) = v_5\ (P_3 - P_2)$

$= 0.0010102\ (3.9)\ 10^2 = 0.394$ kJ/kg

$\therefore h_{6s} = 191.8 + 0.394 = 192.19$ kJ/kg

Similarly, $h_{1s} = h_7 + v_7\ (P_1 - P_2)$

$= 604.7 + -.0010839\ (16)\ 10^2 = 606.43$ kJ/kg

$$\therefore m = \frac{604.7 - 192.19}{2737.6 - 192.19} = 0.162$$

c) Power output or $W_T = (h_2 - h_3) + (1 - m)\ (h_3 - h_4)$

$= (3025 - 2737.6) + (1 - 0.162)\ (2737.6 - 2268.54)$

$= 680.44$ kJ/kg

For 1kg/s of steam, $\quad W_T = 680.44$ kW

d) Overall thermal efficiency, $\eta_0 = \dfrac{W_{net}}{Q_H}$

$W_P \quad = (1 - m)\ (h_{6s} - h_5) + 1\ (h_{1s} - h_7)$

$= (1 - 0162)\ (192.19 - 191.8) + 1\ (606.43 - 604.7)$

$= 2.057$ kJ/kg

$W_{net} = 680.44 - 2.057 = 678.38$ kJ/kg

$Q_H = 1\ (h_2 - h_{1s}) = (3025 - 606.43) = 2418.57$ kJ/kg

$$\therefore \eta_0 = \frac{678.38}{2418.57} = 0.2805$$

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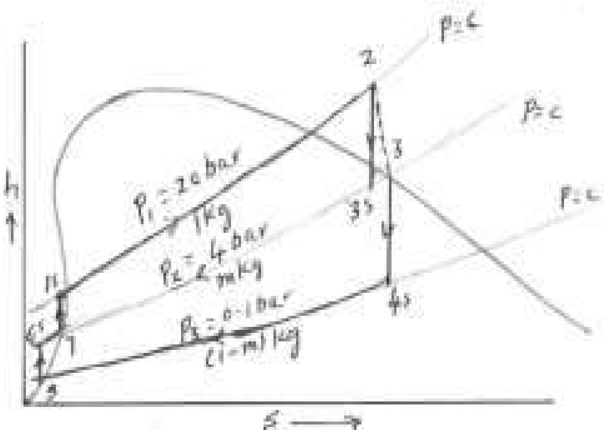
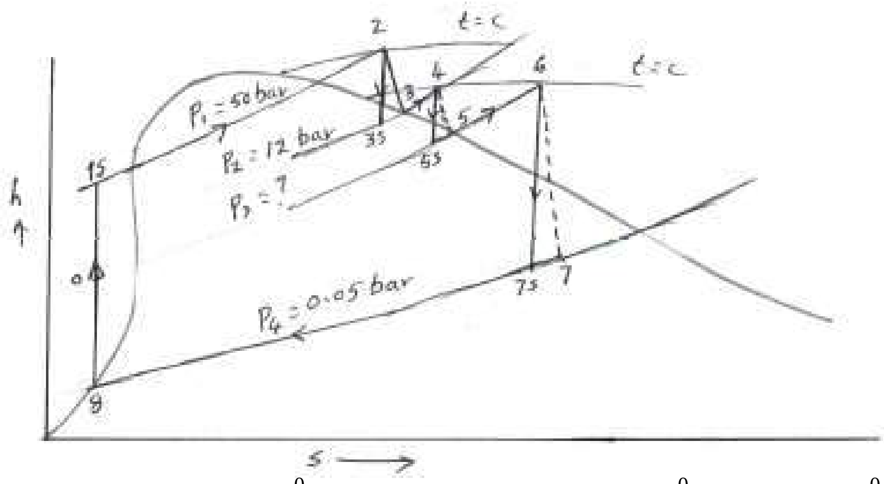
Using Moiller Diagram



4. **Steam at 50 bar, 350^{0}C expands to 12 bar in a HP stage, and is dry saturated at the stage exit. This is now reheated to 280^{0}C without any pressure drop. The reheat steam expands in an intermediate stage and again emerges dry and saturated at a low pressure, to be reheated a second time to 280^{0}C. Finally, the steam expands in a LP stage to 0.05 bar. Assuming the work output is the same for the high and intermediate stages, and the efficiencies of the high and low pressure stages are equal, find: (a) η of the HP stage (b) Pressure of steam at the exit of the intermediate stage, (c) Total power output from the three stages for a flow of 1kg/s of steam, (d) Condition of steam at exit of LP stage and (e) Then η of the reheat cycle. Also calculate the thermodynamic mean temperature of energy addition for the cycle.**

Solution:



P_1 = 50 bar t_2 = 350^{0}C P_2 = 12 bar t_4 = 280^{0}C, t_6 = 280^{0}C

P_3 = ? P_4 = 0.05 bar

From Mollier diagram

h_2 = 3070kJ/kg h_{3s} = 2755 kJ/kg h_3 = 2780 kJ/kg h_4 = 3008 kJ/kg

(a) η_t for HP stage $= \dfrac{h_2 - h_3}{h_2 - h_{3s}} = \dfrac{3070 - 2780}{3070 - 2755}$

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= 0.921

(b) Since the power output in the intermediate stage equals that of the HP stage, we have

$h_2 – h_3 = h_4 – h_5$

i.e., $3070 – 2780 = 3008 – h_5$

$\therefore h_5 = 2718$ kJ/kg

Since state 5 is on the saturation line, we find from Mollier chart, $P_3 = 2.6$ bar,
Also from Mollier chart, $h_{5s} = 2708$ kJ/kg, $h_6 = 3038$ kJ/kg, $h_{7s} = 2368$ kJ/kg

Since η_t is same for HP and LP stages,

$$\eta_t = \frac{h_6 - h_7}{h_6 - h_{7s}} = 0.921 = \frac{3038 - h_7}{3038 - 2368} \quad \therefore h_7 = 2420.93 \text{kJ/kg}$$

$\therefore$ At a pressure 0.05 bar, $h_7 = h_{f7} + x_7 h_{fg7}$

$2420.93 = 137.8 + x_7 (2423.8)$

$\therefore x_7 = 0.941$

Total power output $= (h_2 – h_3) + (h_4 – h_5) + (h_6 – h_7)$

$= (3070 – 2780) + (3008 – 2718) + (3038 – 2420.93)$

$= 1197.07$ kJ/kg

$\therefore$ Total power output /kg of steam = 1197.07 kW

For $P_4 = 0.05$ bar from steam tables, $h_8 = 137.8$ kJ/kg;

$W_P = 0.0010052 (50 – 0.05) 10^2 = 5.021$ kJ/kg

$= h_8 – h_{1s}$

$\therefore h_{1s} = 142.82$ kJ/kg

Heat supplied, $Q_H = (h_2 – h_{1s}) + (h_4 – h_3) + (h_6 – h_5)$

$= (3070 – 142.82) + (3008 – 2780) + (3038 – 2718)$

$= 3475.18$ kJ/kg

$W_{net} = W_T – W_P = 1197.07 – 5.021 = 1192.05$ kJ/kg

$$\therefore \eta_{th} = \frac{W_{net}}{Q_H} = \frac{1192.05}{3475.18} = 0.343$$

$$\eta_{th} = 1 - \frac{T_o}{T_m} = 1 - \frac{(273 + 32.9)}{T_m} = 0.343,$$

$$0.657 = \frac{305.9}{T_m}$$

$\therefore T_m = 465.6$ K

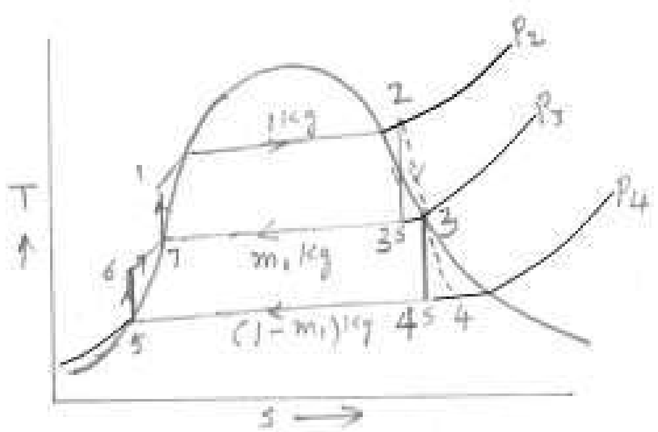
Or

$$T_m = \frac{h_2 - h_{1s}}{S_2 - S_{1s}} = \frac{3070 - 142.82}{6.425 - 0.4763} = 492K$$

$$SSC = \frac{3600}{1192.05} = 3.02kg / kWh$$

5. Steam at 30 bar and 350^{0}C is supplied to a steam turbine in a practical regenerative cycle and the steam is bled at 4 bar. The bled steam comes out as dry saturated steam and heats the feed water in an open feed water heater to its saturated liquid state. The rest of the steam in the turbine expands to condenser pressure of 0.1 bar. Assuming the turbine η to be same before and after bleeding determine (i) the turbine η, (ii) steam quality at inlet to condenser, (iii) mass flow rate of bled steam per unit mass rate at turbine inlet and (iv) the cycle η.

Solution:



P_2 = 30 bar t_2 = 350^{0}C P_3 = 4 bar P_4 = 0.1 bar

$h_3 = h_g$ at P_3 = 4 bar, = 2737.6 kJ/kg

From superheated steam tables,

$$h_2 = h_3 = h_g\Big|_{P_3=4bar} = 2737.6\ kJ / kg$$

$$h_2 = h\Big|_{P_2=30bar\ \&\ t_2=350^0C} = 3117.5\ kJ / kg \text{ and } S_2 = 6.7471 \text{ kJ/kg-K}$$

$$h_5 = h_f\Big|_{P_5=0.1bar} = 191.8\ kJ / kg$$

$$h_7 = h_f \Big|_{P_7=4bar} = 604.7\ kJ/kg$$

Process 2-3s is isentropic, i.e., $S_2 = S_{3S}$

$$6.7471 = 1.7764 + x_{3S}(5.1179)$$

$$\therefore x_{3S} = 0.971$$

$\therefore h_{3S} = h_{f3} + x_{3S}\, h_{fg3}$

$= 604.7 + 0.971\,(2132.9)$

$= 2676.25$ kJ/kg

Process 3-4s is isentropic i.e., $S_3 = S_{4S}$

i.e., $6.8943 = 0.6493 + x_{4S}(7.5018)$

$\therefore x_{4S} = 0.832$

$\therefore h_{4S} = 191.8 + 0.832\,(2392.9) = 2183.8$ kJ/kg

Given, η_t (before bleeding) = η_t (after bleeding)

We have, η_t (before bleeding) $= \dfrac{h_2 - h_3}{h_2 - h_{3S}} = \dfrac{3117.5 - 2737.6}{3117.5 - 2676.25} = 0.86$

$$\therefore 0.86 = \frac{h_3 - h_4}{h_3 - h_{4S}} = \frac{2737.6 - h_4}{2737.6 - 2183.8} \quad \therefore h_4 = 2261.33\,kJ/kg$$

But $h_4 = h_{f4} + x_4\, h_{fg4}$

$2261.33 = 191.8 + x_4\,(2392.9)$

$\therefore x_4 = 0.865$

i.e., Dryness fraction at entry to condenser = x_4 = **0.865**

iii) Let m kg of steam is bled. Applying energy balance to FWH,

$mh_3 + (1 - m)\, h_6 = h_7$

We have $W_{P1} = (h_6 - h_5) \quad = v\int dP$

$= 0.0010102\ (4 - 0.1)\ 10^5/10^3$

$= 0.394$ kJ/kg

$\therefore h_6 = 0.394 + 191.8 = 192.19$ kJ/kg

Substituting,

$m\ (2737.6) + (1 - m)\ 192.19 = 604.7$

$\therefore m = 0.162$ kg

Also, $W_{P2} = (h_1 - h_7) = v\int dP$

$= 0.0010839 \times (30 - 4)\ 10^2$

$= 2.82$ kJ/kg

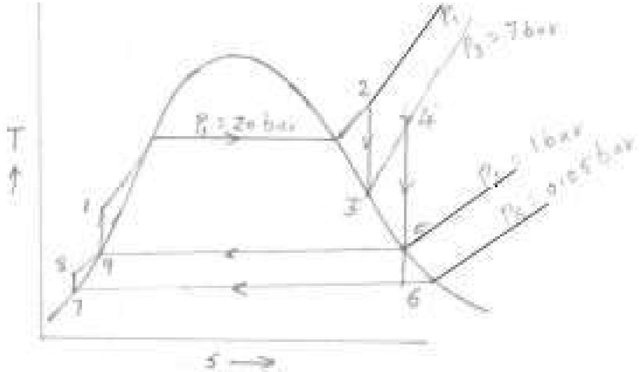
$\therefore h_1 = 2.82 + 604.7 = 607.52$ kJ/kg

$$\therefore \eta_{cycle} = \frac{W_T = W_P}{Q_H} = \frac{[(h_2 - h_3) + (1 - m)(h_3 - h_4)] - [(1 - m)(h_6 - h_5) + (h_1 - h_2)]}{(h_2 - h_1)}$$

$\eta_{cycle} = 0.31$

6. **In an ideal reheat regenerative cycle, the high pressure turbine receives steam at 20 bar, 300^{0}C. After expansion to 7 bar, the steam is reheated to 300^{0}C and expands in an intermediate pressure turbine to 1 bar. A fraction of steam is now extracted for feed water heating in an open type FWH. The remaining steam expands in a low pressure turbine to a final pressure of 0.05 bar. Determine (i) cycle thermal η, (ii) specific steam consumption, (iii) quality of steam entering condenser.**

Solution:



$h_2 = h\Big|_{20\,bar,\,300^0 C} = 3025 kJ/kg$ and s_2 = 6.7696kJ/kg-K

Process 2-3 is isentropic

i.e., $S_2 = S_3$

6.7696 = 1.9918 + x_3 (4.7134)

$\therefore x_3 = 1.014$

i.e., state 3 can be approximated as dry saturated.

$\therefore h_3 = h\Big|_{7\,bar,\,dry\,sat.}$ = 2762kJ/kg

$\therefore h_4 = h\Big|_{7\,bar,\,300^0 C} = 3059.8 kJ/kg$ and s_4 = 7.2997 kJ/kg-K

Process 4-5 is isentropic i.e., $S_4 = S_5$

7.2997 = 1.3027 + x_5 (6.0571)

$\therefore x_5 = 0.99$

$\therefore h_5 = h_{f5} + x_5\, h_{fg5}$ = 417.5 + 0.99 (2257.9) = 2652.9 kJ/kg

Process 5-6 is isentropic i.e., $S_5 = S_6$

7.2997 = 0.4763 + x_6 (7.9197)

$\therefore x_6 = 0.862$

$\therefore h_6$ = 137.8 + 0.862 (2423.8) = 2226.1 kJ/kg

$h_7 = h_7 = h_f\Big|_{0.05\,bar}$ = 137.8 kJ/kg

Neglecting W_{P1}, $h_8 = h_7$, Also neglecting W_{P2}, $h_9 = h_1$

$\therefore h_9 = h_f\Big|_{1\,bar}$ = 417.5 kJ/kg

Applying energy balance to FWH

$mh_5 + (1 – m) h_8 = h_9$

i.e., m (2652.9) + (1 – m) 137.8 = 417.5 $\therefore$ m = 0.111 kg/kg of steam

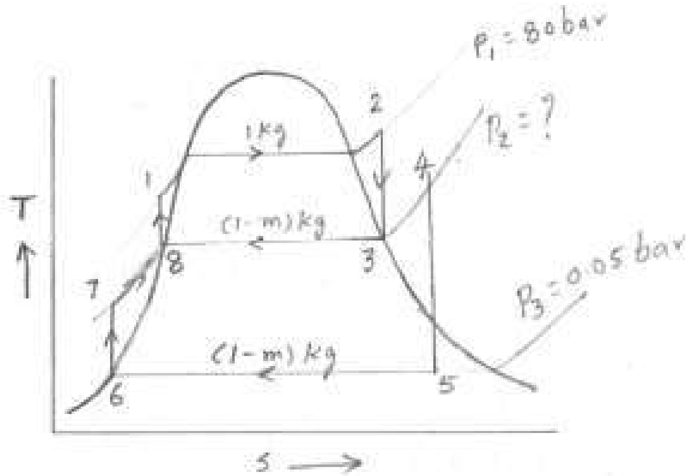
$$(i) \quad \eta_C = \frac{(h_2 - h_3) + (h_4 - h_5) + (1-m)(h_5 - h_6)}{(h_2 - h_1) + (h_4 - h_3)} = 0.35$$

$$(ii) \quad SSC = \frac{3600}{W_{net}} = 3.57 kg / kWh$$

(iii) Quality of steam entering condenser, x_6 = 0.862

7. The net power output of a regenerative – reheat cycle power plant is 80mW. Steam enters the high pressure turbine at 80 bar, 500^{0}C and expands to a pressure P_2 and emerges as dry vapour. Some of the steam goes to an open feed water heater and the balance is reheated at 400^{0}C at constant pressure P_2 and then expanded in the low pressure turbine to 0.05 bar. Determine (i) the reheat pressure P_2, (ii) the mass of bled steam per kg boiler steam, (iii) the steam flow rate in HP turbine, (iv) cycle η. Neglect pump work. Sketch the relevant lines on h-s diagram. Assume expansion in the turbines as isentropic.

Solution:



P = 80000 kW P$_1$ = 80 bar $t_2 = 500^0$C P_2 = ? $t_3 = 400^0$C

P_3 = 0.05 bar   m = ? $\dot{m}_S$ = ? η_{cycle} = ?

$h_2 = h\big|_{80\,bar,\,500^0C} = 3398.8 kJ/kg$ and s_2 = 6.7262

Process 2-3 is isentropic i.e., $S_2 = S_3$ = 6.7262 kJ/kg-K

Given state 3 is dry saturated i.e., S_3 = 6.7262 = $S_g\big|_{P_2}$

From table A – 1, for dry saturated steam, at P = 6.0 bar, S_g = 6.7575

and at P = 7.0 bar, S_g = 6.7052

Using linear interpolation,

$$\Delta P = \frac{6.0-7.0}{(6.7575-6.7052)} x(6.7262-6.7052) = 0.402\ bar$$

$\therefore$(i) P_2 = 6 + 0.402 = 6.402 bar

$\therefore h_3 = h\big|_{P_2=6.4bar}$

From table A – 1, For P = 6 bar h_g = 2755.5 S_g = 6.7575

For P = 7 bar, h_g = 2762.0 S_g = 6.7052

$$\therefore For\ P = 6.4\ bar \Rightarrow \frac{2762-2755.5}{1} x(0.4) + 2755.5 = 2758.1 kJ/kg$$

$\therefore h_3$ = 2758.1 kJ/kg

$h_4 = h\big|_{6.4\,bar,\,400^0C}$

From superheated steam tables, For   P = 6.0 bar,   h = 3270.6   s = 7.709

P = 7.0 bar,   h = 3269.0   s = 7.6362

$\therefore$For 6.4 bar, $h_4 \Rightarrow$ 3269.96 kJ/kg

$S_4 \Rightarrow$ 7.6798 kJ/kg-K

Process 4-5 is isentropic, $S_4 = S_5$

i.e., $7.6798 = 0.4763 + x_5 (7.9197)$

$\therefore x_5 = 0.909$

$\therefore h_5 = 137.8 + 0.909 (2423.8) = 2342.41$ kJ/kg

$$h_6 = h_f \Big|_{0.05\,bar} = 137.8 kJ/kg$$

$h_7 = h_6$ (since W_{P1} is neglected)

$$h_8 = h_f \Big|_{6.4\,bar} = 681.1 kJ/kg$$

$h_1 = h_8$ (since W_{P2} is neglected)

(ii) Applying energy balance to FWH,

$mh_3 + (1 – m) h_7 = h_8$

$m (2758.1) + (1 – m) 137.8 = 681.1$

$\therefore m = 0.313$ kg/kg of steam

(iii) $W_1 = W_{HP} = (h_2 – h_3) = (3398.8 – 2758.1)$

$= 640.7$ kJ/kg

$W_2 = W_{LP} = (1 – m) (h_4 – h_5)$

$= (1 – 0.313) (3269.96 – 2342.41)$

$= 637.2$ kJ/kg

$\therefore W_{net} = W_1 + W_2 = 1277.9$ kJ/kg

$$\therefore \text{Steam flow rate through HP turbine} = \frac{Power}{W_{net}} = \frac{80000}{1277.9} = 62.6 kg/s$$

(iv) $\eta_{cycle} = ?$ $\quad Q_H = (h_2 – h_1) + (1 – m) (h_4 – h_3) = 3069.35$ kJ/kg

$$\therefore \eta_{cycle} = \frac{W_{net}}{Q_H} = \frac{1277.9}{3069.35} = 0.42$$

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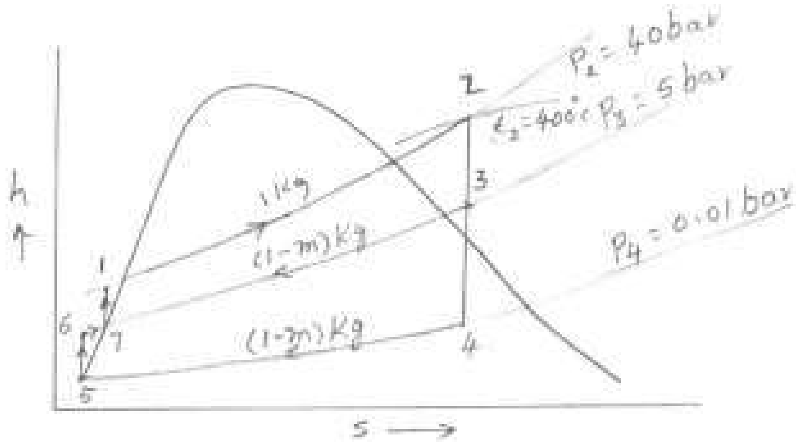
8. **In a single heater regenerative cycle, the steam enters the turbine at 30 bar, 400^{0}C and the exhaust pressure is 0.01 bar. The feed water heater is a direct contact type which operates at 5 bar. Find (i) thermal η and the steam rate of the cycle, (ii) the increase in mean temperature of heat addition, η and steam rate as compared to the Rankine cycle without regeneration. Pump work may neglected.**

Solution:



$P_2 = 40$ bar $t_2 = 400^0$C $P_4 = 0.01$ bar $P_3 = 5$ bar

From h-s diagram,

$$h_2 = h\Big|_{30\,bar,\,400^0 C} = 3230 kJ/kg$$

$h_3 = 2790$ kJ/kg

$h_4 = 1930$ kJ/kg

$h_5 = 29.3$ kJ/kg

$h_7 = 640.1$ kJ/kg

Since pump work may neglect, $h_6 = h_5$ & $h_1 = h_7$

(i) η_{cycle} = ?

Let m = mass of steam bled per kg boiler steam

Applying SFEE to FWH,

$mh_3 + (1 - m)\, h_6 = h_7$

$m\,(2790) + (1 - m)\, 29.3 = 640.1$

$\therefore$ m = 0.221 kg/kg of boiler steam

W_T = $(h_2 – h_3) + (1 – m) (h_3 – h_4)$

= (3230 – 2790) + (1 – 0.221) (2790 – 1930)

= 1109.73 kJ/kg

Q_H = $(h_2 – h_1)$ = (3230 – 640.1)

= 2589.9 kJ/kg

$\therefore \eta_{cycle} = \frac{W_T}{Q_H} = 0.428$ Since W_P is neglected

$$(ii)\ steam\ rate = \frac{3600}{W_T} = 3.24 kg/kWh$$

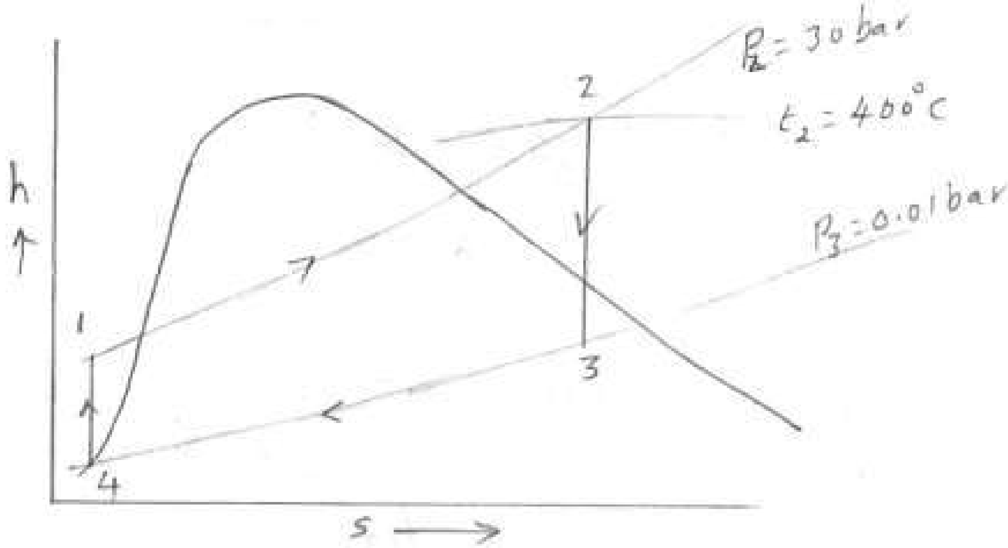
(iii) Mean temperature of heat addition, $\Delta T_m = \frac{Q_H}{s_2 - s_5}$

From h-s diagram, s_2 = 6.83 kJ/kg-K

From steam tables, s_5 = 0.1060 kJ/kg-K

$$\therefore \Delta T_m = \frac{2589.9}{(6.83 - 0.106)} = 385.2^0 K$$

Case (ii) Rankine cycle without Regeneration:



From h-s diagram,

$h_2 = 3230$ kJ/kg

$h_3 = 1930$ kJ/kg

$h_4 = 29.3$ kJ/kg

$h_1 = h_4$

$S_2 = 6.83$ kJ/kg-K

$S_4 = 0.1060$ kJ/kg-K

$$(i)\quad \eta_{cycle} = \frac{W_T}{Q_H} = \frac{(h_2 - h_3)}{(h_2 - h_1)}$$

$$= \frac{1300}{3200.7} = 0.41$$

$$(ii)\quad Steam\ rate = \frac{3600}{W_T} = 2.76 kg/kWh$$

(iii) Mean temperature of heat addition, $\Delta T_m = \dfrac{3200.7}{(6.83 - 0.106)} = 476^0 K$

Comparison	ΔT_m	η_{cycle}	Steam rate
Rankine cycle with regeneration	385.2 K	0.428	3.24 kg/kWh
Rankine cycle without regeneration	476^0K	0.41	2.76kg/kWh
∴ Increase w.r.t Rankine cycle	- 0.19 i.e., (-19%)	0.044 (4.4%)	0.174 (17.4%)

VAPOUR POWER CYCLE

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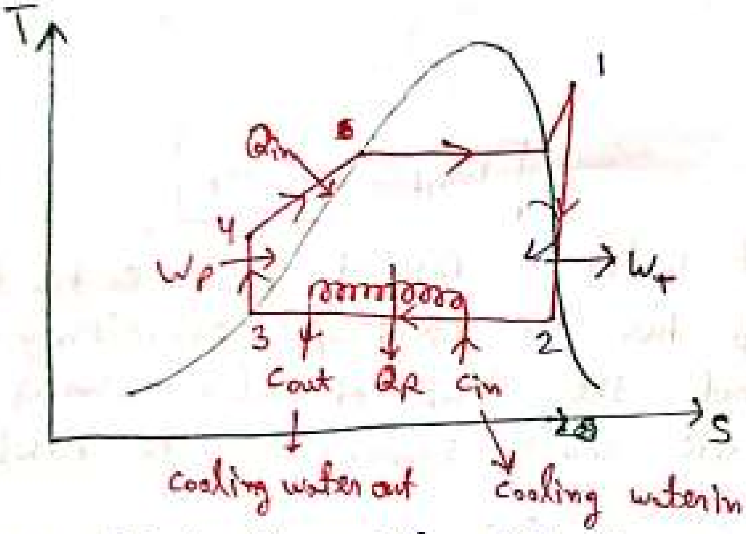
Vapour Power cycle ⇒

Rankine cycle is the practical working cycle for thermal Power plant & it consist of four process.



Adiabatic
Rev Adiabatic
$dS = \frac{dQ}{T} = 0$
Irr. Adiabatic
$dS = \frac{dQ}{T} + S_{gen}$
$dS = S_{gen}$

Process 1-2 ⇒ Reversible Adiabatic expansion of steam from Boiler to condensor pressure in turbine.

Process 2-3 ⇒ Constant Pressure Heat rejection process in the condensor to convert steam in to saturated liquid.

Process 3-4 ⇒ Reversible Adiabatic compression of liquid from condenser to Boiler pressure in Boiler feed pump.

Process 4-1 ⇒ Reversible constant Heat addition process to convert water into steam in Boiler.

By SFEE -

$$h_1 + \frac{C_1^2}{2000} + \frac{gz_1}{1000} + \frac{dQ}{dm} = h_2 + \frac{C_2^2}{2000} + \frac{gz_2}{1000} + \frac{dW_t}{dm}$$

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$\Delta K.E = 0$, $\Delta PE = 0$, $Q = 0$

(i) Turbine work -

$$\boxed{W_T = \dot{m}(h_1 - h_2)} \quad \frac{KJ}{Kg} \times Kg \ (KJ)$$

(ii) Pump work -

$$\boxed{W_P = h_4 - h_3} = -\int v\,dP$$

(iii) Heat supplied -

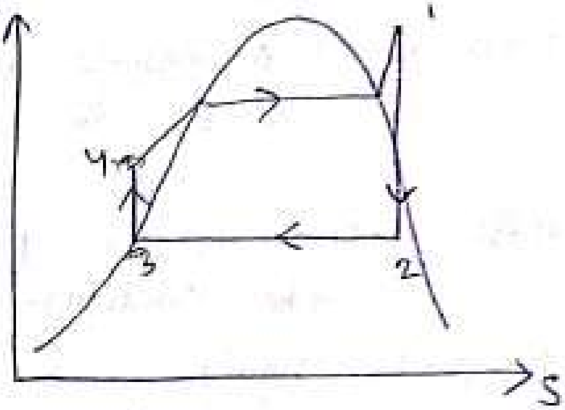
$$Q_S = h_1 - h_4$$

(iv) Heat Rejected -

$$Q_R = h_2 - h_3$$

$$\boxed{W_{net} = W_T - W_P}$$

Q - In a Power Plant water is pumped from 80 KPa to 3 MPa. The Pump has an isentropic efficiency of 0.85. Assuming that the temp of water remains same calculate the specific work supplied to the Pump.



$$W_P = h_4 - h_3$$
$$= \int v\,dP$$
$$= \int \frac{dP}{\rho}$$
$$= \frac{\Delta P}{\rho}$$
$$= \frac{3 \times 10^3 - 80}{1000}$$
$$W_{ideal} = 2.92 \text{ KJ}$$
$$W_{actual} = \frac{2.92}{\eta_P} = \frac{2.92}{0.85} = 3.435 \text{ KJ}$$

Note :-

For Power consuming devices $W_{actual} = \frac{W_{idle}}{\eta_{isentropic}}$

For Power devolping devices $W_{actual} = W_{idle} \times \eta_{isentropic}$

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Sol
2011

Q The Rankine cycle operates b/w Pressure of 80 bar & 0.1 bar the max^m cycle temp is 600°C. Relevant steam table is extracted given below. calculate the thermal efficiency of cycle.

P (bar)	T_{sat} (C)	Specific Volume (m^3/kg) v_f	v_g	Sp. enthalpy h_f	h_{fg}	h_g	Sp. entropy S_f	S_{fg}
0.1	45.84	0.0010103	14.68	191.9	2392.3	2584.2	0.648	7.5006
80	295.1	0.001385	0.0235	131.7	1440.5	2757.7	3.207	

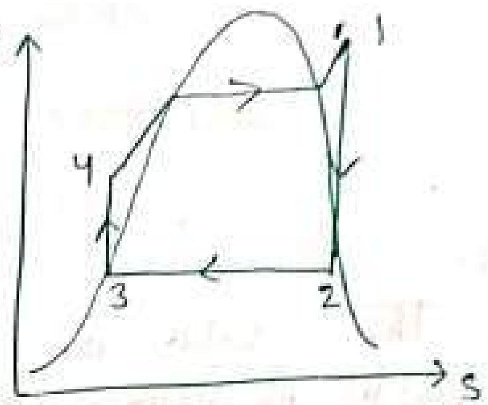
Sol

80 bar – 600°C superheat table

$v = 0.486$

$h = 3642$

$s = 7.0206$



$$\eta = \frac{W_{net}}{Q_{in}} = \frac{W_T - W_P}{Q_{in}}$$

$W_T = h_1 - h_2$, $W_P = h_4 - h_3$, $Q_S = h_1 - h_4$

$S_1 = S_2$

$7.0206 = 0.648 + x \times 7.5006$

$x = 0.8496$

$h_2 = h_f + x h_{fg} = 191.9 + 0.8496 \times 2392.3$

$h_2 = 2224.4268$ kJ/kg

$W_P = \int v\,dP = \frac{(80 - 0.1) \times 10^5}{1000} = 7990$ kJ

$W_P = 0.0010103 [80 - 0.1] \times 100 = \cancel{8.0925}$

$h_4 - h_3$ $W_P = 8.072297$ kJ

$h_4 = 8.072297 + 191.9 = 199.972297$ kJ/kg

$$\eta = \frac{h_1 - h_2 - 8.072297}{h_1 - h_4} = \frac{3642 - 2224.4268 - 8.0722}{3642 - 199.972297}$$

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$$\boxed{\eta = 40.95\%}$$

Steam Rate :- It is the rate of steam required to produce 1 KW power output.

$$SR = \frac{\dot{m}_s \left(\frac{kg}{sec}\right)}{P \quad KW}$$

$$SR = \frac{\dot{m}_s}{\dot{m}_s \times W_{net}}$$

Imp

$$\boxed{SR = \frac{1 \times 3600}{W_{net}} \left(\frac{kg}{KW\text{-}h}\right)}$$

Gate-2011

Q The value of enthalpy of steam at the inlet & Outlet of steam turbine in a rankine cycle are 2800 KJ/Kg & 1800 KJ/Kg resp. Neglecting pump work. calculate the specific steam consumption in kg/KW-h.

$$S.R = \frac{3600}{W_{net}}$$

$$S.R = \frac{3600}{2800 - 1800} = 3.6 \quad kg/KW\text{-}h$$

Gate-2013

Q- Specific enthalpy & velocity of steam at inlet & exit of a steam turbine running under steady state are as given below-

	Speci. Enthalpy (KJ/Kg)	velocity (m/sec)
Inlet steam condition	3250	180
exit steam condition	2360	5

The rate of heat loss from the turbine per kg of steam flow rate is 5 KW. Neglecting changes of potential energy of steam. Calculate the Power development in KW by the steam turbine per kg of steam flow rate.

$$h_1 + \frac{C_1^2}{2000} + \frac{dQ}{dm} = h_2 + \frac{C_2^2}{2000} + \frac{dW}{dm}$$

$$3250 + \frac{(180)^2}{2000} + 5 = 2360 + \frac{5^2}{2000} + \frac{dW}{dm}$$

$$\boxed{\frac{dW}{dm} = 901.1875 \text{ KW}}$$

Heat rate ⇒ It is the rate of heat input required to produce 1 KW Power output.

$$HR = \frac{\text{Heat Supplied}}{\text{Power Output}}$$

$$HR = \frac{\dot{m}_s Q_s}{\dot{m}_s W_{net}}$$

$$HR = \frac{Q_s}{W_{net}}$$

$$\boxed{HR = \frac{1}{\frac{W_{net}}{Q_s}} = \frac{1}{\eta}}$$

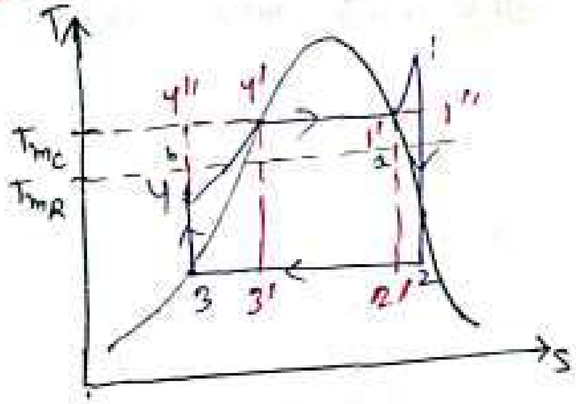
Carnot Vs Rankine cycle ⇒

Rankine ⇒ 1-2-3-4

Carnot ⇒ 1'-2'-3'-4'

⇒ 1'-2'-3-4''

⇒ 1''-2-3-4''



$$(q_R)_S = h_1' - h_4 = T_{mR}(S_a - S_b) = \cancel{T_{mC}(T_1' - T}$$

$$= T_{mC}(S_1' - S_4)$$

$$\eta = \frac{W_{net}}{\partial q_S} = 1 - \frac{T_2}{T_1}$$

T_2 is same for Rankine & carnot.

But T_m or T_1 is more than ~~can~~ Rankine in carnot.

$$\boxed{\eta_{carnot} > \eta_{Rankine}}$$

Carnot cycle in not practical in thermal Power Plant due to following reason.

(i) Problem of

(ii) It is not Possible to control condensation Process in such a manner that desired quality of steam is found at state 3'.

(iii) It is not Possible to design a bump which can handle a liquid & vapour phase together.

(iv) The Pump work will be very High for carnot cycle.

(v) It is impossible Add heat at constant temp with decreasing pressure (expansion in the boiler).

Q- Prove that efficiency of carnot cycle is more than that of rankine cycle whereas the work done ~~&~~ by the rankine cycle is more than that of carnot cycle.

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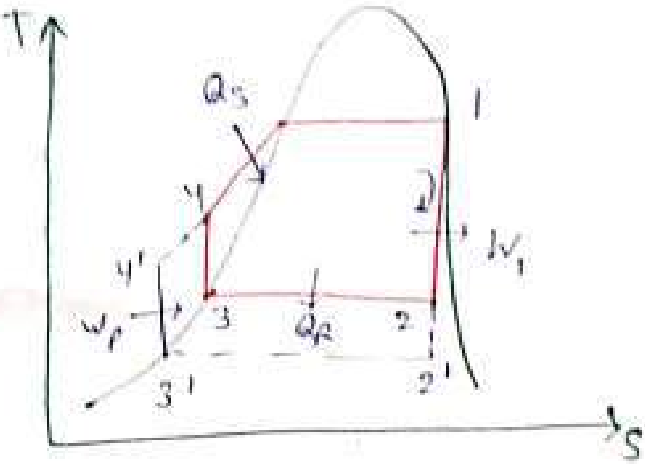
Variables required to increase the Rankine cycle efficiency:-

1. Decreasing Condenser Pressure:-

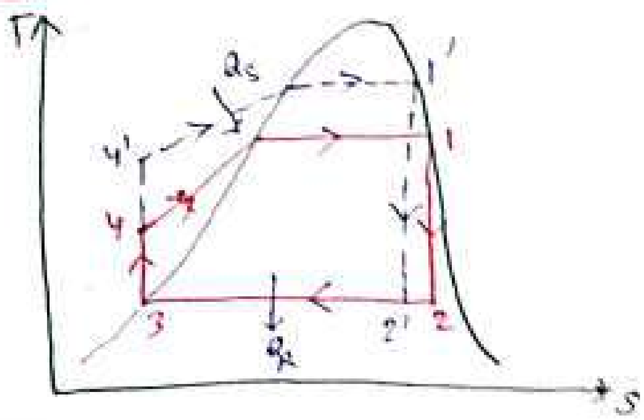
1. W_T ↑
2. W_P ↑
3. W_N ↑
4. Q_S ↑
5. Q_R ↑
6. $(T_{mean})_{Q_{in}}$ ↓
7. $(T_{mean})_{Q_R}$ ↓
8. η ↑ can not be said theoretically
   ↑ (Practically)
9. x ↑



Note:-
(i) Supercritical boiler is called Benson Boiler with No boiler drums. The cost of Benson Boiler is 2-3 times of general Boiler.
(ii) Left section of boiler is at Negative Pressure & Right section of Boiler is at Positive Pressure.
(iii) SEAS are accessories of boiler.
(iv) We can control the pressure in Boiler but in cooler it is not possible.

2. Increasing the Boiler Pressure:-

1. W_T ↑
2. W_P ↑
3. W_N ↑
4. Q_S ↑
5. Q_R ↓
6. $(T_{mean})_{Q_{in}}$ ↑

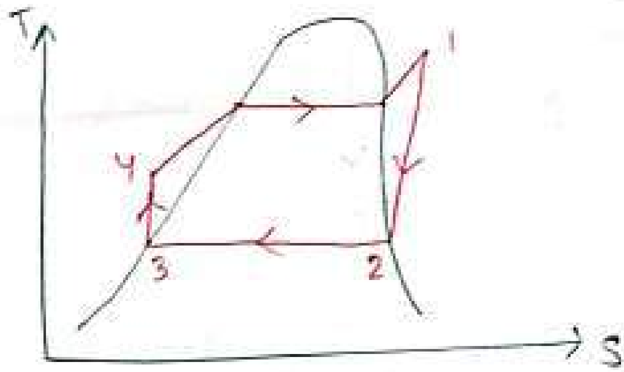


7. $(T_{mean})_{Q_R}$ = constant

8. η ↑

9. x ↓

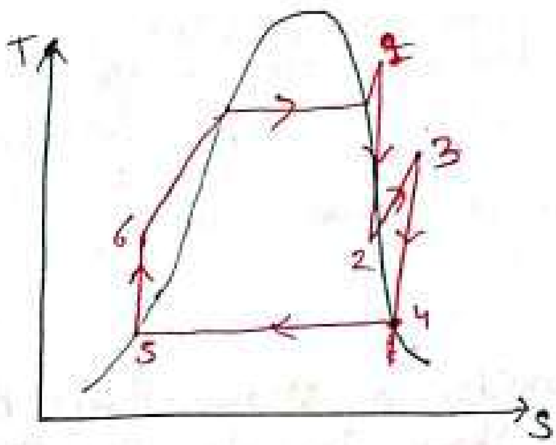
3. Superheating the Steam ⇒



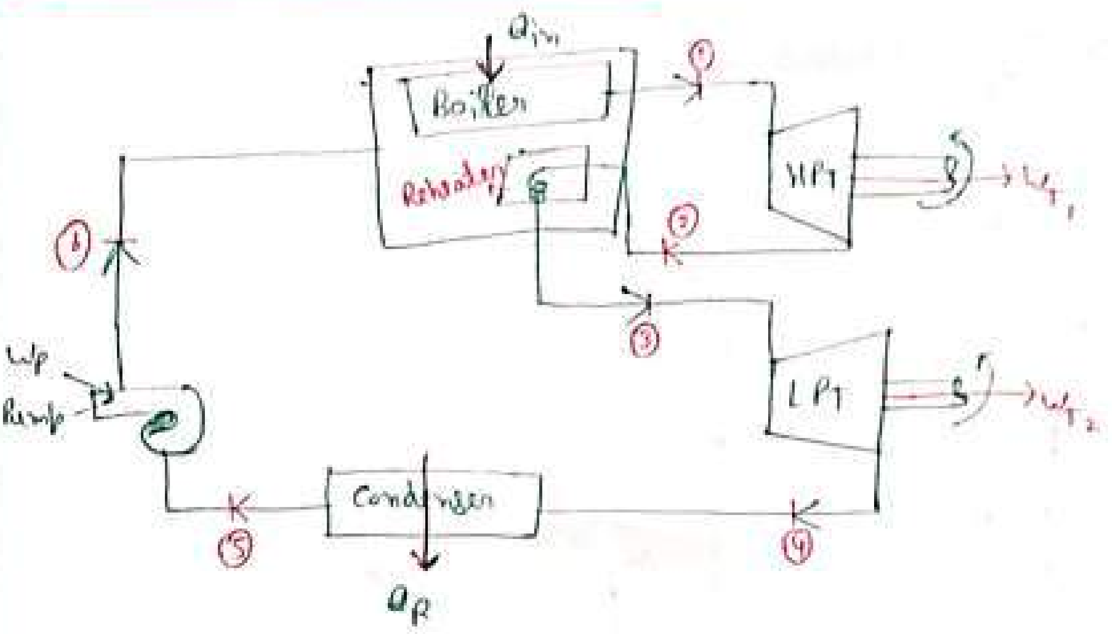
1. W_T ↑
2. W_P = constant
3. W_N ↑
4. Q_S ↑
5. Q_R ↑
6. $(T_{mean})_{Q_{in}}$ ↑
7. $(T_{mean})_{Q_R}$ = const
8. η ↑
9. x ↑

Imp

4. Reheating ⇒



1. W_T
2. W_P
3. W_N
4. Q_S
5. Q_R
6. $(T_{mean})_{Q_{in}}$
7. $(T_{mean})_{Q_R}$
8. η
9. x



$$\eta = \frac{W_{T_1} + W_{T_2} - W_P}{Q_S + Q_R}$$

$$\boxed{\eta = \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)}}$$

Notes:
In Reheat cycle the expansion of steam from boiler to condenser pressure is carried out in more than one stages with reheating of steam in b/w the stages.
The net work output of cycle increases ~~be~~ but main advantages is that it increases the dryness fraction at the exit of turbine.

Q- Consider a steam Power Plant using a reheat cycle as shown in the fig. Find the efficiency of Plant when the Pump work is 8 kJ/kg.

$h_1 = 3240$ kJ/kg
$h_2 = 2890$ kJ/kg

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$h_3 = 3410 \text{ kJ/kg}$

$h_4 = 2610 \text{ kJ/kg}$

$h_5 = 41 \text{ kJ/kg}$

$W_P = \{ h_6 - h_5$

$h_6 = 41 + 8 = 49 \text{ kJ/kg}$

$$\eta = \frac{(h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)}{(h_1 - h_6) + (h_3 - h_2)}$$

$$\eta = \frac{3240 - 2890 + 3410 - 2610 - \overset{49 + 41}{\cancel{40} + \cancel{38}}\ 49}{3240 - 49 + 3240 - 2890}$$

$$\boxed{\eta = 30.77\ \%}$$

Q ~~II~~ An ideal Reheat rankine cycle operates b/w the Pressure limit of 10 KPa & 8 MPa with reheat being done at 4 MPa. The temp of steam at the inlet of both turbine is 500°C & the enthalpy of steam is 3185 KJ/kg at the exit of High Pressure turbine & 2247 KJ/kg at the exit of low Pressure turbine. the enthalpy of water at exit from pump is 191 KJ/kg. Use the following table for relevant data

superheated steam temp	Pressure (MPa)	Sp. volume (m^3/kg)	h (KJ/kg)	s (KJ/kg)
500	4	0.08644	3446	7.6922
500	8	0.04177	3399	6.7266

This regarding the pump work calculate the efficiency of cycle in %.

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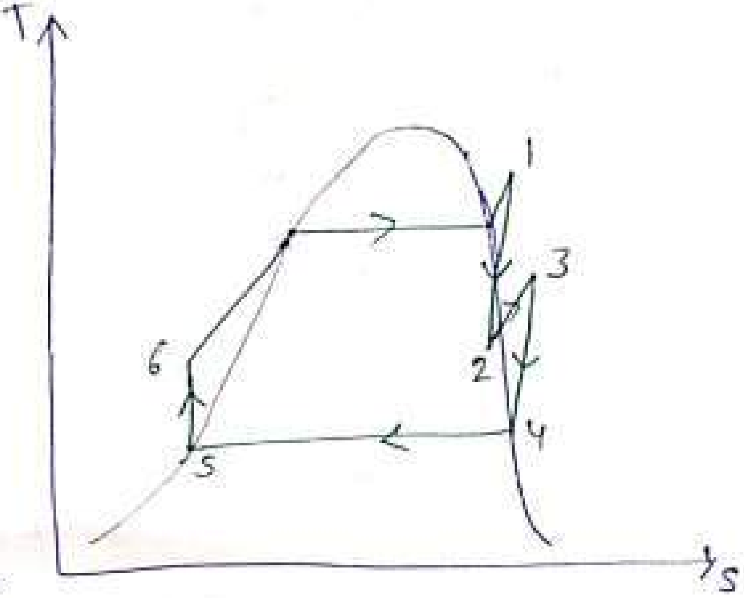
$h_1 = 3498$ KJ/kg

$h_2 = 3185$ KJ/kg

$h_3 = 3446$ ~~$h_3 = 3399$~~ KJ/kg

$h_4 = 2247$ KJ/kg

$h_5 = 191$ KJ/kg



$$\eta = \frac{\overset{3399}{\cancel{3446}} - 3185 + \overset{3446}{\cancel{3399}} - 2247}{\underset{3399}{\cancel{3446}} - 191 + 3446 - 3185}$$

$$\boxed{\eta = 40.187\ \%}$$

(Bleading out stream from Feed water heating)

5. Regenration ⇒ In Regenrative cycle the heat energy exchange internally b/w the expanding steam in turbine & the compressed fluid coming out from the pump. The ideal regenrative cycle is not practical because -

(Bleading out steam for feed water heating)

(i) The moisture content at the exit of turbine will be very high.

(ii) Reversible heat transfer can not be obtain in finite time.

(iii) Heat exchanger in the turbine is not mechanically is not practical.

Therefore in actual thermal power plant steam is extracted from intermediate stages for feed water heating.

Types of feed water heater ⇒

1. open feed water heater

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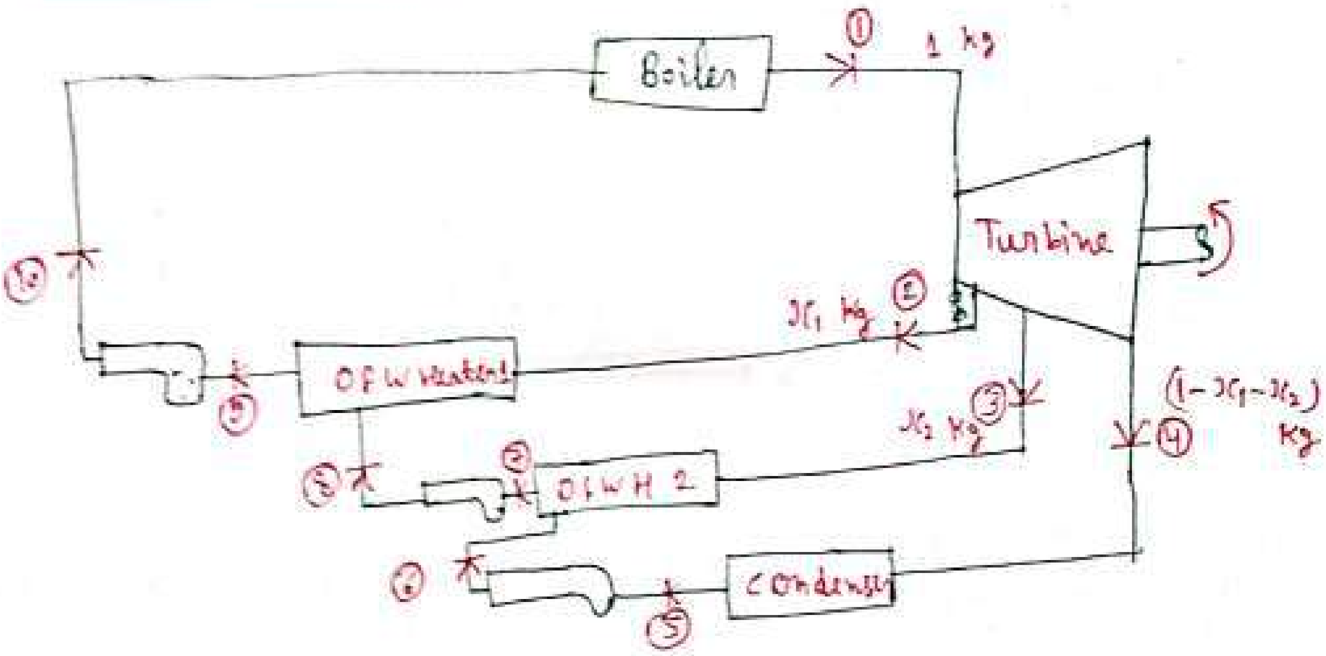
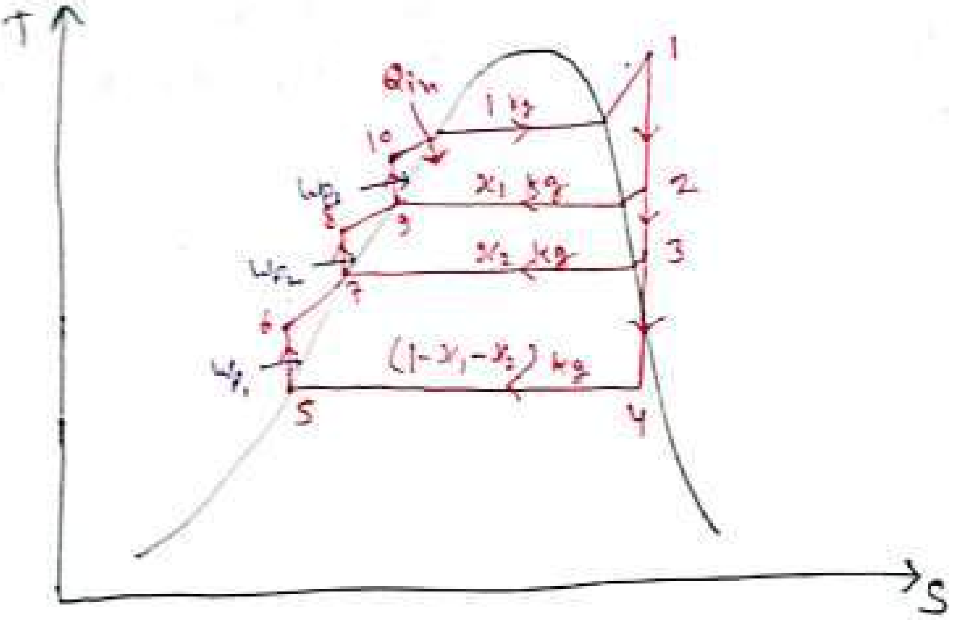
2- Close feed water Heater

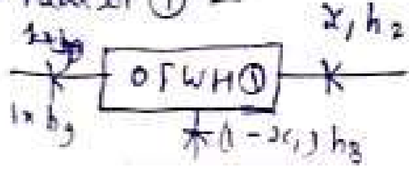
1- Open Feed water Heater → (OFWH)





$$\eta = \frac{\{1(h_1-h_2) + (1-x_1)(h_2-h_3) + (1-x_1-x_2)(h_3-h_4)\} - (w_1+w_2+w_3)}{(h_1-h_{10})}$$

Energy Balance eqn for OFW Heater ① -



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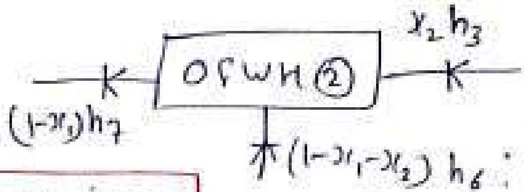
$$(\text{Energy})_{in} = (\text{Energy})_{out}$$

$$\boxed{x_1 h_2 + (1-x_1) h_8 = h_2}$$

Energy Balance eqn for OFW heater (2) –



$$\boxed{(1-x_1)\,h_7 = x_2 h_3 + (1-x_1-x_2)\,h_6}$$

An open feed water Heater is basically a mixing chamber where the bleed out stream from the turbine is mixed with the feed water in feed water Heater at the same ~~Ex~~ Pressure. The Pressure for the stream must be same in order to decrease energy loss due to mixing (Flash loss).

Gate-2008

Q –

INTRODUCTION OF STEAM NOZZLES

- Nozzle is a duct by flowing through which the velocity of a fluid increases at the expense of pressure drop. if the fluid is steam, then the nozzle is called as Steam nozzle.
- The flow of steam through nozzles may be taken as adiabatic expansion. The steam possesses a very high velocity at the end of the expansion, and the enthalpy decreases as expansion occurs.
- The major function of nozzle is to produce steam jet with high velocity to drive a turbine.
- Friction exists between the steam and the sides of the nozzle; heat is produced as the result of the resistance to the flow.
- The phenomenon of super saturation occurs in the steam flow through nozzles. This is because of the time lag in the condensation of the steam during the expansion.

INTRODUCTION OF STEAM NOZZLES

- The area of such duct having minimum cross-section is known as throat.
- A fluid is called compressible if its density changes with the change in pressure brought about by the flow.
- If the density changes very little or does not changes, the fluid is said to be incompressible.
- Generally the gases and vapors are compressible, whereas liquids are incompressible.

TYPES OF NOZZLES

- There are three types of nozzles
  1. Convergent nozzle
  2. Divergent nozzle
  3. Convergent-divergent nozzle

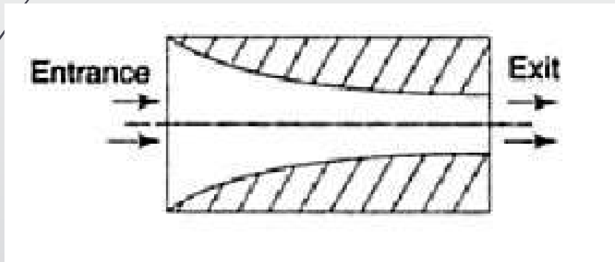


Fig1: Convergent Nozzle

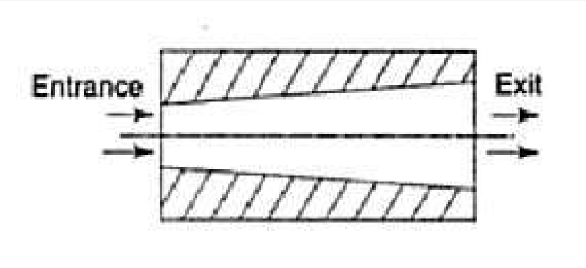


Fig2: Divergent Nozzle

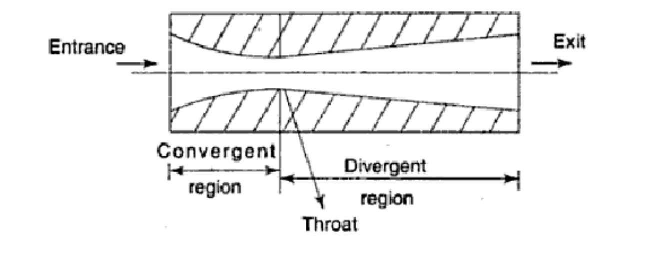


Fig3: Convergent-Divergent Nozzle

1.Convergent Nozzle

- A typical convergent nozzle is shown in the Fig.1.
- In a convergent nozzle, the cross sectional area decreases continuously from its entrance to exit.

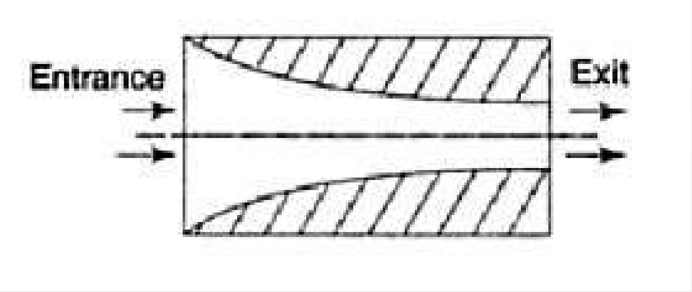


Fig.1. Convergent nozzle

- It is used in a case where the back pressure is equal to or greater than the critical pressure ratio.

2.Divergent Nozzle

- The cross sectional area of divergent nozzle increases continuously from its entrance to exit.

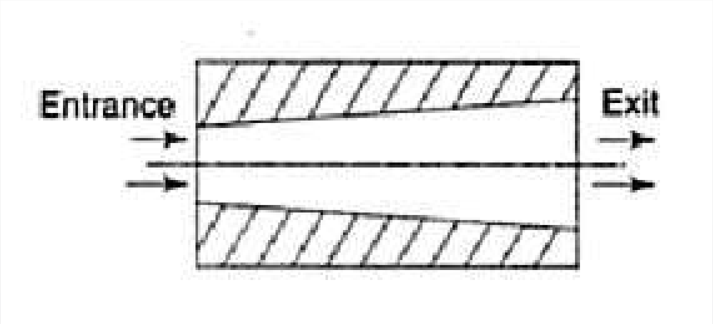


Fig.2.Divergent nozzle

- It is used in a case where the back pressure is less than the critical pressure ratio.

3.Convergent-Divergent Nozzle

- In this condition, the cross sectional area first decreases from its entrance to the throat and then again increases from throat to the exit.
- This case is used in the case where the back pressure is less than the critical pressure.
- Also, in present day application, it is widely used in many types of steam turbines.

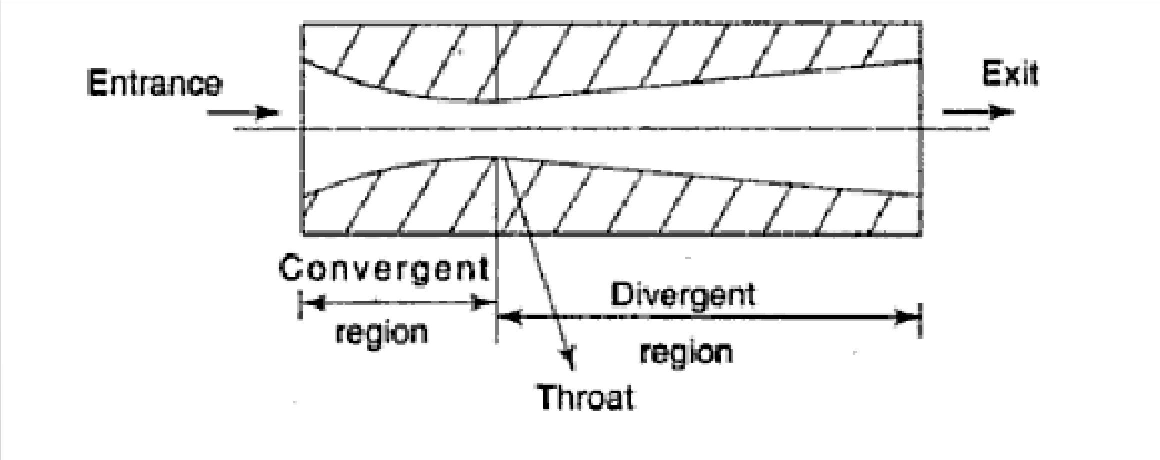


Fig.3. convergent-divergent nozzle

Flow of Steam through Nozzle

- **Super-saturated flow or metastable flow of in Nozzles:**
  - As steam expands in the nozzle, the pressure and temperature in it drop, and it is likely that the steam start condensing when it strikes the saturation line. But this is not always the situation.
  - Due to the high velocities, the time up to which the steam resides in the nozzle is small, and there may not be sufficient time for the needed heat transfer and the formation of liquid droplets due to condensation. As a result, the condensation of steam is delayed for a while.
  - This phenomenon is known as super saturation, and the steam that remains in the wet region without holding any liquid is known as supersaturated steam.
  - The locus of points where condensation occurs regardless of the initial temperature and pressure at the entrance of the nozzle is called the Wilson line.
  - The Wilson line generally lies between 4 and 5 percent moisture curves in the saturation region on the h-s diagram in case of steam, and is often taken as 4 percent moisture line.

Flow of Steam through Nozzle

The phenomenon of super saturation is shown on the h-s chart below:

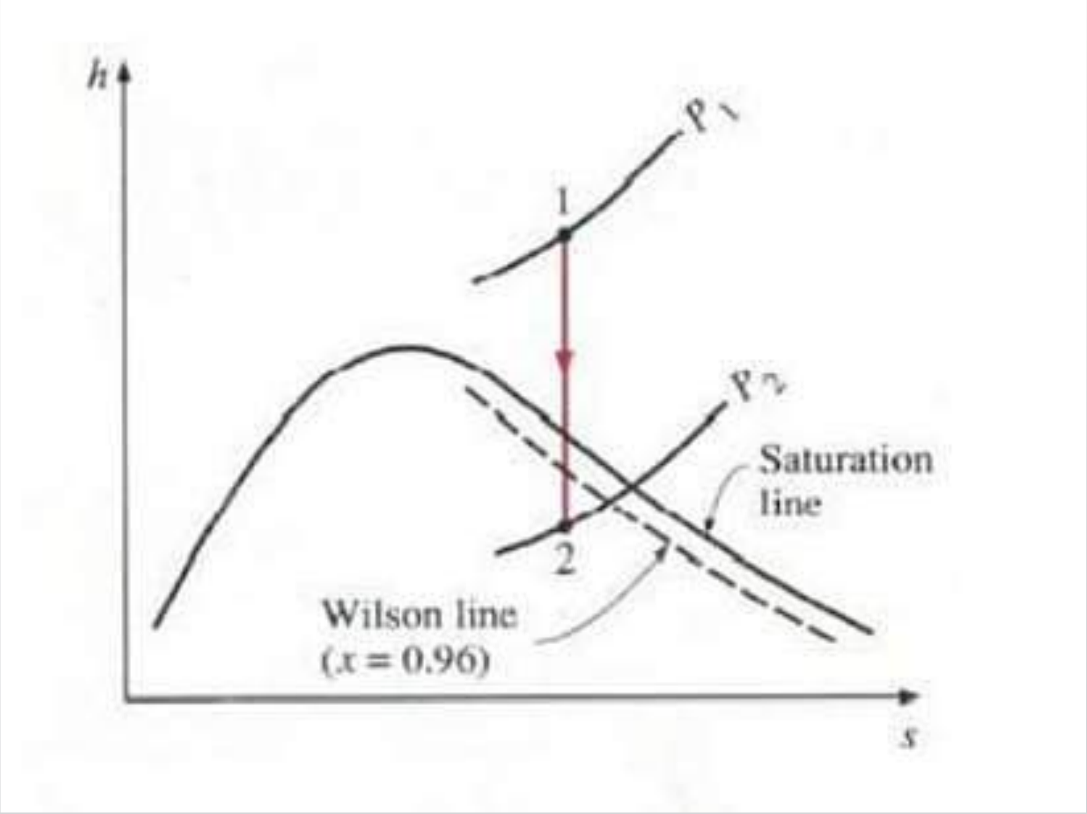


Fig 4. The h-s diagram for the expansion of steam in the nozzle

Flow of Steam through Nozzle

- **Effects of Super-saturation:**

The following are the effects of super-saturation in a nozzle.

1. The temperature at which the steam becomes supersaturated will be less than the saturation temperature corresponding to that pressure. Therefore, supersaturated steam will have the density more than that of equilibrium condition which results in the increase in the mass of steam discharged.
2. Super-saturation causes the specific volume and entropy of the steam to increase.
3. Super-saturation reduces the heat drop. Thus the exit velocity of the steam is reduced.
4. Super-saturation increases the dryness fraction of the steam.

Flow of Steam through Nozzle

- **Effect of Friction on Nozzles:**
  - Entropy is increased.
  - The energy available decreases.
  - Velocity of flow at the throat get decreased.
  - Volume of flowing steam is decreased.
  - Throat area required to discharge a given mass of steam is increased.

Continuity and steady flow energy equations through a certain section of the nozzle:

- Where m denotes the mass flow rate, v is the specific volume of the steam, A is the area of cross-section and C is the velocity of the steam.
- For steady flow of the steam through a certain apparatus, principle of conservation of energy states:

$$h_1 + \frac{C_1^2}{2} + gz_1 + q = h_2 + \frac{C_2^2}{2} + gz_2 + \mathrm{w}$$

- For nozzles, changes in potential energies are negligible, w = 0 and q = 0.

$$\boxed{h_1 + \frac{C_1^2}{2} = h_1 + \frac{C_2^2}{2}}$$

- which is the expression for the steady state flow energy equation.

INTRODUCTION

What is a steam nozzle????

A nozzle is a device designed to control the direction or characteristics of a fluid flow (especially to increase velocity) as it exits (or enters) an enclosed chamber or pipe.
A nozzle is often a pipe or tube of varying cross sectional area, and it can be used to direct or modify the flow of a fluid (liquid or gas). Nozzles are frequently used to control the rate of flow, speed, direction, mass, shape, and/or the pressure of the stream that emerges from them.Finally the goal of a nozzle is to increase the kinetic energy of the flowing medium at the expense of its pressure and internal energy.

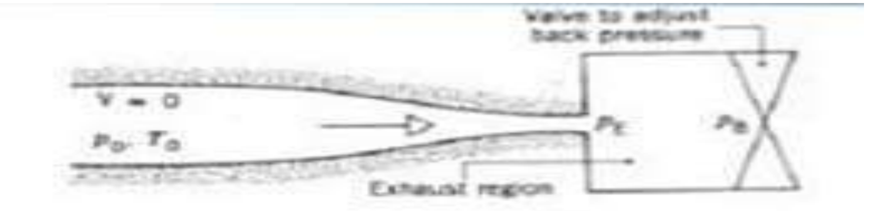
Types of nozzles

<u>Three types of nozzles:-</u>

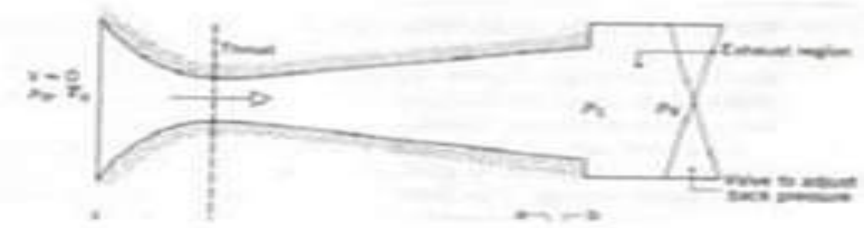
- **Convergent**:- The cross section of nozzle tapers to a smaller section allow for changes which occur due changes in velocity, specific volume dryness fraction – as the flow expands, it has lower expansion ratio and hence lower outlet velocities.

- **Convergent-----Divergent**:- The nozzle which converges to throat and diverges afterwards. It has higher expansion ratio – as addition of divergent portion produces steam of higher velocities.Eg– De–Laval Nozzle

- **Divergent**:- A nozzle whose cross section becomes larger in the direction of flow is known as divergent nozzle.



CONVERGENT NOZZLE



CONVERGENT-DIVERGENT NOZZLE

DIVERGENT NOZZLE

WORKING PRINCIPLE OF A DE LAVAL NOZZLE

DE LAVAL NOZZLE

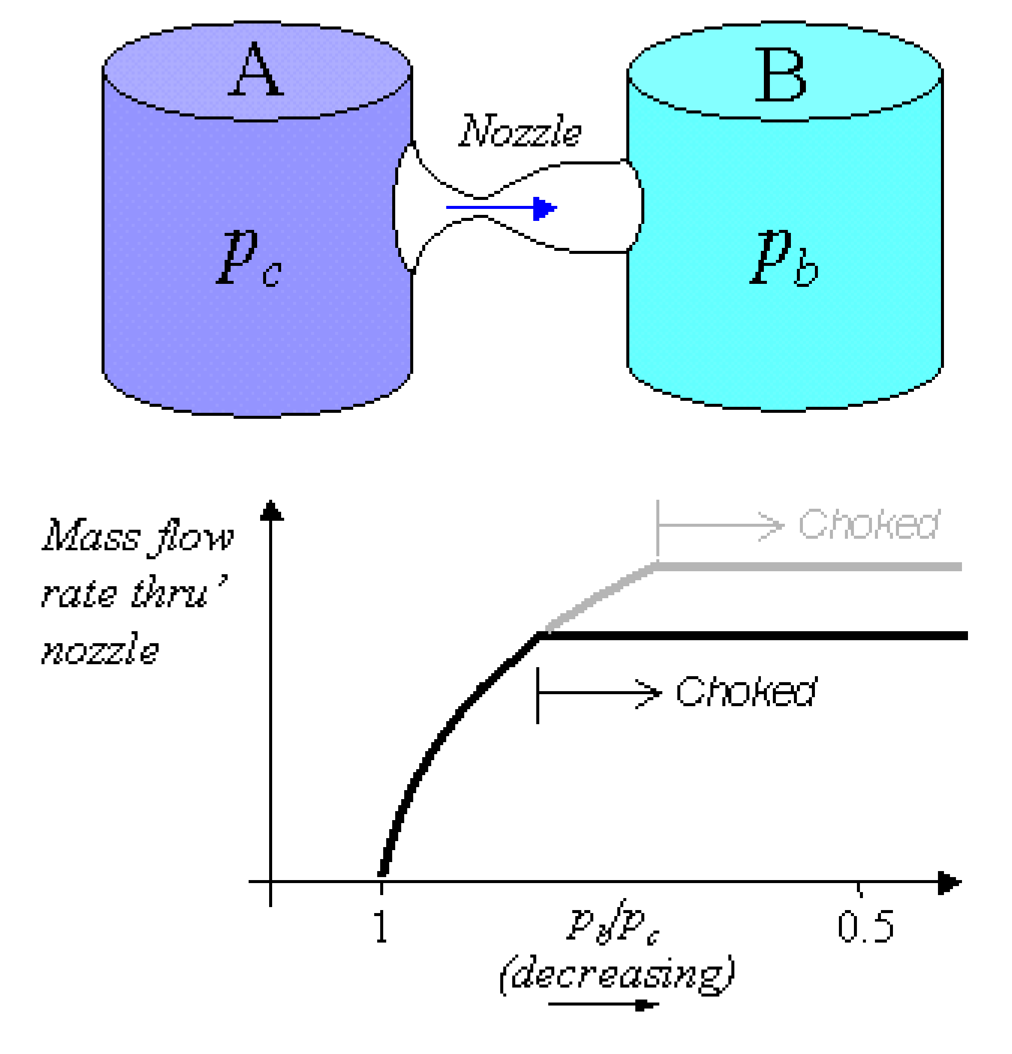


Figure 2. A simple experiment

... of the nozzle (in the diverging section and jet) can still change if you lower the back pressure further, but the mass flow rate is now fixed because the flow in the throat (and for that matter in the entire converging section) is now fixed too.

Convergent Section

Divergent Section

Throat

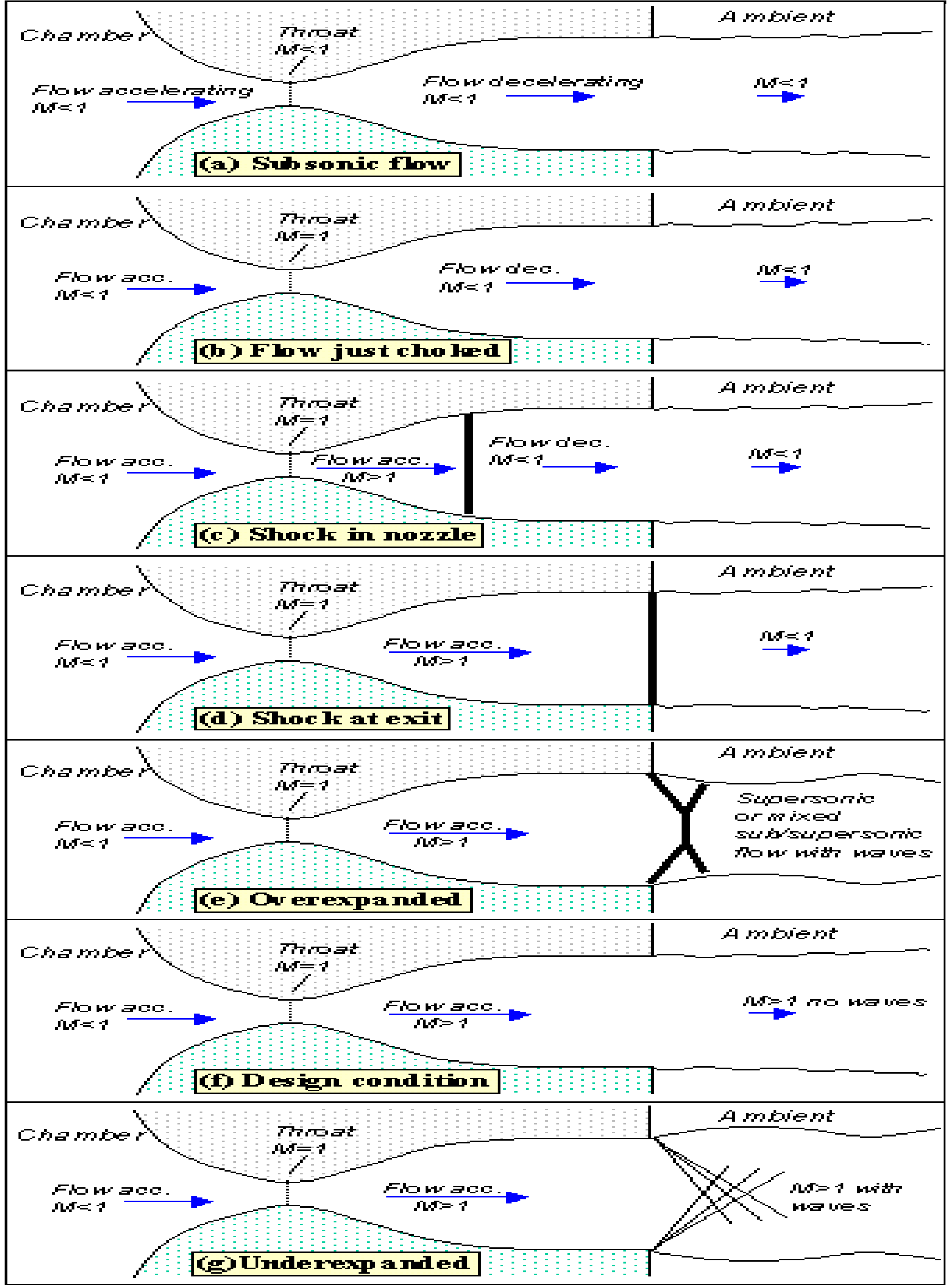


Figure 3. Flow patterns

A further lowering of the back pressure changes and weakens the wave pattern in the jet. Eventually we will have lowered the back pressure enough so that it is now equal to the pressure at the nozzle exit. In this case, the waves in the jet disappear altogether (figure 3f), and the jet will be uniformly supersonic. This situation, since it is often desirable, is referred to as the 'design condition'.

Finally, if we lower the back pressure even further we will create a new imbalance between the exit and back pressures (exit pressure greater than back pressure), figure 3g. In this situation (called 'underexpanded') what we call expansion waves (that produce gradual turning and acceleration in the jet) form at the nozzle exit, initially turning the flow at the jet edges outward in a plume and setting up a different type of complex wave pattern.

CRITICAL PRESSURE RATIO

For a perfect gas undergoing an adiabatic process the index - *n* - is the ratio of specific heats - $k = c_p / c_v$. There is no unique value for - *n*. Values for some common gases are

- Steam where most of the process occurs in the wet region : $n = 1.135$
- Steam superheated : $n = 1.30$
- Air : $n = 1.4$
- Methane : $n = 1.31$
- Helium : $n = 1.667$

Example - Air Nozzles and Critical Pressure Ratios

The critical pressure ratio for an air nozzle can be calculated as

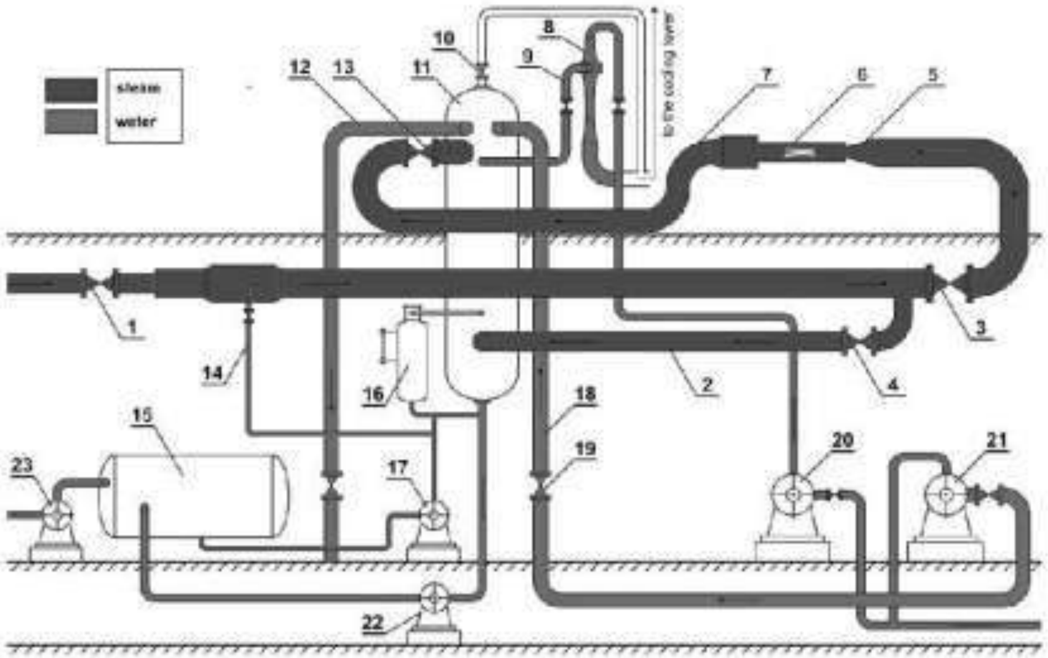
$$p_c / p_1 = (2 / (1.4 + 1))^{1.4 / (1.4 - 1)}$$

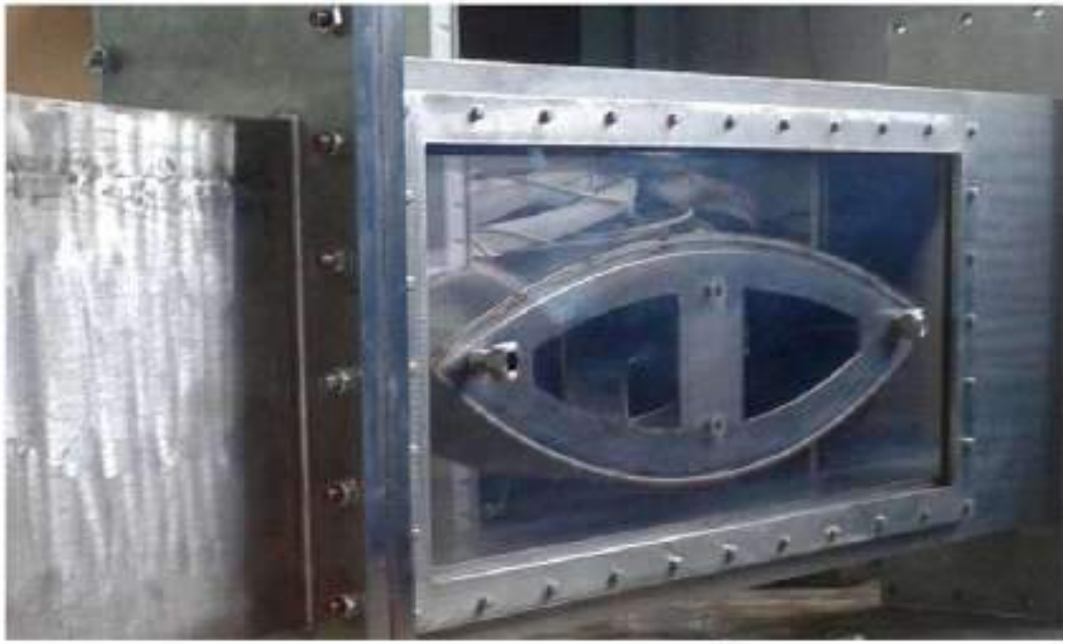
$$= \underline{0.528}$$

Critical pressures for other values of - *n:*

n	1.135	1.300	1.400	1.667
p_c / p_1	0.577	0.546	0.528	0.487

steam transonic flows in Laval nozzles





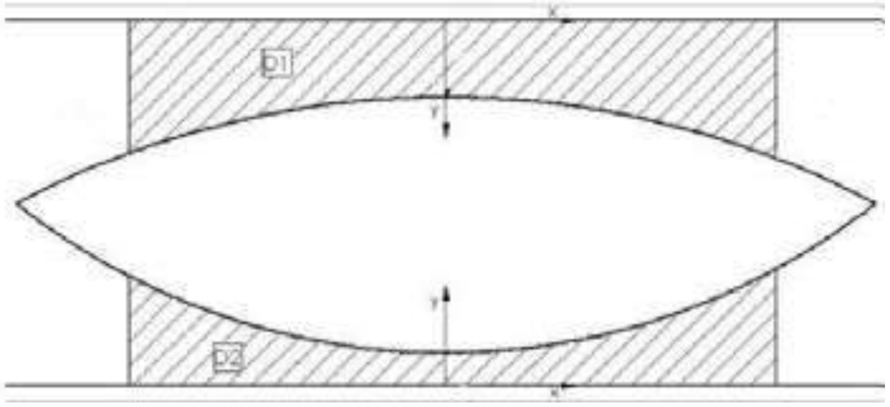
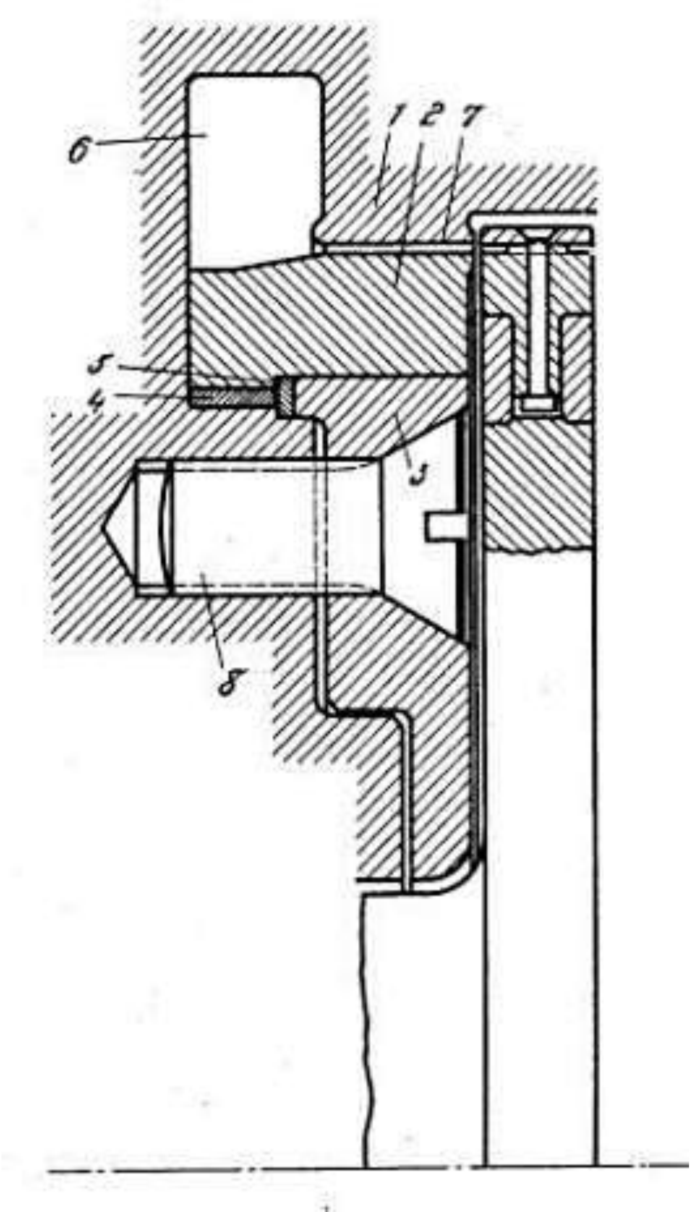


Figure 1: Steam tunnel with auxiliary devices: 1) Control valve, 2) By-pass, 3) Stop gate valve, 4) Stop gate valve at by-pass, 5) Inlet nozzle, 6) Test section, 7) Outlet elbow, 8) Water injec-tor, 9) Pipe, 10) Safety valve, 11) Condenser, 12) Suction line, 13) Throttle valve, 14) Desuperheater, 15) Condensate tank, 16) Control system of condensate level, 17) Condensate pump, 18) Discharge line, 19) Stop valve, 20) Water injector pump, 21) Cooling water pump, 22) Condensate pump, 23) Pump

Construction of steam nozzle



The flow nozzle was constructed based on the following specifications and dimensions;
Throat Diameter60mm
The diameter of the duct pipe 140mm
The length of the up stream pipe 200mm
The thickness of the nozzle3.8mm
The length of the down stream pipe 200mm
The height of the nozzle820mm
The length of the nozzle690mm
The size of the pressure valves ½ inch
Furthermore, the selection of materials for the construction of this flow nozzle was based on the factors which includes; ductility, malleability, fabricability, mechanical strength and stability, availability, corrosion resistance and lastly cost factor.

Effects of friction on nozzle efficiency

For stream flowing through a nozzle, its final velocity for a given pressure drop is reduces to:

- Friction between nozzle surface and stream
- Internal friction of stream itself.
- Shock losses.

Most of the frictional losses occur between the throat and exit in nozzle, producing following effect.

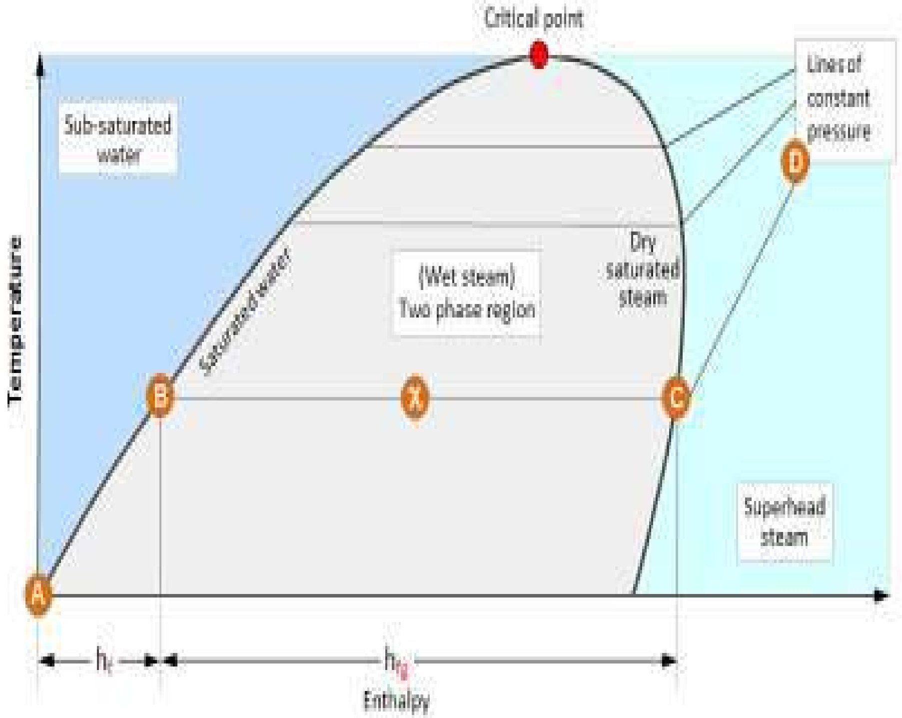
- Expansion is no more isentropic.
- Enthalpy drop is reduced.
- Final dryness fraction of steam increases.(kinetic energy– heat, due to friction and gets absorbed.)
- Specific volume of steam increases.(steam becomes more dry due to friction reheating)

Parameters of steam nozzle

- ❖ Foundation
- ❖ Rotor or shaft
- ❖ Cylinder or Casing
- ❖ Blades
- ❖ Diaphragm
- ❖ Steam Chest
- ❖ Coupling
- ❖ Bearings
- ❖ Labyrinth seal
- ❖ Front pedestal
- ❖ TSI
- ❖ D-EHC(governor)
- ❖ MSV(main steam stop value)
- ❖ CV(control value)
- ❖ IV(intercept value)
- ❖ CRV(combined reheat value)
- ❖ Turbine Turning Gear
- ❖ Turbine Bypass and Drains
- ❖ Lube oil system
- ❖ EHC oil system
- ❖ Gland steam systems
- ❖ Condenser
- ❖ Steam jet Ejector
- ❖ Vacuum Breaker

SUPER SATURATED FLOW

When dry and saturated steam is caused to expand in a nozzle, the actual measured steam flow is found to be greater than the theoretical calculated flow. This is due to the time lag in the condensation of steam and the steam remains in dry state instead of wet. Such a steam is called supersaturated steam. This time lag is caused due to the fact that, the converging part of the nozzle is too short and the steam velocity is too high that the molecules of steam have insufficient time to form droplets.

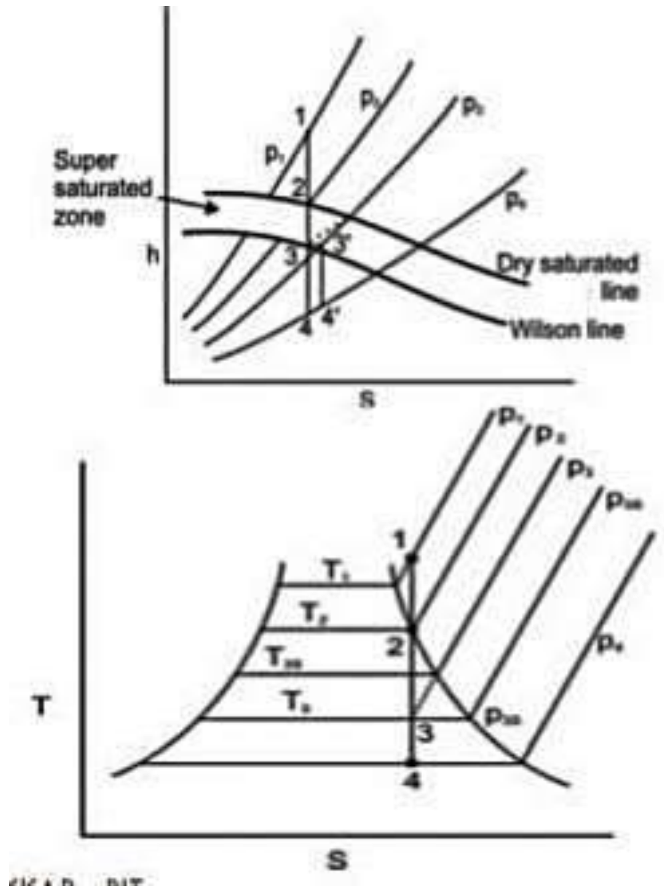


EFFECTS OF SUPERSATURATED FLOW

✓ Final dryness fraction increases.

✓ Density of supersaturated steam is more than that for equilibrium conditions(As no condensation during supersaturated expansion => supersaturation temperature < saturation temperature corresponding to the pressure).

✓ Thus , measured discharge (=>mass) is greater than that theoretically calculated.



MACH NUMBER

Mach number is the ratio of flow velocity passed the boundary to the local speed of sound. It is a dimensionless quantity:- M=u/c

Where,

M= Mach Number.

u= Local flow velocity with respect to the boundaries.

c= Speed of the sound in the medium.

If ,

M>1, the flow is supersonic

M<1, the flow is subsonic

M=1, the flow is sonic

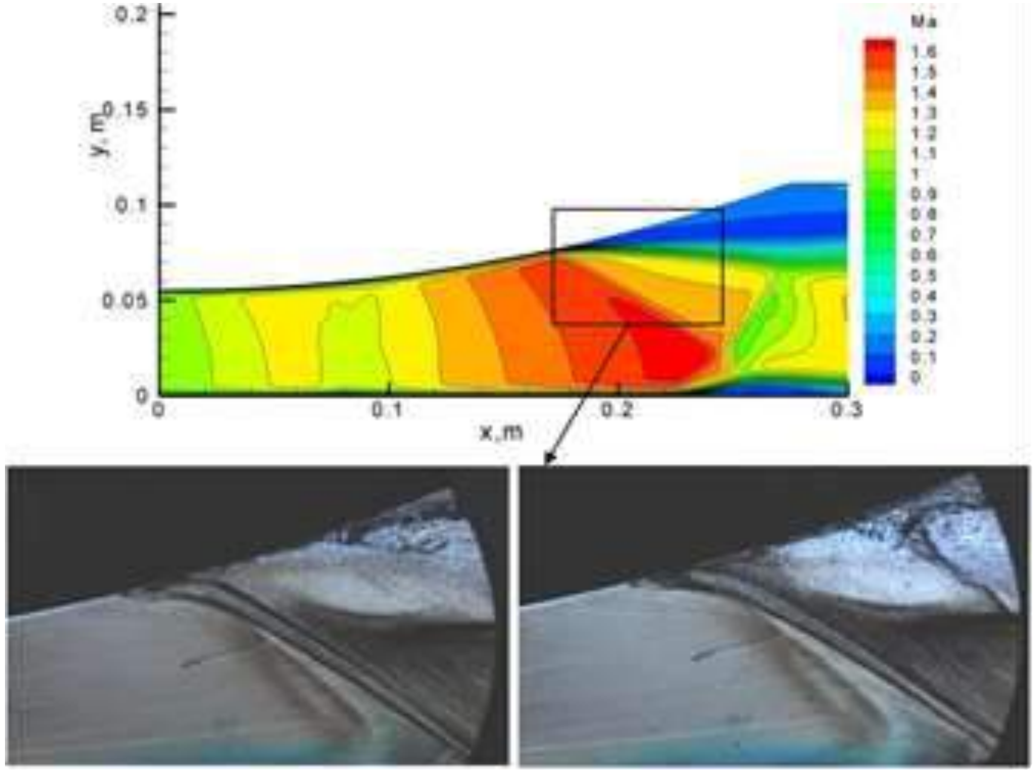


Figure 4: Calculated Mach number distribution (top) and Schlieren pictures from experiment for D1 nozzle

APPLICATIONS OF STEAM NOZZLE

- ❑ To rotate steam turbine.
- ❑ Thermal power plant.
- ❑ Steam nozzle are also used for cleaning purpose.
- ❑ To produce a very fine jet spray.

21

Steam Nozzles

1. Introduction. 2. Types of Steam Nozzles. 3. Flow of Steam through Convergent-divergent Nozzle. 4. Friction in a Nozzle or Nozzle Efficiency. 5. Velocity of Steam Flowing through a Nozzle. 6. Mass of Steam Discharged through a Nozzle. 7. Condition for Maximum Discharge through a Nozzle (Critical Pressure Ratio). 8. Values for Maximum Discharge through a Nozzle. 9. Values for Critical Pressure Ratio. 10. Physical Significance of Critical Pressure Ratio. 11. Diameters of Throat and Exit for Maximum Discharge. 12. Supersaturated Flow or Metastable Flow through Nozzles. 13. Effect of Supersaturation. 14. Steam Injector. 15. Steam Injector Calculations.

21.1. Introduction

A steam nozzle is a passage of varying cross-section, which converts heat energy of steam into kinetic energy. During the first part of the nozzle, the steam increases its velocity. But in its later part, the steam gains more in volume than in velocity. Since the mass of steam, passing through any section of the nozzle remains constant, the variation of steam pressure in the nozzle depends upon the velocity, specific volume and dryness fraction of steam. A well designed nozzle converts the heat energy of steam into kinetic energy with a minimum loss.

The main use of steam nozzle in steam turbines, is to produce a jet of steam with a high velocity. The smallest section of the nozzle is called throat.

21.2. Types of Steam Nozzles

Following three types of nozzles are important from the subject point of view :

1. *Convergent nozzle.* When the cross-section of a nozzle decreases continuously from entrance to exit, it is called a convergent nozzle as shown in Fig. 21.1 (*a*).

(*a*) Convergent. (*b*) Divergent. (*c*) Convergent-divergent.

Fig. 21.1. Types of steam nozzles.

2. *Divergent nozzle.* When the cross-section of a nozzle increases continuously from entrance to exit, it is called a divergent nozzle, as shown in Fig. 21.1 (*b*).

3. *Convergent-divergent nozzle.* When the cross-section of a nozzle first decreases from its entrance to throat, and then increases from its throat to exit, it is called a convergent-divergent nozzle as shown in Fig. 21.1 (*c*). This type of nozzle is widely used these days in various types of steam turbines.

21.3. Flow of Steam through Convergent-divergent Nozzle

The steam enters the nozzle with a high pressure, but with a negligible velocity. In the converging portion (*i.e.* from the inlet to the throat), there is a drop in the steam pressure with a rise in its velocity. There is also a drop in the enthalpy or total heat of the steam. This drop of heat is not utilised in doing some external work, but is converted into kinetic energy. In the divergent portion (*i.e.* from the throat to outlet), there is further drop of steam pressure with a further rise in its velocity. Again, there is a drop in the enthalpy or total heat of steam, which is converted into kinetic energy.

It will be interesting to know that the steam enters the nozzle with a high pressure and negligible velocity. But leaves the nozzle with a high velocity and small pressure. The pressure, at which the steam leaves the nozzle, is known as back pressure. Moreover, no heat is supplied or rejected by the steam during flow through a nozzle. Therefore, it is considered as isentropic flow, and the corresponding expansion is considered as an isentropic expansion.

21.4. Friction in a Nozzle or Nozzle Efficiency

As a matter fact, when the steam flows through a *nozzle*, some loss in its enthalpy or total heat takes place due to friction between the nozzle surface and the flowing steam. This can be best understood with the help of *h-s* diagram or Mollier chart, as shown in Fig. 21.2., which can be completed as discussed below :

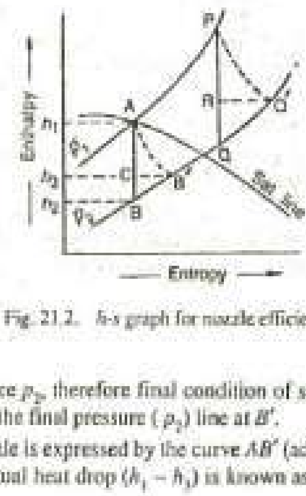


Fig. 21.2. *h-s* graph for nozzle efficiency.

1. First of all, locate the point *A* for the initial conditions of the steam. It is a point, where the saturation line meets the initial pressure (p_1) line.

2. Now draw a vertical line through *A* to meet the final pressure (p_2) line. This is done as the flow through the nozzle is isentropic, which is expressed by a vertical line *AB*. The heat drop ($h_1 - h_2$) is known as *isentropic heat drop*.

3. Due to friction in the *nozzle* the actual heat drop in the steam will be less than ($h_1 - h_2$). Let this heat drop be shown as *AC* instead of *AB*.

4. As the expansion of steam ends at the pressure p_2, therefore final condition of steam is obtained by drawing a horizontal line through *C* to meet the final pressure (p_2) line at *B'*.

5. Now the actual expansion of steam in the nozzle is expressed by the curve *AB'* (adiabatic expansion) instead of *AB* (isentropic expansion). The actual heat drop ($h_1 - h_3$) is known as *useful heat drop*.

Now the coefficient of nozzle or nozzle efficiency (usually denoted by *K*) is defined as *the ratio of useful heat drop to the isentropic heat drop*. Mathematically,

$$K = \frac{\text{Useful heat drop}}{\text{Isentropic heat drop}} = \frac{AC}{AB} = \frac{h_1 - h_3}{h_1 - h_2}$$

Notes : 1. We see from Fig. 21.2, that the dryness fraction of steam at *B'* is greater than that at *B*. It is thus obvious, that the effect of friction is to increase the dryness fraction of steam. This is due to the fact that the energy lost in friction is transferred into heat, which tends to dry or superheat the steam.

2. A similar effect is produced when the steam is superheated at the entrance of the nozzle.

Let PR = Useful heat drop, and PQ = Isentropic heat drop.

$\therefore$ Nozzle efficiency, $K = \frac{PR}{PQ}$

3. In general, if 15% of the heat drop is lost in friction, then efficiency of the nozzle is equal to $100 - 15 = 85\% = 0.85$.

21.5. Velocity of Steam Flowing through a Nozzle

Consider a unit mass flow of steam through a nozzle.

Let V_1 = Velocity of steam at the entrance of nozzle in m/s

V_2 = Velocity of steam at any section considered in m/s,

h_1 = Enthalpy or total heat of steam entering the nozzle in kJ/kg, and

h_2 = Enthalpy or total heat of steam at the section considered in kJ/kg.

We know that for a steady flow process in a nozzle,

$$h_1 + \frac{*1}{1000}\left(\frac{V_1^2}{2}\right) = h_2 + \frac{1}{1000}\left(\frac{V_2^2}{2}\right) + \text{Losses}$$

Neglecting losses in a nozzle,

$$\frac{1}{1000}\left(\frac{V_2^2}{2} - \frac{V_1^2}{2}\right) = h_1 - h_2$$

$$\therefore \quad V_2 = \sqrt{V_1^2 + 2000(h_1 - h_2)} = \sqrt{V_1^2 + 2000\, h_d} \quad \ldots (i)$$

where h_d = Enthalpy or heat drop during expansion of steam in a nozzle

$= h_1 - h_2$

Since the entrance velocity or velocity of approach (V_1) is negligible as compared to V_2, therefore from equation (*i*),

$$V_2 = \sqrt{2000\, h_d} = 44.72\sqrt{h_d} \quad \ldots (ii)$$

Note : In actual practice, there is always a certain amount of friction present between the steam and nozzle surfaces. This reduces the heat drop by 10 to 15 percent and thus the exit velocity of steam is also reduced correspondingly. Thus the above relation may be written as :

$$V_2 = 44.72\sqrt{K\, h_d}$$

where K is the nozzle coefficient or nozzle efficiency.

Example 21.1. *Dry saturated steam at 5 bar with negligible velocity expands isentropically in a convergent nozzle to 1 bar and dryness fraction 0.94. Determine the velocity of steam leaving the nozzle.*

Solution. Given : $p_1 = 5$ bar ; $p_2 = 1$ bar ; $x_2 = 0.94$

From steam tables, corresponding to a pressure of 5 bar, we find that enthalpy or total heat of dry saturated steam,

$$h_1 = h_{g1} = 2747.5 \text{ kJ/kg}$$

and corresponding to a pressure of 1 bar, we find that

$$h_{f2} = 417.5 \text{ kJ/kg, and } h_{fg2} = 2257.9 \text{ kJ/kg}$$

$\therefore$ Enthalpy or total heat of final steam,

$$h_2 = h_{f2} + x_2 h_{fg2} = 417.5 + 0.94 \times 2257.9 = 2540 \text{ kJ/kg}$$

and enthalpy or heat drop, $h_d = h_1 - h_2 = 2747.5 - 2540 = 207.5$ kJ/kg

We know that velocity of steam leaving the nozzle,

$$V_2 = 44.72\sqrt{h_d} = 44.72\sqrt{207.5} = 644 \text{ m/s} \quad \textbf{Ans.}$$

* We know that K.E. $= \frac{1}{2}mV^2 = \frac{1}{2} \times 1 \times V^2 = \frac{V^2}{2}\,\text{J} = \frac{1}{1000}\left(\frac{V^2}{2}\right)\text{kJ}$ $\quad \ldots (\because m = 1 \text{ kg})$

Example 21.2. *Dry saturated steam at a pressure of 15 bar enters in a nozzle and is discharged at a pressure of 1.5 bar. Find the final velocity of the steam, when the initial velocity of the steam is negligible.*

If 10% of the heat drop is lost in friction, find the percentage reduction in the final velocity.

Solution. Given : $p_1 = 15$ bar ; $p_2 = 1.5$ bar

Final velocity of the steam

From steam tables, corresponding to a pressure of 15 bar, we find that enthalpy of dry saturated steam,

$$h_1 = 2789.9 \text{ kJ/kg}$$

and corresponding to a pressure of 1.5 bar, enthalpy of dry saturated steam,

$$h_2 = 2693.4 \text{ kJ/kg}$$

$\therefore$ **Heat drop,** $\quad h_d = h_1 - h_2 = 2789.9 - 2693.4 = 96.5$ kJ/kg

We know that final velocity of the steam,

$$V_2 = 44.72\sqrt{h_d} = 44.72\sqrt{96.5} = 439.3 \text{ m/s} \quad \textbf{Ans.}$$

Percentage reduction in the final velocity

We know that heat drop lost in friction

$$= 10\% = 0.1 \qquad \ldots \text{(Given)}$$

$\therefore$ Nozzle coefficient or nozzle efficiency

$$K = 1 - 0.1 = 0.9$$

We know that final velocity of the steam,

$$V_2 = 44.72\sqrt{K\,h_d} = 44.72\sqrt{0.9 \times 96.5} = 416.8 \text{ m/s}$$

$\therefore$ Percentage reduction in final velocity

$$= \frac{439.3 - 416.8}{439.3} = 0.051 \text{ or } 5.1\% \quad \textbf{Ans.}$$

Example 21.3. *Dry saturated steam at 10 bar is expanded isentropically in a nozzle to 0.1 bar. Using steam tables only, find the dryness fraction of the steam at exit. Also find the velocity of steam leaving the nozzle when 1. initial velocity is negligible, and 2. initial velocity of the steam is 135 m/s.*

Solution. Given : $p_1 = 10$ bar ; $p_2 = 0.1$ bar

Dryness fraction of the steam at exit

Let $\quad x_2$ = Dryness fraction of the steam at exit.

From steam tables, corresponding to a pressure of 10 bar, we find that entropy of dry saturated steam,

$$s_1 = s_{g1} = 6.583 \text{ kJ/kg K}$$

and corresponding to a pressure of 0.1 bar, we find that

$$s_{f2} = 0.649 \text{ kJ/kg K}, \quad \text{and} \quad s_{fg2} = 7.502 \text{ kJ/kg K}$$

Since the expansion of steam is isentropic, therefore

Entropy of steam at inlet (s_1) = Entropy of steam at exit (s_2)

$$6.583 = s_{f2} + x_2 s_{fg2} = 0.649 + x_2 \times 7.502$$

$$x_2 = 0.791 \quad \textbf{Ans.}$$

1. Velocity of steam leaving the nozzle when initial velocity is negligible

From steam tables, corresponding to a pressure of 10 bar, we find that enthalpy or total heat of dry saturated steam,

$$h_1 = h_{g1} = 2776.2 \text{ kJ/kg}$$

and corresponding to a pressure of 0.1 bar,

$$h_{f2} = 191.8 \text{ kJ/kg, and } h_{fg2} = 2392.9 \text{ kJ/kg}$$

$\therefore$ Enthalpy or total heat of steam of exit,

$$h_2 = h_{f2} + x_2 h_{fg2}$$
$$= 191.8 + 0.791 \times 2392.9 = 2084.6 \text{ kJ/kg}$$

and heat drop, $h_d = h_1 - h_2 = 2776.2 - 2084.6 = 691.6 \text{ kJ/kg}$

We know that velocity of steam leaving the nozzle,

$$V_2 = 44.72\sqrt{h_d} = 44.72\sqrt{691.6} = 1176 \text{ m/s} \quad \textbf{Ans.}$$

2. Velocity of steam leaving the nozzle when initial velocity, $V_1 = 135$ m/s

We know that velocity of steam leaving the nozzle,

$$V_2 = \sqrt{V_1^2 + 2000\,h_d} = \sqrt{(135)^2 + 2000 \times 691.6} = 1184 \text{ m/s} \quad \textbf{Ans.}$$

Example 21.4. *Dry saturated steam at a pressure of 10 bar is expanded in a nozzle to a pressure of 0.7 bar. With the help of Mollier diagram find the velocity and dryness fraction of steam issuing from the nozzle, if the friction is neglected.*

Also find the velocity and dryness fraction of the steam, if 15% of the heat drop is lost in friction.

Solution. Given : $p_1 = 10$ bar ; $p_2 = 0.7$ bar

Velocity and dryness fraction of steam issuing from the nozzle, if friction is neglected

The process on the Mollier diagram, as shown in Fig. 21.3, is drawn as discussed below :

1. First of all, locate the point A on the saturation line (because the steam is initially dry saturated) where the initial pressure line (10 bar) meets it.

2. Since the expansion in the nozzle is isentropic, therefore draw a vertical line through A to meet the final pressure line (0.7 bar) at point B.

Now from the Mollier diagram, we find that

$$h_1 = 2772 \text{ kJ/kg,}$$

and $$h_2 = 2310 \text{ kJ/kg}$$

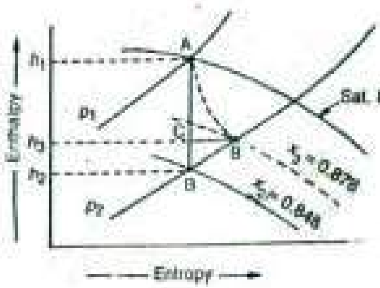


Fig. 21.3

$\therefore$ Heat drop,

$$h_d = h_1 - h_2 = 2772 - 2310 = 462 \text{ kJ/kg}$$

We know that velocity of steam issuing from the nozzle,

$$V_2 = 44.72\sqrt{h_d} = 44.72\sqrt{462} = 961 \text{ m/s} \quad \textbf{Ans.}$$

From Mollier diagram, we also find that the dryness fraction of steam issuing from the nozzle (*i.e.* at point B) is $x_2 = 0.848$. **Ans.**

Velocity and dryness fraction of steam issuing from the nozzle if 15% of the heat drop is lost in friction.

Since 15% heat drop is lost in friction, therefore nozzle coefficient or nozzle efficiency,

$$K = 100 - 15 = 85\% = 0.85$$

and heat drop due to friction $= 462 \times 0.15 = 69.3$ kJ/kg

We know that velocity of steam issuing from the nozzle,

$$V_2 = 44.72\sqrt{K\,h_d} = 44.72\sqrt{0.85 \times 462} = 886 \text{ m/s} \quad \textbf{Ans.}$$

Now let us complete the Mollier diagram as discussed below :

1. Locate point *C* on the vertical line *AB*, such that *BC* = 69.3 kJ/kg.
2. Now through *C*, draw a horizontal line *CB'* to meet the final pressure line (0.7 bar) at *B'*.

From the Mollier diagram, we find that the dryness fraction of steam issuing from the nozzle, (*i.e.* at point *B'*) is $x_3 = 0.878$ **Ans.**

21.6. Mass of Steam Discharged through Nozzle

We have already discussed that the flow of steam, through the nozzle is isentropic, which is approximately represented by the general law:

$$pv^n = \text{Constant}$$

We know that gain in kinetic energy

$$= \frac{V_2^2}{2} \qquad \text{...(Neglecting initial velocity of steam)}$$

and Heat drop = Work done during Rankine cycle

$$= \frac{n}{n-1}(p_1 v_1 - p_2 v_2)$$

Since gain in kinetic energy is equal to heat drop, therefore

$$\frac{V_2^2}{2} = \frac{n}{n-1}(p_1 v_1 - p_2 v_2)$$

$$= \frac{n}{n-1} \times p_1 v_1 \left(1 - \frac{p_2 v_2}{p_1 v_1}\right) \qquad \text{...(i)}$$

We know that $p_1 v_1^n = p_2 v_2^n$

$$\frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = \left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}$$

$$\therefore \quad v_2 = v_1\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}} \qquad \text{...(ii)}$$

Substituting, the value of v_2 / v_1 in equation (*i*),

$$\frac{V_2^2}{2} = \frac{n}{n-1} \times p_1 v_1 \left[1 - \frac{p_2}{p_1}\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}}\right]$$

$$= \frac{n}{n-1} \times p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]$$

or
$$V_2 = \sqrt{2 \times \frac{n}{n-1} \times p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]} \quad \ldots (iii)$$

Now the volume of steam flowing per second

= Cross-sectional area of nozzle × Velocity of steam $= A\, V_2$

and volume of 1 kg of steam *i.e.* specific volume of steam at pressure p_2

$= v_2$ m³/kg

∴ Mass of steam discharged through nozzle per second,

$$m = \frac{\text{Volume of steam flowing per second}}{\text{Volume of 1 kg of steam at pressure } p_2}$$

$$= \frac{A V_2}{v_2} = \frac{A}{v_2} \sqrt{2 \times \frac{n}{n-1} \times p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]}$$

Substituting the value of v_2 from equation (*ii*),

$$m = \frac{A}{v_1} \left(\frac{p_1}{p_2} \right)^{-\frac{1}{n}} \sqrt{\frac{2n}{n-1} \times p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]}$$

$$= \frac{A}{v_1} \left(\frac{p_2}{p_1} \right)^{\frac{1}{n}} \sqrt{\frac{2n}{n-1} \times p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]}$$

$$= A \sqrt{\left(\frac{p_2}{p_1} \right)^{\frac{2}{n}} \times \frac{2n}{n-1} \times \frac{p_1}{v_1} \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]}$$

$$= A \sqrt{\frac{2n}{n-1} \times \frac{p_1}{v_1} \left[\left(\frac{p_2}{p_1} \right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1} \right)^{\frac{n+1}{n}} \right]} \quad \ldots (iv)$$

Example 21.5. *Dry air at a temperature of 27° C and pressure of 20 bar enters a nozzle and leaves at a pressure of 4 bar. Find the mass of air discharged, if the area of the nozzle is 200 mm².*

Solution. Given : $T_1 = 27°\text{C} = 27 + 273 = 300$ K ; $p_1 = 20$ bar $= 20 \times 10^5$ N/m² ; $p_2 = 4$ bar $= 4 \times 10^5$ N/m² ; $A = 200$ mm² $= 200 \times 10^{-6}$ m²

Let v_1 = Specific volume of air in m³/kg.

We know that $p_1 v_1 = m R T_1$

$$\therefore \quad v_1 = \frac{m R T_1}{p_1} = \frac{1 \times 287 \times 300}{20 \times 10^5} = 0.043 \text{ m}^3/\text{kg} \quad \ldots (\because R \text{ for air} = 287 \text{ J/kg K})$$

We know that mass of steam discharged through the nozzle,

$$m = A\sqrt{\frac{2n}{n-1}\times\frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}}-\left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]}$$

$$= 200\times10^{-6}\sqrt{\frac{2\times1.4}{1.4-1}\times\frac{20\times10^5}{0.043}\left[\left(\frac{4}{20}\right)^{\frac{2}{1.4}}-\left(\frac{4}{20}\right)^{\frac{1.4+1}{1.4}}\right]}$$

$$= 200\times10^{-6}\sqrt{3256\times10^5\,[\,0.1-0.06\,]} = 0.72\text{ kg/s} \quad \textbf{Ans.}$$

21.7. Condition for Maximum Discharge through a Nozzle (Critical Pressure Ratio)

A nozzle is, normally, designed for maximum discharge by designing a certain throat pressure which produces this condition.

Let p_1 = Initial pressure of steam in N/m²,

p_2 = Pressure of steam at throat in N/m²,

v_1 = Volume of 1 kg of steam at pressure (p_1) in m³,

v_2 = Volume of 1 kg of steam at pressure(p_2) in m³, and

A = Cross-sectional area of nozzle at throat, in m².

We have derived an equation in the previous article that the mass of steam discharged through nozzle,

$$m = A\sqrt{\frac{2n}{n-1}\times\frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}}-\left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]} \quad \ldots(i)$$

There is only one value of the ratio p_2/p_1, which produces maximum discharge from the nozzle. This ratio p_2/p_1, is obtained by differentiating the right hand side of the equation. We see from this equation that except p_2/p_1, all other values are constant. Therefore, only that portion of the equation which contains p_2/p_1, is differentiated and equated to zero for maximum discharge.

$$\therefore \quad \frac{d}{d\left(\frac{p_2}{p_1}\right)}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}}-\left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right] = 0$$

or

$$\frac{2}{n}\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}-1} - \frac{n+1}{n}\left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}-1} = 0$$

$$\frac{2}{n}\left(\frac{p_2}{p_1}\right)^{\frac{2-n}{n}} = \frac{n+1}{n}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}}$$

$$\left(\frac{p_2}{p_1}\right)^{\frac{2-n}{n}}\times\left(\frac{p_2}{p_1}\right)^{-\frac{1}{n}} = \frac{n+1}{n}\times\frac{n}{2}$$

$$\left(\frac{p_2}{p_1}\right)^{\frac{2-n}{n}-\frac{1}{n}} = \frac{n+1}{2}$$

$$\left(\frac{p_2}{p_1}\right)^{\frac{1-n}{n}} = \frac{n+1}{2}$$

$$\frac{p_2}{p_1} = \left(\frac{n+1}{2}\right)^{\frac{n}{1-n}} = \left(\frac{n+1}{2}\right)^{\frac{-n}{-(1-n)}}$$

$$= \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}} \qquad \ldots (ii)$$

Notes: 1. The ratio p_2 / p_1 is known as *critical pressure ratio*, and the pressure p_2 at the throat is known as *critical pressure*.

2. The maximum value of the discharge per second is obtained by substituting the value of p_2 / p_1 in equation (*i*).

$$\therefore \text{ Maximum discharge, } m_{max} = A\sqrt{\frac{2n}{n-1}\times\frac{p_1}{v_1}\left[\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}-\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}}\right]}$$

$$= A\sqrt{\frac{2n}{n-1}\times\frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}\left[1-\left(\frac{2}{n+1}\right)^{\frac{n+1}{n-1}-\frac{2}{n-1}}\right]}$$

$$= A\sqrt{\frac{2n}{n-1}\times\frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}\left[1-\left(\frac{2}{n+1}\right)\right]}$$

$$= A\sqrt{\frac{2n}{n-1}\times\frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}\left[\frac{n-1}{n+1}\right]}$$

$$= A\sqrt{\frac{2n}{n+1}\times\frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}} \qquad \ldots (iii)$$

3. We see from the above equation that in a convergent-divergent nozzle, the discharge depends upon the area of nozzle at throat and the initial conditions of the steam (*i.e.* pressure p_1 and volume v_1). It is independent of the exit conditions of the steam. It is thus obvious, that the discharge remains constant after the throat (*i.e.* in the divergent portion of the nozzle).

4. The equations derived above are true for gases also.

21.8. Values for Maximum Discharge through a Nozzle

In the last article we have derived a relation for the maximum discharge through a nozzle, *i.e.*

$$m_{max} = A\sqrt{\frac{2n}{n+1}\times\frac{p_1}{v_1}\left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}}$$

Now we shall discuss the values of maximum discharge for the following three conditions :

1. *When the steam is initially dry saturated*

We know that for dry saturated steam, $n = 1.135$. Therefore substituting the value of n in the relation for maximum discharge, we have

$$m_{max} = 0.637\, A \sqrt{\frac{p_1}{v_1}}$$

2. *When the steam is initially superheated*

We know that for superheated steam, $n = 1.3$. Therefore substituting the value of n in the relation for maximum discharge, we have

$$m_{max} = 0.666\, A \sqrt{\frac{p_1}{v_1}}$$

3. *For gases*

We know that for gases, $n = 1.4$. Therefore substituting the value of n in the relation for maximum discharge, we have

$$m_{max} = 0.685\, A \sqrt{\frac{p_1}{v_1}}$$

Example 21.6. *Dry air at a pressure of 12 bar and 300°C is expanded isentropically through a nozzle at a pressure of 2 bar. Determine the maximum discharge through the nozzle of 150 mm² area.*

Solution. Given : $p_1 = 12$ bar $= 12 \times 10^5$ N/m² ; $T_1 = 300°C = 300 + 273 = 573$ K ; $p_2 = 2$ bar ; $A = 150$ mm² $= 150 \times 10^{-6}$ m²

Let v_1 = Specific volume of air in m³/kg.

We know that $p_1 v_1 = m R T_1$

$$\therefore \quad v_1 = \frac{m R T_1}{p_1} = \frac{1 \times 287 \times 573}{12 \times 10^5} = 0.137 \text{ m}^3/\text{kg}$$

We know that maximum discharge through the nozzle,

$$m_{max} = 0.685\, A \sqrt{\frac{p_1}{v_1}} = 0.685 \times 150 \times 10^{-6} \sqrt{\frac{12 \times 10^5}{0.137}} \text{ kg/s}$$

$$= 0.304 \text{ kg/s } \textbf{Ans.}$$

Example 21.7. *Steam at a pressure of 10 bar and 210°C is supplied to a convergent divergent nozzle with a throat area of 1500 mm². The exit is below critical pressure. Find the coefficient of discharge, if the flow is 7200 kg of steam per hour.*

Solution. Given : $p_1 = 10$ bar $= 10 \times 10^5$ N/m² ; $T_1 = 210°$ C ; $A = 1500$ mm² $= 1500 \times 10^{-6}$ m² ; $m = 7200$ kg/h $= 2$ kg/s

From steam tables, for superheated steam, corresponding to a pressure of 10 bar and 210° C, we find that specific volume of steam,

$$v_1 = 0.2113 \text{ m}^3/\text{kg}$$

We know that for superheated steam, $n = 1.3$.

$$\therefore \text{ Maximum discharge, } m_{max} = A \sqrt{\frac{2n}{n+1} \times \frac{p_1}{v_1} \left(\frac{2}{n+1}\right)^{\frac{2}{n-1}}}$$

$$= 1500 \times 10^{-6} \sqrt{\frac{2 \times 1.3}{1.3+1} \times \frac{10 \times 10^5}{0.2113} \left(\frac{2}{1.3+1}\right)^{\frac{2}{1.3-1}}} \text{ kg/s}$$

$$= 2.17 \text{ kg/s}$$

We know that coefficient of discharge

$$= \frac{\text{Actual discharge}}{\text{Maximum discharge}} = \frac{2}{2.17} = 0.922 \text{ Ans.}$$

Note : The maximum discharge for superheated steam may also be calculated by using the relation,

$$m_{max} = 0.666 A \sqrt{\frac{p_1}{v_1}}$$

21.9. Values for Critical Pressure Ratio

We have also discussed in Art. 21.7 that the critical pressure ratio,

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$

where p_2 / p_1 = Critical pressure ratio.

We shall now discuss the values of critical pressure ratio for the following conditions :

1. *When the steam is initially saturated*

We know that for dry saturated steam, n = 1.135.

$$\therefore \quad \frac{p_2}{p_1} = \left(\frac{2}{1.135+1}\right)^{\frac{1.135}{1.135-1}} = 0.577 \quad \text{or} \quad p_2 = 0.577\, p_1$$

2. *When the steam is initially superheated*

We know that for superheated steam, n = 1.3.

$$\therefore \quad \frac{p_2}{p_1} = \left(\frac{2}{1.3+1}\right)^{\frac{1.3}{1.3-1}} = 0.546 \quad \text{or} \quad p_2 = 0.546\, p_1$$

3. *When the steam is initially wet*

It has been experimentally found that the critical pressure ratio for wet steam,

$$\frac{p_2}{p_1} = 0.582 \quad \text{or} \quad p_2 = 0.582\, p_1$$

4. *For gases*

We know that for gases, n = 1.4.

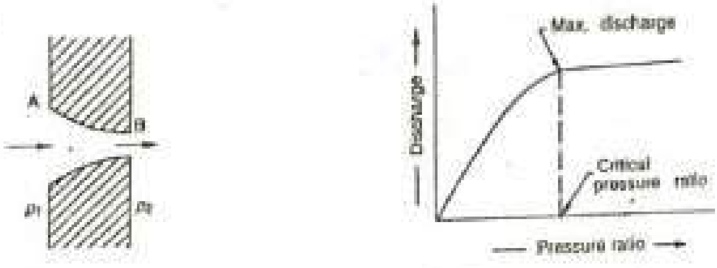
$$\therefore \quad \frac{p_2}{p_1} = \left(\frac{2}{1.4+1}\right)^{\frac{1.4}{1.4-1}} = 0.528 \quad \text{or} \quad p_2 = 0.528\, p_1$$

21.10. Physical Significance of Critical Pressure Ratio

In the previous article, we discussed the values of critical pressure ratio for various forms of steam. But now we shall discuss the physical significance of the critical pressure ratio.

Now consider two vessels A and B connected by a convergent nozzle as shown in Fig. 21.4 (*a*). Let the vessel A contains steam at a high and steady pressure (p_1), and the vessel B contains steam at another pressure (p_2) which may be varied at will.

First of all, let the pressure (p_2) in the vessel *B* be made equal to the pressure (p_1) in the vessel *A*. In this case, there will be no flow of steam through the nozzle. Now if the pressure (p_2) in the vessel *B* is gradually reduced, the discharge through the nozzle will increase accordingly as shown in Fig. 21.4 (*b*). As the pressure (p_2) in the vessel *B* approaches the critical value, the rate of discharge will also approach its maximum value. If the pressure (p_2) in the vessel *B* is further reduced, it will not increase the rate of discharge. But the discharge will remain the same as that at critical pressure as shown in Fig. 21.4 (*b*). The ratio of exit pressure to the inlet pressure is called *critical pressure ratio*.



(*a*) Nozzle section. (*b*) Discharge-pressure ratio curve.

Fig. 21.4

We know that the velocity of steam at any section in the nozzle [Refer Art. 21.6, equation (*iii*)],

$$V_2 = \sqrt{\frac{2n}{n-1} \times p_1 v_1 \left[1 - \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]} \qquad \ldots (i)$$

and the critical pressure ratio for maximum discharge,

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1} \right)^{\frac{n}{n-1}} \quad \text{or} \quad \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} = \frac{2}{n+1}$$

Substituting this value in equation (*i*),

$$V_2 = \sqrt{\frac{2n}{n-1} \times p_1 v_1 \left[1 - \frac{2}{n+1} \right]} = \sqrt{\frac{2n}{n-1} \times p_1 v_1 \left[\frac{n+1-2}{n+1} \right]}$$

$$= \sqrt{\frac{2n}{n+1} \times p_1 v_1} = \sqrt{\frac{2n}{n+1} \times \frac{p_1}{\rho_1}} \qquad \ldots (ii)$$

$$\ldots \left[\because \text{Volume } (v) = \frac{1}{\text{Density } (\rho)} \right]$$

We also know that for isentropic expansion,

$$p_1 v_1^n = p_2 v_2^n$$

or

$$\frac{p_1}{\rho_1^n} = \frac{p_2}{\rho_2^n} \qquad \ldots \left(\because v = \frac{1}{\rho} \right)$$

or

$$\frac{1}{\rho_1} = \frac{1}{\rho_2} \left(\frac{p_2}{p_1} \right)^{\frac{1}{n}}$$

$$\therefore \qquad \frac{p_1}{\rho_1} = \frac{p_1}{\rho_2}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} \qquad \text{(Multiplying by } p_1\text{)}$$

$$= \frac{p_2}{\rho_2}\times\frac{p_1}{p_2}\left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} = \frac{p_2}{\rho_2}\left(\frac{p_2}{p_1}\right)^{\frac{1-n}{n}}$$

$$= \frac{p_2}{\rho_2}\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}\times\frac{1-n}{n}} = \frac{p_2}{\rho_2}\left(\frac{n+1}{2}\right) \quad \ldots\left[\because \frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}\right]$$

Substituting the value of p_1/ρ_1, in equation (*ii*),

$$V_2 = \sqrt{2\left(\frac{n}{n+1}\right)\frac{p_2}{\rho_2}\left(\frac{n+1}{2}\right)} = \sqrt{\frac{n\,p_2}{\rho_2}} \qquad \ldots (iii)$$

This is the value of velocity of sound in the medium at pressure p_2 and is known as *sonic velocity*.

Notes : 1. The critical pressure gives the velocity of steam at the throat equal to the velocity of sound.

2. The flow in the convergent portion of the nozzle is sub-sonic and in the divergent portion it is supersonic.

3. To increase the velocity of steam above sonic velocity (supersonic) by expanding steam below the critical pressure, the divergent portion for the nozzle is necessary.

21.11. Diameters of Throat and Exit for Maximum Discharge

Consider a convergent-divergent nozzle discharging steam, as shown in Fig. 21.5 (*a*).

(*a*) Convergent-divergent nozzle. (*b*) *h-s* graph for a convergent-divergent nozzle.

Fig. 21.5

Let p_1 = Initial pressure of steam,

h_1 = Enthalpy or total heat of steam at inlet,

p_2, p_3, h_2, h_3 = Corresponding values at throat and outlet,

x_2 = Dryness fraction of steam at throat,

V_2 = Velocity of steam at throat,

v_{g2} = Specific volume of steam at throat corresponding to pressure p_2 (from steam tables),

A_2 = Area of throat,

x_3, V_3, v_{g3}, A_3 = Corresponding values at exit, and

m = Mass of steam discharged.

First of all, find the value of critical pressure (p_2) as discussed in Art. 21.7.

Now complete the h-s diagram, as shown in Fig. 21.5 (b), for the expansion of steam through the convergent-divergent nozzle as discussed below :

1. First of all, locate the point A for the initial conditions of steam. It is a point, where the saturation line meets the initial pressure (p_1) line.
2. Now draw a vertical line through A to meet the critical pressure (p_2) line at B. This represents the throat of the nozzle.
3. Now extend the vertical line AB to meet the outlet pressure (p_3) line at C. This represents the outlet of the nozzle.
4. Now find the values of h_1, h_2, h_3, x_2 and x_3 from the h-s graph.

First of all, consider the flow of steam from the inlet to the throat. We know that

Enthalpy or heat drop, $\quad h_{d2} = h_1 - h_2$

∴ Velocity of steam at throat,

$$V_2 = 44.72\sqrt{h_{d2}} \qquad \text{... (Neglecting friction)}$$

We know that mass of steam discharged per second,

$$m = \frac{\text{Volume of steam flowing at throat}}{\text{Volume of 1 kg of steam at pressure } p_2}$$

$$= \frac{A_2\, V_2}{v_2} = \frac{A_2\, V_2}{x_2\, v_{g2}} \qquad \text{... } (\because v_2 = x_2\, v_{g2})$$

Similarly, for exit conditions,

$$m = \frac{A_3\, V_3}{x_3\, v_{g3}} = \frac{A_2\, V_2}{x_2\, v_{g2}}$$

Now knowing the value of m, we can determine the area or diameter of throat and exit.

Example 21.8. *Steam enters a group of nozzles of a steam turbine at 12 bar and 220°C and leaves at 1.2 bar. The steam turbine develops 220 kW with a specific steam consumption of 13.5 kg/kWh. If the diameter of nozzles at throat is 7 mm, calculate the number of nozzles.*

Solution. Given : $p_1 = 12$ bar ; $T_1 = 220°C$; $p_3 = 1.2$ bar ; Power developed = 220 kW ; $m_s = 13.5$ kg/kWh ; $d_2 = 7$ mm

We know that for superheated steam, pressure of steam at throat,

$$p_2 = 0.546\, p_1 = 0.546 \times 12 = 6.552 \text{ bar}$$

The Mollier diagram for the expansion of steam through the nozzle is shown in Fig. 21.6.

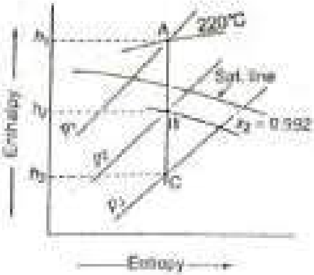


Fig. 21.6

From the Mollier diagram, we find that enthalpy of steam at entrance (*i.e.* at 12 bar and 220° C),

$$h_1 = 2860 \text{ kJ/kg}$$

Enthalpy of steam at throat (*i.e.* at pressure 6.552 bar),

$$h_2 = 2750 \text{ kJ/kg}$$

and dryness fraction of steam at throat,

$$x_2 = 0.992$$

From steam tables, we find that specific volume of dry saturated steam at throat (*i.e.* at pressure 6.552 bar),

$$v_{g2} = 0.29 \text{ m}^3/\text{kg}$$

We know that heat drop from entrance to throat,

$$h_{d2} = h_1 - h_2 = 2860 - 2750 = 110 \text{ kJ/kg}$$

∴ Velocity of steam at throat,

$$V_2 = 44.72\sqrt{h_{d2}} = 44.72\sqrt{110} = 470 \text{ m/s}$$

Area of nozzle at throat,

$$A_2 = \frac{\pi}{4}(d_2)^2 = \frac{\pi}{4} \times 7^2 = 38.5 \text{ mm}^2 = 38.5 \times 10^{-6} \text{ m}^2$$

∴ Mass flow rate per nozzle,

$$m = \frac{A_2 V_2}{v_2} = \frac{A_2 V_2}{x_2 v_{g2}} = \frac{38.5 \times 10^{-6} \times 470}{0.992 \times 0.29} = 0.063 \text{ kg/s}$$

We know that total mass flow rate

$$= 13.5 \times 220 = 2970 \text{ kg/h} = 0.825 \text{ kg/s}$$

$$\therefore \text{Number of nozzles} = \frac{\text{Total mass flow rate}}{\text{Mass flow rate per nozzle}} = \frac{0.825}{0.063} = 13.1 \text{ say } 14 \textbf{ Ans.}$$

Example 21.9. *Estimate the mass flow rate of steam in a nozzle with the following data :*

Inlet pressure and temperature = 10 bar and 200° C ; Back pressure = 0.5 bar ; Throat diameter = 12 mm.

Solution. Given : $p_1 = 10$ bar ; $T_1 = 200°$ C ; $p_3 = 0.5$ bar ; $d_2 = 12$ mm

We know that for superheated steam, pressure of steam at throat,

$$p_2 = 0.546\, p_1 = 0.546 \times 10 = 5.46 \text{ bar}$$

The Mollier diagram for the expansion of steam through the nozzle is shown in Fig. 21.7. From the Mollier diagram, we find that

$$h_1 = 2825 \text{ kJ/kg}$$

$$h_2 = 2710 \text{ kJ/kg}$$

and $$x_2 = 0.982$$

We know that heat drop,

$$h_d = h_1 - h_2 = 2825 - 2710 = 115 \text{ kJ/kg}$$

∴ Velocity of steam at throat,

$$V_2 = 44.72\sqrt{h_d} = 44.72\sqrt{115} = 480 \text{ m/s}$$

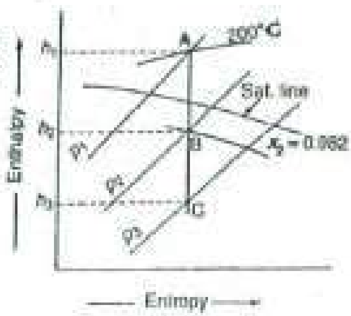


Fig. 21.7

From steam tables, corresponding to a pressure of 5.46 bar, we find that specific volume of steam at throat,

$$v_{g2} = 0.345 \text{ m}^3/\text{kg}$$

Area of nozzle at throat,

$$A_2 = \frac{\pi}{4}(d_2)^2 = \frac{\pi}{4}(12)^2 = 113 \text{ mm}^2 = 113 \times 10^{-6} \text{ m}^2$$

∴ Mass flow rate of steam,

$$m = \frac{A_2 V_2}{x_2 v_{g2}} = \frac{113 \times 10^{-6} \times 480}{0.982 \times 0.345} = 0.16 \text{ kg/s}$$

$$= 0.16 \times 3600 = 576 \text{ kg/h} \textbf{ Ans.}$$

Example 21.10. *Dry saturated steam enters a nozzle at a pressure of 10 bar and with an initial velocity of 90 m/s. The outlet pressure is 6 bar and the outlet velocity is 435 m/s. The heat loss from the nozzle is 9 kJ/kg of steam flow.*

Calculate the dryness fraction and the area at the exit, if the area at the inlet is 1256 mm².

Solution. Given : $p_1 = 10$ bar ; $V_1 = 90$ m/s ; $p_3 = 6$ bar ; $V_3 = 435$ m/s ; Loses = 9 kJ/kg ; $A_1 = 1256 \text{ mm}^2 = 1256 \times 10^{-6} \text{ m}^2$

Dryness fraction of steam

Let x_3 = Dryness fraction of steam at the exit.

From steam tables, corresponding to a pressure of 10 bar, we find that enthalpy of dry saturated steam,

$$h_1 = 2776.2 \text{ kJ/kg} \text{ ; and } v_{g1} = 0.1943 \text{ m}^3\text{/kg}$$

and corresponding to a pressure of 6 bar, we find that

$$h_{f3} = 670.4 \text{ kJ/kg} \text{ ; } h_{fg3} = 2085 \text{ kJ/kg} \text{ ; and } v_{g3} = 0.3155 \text{ m}^3\text{/kg}$$

We know that for a steady flow through the nozzle,

$$h_1 + \frac{1}{1000}\left(\frac{V_1^2}{2}\right) = h_3 + \frac{1}{1000}\left(\frac{V_3^2}{2}\right) + \text{Losses}$$

$$\therefore \quad h_3 = h_1 + \frac{1}{2000}(V_1^2 - V_3^2) - \text{Losses}$$

$$= 2776.2 + \frac{1}{2000}\left[(90)^2 - (435)^2\right] - 9$$

$$= 2776.2 - 99.6 = 2676.6 \text{ kJ/kg}$$

We also know that enthalpy of wet steam (h_3),

$$2676.6 = h_{f3} + x_3 h_{fg3} = 670.4 + x_3 \times 2085$$

$$\therefore \quad x_3 = 0.962 \textbf{ Ans.}$$

Area at exit

Let A_3 = Area at exit in m².

We know that $$\frac{A_1 V_1}{x_1 v_{g1}} = \frac{A_3 V_3}{x_3 v_{g3}} \quad \text{or} \quad \frac{1256 \times 10^{-6} \times 90}{1 \times 0.1943} = \frac{A_3 \times 435}{0.962 \times 0.3155}$$

... (For dry saturated steam, $x_1 = 1$)

$$\therefore \quad A_3 = 406 \times 10^{-6} \text{ m}^2 = 406 \text{ mm}^2 \textbf{ Ans.}$$

Example 21.11. *Dry saturated steam at a pressure of 8 bar enters a convergent-divergent nozzle and leaves it at a pressure of 1.5 bar. If the flow is isentropic, and the corresponding expansion index is 1.135 ; find the ratio of cross-sectional area at exit and throat for maximum discharge.*

Solution. Given : $p_1 = 8$ bar ; $p_3 = 1.5$ bar ; $n = 1.135$

Let A_2 = Cross-sectional area at throat,

A_3 = Cross-sectional area at exit, and

m = Mass of steam discharged per second.

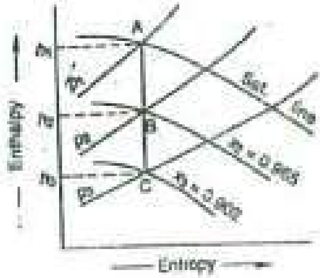


Fig. 21.8

We know that for dry saturated steam (or when n = 1.135), critical pressure ratio,

$$\frac{p_2}{p_1} = 0.577$$

$$\therefore \quad p_2 = 0.577\, p_1 = 0.577 \times 8 = 4.616 \text{ bar}$$

Now complete the Mollier diagram for the expansion of steam through the nozzle, as shown in Fig. 21.8.

From Mollier diagram, we find that

$h_1 = 2775$ kJ/kg ; $h_2 = 2650$ kJ/kg ; $h_3 = 2465$ kJ/kg ; $x_2 = 0.965$; and $x_3 = 0.902$

From steam tables, we also find that the specific volume of steam at throat corresponding to 4.616 bar,

$$v_{g2} = 0.405 \text{ m}^3\text{/kg}$$

and specific volume of steam at exit corresponding to 1.5 bar,

$$v_{g3} = 1.159 \text{ m}^3\text{/kg}$$

Heat drop between entrance and throat,

$$h_{d2} = h_1 - h_2 = 2775 - 2650 = 125 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at throat,

$$V_2 = 44.72\sqrt{h_{d2}} = 44.72\sqrt{125} = 500 \text{ m/s}$$

and

$$m = \frac{A_2 V_2}{x_2 v_{g2}}$$

or

$$A_2 = \frac{m x_2 v_{g2}}{V_2} = \frac{m \times 0.965 \times 0.405}{500} = 0.000\,786\, m \quad \ldots (i)$$

Heat drop between entrance and exit,

$$h_{d3} = h_1 - h_3 = 2775 - 2465 = 310 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at exit,

$$V_3 = 44.72\sqrt{h_{d3}} = 44.72\sqrt{310} = 787.4 \text{ m/s}$$

and

$$m = \frac{A_3 V_3}{x_3 v_{g3}}$$

or

$$A_3 = \frac{m x_3 v_{g3}}{V_3} = \frac{m \times 0.902 \times 1.159}{787.4} = 0.001\,33\, m \quad \ldots (ii)$$

$\therefore$ Ratio of cross-sectional area at exit and throat,

$$\frac{A_3}{A_2} = \frac{0.001\,33\, m}{0.000\,786\, m} = 1.7 \text{ Ans.}$$

Example 21.12. *A convergent-divergent nozzle is required to discharge 2 kg of steam per second. The nozzle is supplied with steam at 7 bar and 180° C and discharge takes place against a back pressure of 1 bar. The expansion upto throat is isentropic and the frictional resistance between the throat and exit is equivalent to 63 kJ/kg of steam. Taking approach velocity of 75 m/s and throat pressure of 4 bar, estimate :*

1. Suitable areas for the throat and exit ; and 2. Overall efficiency of the nozzle based on the enthalpy drop between the actual inlet pressure and temperature and the exit pressure.

Solution. Given : $m = 2$ kg/s ; $p_1 = 7$ bar ; $T_1 = 180°$ C ; $p_3 = 1$ bar ; Frictional resistance $= 63$ kJ/kg of steam ; $V_1 = 75$ m/s ; $p_2 = 4$ bar

1. *Suitable areas for the throat and exit*

Let A_2 = Area at the throat, and

A_3 = Area at the exit.

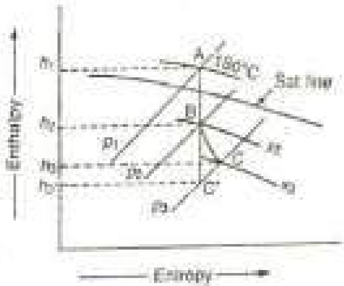


Fig. 21.9

The expansion of steam through the nozzle on the Mollier diagram is shown in Fig. 21.9. From the Mollier diagram, we find that

$h_1 = 2810$ kJ/kg ; $h_2 = 2680$ kJ/kg ; $h_{3'} = 2470$ kJ/kg ; $x_2 = 0.97$; $x_3 = 0.934$

From steam tables, we also find that the specific volume of steam at throat corresponding to 4 bar,

$$v_{g2} = 0.462 \text{ m}^3/\text{kg}$$

and specific volume of steam corresponding to 1 bar,

$$v_{g3} = 1.694 \text{ m}^3/\text{kg}$$

We know that heat drop between entrance and throat,

$$h_{d2} = h_1 - h_2 = 2810 - 2680 = 130 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at throat,

$$V_2 = \sqrt{V_1^2 + 2000\, h_{d2}} = \sqrt{(75)^2 + 2000 \times 130} = 515 \text{ m/s}$$

and

$$m = \frac{A_2\, V_2}{x_2\, v_{g2}}$$

or

$$A_2 = \frac{m\, x_2\, v_{g2}}{V_2} = \frac{2 \times 0.97 \times 0.462}{515} = 1.74 \times 10^{-3} \text{ m}^2$$

$$= 1740 \text{ mm}^2 \quad \textbf{Ans.}$$

Since there is a frictional resistance of 63 kJ/kg of steam between the throat and exit, therefore

$$h_3 - h_{3'} = 63 \text{ or } h_3 = h_{3'} + 63 = 2470 + 63 = 2533 \text{ kJ/kg}$$

and heat drop between entrance and exit,

$$h_{d3} = h_1 - h_3 = 2810 - 2533 = 277 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at exit,

$$V_3 = \sqrt{V_1^2 + 2000\, h_{d3}} = \sqrt{(75)^2 + 2000 \times 277} = 748 \text{ m/s}$$

and $$m = \frac{A_3 V_3}{x_3 v_{g3}}$$

or $$A_3 = \frac{m\, x_3\, v_{g3}}{V_3} = \frac{2 \times 0.934 \times 1.694}{748} = 4.23 \times 10^{-3}\ \text{m}^2$$

$$= 4230\ \text{mm}^2\ \textbf{Ans.}$$

2. Overall efficiency of the nozzle

We know that overall efficiency of the nozzle,

$$\eta = \frac{\text{Useful heat drop}}{\text{Isentropic heat drop}} = \frac{h_1 - h_3}{h_1 - h_{3'}}$$

$$= \frac{2810 - 2533}{2810 - 2470} = 0.815 \text{ or } 81.5\%\ \textbf{Ans.}$$

Example 21.13. *Steam at a pressure of 10 bar and 0.9 dry discharges through a nozzle having throat area of 450 mm². If the back pressure is 1 bar, find 1. final velocity of the steam, and 2. cross-sectional area of the nozzle at exit for maximum discharge.*

Solution. Given : $p_1 = 10$ bar ; $x_1 = 0.9$; $A_2 = 450\ \text{mm}^2 = 450 \times 10^{-6}\ \text{m}^2$; $p_3 = 1$ bar

Final velocity of steam

Let V_3 = Final velocity of steam.

We know that for maximum discharge, pressure of steam at throat (for wet steam),

$$p_2 = 0.582\, p_1 = 0.582 \times 10 = 5.82 \text{ bar}$$

Now complete the Mollier diagram for the expansion of steam through the nozzle as shown in Fig. 21.10.

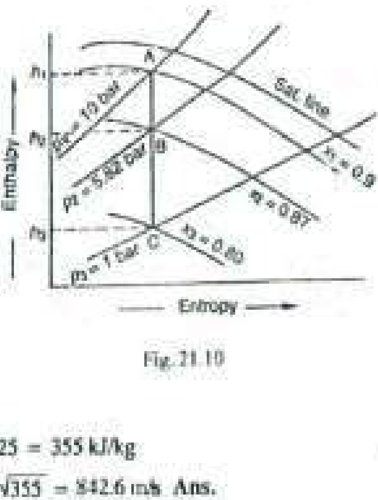


Fig. 21.10

From the Mollier diagram, we find that

$$h_1 = 2580 \text{ kJ/kg} ; \ h_2 = 2485 \text{ kJ/kg} ;$$

$$h_3 = 2225 \text{ kJ/kg} ; \ x_2 = 0.87 ;$$

and $$x_3 = 0.8$$

$\therefore$ Heat drop from entrance to exit,

$$h_{d3} = h_1 - h_3 = 2580 - 2225 = 355 \text{ kJ/kg}$$

and velocity of steam, $$V_3 = 44.72\sqrt{h_{d3}} = 44.72\sqrt{355} = 842.6 \text{ m/s}\ \textbf{Ans.}$$

Cross-sectional area of the nozzle at exit

Let A_3 = Cross-sectional area of the nozzle at exit.

From steam tables, we find that specific volume of steam at throat corresponding to a pressure of 5.82 bar,

$$v_{g2} = 0.3254\ \text{m}^3/\text{kg}$$

and specific volume of steam at exit corresponding to a pressure of 1 bar,

$$v_{g3} = 1.694\ \text{m}^3/\text{kg}$$

We know that heat drop from entrance to throat,

$$h_{d2} = h_1 - h_2 = 2580 - 2485 = 95 \text{ kJ/kg}$$

∴ Velocity of steam at throat,

$$V_2 = 44.72\sqrt{h_{d2}} = 44.72\sqrt{95} = 436 \text{ m/s}$$

*Since the mass flow rate is same at throat and exit, therefore

$$\frac{A_2 V_2}{x_2 v_{g2}} = \frac{A_3 V_3}{x_3 v_{g3}}$$

$$\frac{450 \times 10^{-6} \times 436}{0.87 \times 0.3254} = \frac{A_3 \times 842.6}{0.8 \times 1.694}$$

$$\therefore \qquad A_3 = 1114 \times 10^{-6} \text{ m}^2 = 1114 \text{ mm}^2 \text{ Ans.}$$

Example 21.14. *A gas expands in a convergent-divergent nozzle from 5 bar to 1.5 bar, the initial temperature being 700° C and the nozzle efficiency is 90%. All the losses take place after the throat. For 1 kg/s mass flow rate of the gas, find throat and exit areas.*

Take n = 1.4 and R = 287 J/kg K.

Solution. Given : $p_1 = 5$ bar ; $p_3 = 1.5$ bar ; $T_1 = 700°$ C $= 700 + 273 = 973$ K ; $K = 90\%$ $= 0.9$; $m = 1$ kg/s ; $n = 1.4$; $R = 287$ J/kg K

Throat area

We know that pressure at the throat,

$$p_2 = 0.528 p_1 = 0.528 \times 5 = 2.64 \text{ bar}$$

and heat drop between entrance and throat,

$$h_{d2} = h_1 - h_2 = \frac{n}{n-1} \times p_1 v_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right] \qquad \text{...(Refer Art. 21.6)}$$

$$= \frac{n}{n-1} \times m R T_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]$$

$$= \frac{1.4}{1.4-1} \times 1 \times 287 \times 973 \left[1 - \left(\frac{2.64}{5}\right)^{\frac{1.4-1}{1.4}}\right] \text{J/kg}$$

$$= 163\,220 \text{ J/kg} = 163.22 \text{ kJ/kg}$$

∴ Velocity of gas at the throat,

$$V_2 = 44.72\sqrt{h_{d2}} = 44.72\sqrt{163.22} = 571.3 \text{ m/s}$$

Let T_2 = Temperature of gas at the throat,

v_2 = Volume of gas at the throat, and

---

* This may also be found as discussed below :

We know that mass of steam discharged per second,

$$m = \frac{A_2 V_2}{x_2 v_{g2}} = \frac{450 \times 10^{-6} \times 436}{0.87 \times 0.3254} = 0.693 \text{ kg}$$

Similarly, $$0.693 = \frac{A_3 V_3}{x_3 v_{g3}} = \frac{A_3 \times 842.6}{0.8 \times 1.694} = 621.5 A_3$$

$$A_3 = 1114 \times 10^{-6} \text{ m}^2 = 1114 \text{ mm}^2 \text{ Ans.}$$

$$A_2 = \text{Area at the throat.}$$

We know that $\quad \dfrac{T_1}{T_2} = \left(\dfrac{p_1}{p_2}\right)^{\frac{n-1}{n}} = \left(\dfrac{5}{2.64}\right)^{\frac{1.4-1}{1.4}} = 1.2$

$\therefore \quad T_2 = T_1/1.2 = 973/1.2 = 810.8 \text{ K}$

and $\quad p_2 v_2 = m R T_2$

or $\quad v_2 = \dfrac{m R T_2}{p_2} = \dfrac{1 \times 287 \times 810.8}{2.64 \times 10^5} = 0.88 \text{ m}^3/\text{kg}$

We know that $\quad m = \dfrac{A_2 V_2}{v_2}$

or $\quad A_2 = \dfrac{m v_2}{V_2} = \dfrac{1 \times 0.88}{571.3} = 1.54 \times 10^{-3} \text{ m}^2 = 1540 \text{ mm}^2$ **Ans.**

Exit area

We know that heat drop between entrance and exit,

$$h_{d1} = h_1 - h_3 = \frac{n}{n-1} \times m R T_1 \left[1 - \left(\frac{p_3}{p_1}\right)^{\frac{n-1}{n}}\right]$$

$$= \frac{1.4}{1.4-1} \times 1 \times 287 \times 973 \left[1 - \left(\frac{1.5}{5}\right)^{\frac{1.4-1}{1.4}}\right] \text{J/kg}$$

$$= 284\,420 \text{ J/kg} = 284.42 \text{ kJ/kg}$$

$\therefore$ Velocity of gas at exit,

$$V_3 = 44.72\sqrt{K\,h_{d1}} = 44.72\sqrt{0.9 \times 284.42} = 715.5 \text{ m/s}$$

... ($\because$ losses takes place after throat)

Let $\quad T_{3'}$ = Temperature of gas at exit when friction is neglected,

T_3 = Temperature of gas at exit when friction is considered,

v_3 = Volume of gas at exit, and

A_3 = Area at exit.

We know that $\quad \dfrac{T_1}{T_{3'}} = \left(\dfrac{p_1}{p_3}\right)^{\frac{n-1}{n}} = \left(\dfrac{5}{1.5}\right)^{\frac{1.4-1}{1.4}} = 1.41$

$\therefore \quad T_{3'} = T_1/1.41 = 973/1.41 = 690 \text{ K}$

Since the nozzle efficiency is 90% (*i.e.* $K = 0.9$), therefore heat drop lost in friction is

$$= (1 - K)\,h_{d1} = (1 - 0.9)\,284.42 = 28.442 \text{ kJ/kg}$$

∴ Increase in temperature due to friction

$$= {}^{*}28.442 \text{ K}$$

$$\therefore \qquad T_3 = 690 + 28.442 = 718.442 \text{ K}$$

and $$p_3 v_3 = m R T_3$$

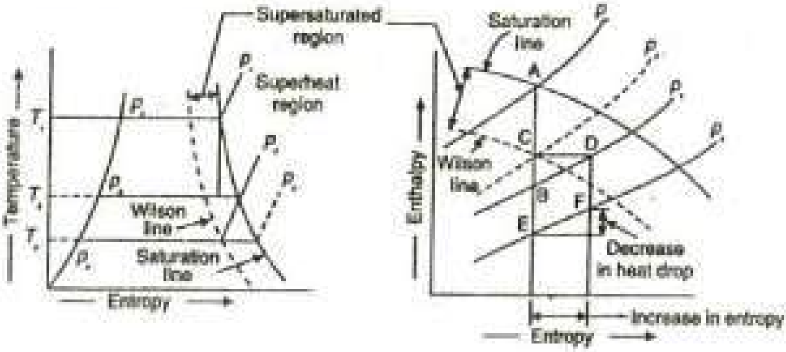
or $$v_3 = \frac{m R T_3}{p_3} = \frac{1 \times 287 \times 718.442}{1.5 \times 10^5} = 1.375 \text{ m}^3/\text{kg}$$

We know that $$m = \frac{A_3 V_3}{v_3}$$

or $$A_3 = \frac{m v_3}{V_3} = \frac{1 \times 1.375}{715.5} = 1.92 \times 10^{-3} \text{ m}^2 = 1920 \text{ mm}^2 \text{ Ans.}$$

21.12. Supersaturated Flow or Metastable Flow through Nozzle

When dry saturated steam is expanded adiabatically or isentropically, it becomes wet and is shown by a vertical line on Mollier diagram.



(a) T-s diagram (b) h-s diagram

Fig. 21.11. Supersaturated flow on T-s and h-s diagram

We have already discussed that expansion of steam in an ideal nozzle is isentropic, which is accompanied by condensation process. If the steam is initially superheated, the condensation should start after it has become dry saturated. This is possible when the steam has proceeded through some distance in the nozzle and in a short interval of time. But from practical point of view, the steam has a great velocity (sometimes sonic and even supersonic). Thus the phenomenon of condensation does not take place at the expected rate. As a result of this, equilibrium between the liquid and vapour phase is delayed and the steam continues to expand in a dry state. The steam in such a set of conditions, is said to be *supersaturated* or in *metastable state*. It is also called *supercooled steam*, as its temperature at any pressure is less than the saturation temperature corresponding to the pressure. The flow of supersaturated steam, through the nozzle is called *supersaturated flow* or *metastable flow*.

---

* We know that heat drop lost in friction

$$= \text{Mass} \times \text{Sp. heat} \times \text{Increase in temp.}$$

$$\text{Increase in temp} = \frac{\text{Heat drop lost in friction}}{\text{Mass} \times \text{Sp. heat}} = \frac{28.442}{1 \times 1} = 28.442 \text{ K}$$

(∵ Sp. heat is taken as 1 kJ/kg K)

Experiments on supersaturated flow of steam have shown that there is a limit to which the supersaturated flow is possible. This limit is represented by *Wilson line on $T-s$ and h-s diagram as shown in Fig. 21.11 (*a*) and (*b*) respectively. It may be noted that the Willson line closely follows the 0.97 dryness fraction line. Beyond this Wilson line, there is no supersaturation. The steam suddenly condenses and restores its normal equilibrium state.

In Fig. 21.11 (*b*) is shown the isentropic expansion of steam in a nozzle. The point A represents the position of initial dry saturated steam at pressure p_1. The line AC represents the isentropic expansion of steam in the supersaturated region. The metastable state (point C) is obtained by drawing a vertical line through A to meet the Wilson line. At C, the steam condenses suddenly. The line CD represents the condensation of steam at constant enthalpy. The point D is obtained by drawing a horizontal line through C to meet the throat pressure (p_2) of the nozzle. The line DF represents the isentropic expansion of steam in the divergent portion.

Notes : 1. The same theory is applicable, if the steam is initially superheated.

2. The difference of supersaturated temperature and saturation temperature at that pressure is known as *degree of undercooling*. Mathematically, degree of undercooling

$$= T_2 - T_2'$$

3. The ratio of pressures corresponding to temperatures T_2 and T_2' is known as *degree of supersaturation*. Mathematically, degree of supersaturation

$$= \frac{\text{Pressure corresponding to } T_2}{\text{Pressure corresponding to } T_2'} = \frac{p_2}{p_2'}$$

4. The following relations may be used in solving problem on supersaturated flow.

(*i*) $v = \dfrac{0.0023\,(h-1943)}{p}$ (*ii*) $pv^{1.3}$ = Constant ; and (*iii*) $\dfrac{p}{T^{13/3}}$ = Constant

where v = Volume of steam in m^3/kg,

p = Pressure of steam in bar,

h = Enthalpy or total heat of steam in kJ/kg, and

T = Absolute temperature of supersaturated steam in K.

21.13. Effects of Supersaturation

The following effects in a nozzle, in which supersaturation occurs, are important from the subject point of view ;

1. Since the condensation does not take place during supersaturated expansion , so the temperature at which the supersaturation occurs will be *less* than the saturation temperature corresponding to the pressure. Therefore, the density of supersaturated steam will be more** than for the equilibrium conditions, which gives the increase in the mass of steam discharged.

2. The supersaturation increases the entropy and specific volume of the steam.

3. The supersaturation reduces the heat drop (for the same pressure limits) below that for thermal equilibrium. Hence the exit velocity of the steam is reduced.

4. The supersaturation increases dryness fraction of steam.

Example 21.15. *The dry saturated steam is expanded in a nozzle from pressure of 10 bar to a pressure of 5 bar. If the expansion is supersaturated, find : 1. the degree of undercooling ; and 2. the degree of supersaturation.*

---

* The limit of supersaturated expansion was first shown by the experiments done by C.T.R. Wilson in 1857. The subsequent work by H.M. Martin has enabled a curve which was termed by him as the Wilson line.

** It has been found that the density of supersaturated steam is about *eight times* that of the ordinary saturated vapour at the corresponding pressure.

Solution. Given : $p_1 = 10$ bar ; $p_2 = 5$ bar

1. *Degree of undercooling*

From steam tables, corresponding to a pressure of 10 bar, we find that the initial temperature of steam,

$$T_1 = 179.9^\circ C = 179.9 + 273 = 452.9 \text{ K}$$

Let $\quad T_2' =$ Temperatureat which supersaturation occurs.

We know that for supersaturated expansion,

$$\frac{p_1}{(T_1)^{1.3/3}} = \frac{p_2}{(T_2')^{1.3/3}} \quad \text{or} \quad \frac{T_2'}{T_1} = \left(\frac{p_2}{p_1}\right)^{3/13} = \left(\frac{5}{10}\right)^{3/13} = 0.852$$

$$\therefore \quad T_2' = T_1 \times 0.852 = 452.9 \times 0.852 = 385.9 \text{ K}$$

$$= 385.9 - 273 = 112.9^\circ C$$

From steam tables, corresponding to a pressure of 5 bar, we find that the saturation temperature,

$$T_2 = 151.9^\circ C$$

$\therefore$ Degree of undercooling $= T_2 - T_2' = 151.9 - 112.9 = 39^\circ C$ **Ans.**

2. *Degree of supersaturation*

From steam tables, corresponding to a temperature of 112.9° C, we find that

$$p_2' = 1.584 \text{ bar}$$

$\therefore$ Degree of supersaturation

$$= p_2 / p_2' = 5 / 1.584 = 3.16 \text{ Ans.}$$

Example 21.16. *Find the percentage increase in discharge from a convergent-divergent nozzle expanding steam from 8.75 bar dry to 2 bar, when ; 1. the expansion is taking place under thermal equilibrium, and 2. the steam is in metastable state during part of its expansion.*

Take area of nozzle as 2500 mm².

Solution. Given : $p_1 = 8.75$ bar ; $p_2 = 2$ bar ; $A_2 = 2500 \text{ mm}^2 = 2500 \times 10^{-6} \text{ m}^2$

1. *Mass of steam discharged when the expansion is under thermal equilibrium*

Let $\quad m_1 =$ Mass of steam discharged.

The expansion of steam under conditions of *thermal equilibrium is shown on Mollier diagram as in Fig. 21.12.

From Mollier diagram, we find that

$h_1 = 2770$ kJ/kg ; $h_2 = 2515$ kJ/kg ; and $x_2 = 0.91$

From steam tables, at a pressure of 2 bar, we find that the specific volume of steam at exit,

$$v_{g2} = 0.885 \text{ m}^3/\text{kg}$$

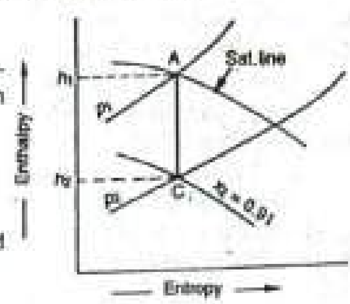


Fig. 21.12

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* Thermal equilibrium means that the flow of steam of isentropic.

We know that heat drop from inlet to exit,

$$h_{d2} = h_1 - h_2 = 2770 - 2515 = 255 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at exit,

$$V_2 = 44.72\sqrt{h_{d2}} = 44.72\sqrt{255} = 714 \text{ m/s}$$

and
$$m_1 = \frac{A_2 V_2}{v_2} = \frac{A_2 V_2}{x_2 v_{g2}} = \frac{2500 \times 10^{-6} \times 714}{0.91 \times 0.885} = 2.21 \text{ kg/s} \quad \text{Ans.}$$

2. *Mass of steam discharged when it is in *metastable state*

Let m_2 = Mass of steam discharged.

We know that volume of steam at inlet,

$$v_1 = \frac{0.0023(h_1 - 1943)}{p_1} = \frac{0.0023(2770 - 1943)}{8.75} = 0.217 \text{ m}^3\text{/kg}$$

and volume of steam at exit,

$$v_2 = v_1\left(\frac{p_1}{p_2}\right)^{1/1.3} = 0.217\left(\frac{8.75}{2}\right)^{1/1.3} = 0.675 \text{ m}^3\text{/kg}$$

$$\ldots (\because p_1 v_1^{1.3} = p_2 v_2^{1.3})$$

We know that volume of steam at exit (v_2),

$$0.675 = \frac{0.0023(h_2 - 1943)}{p_2} = \frac{0.0023(h_2 - 1943)}{2}$$

$$\therefore \quad h_2 = 2530 \text{ kJ/kg}$$

We know that heat drop from inlet to exit,

$$h_{d2} = h_1 - h_2 = 2770 - 2530 = 240 \text{ kJ/kg}$$

$\therefore$ Velocity of steam at exit,

$$V_2 = 44.72\sqrt{h_{d2}} = 44.72\sqrt{240} = 693 \text{ m/s}$$

and
$$m_2 = \frac{A_2 V_2}{v_2} = \frac{2500 \times 10^{-6} \times 693}{0.675} = 2.57 \text{ kg/s} \quad \text{Ans.}$$

$\therefore$ Percentage increase in discharge

$$= \frac{m_2 - m_1}{m_1} = \frac{2.57 - 2.21}{2.21} = 0.163 \text{ or } 16.3\% \quad \text{Ans.}$$

21.14. Steam Injector

The principle of a steam nozzle may also be applied to a steam injector. It utilises the kinetic energy of a steam jet for increasing the pressure and velocity of water. It is mostly used for forcing the feed water into steam boilers under pressure. The action of a steam injector is shown in Fig. 21.13.

The steam from the boiler is expanded to a high velocity by passing it through a convergent nozzle *A*. The steam jet enters the mixing cone and imparts its momentum to the incoming water supply from the feed tank** The cold water causes the steam to condense. The resulting jet at *B*,

* The problems on metastable flow cannot be solved by Mollier diagram unless Wilson line is drawn.

** The feed tank may be above or below the level of the steam injector.

formed by the steam and water is at atmospheric pressure, and has a large velocity. The mixture then enters delivery pipe at C through a diverging cone or diffuser, in which the kinetic energy is reduced and converted into pressure energy. This pressure energy is sufficient to overcome the boiler pressure

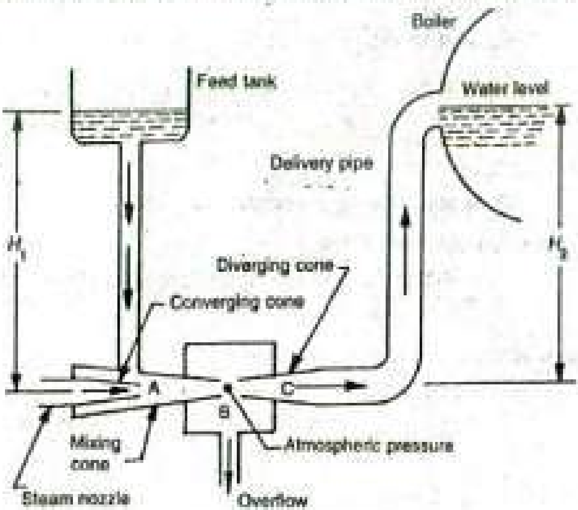


Fig. 21.13. Steam injector

and to lift the water through a height H_2. The pressure of water on leaving the delivery pipe must be about 20% higher than the boiler pressure in order to overcome all resistances. The gap between the mixing cone and diverging cone is provided with an outlet through which any excess water may overflow during the starting of the injector.

21.15. Steam Injector Calculations

The following calculations of a steam injector are important from the subject point of view :

1. *Amount of water injected*

Let m_w = Mass of water entering the mixing cone in kg/kg of steam,

V_s = Velocity of steam leaving the converging cone or nozzle,

V_w = Velocity of water entering the mixing cone, and

V_m = Velocity of the mixture leaving the mixing cone.

According to the principle of conservation of momentum,

Momentum of steam + Momentum of water = Momentum of mixture

$$1 \times V_s + m_w V_w = (1 + m_w) V_m = V_m + m_w V_m \quad \ldots (i)$$

or

$$V_s - V_m = m_w (V_m - V_w)$$

$\therefore$

$$m_w = \frac{(V_s - V_m)}{(V_m - V_w)} \text{ kg/kg of steam}$$

Note : In case the feed tank is below the level of the steam injector, the equation (*i*) may be written as :

$$1 \times V_s - m_w V_w = (1 + m_w) V_m$$

or

$$m_w = \frac{(V_s - V_m)}{(V_m + V_w)} \text{ kg/kg of steam}$$

2. *Velocity of steam leaving the nozzle*

Let p_1 = Initial pressure of steam,

p_2 = Pressure of steam leaving the nozzle at A, and

h_d = Isentropic heat drop.

∴ Velocity of steam leaving the nozzle,

$$V_s = 44.72\sqrt{h_d}$$

3. *Velocity of water entering the mixing cone*

Let H_1 = Height of water in the feed tank from the level of steam injector

∴ Velocity of water entering the mixing cone,

$$V_w = \sqrt{2gH_1}$$

4. *Velocity of mixture leaving the mixing cone*

Let p_1 = Pressure of steam in the boiler in bar,

p_m = Pressure of the mixture leaving the mixing cone at B in bar,

ρ = Density of the mixture at B in kg/m³, and

V_m = Velocity of mixture at B.

∴ Total energy per kg of water at B

$$= \frac{10^5 p_m}{\rho} + \frac{V_m^2}{2} \text{ (in joules)} \quad \ldots (i)$$

This energy must be sufficient to lift the water through a height H_2 metres and inject it into the boiler. The final pressure head on leaving at C, must be somewhat greater than the height H_2 *plus* the boiler pressure. If H is the necessary excess head in metres, then

Total energy per kg of water at B

$$= \frac{10^5 p_1}{\rho} + g\,(H_2 + H) \text{ (in joules)} \quad \ldots (ii)$$

Equating equations (*i*) and (*ii*),

$$\frac{10^5 p_m}{\rho} + \frac{V_m^2}{2} = \frac{10^5 p_1}{\rho} + g\,(H_2 + H)$$

or

$$V_m = \sqrt{\frac{2 \times 10^5\,(p_1 - p_m)}{\rho} + 2g\,(H_2 + H)} \quad \ldots (iii)$$

We have already discussed that the pressure of the mixture leaving the mixing cone is atmospheric. Therefore, taking the value of p_m as 1.013 bar and assuming the density of the mixture as 1000 kg/m³ (equal to density of water), we get

$$V_m = \sqrt{\frac{2 \times 10^5\,(p_1 - 1.013)}{1000} + 2g\,(H_2 + H)}$$

$$= 4.43\sqrt{10.2\,(p_1 - 1.013) + H_2 + H}$$

Note : If V_d is the velocity in the delivery pipe, then

$$H = \frac{V_d^2}{2g}$$

5. *Nozzle areas*

Let A_a = Area of steam nozzle at A,

A_b = Area of combining nozzle (or mixing cone) at B,

V_s = Velocity of steam leaving the converging cone,

v_a = Specific volume of steam after expansion in nozzle at A,

m = Mass of water required to be delivered in kg/s, and

m_w = Mass of water entering the mixing cone in kg/kg of steam.

∴ Mass of steam supplied,

$$m_s = \frac{A_a V_s}{v_a} = \frac{m}{m_w}$$

or

$$A_a = \frac{m}{m_w} \times \frac{v_a}{V_s}$$

Also

$$m_s + m = A_b V_m \rho$$

or

$$A_b = \frac{m_s + m}{1000\, V_m} = \frac{m\left(1 + \frac{1}{m_w}\right)}{1000\, V_m} \qquad \ldots (\because \rho = 1000 \text{ kg/m}^3)$$

6. *Heat balance per kg of steam*

Let h = Enthalpy or total heat of steam entering the injector in kJ,

t_w = Temperature of water in feed tank in °C,

h_{fw} = Sensible heat of water supplied to the injector, corresponding to a temperature of t_w in kJ/kg,

t_m = Temperature of water leaving the mixing cone at B in °C, and

h_{fm} = Sensible heat of water leaving the mixing cone at B, corresponding to a temperature of t_m in kJ/kg.

Then heat supplied in steam + Heat supplied in water + Kinetic energy of water

= Heat in mixture + Kinetic energy of mixture

$$h + m_w h_{fw} \pm \frac{m_w V_w^2}{2000} = (1 + m_w)\, h_{fm} + \frac{(1 + m_w)\, V_m^2}{2000}$$

From this equation, the value of h_{fm} may be determined and hence the temperature of the mixture t_m is known.

Note : In the above equation plus sign is used when feed tank is above the level of the steam injector, while negative sign is used when it is below the level of injector.

Example 21.17. *An injector is required to deliver 120 kg of water per minute from a tank, whose constant water level is 3 m below the level of injector, into a boiler in which the steam pressure is 15 bar. The water level in the boiler is 0.7 metre above the level of the injector. The steam for the injector is taken from the same boiler and it is assumed to be dry and saturated. The pressure of steam leaving steam nozzle is 0.6 times that of the supply pressure. The temperature of the water in the feed tank is 25° C. If the velocity in the delivery pipe is 15 m/s, find :*

1. Mass of water injected per kg of steam, 2. Area of mixing cone, 3. Area of steam nozzle, and 4. Temperature of water leaving the injector.

Solution. Given : $m = 120$ kg/min $= 2$ kg/s ; $H_1 = 3$ m below the level of injector ; $p_1 = 15$ bar ; $H_2 = 0.7$ m above the level of injector ; $p_2 = 0.6\,p_1 = 0.6 \times 15 = 9$ bar ; $t_w = 25°$ C ; $V_d = 15$ m/s

1. *Mass of water injected per kg of steam*

From Mollier chart, the isentropic heat drop between pressure 15 bar dry and 9 bar,

$$h_d = h_1 - h_2 = 2795 - 2700 = 95 \text{ kJ/kg}$$

and dryness fraction of steam after expansion,

$$x_2 = 0.965$$

We know that velocity of steam leaving the nozzle,

$$V_s = 44.72\sqrt{h_d} = 44.72\sqrt{95} = 436 \text{ m/s}$$

We know that velocity of water entering the mixing cone,

$$V_w = \sqrt{2gH_1} = \sqrt{2 \times 9.81 \times 3} = 7.67 \text{ m/s}$$

and velocity of mixture leaving the mixing cone,

$$V_m = 4.43\sqrt{10.2\,(p_1 - 1.013) + H_2 + H}$$

$$= 4.43\sqrt{10.2\,(15 - 1.013) + 0.7 + 11.47} = 55 \text{ m/s}$$

$$\ldots\left[\because H = \frac{V_d^2}{2g} = \frac{15^2}{2 \times 9.81} = 11.47 \text{ m}\right]$$

$\therefore$ Mass of water injected per kg of steam,

$$m_w = \frac{V_s - V_m}{V_m + V_w} = \frac{436 - 55}{55 + 7.67} = 6.08 \text{ kg } \textbf{Ans.}$$

2. *Area of mixing cone*

We know that area of the mixing cone

$$A_k = \frac{m\left(1 + \frac{1}{m_w}\right)}{1000\,V_m} = \frac{2\left(1 + \frac{1}{6.08}\right)}{1000 \times 55} = 42.3 \times 10^{-6} \text{ m}^2$$

$$= 42.3 \text{ mm}^2 \textbf{ Ans.}$$

3. *Area of steam nozzle*

From steam tables, corresponding to a pressure of 9 bar, we find that specific volume of steam,

$$v_{g4} = 0.2148 \text{ m}^3\text{/kg}$$

We know that area of steam nozzle,

$$A_s = \frac{m}{m_w} \times \frac{v_d}{V_s} = \frac{m}{m_w} \times \frac{x_2 v_{g4}}{V_s} = \frac{2}{6.08} \times \frac{0.965 \times 0.2148}{436}$$

$$= 156 \times 10^{-6} \text{ m}^2 = 156 \text{ mm}^2 \textbf{ Ans.}$$

4. *Temperature of water leaving the injector*

From steam tables, corresponding to a temperature of $t_w = 25°$ C, we find that

$$h_{fw} = 104.8 \text{ kJ/kg}$$

We know that $\quad h_1 + m_w h_{fw} - \dfrac{m_w V_w^2}{2000} = (1 + m_w)\,h_{fm} + \dfrac{(1 + m_w)\,V_m^2}{2000}$

$$2795 + 6.08 \times 104.8 - \frac{6.08\,(7.67)^2}{2000} = (1 + 6.08)\,h_{fm} + \frac{(1 + 6.08)\,(55)^2}{2000}$$

$$3432 = 7.08\,h_{fm} + 10.71$$

or $$h_{fm} = 483.2 \text{ kJ/kg}$$

∴ Temperature of water leaving the injector (from steam tables corresponding to 483.2 kJ/kg),

$$t_m = 115°\text{C} \text{ Ans.}$$

EXERCISES

1. The dry and saturated steam at a pressure of 5 bar is expanded isentropically in a nozzle to a pressure of 0.2 bar. Find the velocity of steam leaving the nozzle. [Ans. 1000 m/s]

2. The dry and saturated steam at a pressure of 10.5 bar is expanded isentropically in a nozzle to a pressure of 0.7 bar. Determine the final velocity of the steam issuing from the nozzle, when (a) friction is neglected, and (b) 10% of the heat drop is lost in friction.

The initial velocity of steam may be neglected. [Ans. (a) 905 m/s; (b) 859 m/s]

3. Steam at a pressure of 6.3 bar and 200° C is expanded in a nozzle to a pressure of 0.2 bar. Find the final velocity and dryness fraction of steam, if

(a) friction is neglected ; and (b) 10% of the heat drop is lost in friction.

[Ans. (a) 1030 m/s, 0.83 ; (b) 996 m/s, 0.852]

4. Steam is supplied to a nozzle at 3.5 bar and 0.96 dry. The steam enters the nozzle at 240 m/s. The pressure drops to 0.8 bar. Determine the velocity and dryness fraction of the steam when it leaves the nozzle.

[Ans. 545.5 m/s ; 0.92]

5. Steam expands through an ideally, insulated nozzle following a reversible polytropic law $p\,v^{1.2} = C$. There is no change in potential energy but the pressure drops from 20 bar to 2 bar and the specific volume increases from 0.05 m³ to 0.3 m³. If the entrance velocity is 80 m/s, determine the exit velocity.

[Ans. 697.5 m/s]

6. Calculate the throat area of nozzle supplied with steam at 10 bar and 200°C. The rate of flow of steam is 1.2 kg/s. Neglect friction and assume the velocity at inlet to be small. [Ans. 837 mm²]

7. Steam expands isentropically from the state of 8 bar and 250° C to 1.5 bar in a convergent-divergent nozzle. The steam flow rate is 0.75 kg/s. Find : 1. the velocity of steam at exit from the nozzle ; and 2. the exit area of nozzle. Neglect the inlet velocity of steam. [Ans. 800 m/s ; 1054 mm²]

8. Steam enters a group of convergent-divergent nozzles at a pressure of 22 bar and with a temperature of 240° C. The exit pressure is 4 bar and 9% of the total heat drop is lost in friction. The mass flow rate is 10 kg/s and the flow upto the throat may be assumed friction less. Calculate

1. the throat and exit velocities, and 2. the throat and exit areas.

[Ans. 529 m/s, 775 m/s ; 3000 mm², 3500 mm²]

9. The throat diameter of a nozzle is 5 mm. If dry and saturated steam at 10 bar is supplied to the nozzle, calculate the mass flow per second. The exhaust pressure is 1.5 bar. Assume friction less adiabatic flow and index of expansion, $n = 1.135$.

If 10 percent of the isentropic heat drop is lost in friction, what should be the correct diameter at outlet for steam to issue at the same exhaust pressure ? [Ans. 103.3 kg/h ; 7.12 mm]

10. Calculate the throat and exit diameters of a convergent divergent nozzle which will discharge 820 kg of steam per hour from a pressure of 8 bar superheated to 220° C into a chamber having a pressure of 1.5 bar. The friction loss in the divergent part of the nozzle may be taken as 0.15 of the total enthalpy drop.

[Ans. 15.2 mm ; 20.6 mm]

11. A steam turbine develops 185 kW with a consumption of 16.5 kg/kWh. The pressure and temperature of the steam entering the nozzle are 12 bar and 220° C. The steam leaves the nozzle at 1.2 bar. The diameter of the nozzle at throat is 7 mm. Find the number of nozzles.

If 8% of the total enthalpy drop is lost in friction in the diverging part of the nozzle, determine the diameter at the exit of the nozzle and the exit velocity of the leaving steam.

Sketch the skeleton Mollier diagram and show on it the values of pressure, temperature or dryness fraction, enthalpy and specific volume at inlet, throat and exit. [Ans. 14 ; 11.1 mm ; 847 m/s]

12. Steam expands in a nozzle under the following conditions : Inlet pressure = 15 bar ; Inlet temperature = 250° C; Final pressure = 4 bar ; Mass flow = 1 kg/s.

Calculate the required throat and exit areas, using Mollier diagram, when 1. the expansion is frictionless, and 2. the friction loss at any pressure amounts to 10 percent of the total heat drop down to that pressure. [Ans. 480 mm^2, 606 mm^2 ; 508 mm^2, 650 mm^2]

13. Gases expand in a convergent divergent nozzle from 3.6 bar and 425° C to a back pressure of 1 bar, at the rate of 18 kg/s. If the nozzle efficiency is 0.92, calculate the required throat and exit areas of the nozzle. Neglect inlet velocity and friction in the convergent part. For the gases, take c_p = 1.113 kJ/kg K and γ = 1.33. [Ans. 0.0325 m^2 ; 0.04 m^2]

14. The dry saturated steam expands in a nozzle from a pressure of 2 bar to 1 bar. If the expansion is supersaturated, determine the degree of undercooling and the degree of supersaturation. [Ans. 37.63°C ; 4.58]

15. Steam at 42 bar and 260° C enters a nozzle and leaves at 28 bar. Neglecting initial kinetic energy and considering super-saturation, determine the discharge area for a flow of 10 350 kg/h and a nozzle velocity coefficient of 96%. [Ans. 484 mm^2]

16. Compare the mass of discharge from a convergent-divergent nozzle expanding from 8 bar and 210° C to 2 bar, when

1. the expansion takes place under thermal equilibrium, and 2. the steam is in super-saturated condition during a part of its expansion.

Take area of nozzle as 2400 mm^2. [Ans. 8.3%]

17. An injector is to deliver 100 kg of water per minute from a tank, whose constant water level is 1.2 m below the level of the injector into a boiler in which the steam pressure is 14 bar. The water level in the boiler is 1.5 metre above the level of the injector. The steam for the injector is taken from the same boiler and it is assumed to be dry and saturated. The pressure of steam leaving steam nozzle is 0.5 times that of the supply pressure. If the velocity in the delivery pipe is 13.5 m/s, find :

1. Mass of water pumped per kg of steam ; 2. Area of mixing cone ; 3. Area of steam nozzle ; and 4. Temperature of water leaving the injector, if the temperature of water in the feed tank is 15° C.

[Ans. 7.9 kg ; 35.5 mm^2 ; 107.4 mm^2 ; 87.8° C]

QUESTIONS

1. Explain the function of nozzles used with steam turbines.
2. Discuss the functions of the convergent portion, the throat and the divergent portion of a convergent-divergent nozzle with reference to flow of steam.
3. What is steady flow energy equation as applied to steam nozzles ? Explain its use in the calculation of steam velocity at the exit of a nozzle.
4. Discuss the effect of friction during the expansion of steam through a convergent-divergent nozzle when

   (*i*) the steam at entry to the nozzle is saturated, and

   (*ii*) the steam at entry is superheated.

   Assume the pressure of steam to be initially same in both the cases. Mark the processes on a sketch of enthalpy-entropy diagram.
5. Explain what is meant by critical pressure ratio of a nozzle.
6. Starting from fundamentals, show that for maximum discharge through a nozzle, the ratio of throat pressure to inlet pressure is given by $\left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$ where n is the index for isentropic expansion through the nozzle.

7. Derive an expression for maximum discharge through convergent divergent nozzle for steam.

8. Draw the 'discharge' versus 'ratio of pressures at outlet to inlet' curve for a convergent steam nozzle. Discuss the physical significance of critical pressure ratio.

9. Explain the supersaturated or metastable flow of steam through a nozzle and the significance of Wilson's line.

10. What are the effects of supersaturation on discharge and heat drop ?

OBJECTIVE TYPE QUESTIONS

1. The steam leaves the nozzle at a

   (*a*) high pressure and low velocity (*b*) high pressure and high velocity
   (*c*) low pressure and low velocity (*d*) low pressure and high velocity

2. The effect of friction in a nozzle ...... dryness fraction of steam.

   (*a*) increases (*b*) decreases

3. The velocity of steam leaving the nozzle (V) is given by

   (*a*) $V = 44.72\, K\, h_d$ (*b*) $V = 44.72\, K \sqrt{h_d}$
   (*c*) $V = 44.72 \sqrt{K\, h_d}$ (*d*) $V = 44.72\, h_d \sqrt{K}$

where K = Nozzle coefficient, and
h_d = Enthalpy drop during expansion.

4. The critical pressure ratio for initially dry saturated steam is

   (*a*) 0.528 (*b*) 0.546 (*c*) 0.577 (*d*) 0.582

5. The critical pressure ratio for initially superheated steam is ...... as compared to initially dry saturated steam.

   (*a*) more (*b*) less

6. The flow of steam is super sonic

   (*a*) at the entrance to the nozzle (*b*) at the throat of the nozzle
   (*c*) in the convergent portion of the nozzle (*d*) in the divergent portion of the nozzle

7. The difference of supersaturated temperature and saturation temperature at that pressure is known as

   (*a*) degree of supersaturation (*b*) degree of superheat
   (*c*) degree of undercooling (*d*) none of these

8. In a nozzle, the effect of supersaturation is to

   (*a*) decrease the dryness fraction of steam (*b*) decrease the specific volume of steam
   (*c*) increase the entropy (*d*) increase the enthalpy drop

9. The density of supersaturated steam is about .... that of the ordinary saturated vapour at the corresponding pressure.

   (*a*) same as (*b*) 2 times (*c*) 4 times (*d*) 8 times

10. When the back pressure of a nozzle is below the designed value of pressure at exit of nozzle, the nozzle is said to be

   (*a*) choked (*b*) under damping (*c*) over damping

ANSWERS

1. (*d*)	2. (*a*)	3. (*c*)	4. (*c*)	5. (*b*)
6. (*d*)	7. (*c*)	8. (*c*)	9. (*d*)	10. (*b*)

Definition

A ***steam turbine*** is a *prime mover* in which the *potential energy* of the steam is transformed into *kinetic energy* and later in its turn is transformed into the *mechanical energy* of rotation of the turbine shaft.

Classification of steam turbines

According to the action of steam:

- *Impulse turbine:* In impulse turbine, steam coming out through a fixed nozzle at a very high velocity strikes the blades fixed on the periphery of a rotor. The blades change the direction of steam flow without changing its pressure. The force due to change of momentum causes the rotation of the turbine shaft. Ex: De-Laval, Curtis and Rateau Turbines
- *Reaction turbine:* In reaction turbine, steam expands both in fixed and moving blades continuously as the steam passes over them. The pressure drop occurs continuously over both moving and fixed blades.
- *Combination of impulse and reaction turbine*

According to the number of pressure stages:

- *Single stage turbines:* These turbines are mostly used for driving centrifugal compressors, blowers and other similar machinery.
- *Multistage Impulse and Reaction turbines:* They are made in a wide range of power capacities varying from small to large.

According to the type of steam flow:

- *Axial turbines:* In these turbines, steam flows in a direction parallel to the axis of the turbine rotor.
- *Radial turbines:* In these turbines, steam flows in a direction perpendicular to the axis of the turbine, one or more low pressure stages are made axial.

According to the number of shafts:

- *Single shaft turbines*
- *Multi-shaft turbines*

According to the method of governing:

- *Turbines with throttle governing:* In these turbines, fresh steam enter through one or more (depending on the power developed) simultaneously operated throttle valves.
- *Turbines with nozzle governing:* In these turbines, fresh steam enters through one or more consecutively opening regulators.
- *Turbines with by-pass governing:* In these turbines, the steam besides being fed to the first stage is also directly fed to one, two or even three intermediate stages of the turbine.

According to the heat drop process:

- *Condensing turbines with generators:* In these turbines, steam at a pressure less than the atmospheric is directed to the condenser. The steam is also extracted from intermediate stages for feed water heating). The latent heat of exhaust steam during the process of condensation is completely lost in these turbines.
- *Condensing turbines with one or more intermediate stage extractions:* In these turbines, the steam is extracted from intermediate stages for industrial heating purposes.
- *Back pressure turbines:* In these turbines, the exhaust steam is utilized for industrial or heating purposes. Turbines with deteriorated vacuum can also be used in which exhaust steam may be used for heating and process purposes.
- *Topping turbines:* In these turbines, the exhaust steam is utilized in medium and low pressure condensing turbines. These turbines operate at high initial conditions of steam pressure and temperature, and are mostly used during extension of power station capacities, with a view to obtain better efficiencies.

According to the steam conditions at inlet to turbine:

- *Low pressure turbines:* These turbines use steam at a pressure of 1.2 ata to 2 ata.
- *Medium pressure turbines:* These turbines use steam up to a pressure of 40 ata.
- *High pressure turbines:* These turbines use steam at a pressure above 40 ata.
- *Very high pressure turbines:* These turbines use steam at a pressure of 170 ata and higher and temperatures of 550°C and higher.
- *Supercritical pressure turbines:* These turbines use steam at a pressure of 225 ata and higher.

According to their usage in industry:

- *Stationary turbines with constant speed of rotation:* These turbines are primarily used for driving alternators.
- *Stationary turbines with variable speed of rotation:* These turbines are meant for driving turbo-blowers, air circulators, pumps, etc.
- *Non-stationary turbines with variable speed:* These turbines are usually employed in steamers, ships and railway locomotives.

Advantages of steam turbines over steam engines

1. The thermal efficiency is much higher.
2. As there is no reciprocating parts, perfect balancing is possible and therefore heavy foundation is not required.
3. Higher and greater range of speed is possible.
4. The lubrication is very simple as there are no rubbing parts.
5. The power generation is at uniform rate & hence no flywheel is required.
6. The steam consumption rate is lesser.
7. More compact and require less attention during operation.
8. More suitable for large power plants.
9. Less maintenance cost as construction and operation is highly simplified due to absence of parts like piston, piston rod, cross head, connecting rod.
10. Considerable overloads can be carried at the expense of slight reduction in overall efficiency.

Methods of reducing rotor speed
(Compounding of turbines)

- If high velocity of steam is allowed to flow through one row of moving blades, it produces a rotor speed of about **30000** rpm which is too high for practical use.

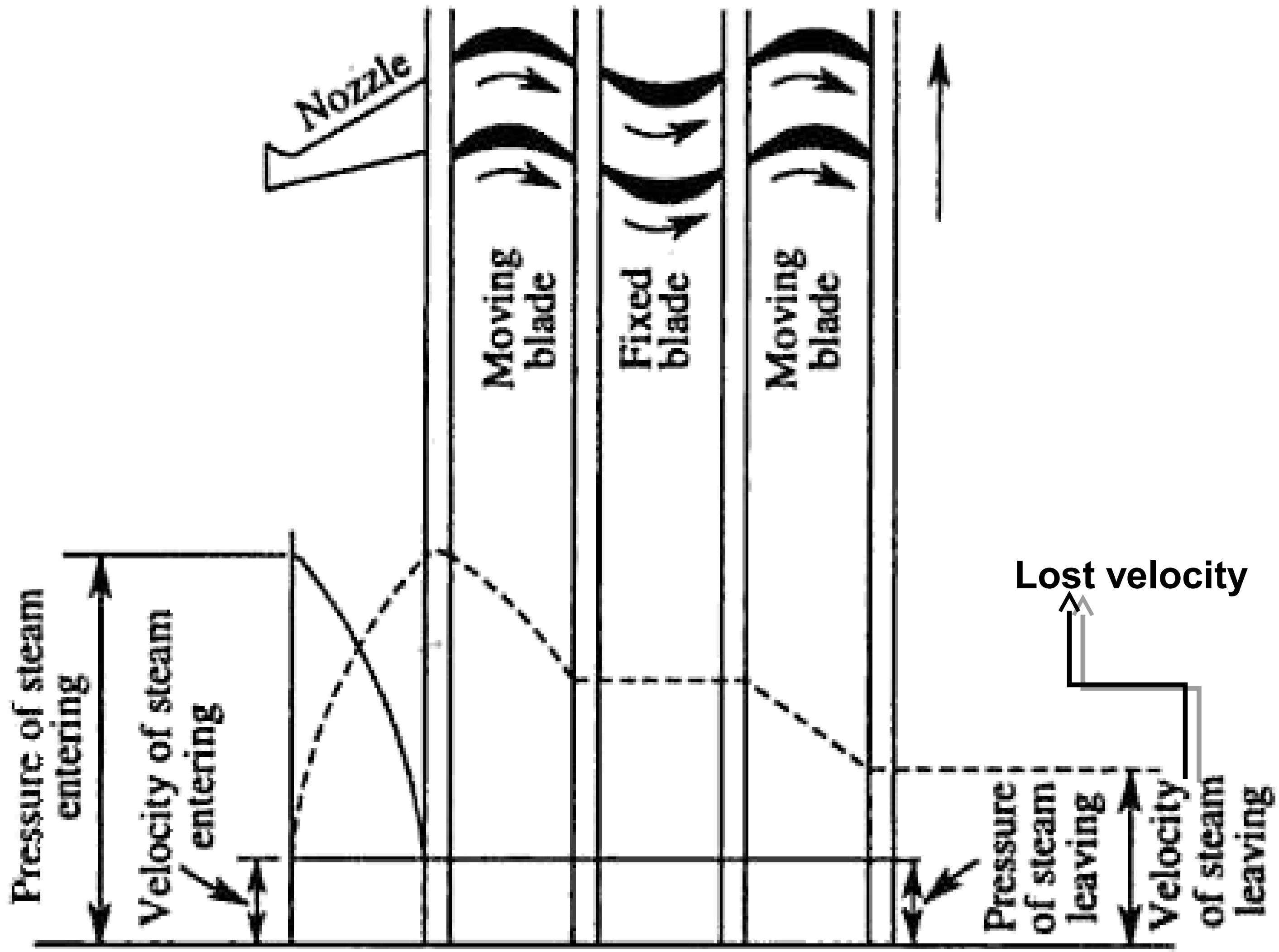
- It is therefore essential to incorporate some improvements for practical use and also to achieve high performance.

- This is possible by making use of more than one set of nozzles, and rotors, in a series, keyed to the shaft so that either the steam pressure or the jet velocity is absorbed by the turbine in stages. This is called *compounding of turbines*.

- The high rotational speed of the turbine can be reduced by the following methods of compounding:

1) Velocity compounding
2) Pressure compounding, and
3) Pressure-Velocity compounding

VELOCITY COMPOUNDING



Velocity compounding

- It consists of a set of nozzles and a few rows of moving blades which are fixed to the shaft and rows of fixed blades which are attached to the casing.
- As shown in figure, the two rows of moving blades are separated by a row of fixed blades.
- The high velocity steam first enters the first row of moving blades, where some portion of the velocity is absorbed.
- Then it enters the ring of fixed blades where the direction of steam is changed to suit the second ring of moving blades. There is no change in the velocity as the steam passes over the fixed blades.
- The steam then passes on to the second row of moving blades where the velocity is further reduced. Thus a fall in velocity occurs every time when the steam passes over the row of moving blades. Steam thus leaves the turbine with a low velocity.
- The variation of pressure and velocity of steam as it passes over the moving and fixed blades is shown in the figure. It is clear from the figure that the pressure drop takes place only in the nozzle and there is no further drop of pressure as it passes over the moving blades.
- This method of velocity compounding is used in Curtis turbine after it was first proposed by C.G. Curtis.

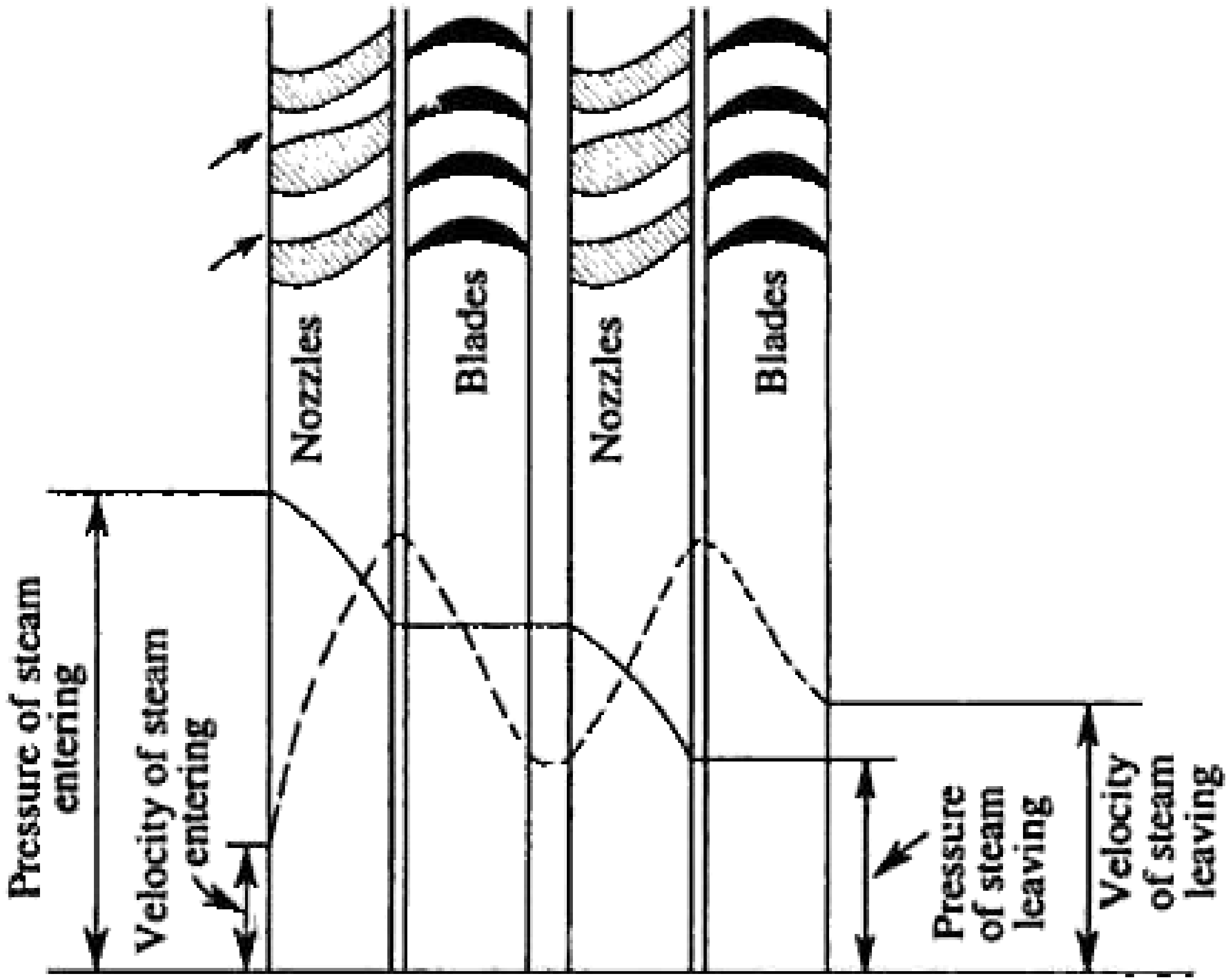
Advantages

1) The arrangement has less number of stages and hence less initial cost
2) The arrangement requires less space
3) The system is reliable and easy to operate
4) The fall of pressure in the nozzle is considerable, so the turbine itself need not work in high pressure surroundings and the turbine housing need not be very strong.

Disadvantages

1) More friction losses due to very high velocity in the nozzles
2) Less efficiency because ratio of blade velocity to steam velocity is not optimum
3) Power developed in the later rows is only fraction of first row. Still all the stages require same space, material and cost.

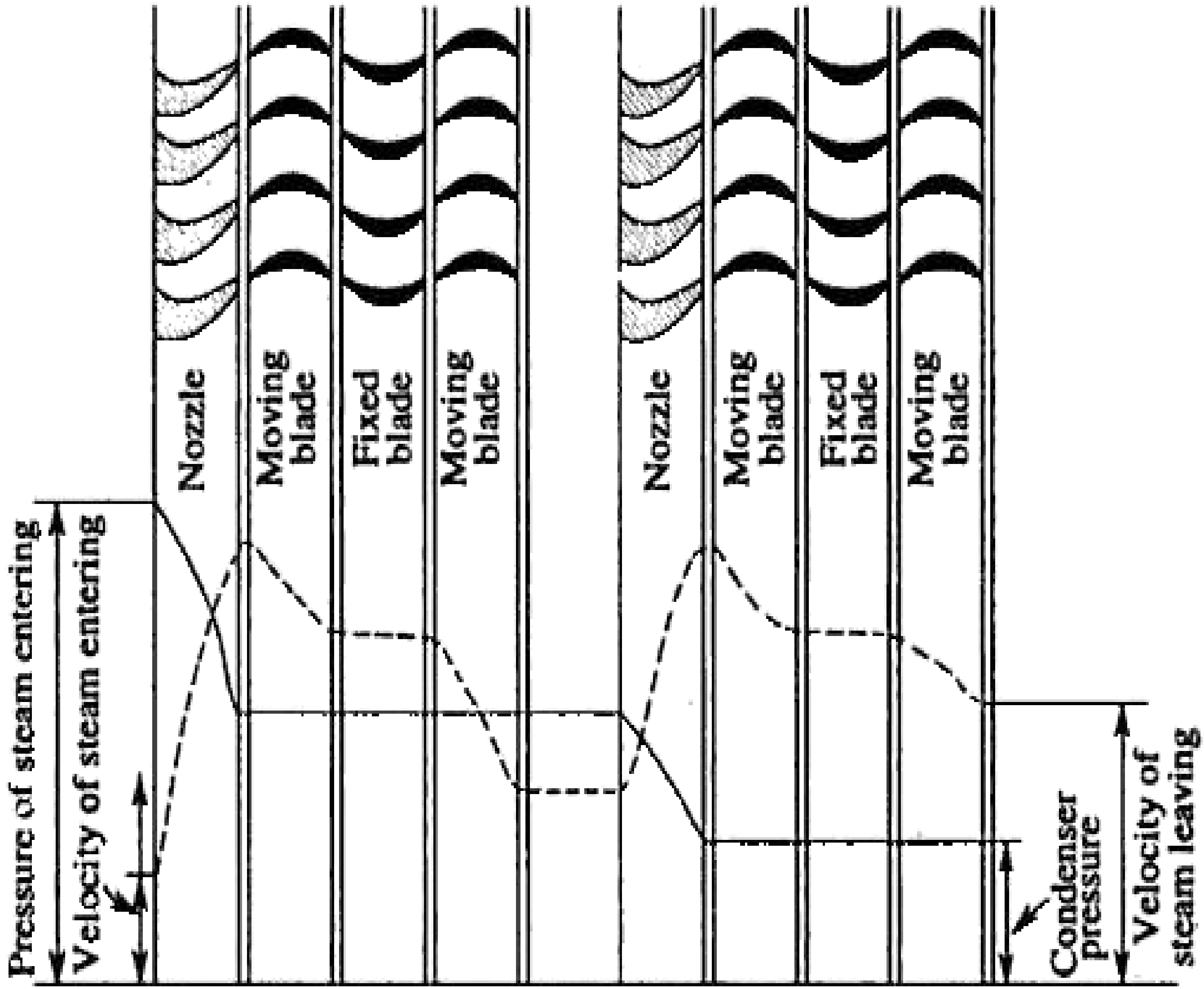
Pressure compounding



- It consists of a number of fixed nozzles which are incorporated between the rings of moving blades. The moving blades are keyed to the shaft.
- Here the pressure drop is done in a number of stages. Each stage consists of a set of nozzles and a ring of moving blades.
- Steam from the boiler passes through the first set of nozzles where it expands partially. Nearly all its velocity is absorbed when it passes over the first set of moving blades.
- It is further passed to the second set of fixed nozzles where it is partially expanded again and through the second set of moving blades where the velocity of steam is almost absorbed. This process is repeated till steam leaves at condenser pressure.
- By reducing the pressure in stages, the velocity of steam entering the moving blades is considerably reduced. Hence the speed of the rotor is reduced. Rateau & Zoelly turbines use this method of compounding.

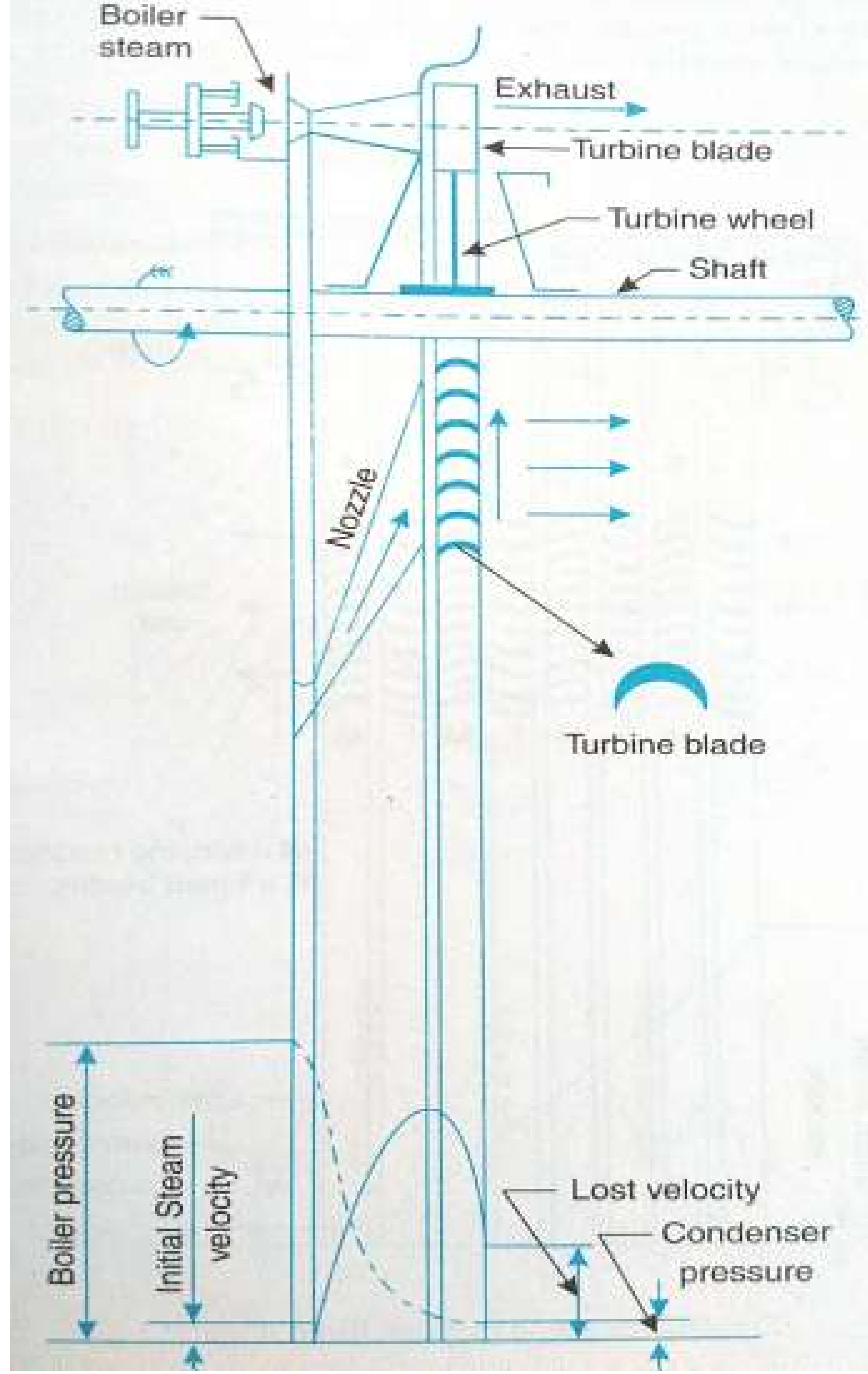
1) In this method of compounding, both pressure and velocity compounding methods are utilized.
2) The total drop in steam pressure is carried out in two stages and the velocity obtained in each stage is also compounded.
3) The ring of nozzles are fixed at the beginning of each stage and pressure remains constant during each stage.
4) This method of compounding is used in *Curtis* and *More* turbines.

Pressure-Velocity compounding



Simple impulse principle

- It primarily consists of a nozzle or a set of nozzles, a rotor mounted on a shaft, one set of moving blades attached to the rotor and a casing.
- A simple impulse turbine is also called De-Laval turbine, after the name of its inventor
- This turbine is called *simple* impulse turbine since the expansion of the steam takes place in one set of nozzles.



Basics of impulse turbine

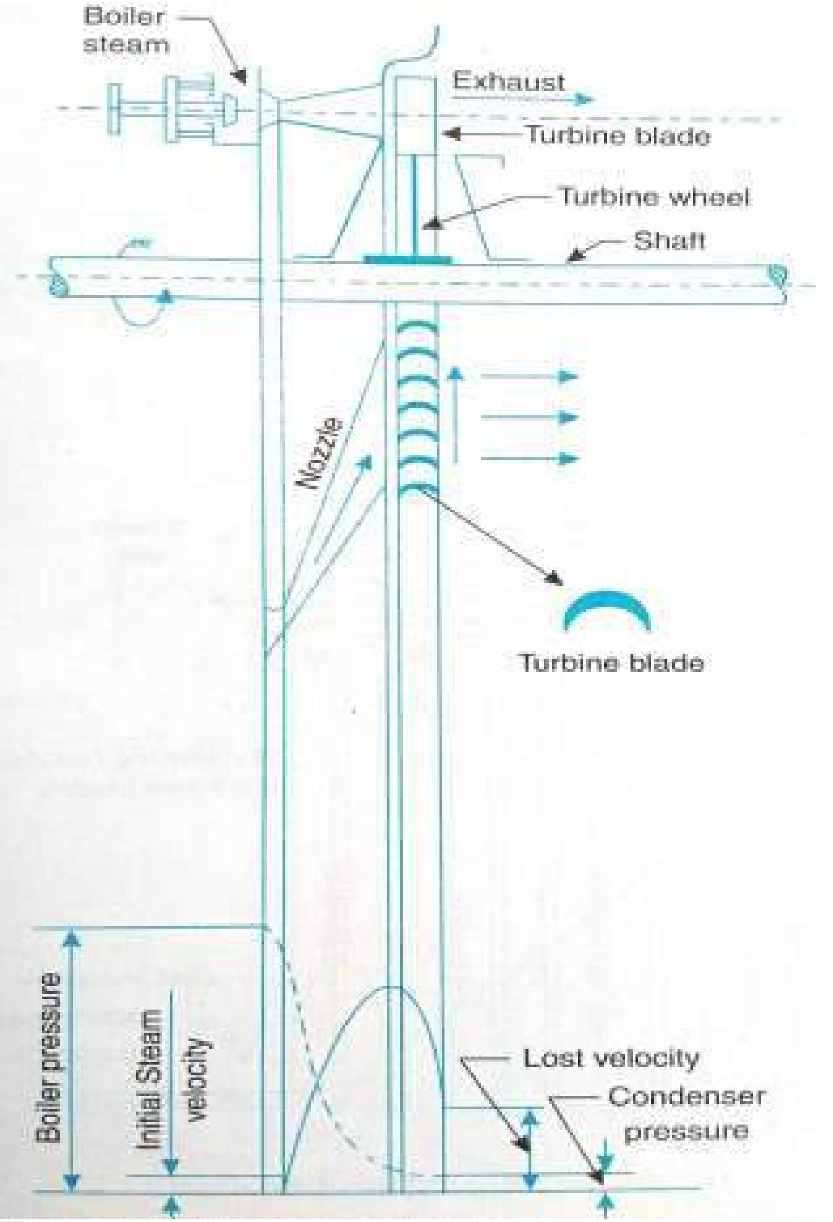
- In impulse turbine, steam coming out through a fixed nozzle at a very high velocity strikes the blades fixed on the periphery of a rotor.
- The blades change the direction of steam flow without changing its pressure.
- The force due to change of momentum causes the rotation of the turbine shaft.
- Examples: De-Laval, Curtis and Rateau turbines.

- The impulse turbine consists basically of a rotor mounted on a shaft that is free to rotate in a set of bearings.
- The outer rim of the rotor carries a set of curved blades, and the whole assembly is enclosed in an airtight case.
- Nozzles direct steam against the blades and turn the rotor. The energy to rotate an impulse turbine is derived from the kinetic energy of the steam flowing through the nozzles.
- The term impulse means that the force that turns the turbine comes from the impact of the steam on the blades.

- The toy pinwheel can be used to study some of the basic principles of turbines. When we blow on the rim of the wheel, it spins rapidly. The harder we blow, the faster it turns.
- The steam turbine operates on the same principle, except it uses the kinetic energy from the steam as it leaves a steam nozzle rather than air.
- Steam nozzles are located at the turbine inlet. As the steam passes through a steam nozzle, potential energy is converted to kinetic energy.
- This steam is directed towards the turbine blades and turns the rotor. The velocity of the steam is reduced in passing over the blades.
- Some of its kinetic energy has been transferred to the blades to turn the rotor.
- Impulse turbines may be used to drive forced draft blowers, pumps, and main propulsion turbines.

Construction and working of impulse turbine

- The uppermost portion of the diagram shows a longitudinal section through the upper half of the turbine.
- The middle portion shows the actual shape of the nozzle and blading.
- The bottom portion shows the variation of absolute velocity and absolute pressure during the flow of steam through passage of nozzles and blades.
- The expansion of steam from its initial pressure (steam chest pressure) to final pressure (condenser pressure) takes place in one set of nozzles.
- Due to high drop in pressure in the nozzles, the velocity of steam in the nozzles increases.



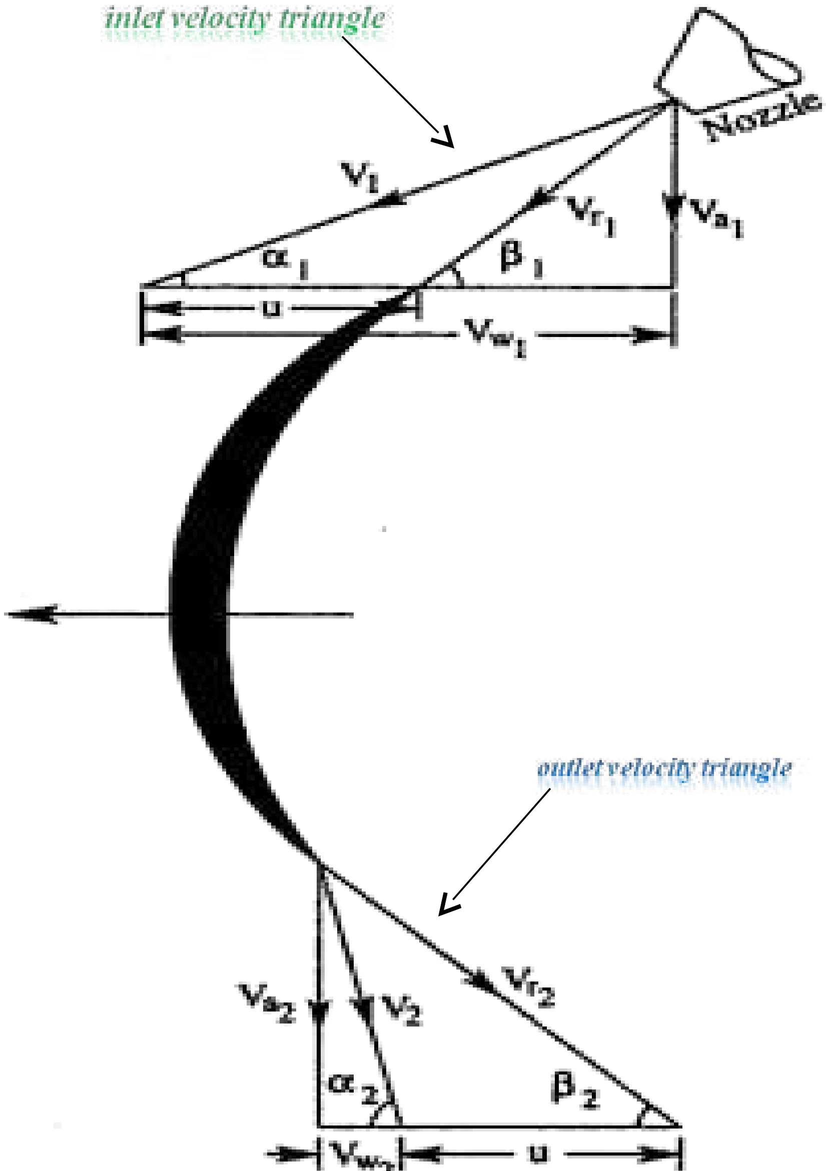
- The steam leaves the nozzle with a very high velocity and strikes the blades of the turbine mounted on a wheel with this high velocity.
- The loss of energy due to this higher exit velocity is commonly known as *carry over loss* (or) *leaving loss*.
- The pressure of the steam when it moves over the blades remains constant but the velocity decreases.
- The exit/leaving/lost velocity may amount to 3.3 percent of the nozzle outlet velocity.
- Also since all the KE is to be absorbed by one ring of the moving blades only, the velocity of wheel is too high (varying from 25000 to 30000 RPM).
- However, this wheel or rotor speed can be reduced by adopting the method of compounding of turbines.

Disadvantages of impulse turbine

1. Since all the KE of the high velocity steam has to be absorbed in only one ring of moving blades, the velocity of the turbine is too high i.e. up to 30000 RPM for practical purposes.

2. The velocity of the steam at exit is sufficiently high which means that there is a considerable loss of KE.

Velocity diagram / velocity triangle



V_1 = Absolute velocity of steam at inlet in m/s

α_1 = Nozzle inlet angle

u = Blade velocity in m/s

V_{r1} = Relative velocity of steam at inlet in m/s

V_{w1} = Tangential velocity of steam at inlet in m/s

V_{a1} = Axial velocity of steam at inlet in m/s

β_1 = Blade inlet angle

β_2 = Blade outlet angle

V_{r2} = Relative velocity of steam at outlet in m/s

V_{w2} = Tangential velocity of steam at outlet in m/s

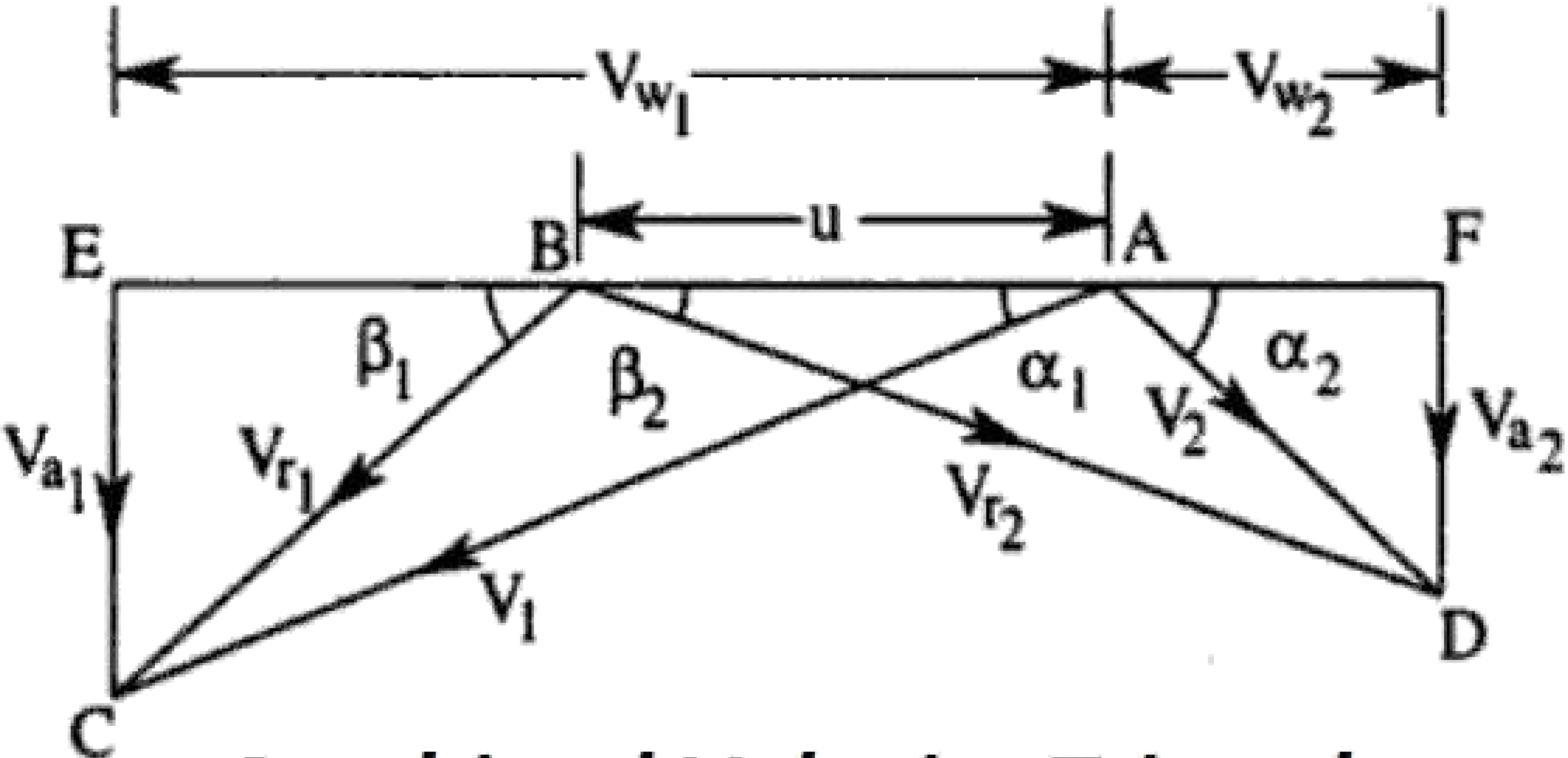
V_{a2} = Axial velocity of steam at outlet in m/s

K = Blade velocity coefficient $= \frac{V_{r2}}{V_{r1}}$

V_2 = Absolute velocity of steam at outlet in m/s

α_2 = Angle made by absolute velocity V_2 with the tangent of the wheel at outlet

Combined velocity triangle



Combined Velocity Triangle

WORK OUTPUT, POWER, BLADE EFFICIENCY & STAGE EFFICIENCY

Force in the tangential direction = Rate of change of momentum in the tangential direction.

$$= \text{Mass per second} \times \text{change in velocity Newtons}$$

$$= m\left(V_{w1} \pm V_{w2}\right) \text{ Newtons}$$

Force in the axial direction = Rate of change of momentum in the axial direction.

(axial thrust) $\qquad = m\left(V_{a1} - V_{a2}\right)$ Newtons

$$\text{Work done by steam on blades} = m\left(V_{w1} \pm V_{w2}\right)u \quad \text{N - m/s}$$

$$\text{Power developed by the turbine} = \frac{m\left(V_{w1} \pm V_{w2}\right)u}{1000} \; kW$$

$$\text{Blade efficiency} = \frac{\text{Work done on the blade(s)}}{\text{Energy supplied to the blade(s)}} = \frac{m\left(V_{w1} \pm V_{w2}\right)u}{\frac{1}{2} m V_1^{\,2}} = \frac{2u\left(V_{w1} \pm V_{w2}\right)u}{V_1^{\,2}}$$

$$\text{Energy lost due to blade friction} = \frac{1}{2} m\left(V_{r1}^{\,2} - V_{r2}^{\,2}\right) \quad \text{N - m/s}$$

$$\text{Stage efficiency} = \frac{\text{Work done on the blade(s)}}{\text{Total energy supplied per stage}} = \frac{m\left(V_{w1} \pm V_{w2}\right)u}{m\left(H_1 - H_2\right)} = \frac{\left(V_{w1} \pm V_{w2}\right)u}{H_d}$$

where $H_d = H_1 - H_2 =$ Heat drop in the nozzle ring

MAXIMUM WORK & MAXIMUM DIAGRAM EFFICIENCY

From the combined velocity triangle (diagram), we have

$$V_{w1} = V_1 \cos\alpha_1 = V_{r1}\cos\beta_1 + u \quad \text{and} \quad V_{w2} = V_2\cos\alpha_2 = V_{r2}\cos\beta_2 - u$$

$$\therefore V_{w1} + V_{w2} = V_{r1}\cos\beta_1 + V_{r2}\cos\beta_2 = V_{r1}\cos\beta_1\left[1 + \frac{V_{r2}\cos\beta_2}{V_{r1}\cos\beta_1}\right] = V_{r1}\cos\beta_1(1+KC)$$

where $K = \dfrac{V_{r2}}{V_{r1}}$ and $C = \dfrac{\cos\beta_2}{\cos\beta_1}$

(or) $$V_{w1} + V_{w2} = (V_1\cos\alpha_1 - u)(1+KC)$$

Rate of doing work per kg of steam per second = $(V_1\cos\alpha_1 - u)(1+KC)u$

$\therefore$ Diagram efficiency, $$\eta_b = \frac{(V_1\cos\alpha_1 - u)(1+KC)}{V_1^2}$$

Let, $\rho = \dfrac{u}{V_1}$ = Blade speed ratio

Then, Diagram efficiency, $$\eta_b = 2\left(\rho\cos\alpha_1 - \rho^2\right)(1+KC)$$

MAXIMUM WORK & MAXIMUM DIAGRAM EFFICIENCY

From the above equation, it is evident that *diagram efficiency* depends on the following factors:

1) Nozzle angle, α_1
2) Blade speed ratio, ρ
3) Blade angles, β_1 & β_2
4) Blade velocity coefficient, K

➢ If the values of α_1, K and C are assumed to be constant, then diagram efficiency depends only on the value of blade speed ratio, ρ

➢ In order to determine the optimum value of for maximum diagram efficiency, $\frac{\partial \eta_b}{\partial \rho} = 0$

➢ Then, ρ becomes, $\rho = \dfrac{\cos \alpha_1}{2}$

MAXIMUM WORK & MAXIMUM DIAGRAM EFFICIENCY

Maximum diagram efficiency =

$$(\eta_b)_{\max} = 2(1+KC)\left[\frac{\cos\alpha_1}{2}.\cos\alpha_1 - \frac{\cos^2\alpha_1}{4}\right] = (1+KC)\frac{\cos^2\alpha_1}{2}$$

Note: If the blade is symmetrical & friction is absent, then, we have $\beta_1 = \beta_2$ and $K = C = 1$

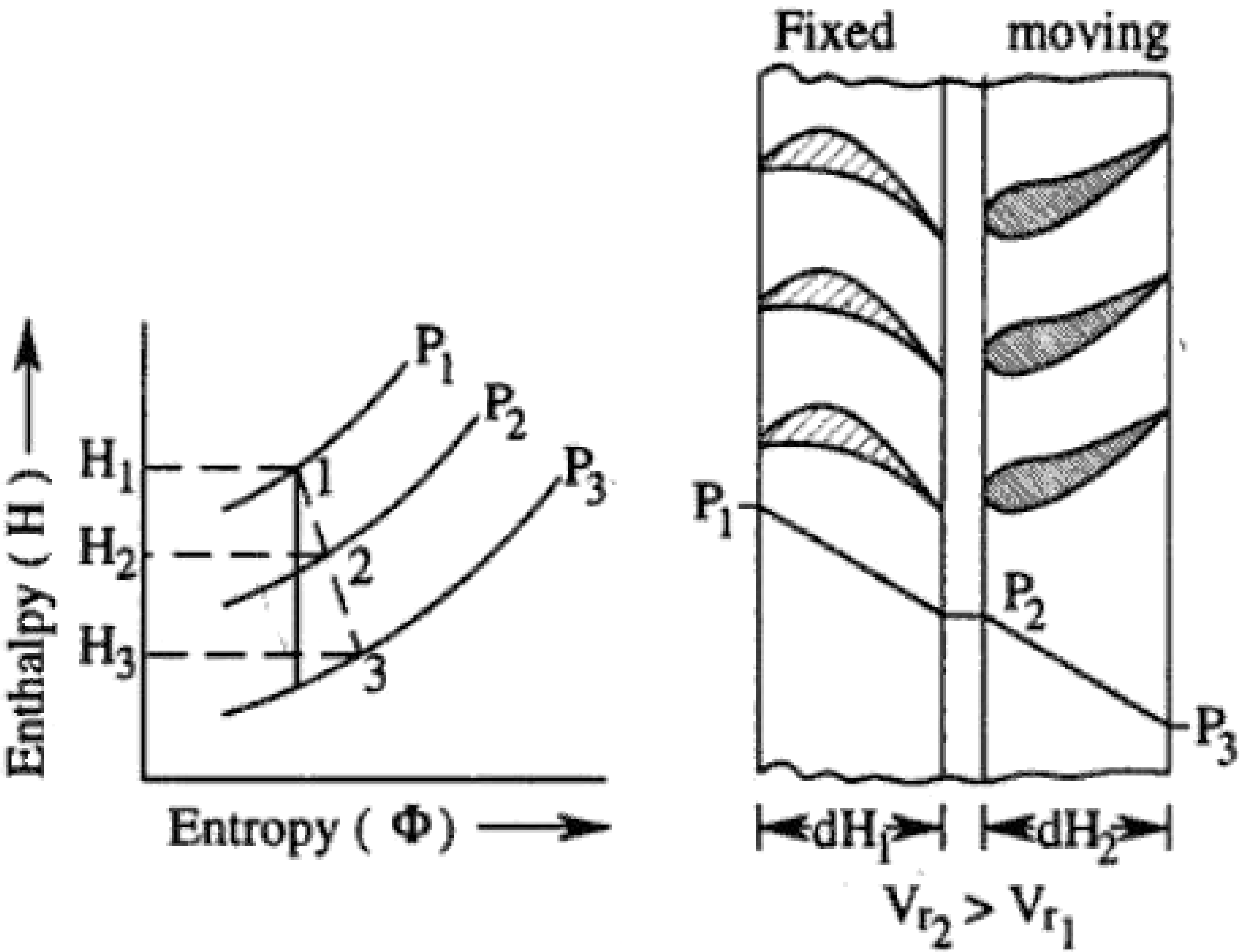
Then, maximum diagram efficiency, $(\eta_b)_{max} = cos^2\alpha_1$

Work done/kg of steam/second = $(V_1\cos\alpha_1 - u)(1+KC)u$

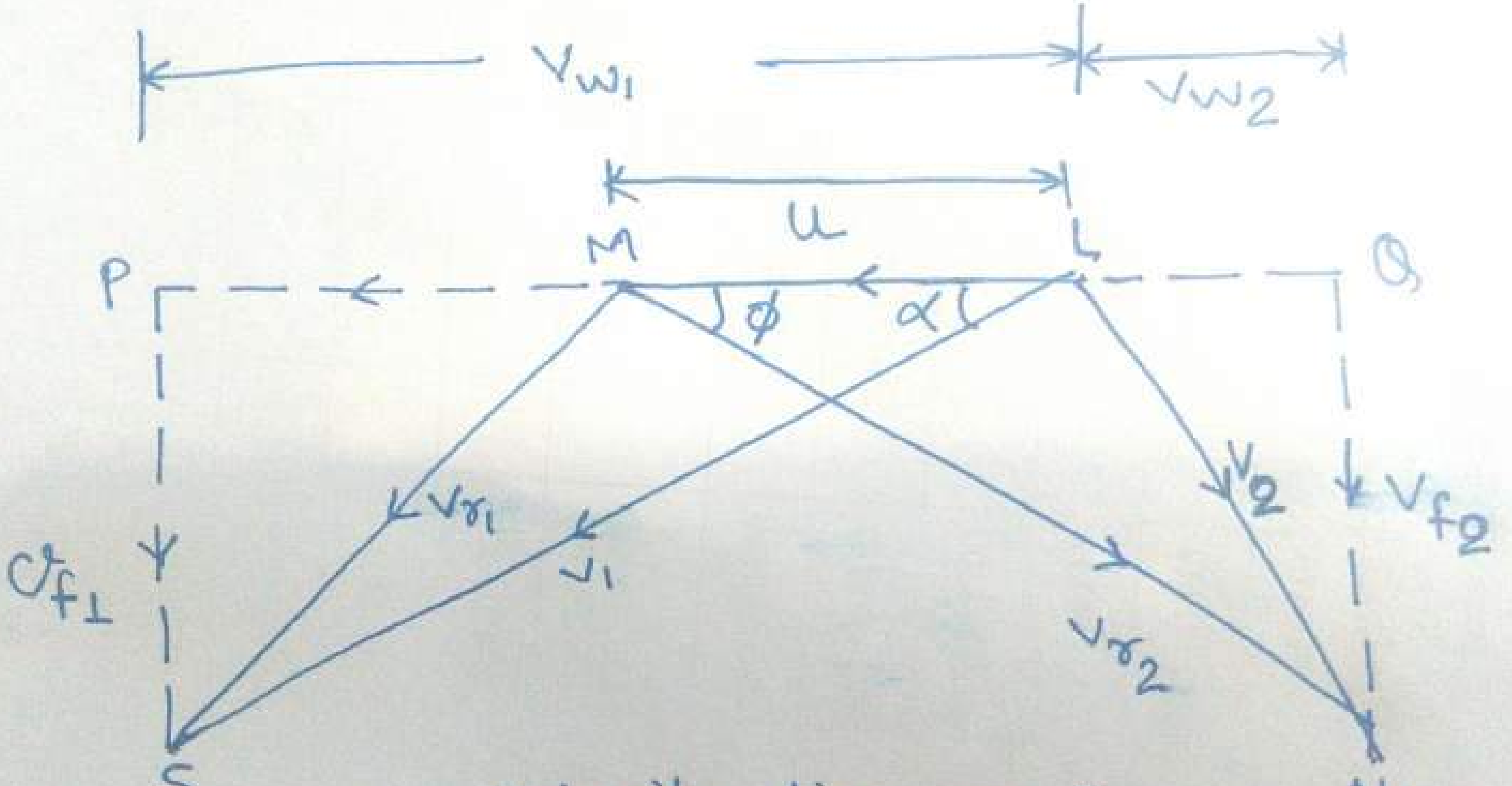
Then maximum rate of doing work/kg of steam/second $= 2u^2$

2. Reaction turbine

- A turbine in which steam pressure decreases gradually while expanding through the moving blades as well as the fixed blades is known as *reaction turbine.*
- It consists of a large number of stages, each stage consisting of set of fixed and moving blades. The heat drop takes place throughout in both fixed and moving blades.
- No nozzles are provided in a reaction turbine. The fixed blades act both as nozzles in which velocity of steam increased and direct the steam to enter the ring of moving blades. As pressure drop takes place both in the fixed and moving blades, all the blades are nozzle shaped.
- The steam expands while flowing over the moving blades and thus gives reaction to the moving blades. Hence the turbine is called *reaction turbine*.
- The fixed blades are attached to the casing whereas moving blades are fixed with the rotor.
- It is also called *Parson's reaction turbine.*



Isentropic expansion in Reaction Turbine



Velocity diagram for Reaction turbine blade

Work output & power in reaction turbine

The work done per kg of steam in the stage (per pair) =

$$u\left(V_{w1}+V_{w2}\right)N-m$$

The work done per kg of steam per second in the stage (per pair) =

$$mu\left(V_{w1}+V_{w2}\right)N-m/s$$

where, m = mass of steam flowing over blades in *kg/s*

Power developed (per pair) = $\dfrac{mu\left(V_{w1}+V_{w2}\right)}{1000}kW$

$$Efficiency,\eta=\frac{work\ done\ per\ kg\ of\ steam\ in\ the\ stage\ per\ pair}{Enthalpy\ drop\ in\ the\ stage\ per\ pair}$$

$$\therefore Efficiency,\eta=\frac{u\left(V_{w1}+V_{w2}\right)}{1000H}$$

where, H = Enthalpy drop in the stage per pair in *kJ/kg*

Degree of reaction in reaction turbine

DEGREE OF REACTION

The degree of reaction is defined as the ratio of isentropic heat drop in the moving blades to the isentropic heat drop in the entire stage of reaction turbine. It is denoted by R.

$$R = \frac{Enthalpy\,drop\,in\,the\,moving\,blade}{Enthalpy\,drop\,in\,the\,stage} = \frac{dH_2}{dH_1 + dH_2}$$

Where, $dH_1 =$ Enthalpy drop in the fixed blade per kg of steam $= \dfrac{V_1^2 - V_2^2}{2} kJ/kg = H_1 - H_2$

$dH_2 =$ Enthalpy drop in the moving blade per kg of steam $= \dfrac{V_{r2}^2 - V_{r1}^2}{2} kJ/kg = H_2 - H_3$

Also, $dH_1 + dH_2$ = Enthalpy drop in the stage per kg of steam

= $H_1 - H_3$

= Work done by the steam in the stage

= $u(V_{w1} + V_{w2})$

$$\boxed{\therefore Degree\ of\ Reaction, R = \frac{V_{r2}^2 - V_{r1}^2}{2u(V_{w1} + V_{w2})}}$$

Note-1: *In Pearson's turbine, the degree of reaction, R=0.5, then,* $\alpha_1 = \beta_2$ *and* $\alpha_2 = \beta_1$. *This means that moving blade and fixed blade have the same shape.*

Note-2: *If degree of reaction, R=0, then the turbine is a simple impulse turbine.*

Note-3: *If degree of reaction, R=1, then the turbine is a pure reaction turbine.*

Work done & efficiency in reaction turbine

BLADE EFFICIENCY AND STAGE EFFICIENCY

The condition for maximum efficiency is calculated considering the following assumptions:

- The degree of reaction, R = 0.5, i.e. $\alpha_1 = \beta_2 \, and \, \alpha_2 = \beta_1$
- The fixed and moving blades are symmetrical, i.e. $V_1 = V_{r2} \,\&\, V_2 = V_{r1}$

The kinetic energy supplied to the fixed blade per kg of steam $= \dfrac{V_1^2}{2}$

The kinetic energy supplied to the moving blade per kg of steam $= \dfrac{V_{r2}^2 - V_{r1}^2}{2}$

Total energy supplied in the stage per kg of steam $= \dfrac{V_1^2}{2} + \dfrac{V_{r2}^2 - V_{r1}^2}{2}$

Since blades are symmetrical, $V_1 = V_{r2} \,\&\, V_2 = V_{r1}$ & from velocity triangles, $V_{r1}^2 = V_1^2 + u^2 - 2 \cdot V_1 \cdot u \cdot \cos\alpha_1$

Therefore, total energy supplied in the stage per kg of steam $= V_1^2 - \dfrac{V_1^2 + u^2 - 2 \cdot V_1 \cdot u \cdot \cos\alpha_1}{2}$

Work done per kg of steam is given by,

$$
\begin{aligned}
\text{Work Done} &= u(V_{w1} + V_{w2}) \\
&= u(V_1 \cos\alpha_1 + V_{r2} \cos\beta_2 - u) \\
&= u(2V_1 \cos\alpha_1 - u) \quad (\because \alpha_1 = \beta_2 \, \text{and} \, V_1 = V_{r2})
\end{aligned}
$$

Work done & efficiency in reaction turbine

$$\text{Diagram efficiency}, \quad = \eta_d = \frac{\text{Work done per kg of steam}}{\text{Total energy supplied per kg of steam}}$$

$$= \frac{u(2V_1\cos\alpha_1 - u)}{V_1^2 - \dfrac{V_1^2 + u^2 - 2\cdot V_1\cdot u\cdot\cos\alpha_1}{2}}$$

$$= \frac{2u(2V_1\cos\alpha_1 - u)}{V_1^2 - u^2 + 2\cdot V_1\cdot u\cdot\cos\alpha_1}$$

$$= \frac{2uV_1\left(2\cos\alpha_1 - \dfrac{u}{V_1}\right)}{V_1^2\left(1 - \dfrac{u^2}{V_1^2} + 2\cdot\dfrac{u}{V_1}\cdot\cos\alpha_1\right)}$$

$$= \frac{2\dfrac{u}{V_1}\left(2\cos\alpha_1 - \dfrac{u}{V_1}\right)}{\left(1 - \dfrac{u^2}{V_1^2} + 2\cdot\dfrac{u}{V_1}\cdot\cos\alpha_1\right)}$$

$$\boxed{\therefore \eta_d = \frac{2\rho(2\cos\alpha_1 - \rho)}{(1 - \rho^2 + 2\cdot\rho\cdot\cos\alpha_1)}}$$

where $\rho = \dfrac{u}{V_1}$ =Blade speed ratio

The efficiency is maximum when the term $\left(1-\rho^2+2\cdot\rho\cdot\cos\alpha_1\right)$ is minimum or when $\dfrac{d\eta_d}{d\rho}=0$

$$\frac{d}{d\rho}\left(1-\rho^2+2\cdot\rho\cdot\cos\alpha_1\right)=0$$

(or) $$\left(-2\rho+2\cos\alpha_1\right)=0$$

(or) $$\boxed{\rho=\cos\alpha_1}$$

Therefore efficiency is maximum when $\boxed{\rho=\cos\alpha_1}$

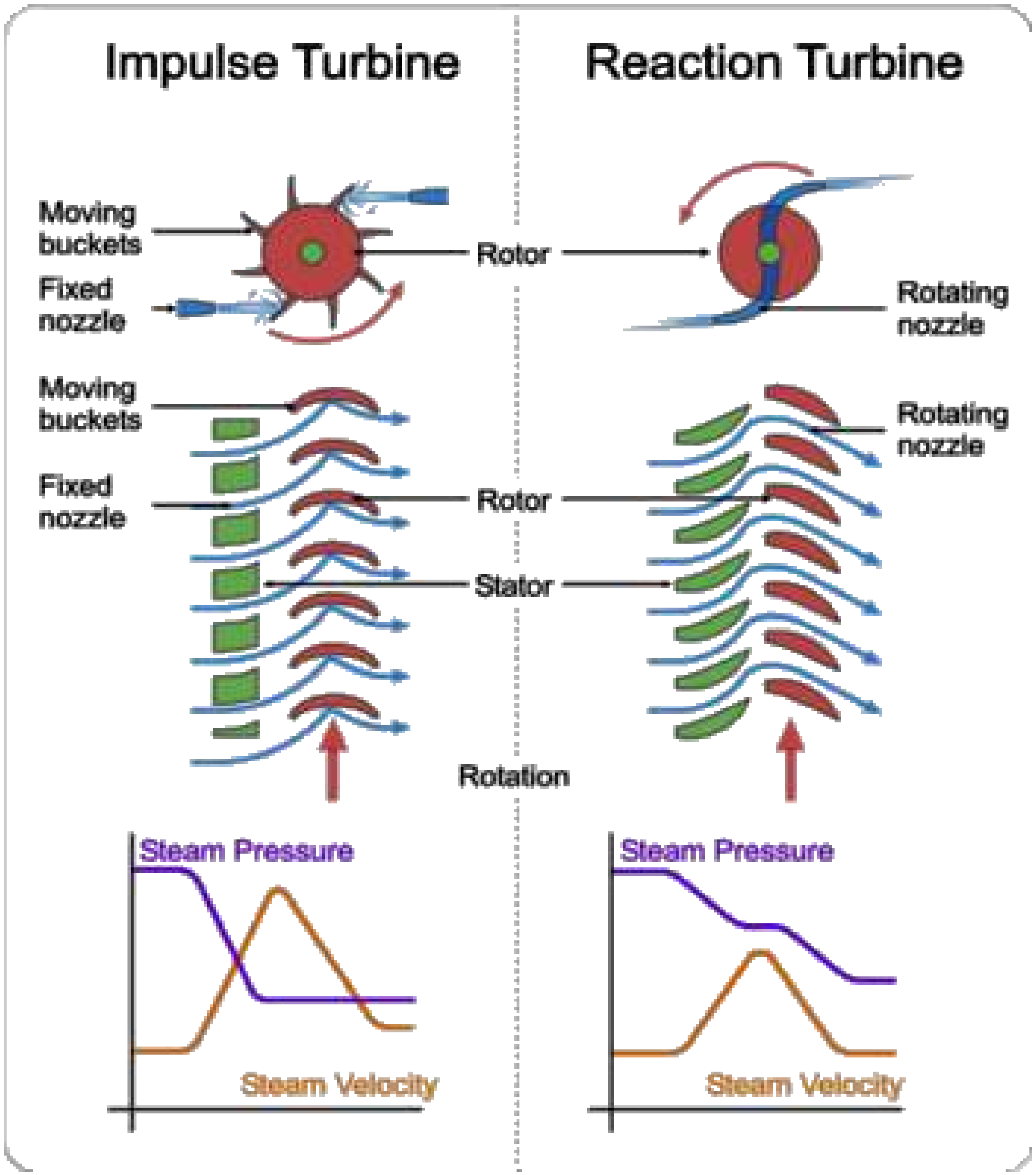
Then, $$\therefore\left(\eta_d\right)_{\max}=\frac{2\cos\alpha_1\left(2\cos\alpha_1-\cos\alpha_1\right)}{\left(1-\cos^2\alpha_1+2\cdot\cos\alpha_1\cdot\cos\alpha_1\right)}=\frac{2\cos^2\alpha_1}{\left(1+\cos^2\alpha_1\right)}$$

$$\boxed{\therefore\left(\eta_d\right)_{\max}=\frac{2\cos^2\alpha_1}{\left(1+\cos^2\alpha_1\right)}}$$

Impulse turbine vs Reaction turbine

Impulse turbine	Reaction turbine
The steam completely expands in the nozzle and its pressure remains constant during its flow through the blade passages	The steam expands partially in the nozzle and further expansion takes place in the rotor blades
The relative velocity of steam passing over the blade remains constant in the absence of friction	The relative velocity of steam passing over the blade increases as the steam expands while passing over the blade
Blades are symmetrical	Blades are asymmetrical
The pressure on both ends of the moving blade is same	The pressure on both ends of the moving blade is different
For the same power developed, as pressure drop is more, the number of stages required are less	For the same power developed, as pressure drop is small, the number of stages required are more
The blade efficiency curve is less flat	The blade efficiency curve is more flat
The steam velocity is very high and therefore the speed of turbine is high.	The steam velocity is not very high and therefore the speed of turbine is low.

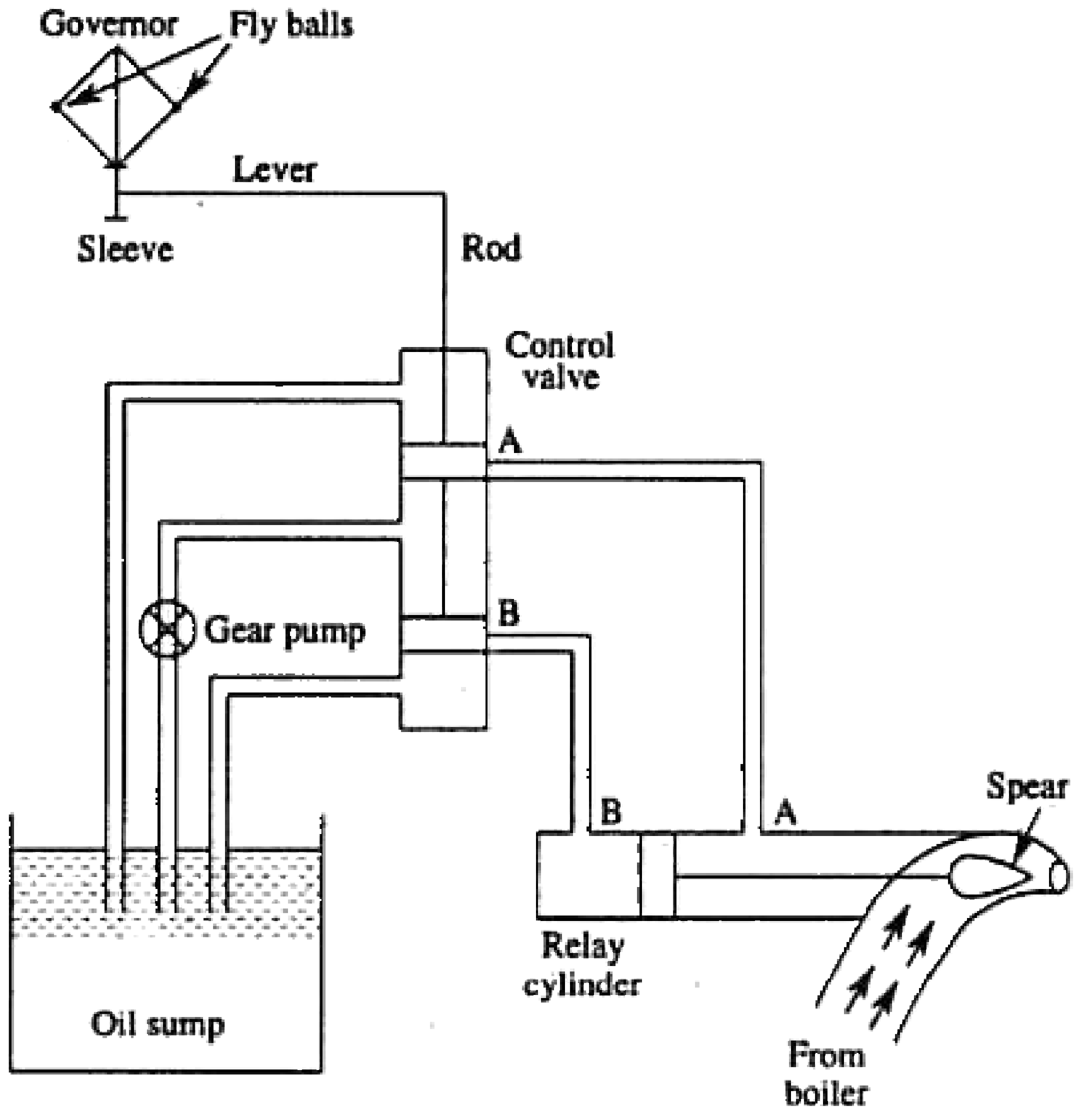
IMPULSE TURBINE VS REACTION TURBINE



Governing of turbines

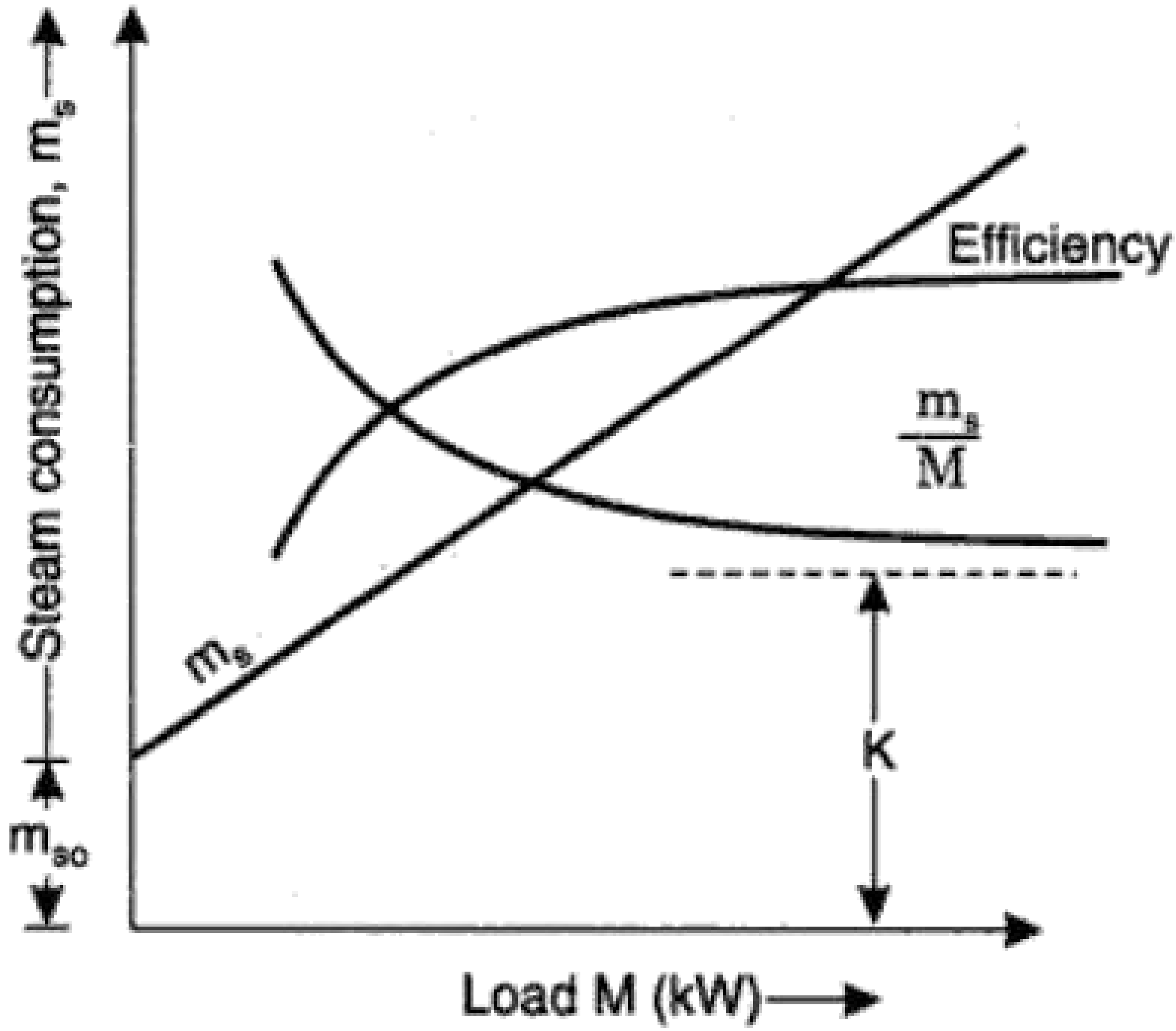
- *Governing* is the method of maintaining the speed of the turbine constant irrespective of variation of the load on the turbine.
- A *governor* is used for achieving this purpose which regulates the supply of steam to the turbine in such a way that the speed of the turbine is maintained as far as possible a constant under varying load conditions.
- The various methods of governing of steam turbines are:
  1) Throttle governing
  2) Nozzle governing
  3) By-pass governing
  4) Combination of (1) & (2) or (2) & (3)

Throttle governing

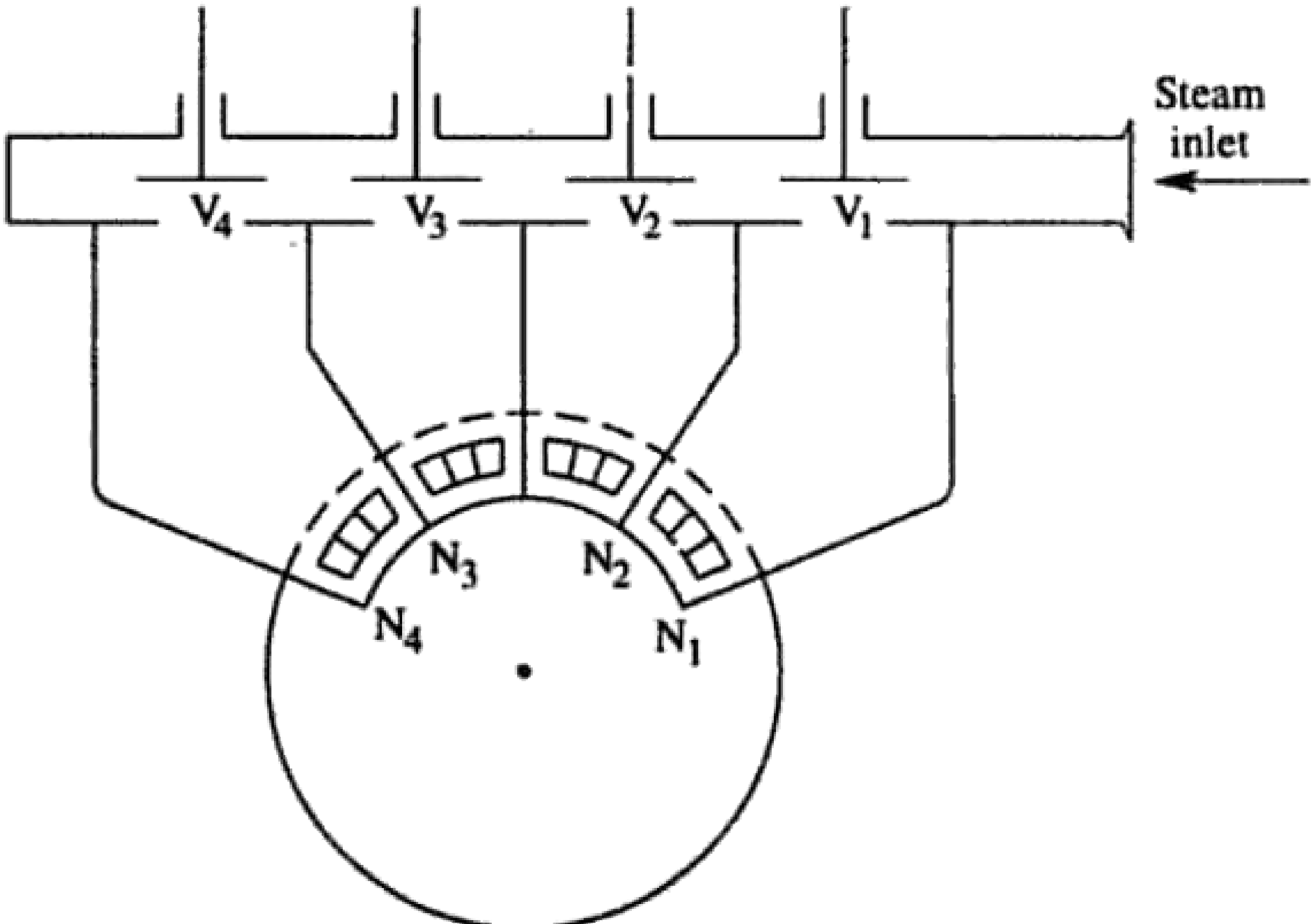


Throttle governing

WILLIAN'S LINE

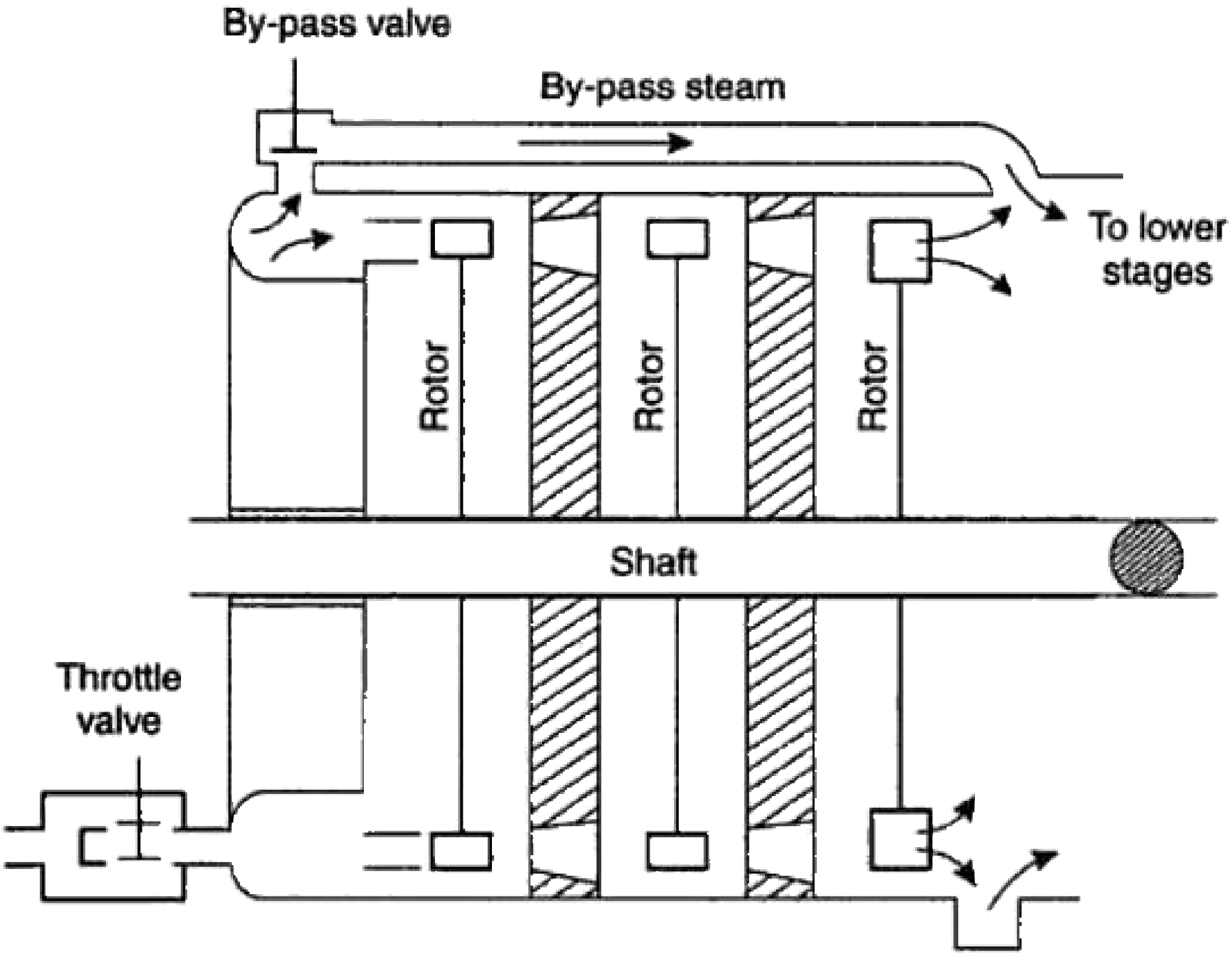


2. Nozzle *governing*



Nozzle Control governing

3. By-pass governing

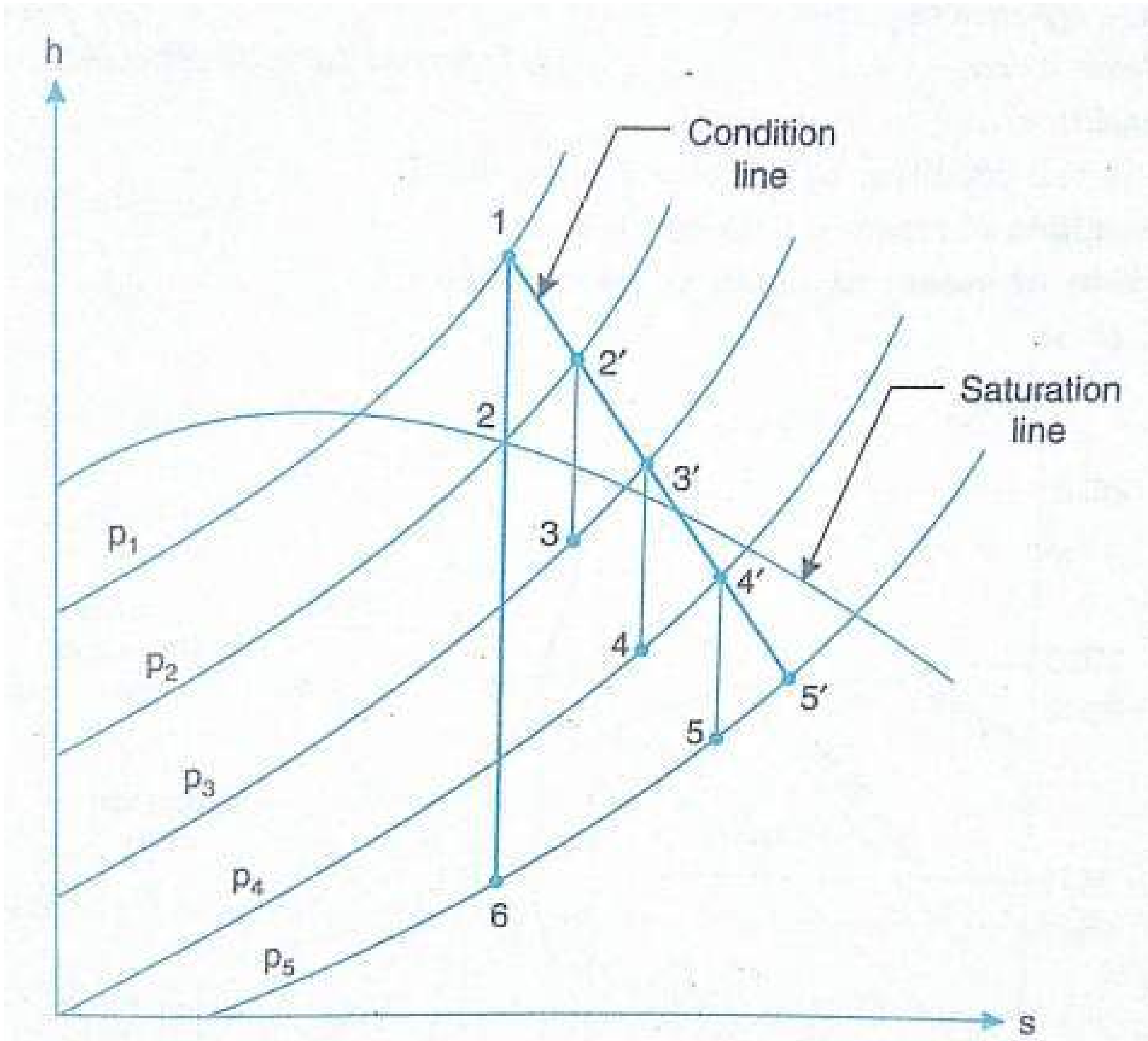


By-Pass Governing

Losses in steam turbines

- Residual velocity loss
- Losses in regulating valves
- Loss due to steam friction in nozzle
- Loss due to leakage
- Loss due to mechanical friction
- Loss due to wetness of steam
- Radiation loss

Effect of blade friction in steam turbines



overall efficiency & reheat factor

Reheat factor:

It is defined as the ratio of cumulative heat drop to the adiabatic heat drop in all the stages of the turbine. The value of reheat factor depends on the type and efficiency of the turbine, the average value being 1.05.

$$\text{Reheat factor} = \frac{\text{Cumulative heat drop}}{\text{Adiabatic heat drop}} = \frac{A_1B_1 + A_2B_2 + A_3B_3}{A_1D}$$

Overall efficiency:

It is defined as the ratio of total useful heat drop to the total heat supplied.

$$\text{Overall efficiency} = \frac{\text{Total useful heat drop}}{\text{Total heat supplied}} = \frac{A_1C_1 + A_2C_2 + A_3C_3}{H_{A1} - h_D}$$

19 Steam Turbines

19.1. INTRODUCTION

*The **steam turbine** is a prime-mover in which the potential energy of the steam is transformed into kinetic energy, and latter in its turn is transformed into the mechanical energy of rotation of the turbine shaft*. The turbine shaft, directly or with the help of a reduction gearing, is connected with the driven mechanism. Depending on the type of the driven mechanism a steam turbine may be *utilised in most diverse fields of industry, for power generation* and *for transport*. Transformation of the potential energy of steam into the mechanical energy of rotation of the shaft is brought about by different means.

19.2. CLASSIFICATION OF STEAM TURBINES

There are several ways in which the steam turbines may be classified. The most important and common division being with respect to the *action of the steam*, as :

(*a*) Impulse.

(*b*) Reaction.

(*c*) Combination of impulse and reaction.

Other classification are :

1. According to the number of pressure stages :

(*i*) *Single stage turbines with one or more velocity stages usually of small power capacities* ; these turbines are mostly used for driving centrifugal compressors, blowers and other similar machinery.

(*ii*) *Multistage impulse and reaction turbines* ; they are made in a wide range of power capacities varying from small to large.

2. According to the direction of steam flow :

(*i*) *Axial turbines* in which steam flows in a direction parallel to the axis of the turbine.

(*ii*) *Radial turbines* in which steam flows in a direction perpendicular to the axis of the turbine ; one or more low-pressure stages in such turbines are made axial.

3. According to the number of cylinders :

(*i*) Single cylinder turbines.

(*ii*) Double cylinder turbines.

(*iii*) Three cylinder turbines.

(*iv*) Four cylinder turbines.

Multi-cylinder turbines which have their rotors mounted on one and the same shaft and coupled to a single generator are known as *single shaft turbines* ; turbines with separate rotor shafts for each cylinder placed parallel to each other are known as **multiaxial turbines.**

4. According to the method of governing :

(*i*) *Turbines with throttle governing* in which fresh steam enters through one or more (depending on the power developed) simultaneously operated throttle valves.

(*ii*) *Turbines with nozzle governing* in which fresh steam enters through two or more consecutively opening regulators.

(*iii*) *Turbines with by pass governing* in which steam turbines besides being fed to the first stage is also directly fed to one, two or even three intermediate stages of the turbine.

5. According to heat drop process :

(*i*) *Condensing turbines with generators* ; in these turbines steam at a pressure less than atmospheric is directed to a condenser ; besides, steam is also extracted from intermediate stages for feed water heating, the number of such extractions usually being from 2–3 to as much 8–9. The latent heat of exhaust steam during the process of condensation is completely lost in these turbines.

(*ii*) *Condensing turbines with one or two intermediate stage extractions* at specific pressures for industrial and heating purposes.

(*iii*) *Back pressure turbines*, the exhaust steam from which is utilised for industrial or heating purposes ; to this type of turbines can also be added (in a relative sense) turbines with deteriorated vacuum, the exhaust steam of which may be used for heating and process purposes.

(*iv*) *Topping turbines* ; these turbines are also of the back pressure type with the difference that the exhaust steam from these turbines is further utilised in medium and low pressure condensing turbines. These turbines, in general, operate at high initial conditions of steam pressure and temperature, and are mostly used during extension of power station capacities, with a view to obtain better efficiencies.

(*v*) *Back pressure turbines with steam extraction from intermediate stages at specific pressure* ; turbines of this type are meant for supplying the consumer with steam of various pressures and temperature conditions.

(*vi*) *Low pressure turbines* in which the exhaust steam from reciprocating steam engines, power hammers, presses, etc., is utilised for power generation purposes.

(*vii*) *Mixed pressure turbines* with two or three pressure stages, with supply of exhaust steam to its intermediate stages.

6. According to steam conditions at inlet to turbine :

(*i*) *Low pressure turbines*, using steam at a pressure of 1.2 to 2 ata.

(*ii*) *Medium pressure turbines*, using steam at pressures of upto 40 ata.

(iii) *High pressure turbines*, utilising pressures above 40 ata.

(iv) *Turbines of very high pressures*, utilising steam at pressures of 170 ata and higher and temperatures of 550°C and higher.

(v) *Turbines of supercritical pressures*, using steam at pressures of 225 ata and above.

7. According to their usage in industry :

(i) *Stationary turbines with constant speed of rotation* primarily used for driving alternators.

(ii) *Stationary steam turbines* with variable speed meant for driving turbo-blowers, air circulators, pumps, etc.

(iii) *Non-stationary turbines with variable speed* ; turbines of this type are usually employed in steamers, ships and railway locomotives.

19.3. ADVANTAGES OF STEAM TURBINE OVER STEAM ENGINES

The following are the *principal advantages of steam turbine over steam engines* :

1. The thermal efficiency of a steam turbine is much higher than that of a steam engine.
2. The power generation in a steam turbine is at a uniform rate, therefore necessity to use a flywheel (as in the case of steam engine) is not felt.
3. Much higher speeds and greater range of speed is possible than in case of a steam engine.
4. In large thermal stations where we need higher outputs, the steam turbines prove very suitable as these can be made in big sizes.
5. With the absence of reciprocating parts (as in steam engine) the balancing problem is minimised.
6. No internal lubrication is required as there are no rubbing parts in the steam turbine.
7. In a steam turbine there is no loss due to initial condensation of steam.
8. It can utilise high vacuum very advantageously.
9. Considerable overloads can be carried at the expense of slight reduction in overall efficiency.

19.4. DESCRIPTION OF COMMON TYPES OF TURBINES

The common types of steam turbines are :

1. Simple impulse turbine.
2. Reaction turbine.

The main difference between these turbines lies *in the way in which the steam is expanded while it moves through them. In the former type steam expands in the nozzles and its pressure does not alter as it moves over the blades while in the latter type the steam expands continuously as it passes over the blades and thus there is gradual fall in the pressure during expansion.*

1. Simple impulse turbines

Fig. 19.1 shows a simple impulse turbine diagrammatically. The top portion of the figure exhibits a longitudinal section through the upper half of the turbine, the middle portion shows one set of nozzles which is followed by a ring of moving blades, while lower part of the diagram indicates approximately changes in pressure and velocity during the flow of steam through the turbine. This turbine is called '*simple*' impulse turbine since the expansion of the steam takes place in *one set of the nozzles*.

As the steam flows through the nozzle its pressure falls from steam chest pressure to condenser pressure (or atmospheric pressure if the turbine is non-condensing). Due to this relatively higher ratio of expansion of steam in the nozzles the steam leaves the nozzle with a very high velocity. Refer Fig. 19.1, it is evident that the velocity of the steam leaving the moving blades is a large portion of the maximum velocity of the steam when leaving the nozzle. The loss of energy due to this higher exit velocity is commonly called the "**carry over loss**" or "**leaving loss**"

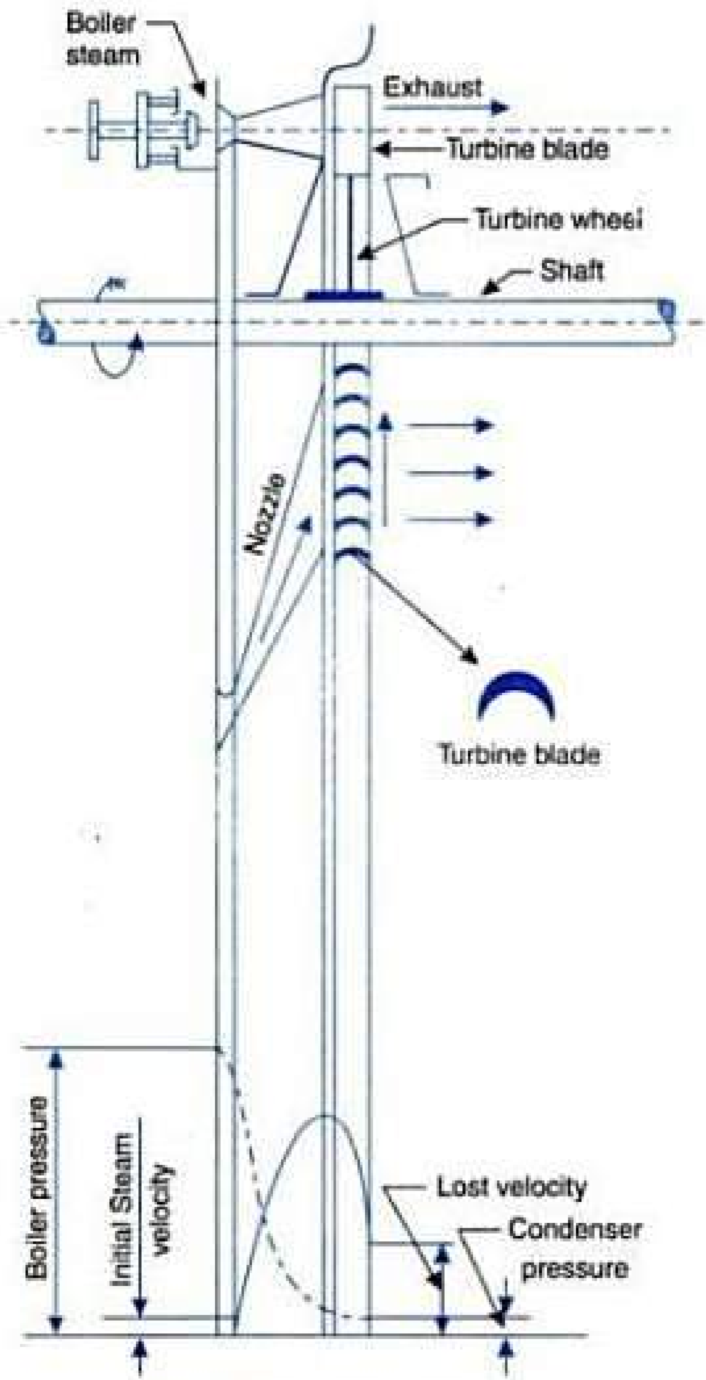


Fig. 19.1. Simple impulse turbine.

The principal example of this turbine is the well known "**De laval turbine**" and in this turbine the 'exit velocity' or 'leaving velocity' or 'lost velocity' may amount to 3.3 per cent of the nozzle outlet velocity. Also since all the kinetic energy is to be absorbed by one ring of the moving blades only, the velocity of wheel is too high (varying from 25000 to 30000 r.p.m.). This wheel or rotor speed however, can be reduced by different methods (discussed in the following article).

2. Reaction turbine

In this type of turbine, *there is a gradual pressure drop and takes place continuously over the fixed and moving blades*. The function of the fixed blades is (the same as the nozzle) that they alter the direction of the steam as well as allow it expand to a larger velocity. As the steam passes over the moving blades its kinetic energy (obtained due to fall in pressure) is absorbed by them. Fig. 19.2. shows a *three stage* reaction turbine. The changes in pressure and velocity are also shown there in.

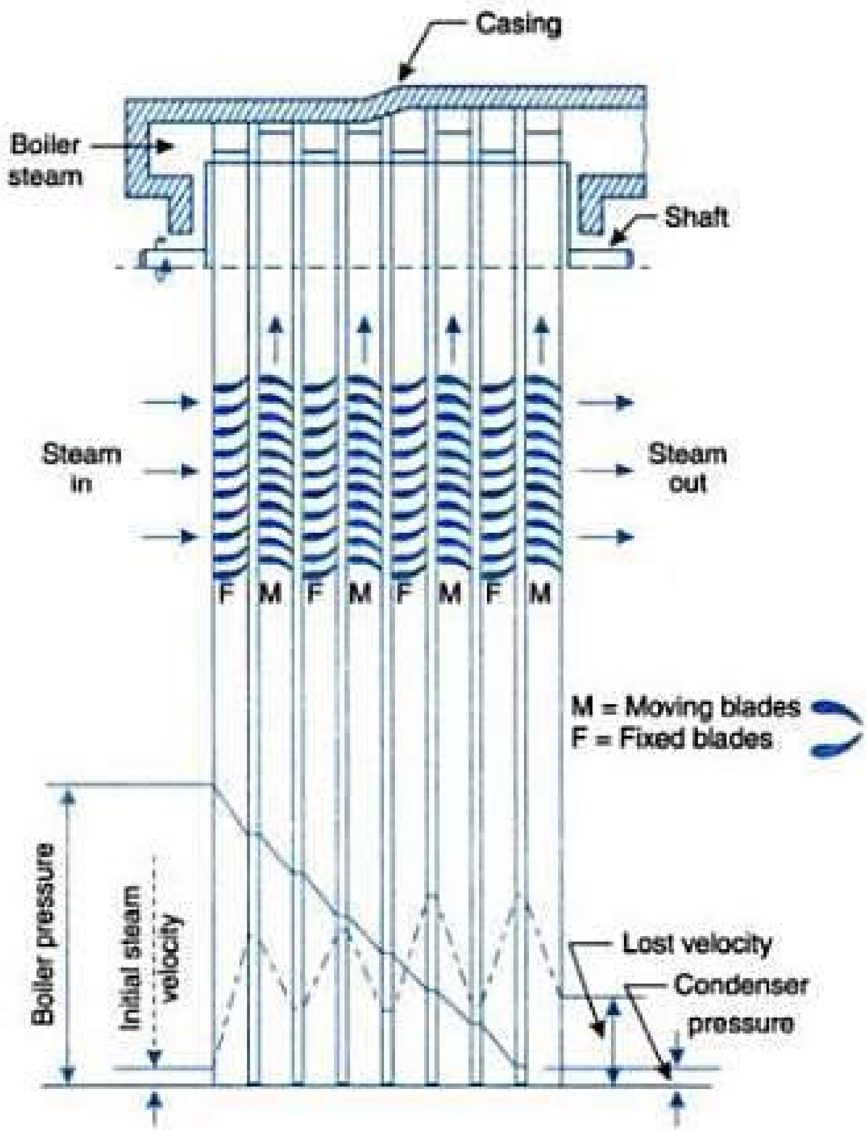


Fig. 19.2. Reaction turbine (three stage).

As the volume of steam increases at lower pressures therefore, the diameter of the turbine must increase after each group of blade rings. It may be noted that in this turbine since the pressure drop per stage is small, therefore the number of stages required is much higher than an impulse turbine of the same capacity.

19.5. METHODS OF REDUCING WHEEL OR ROTOR SPEED

As already discussed under the heading 'simple impulse turbine' that if the steam is expanded from the boiler pressure to condenser pressure in one stage the speed of the rotor becomes *tremendously high which crops up practical complicacies*. There are several methods of reducing this speed to lower value ; all these methods utilise a multiple system of rotor in series, keyed on a common shaft and the steam pressure or jet velocity is absorbed in stages as the steam flows over the blades. This is known as '**compounding**'. The different methods of compounding are :

1. Velocity compounding.
2. Pressure compounding.
3. Pressure velocity compounding.
4. Reaction turbine.

1. Velocity compounding

Steam is expanded through a stationary nozzle from the boiler or inlet pressure to condenser pressure. So the pressure in the nozzle drops, the kinetic energy of the steam increases due to increase in velocity. A portion of this available energy is absorbed by a row of moving blades. The

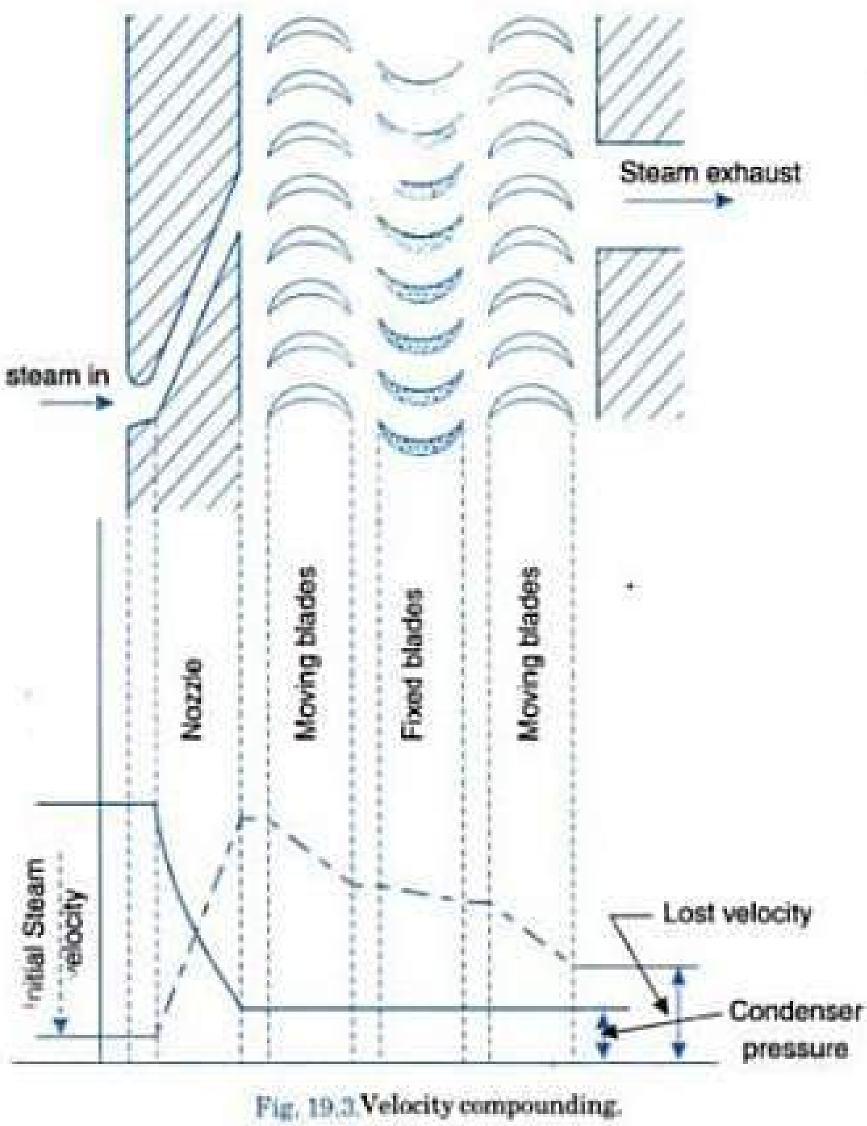


Fig. 19.3. Velocity compounding.

steam (whose velocity has decreased while moving over the moving blades) then flows through the second row of blades which are fixed. The function of these fixed blades is to re-direct the steam flow without altering its velocity to the following next row moving blades where again work is done on them and steam leaves the turbine with a low velocity. Fig. 19.3 shows a cut away section of such a stage and changes in pressure and velocity as the steam passes through the nozzle, fixed and moving blades.

Though this method has the advantage that the initial cost is low due to lesser number of stages yet its efficiency is low.

2. Pressure compounding

Fig. 19.4 shows rings of fixed nozzles incorporated between the rings of moving blades. The steam at boiler pressure enters the first set of nozzles and expands partially. The kinetic energy of the steam thus obtained is absorbed by the moving blades (stage 1). The steam then expands partially in the second set of nozzles where its pressure again falls and the velocity increases ; the

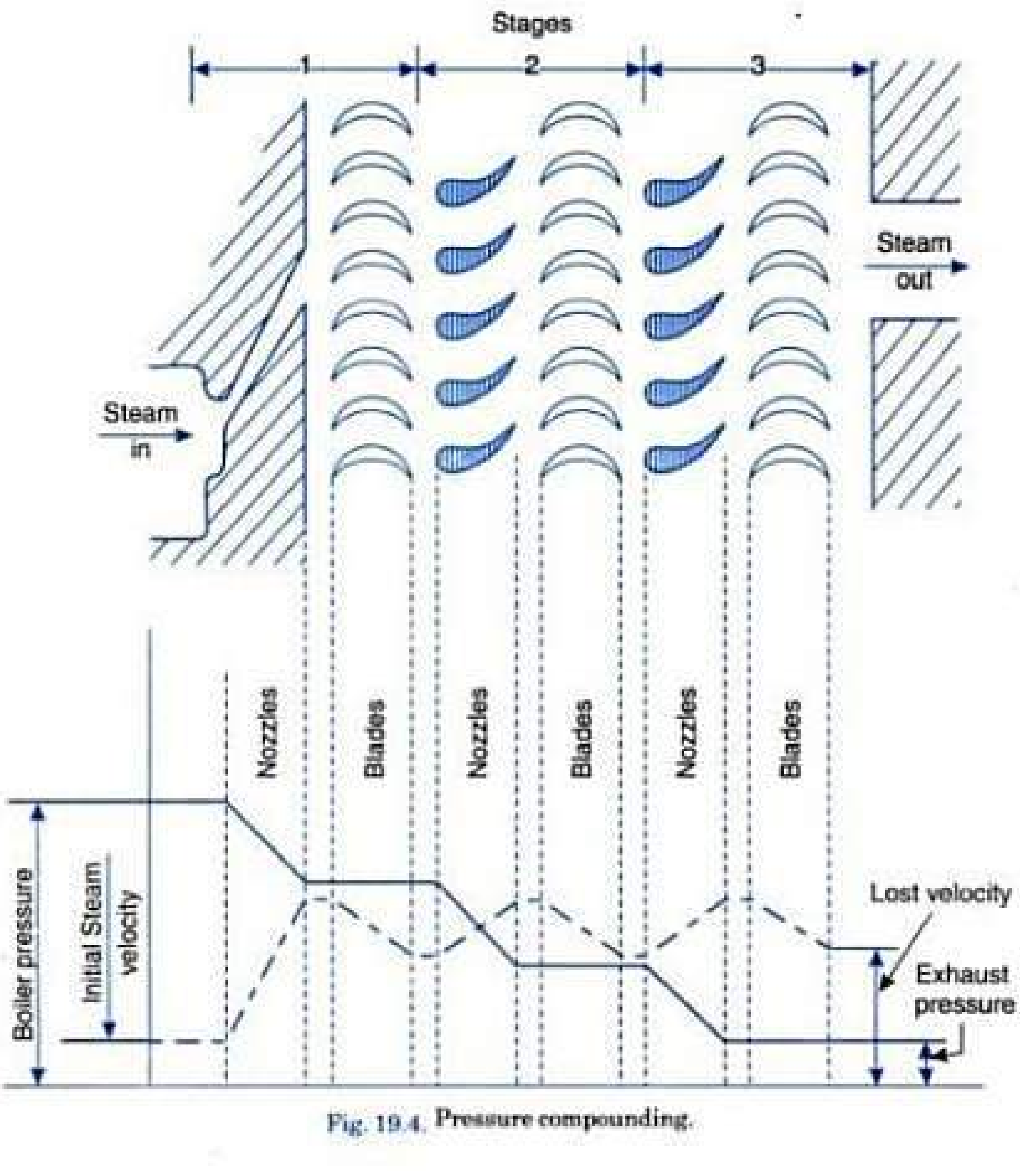


Fig. 19.4. Pressure compounding.

kinetic energy so obtained is absorbed by the second ring of moving blades (stage 2). This is repeated in stage 3 and steam finally leaves the turbine at low velocity and pressure. The number of stages (or pressure reductions) depends on the number of rows of nozzles through which the steam must pass.

This method of compounding is used in *Rateau and Zoelly turbine. This is most efficient turbine since the speed ratio remains constant but it is expensive owing to a large number of stages.*

3. Pressure velocity compounding

This method of compounding is the combination of two previously discussed method. The total drop in steam pressure is divided into stages and the velocity obtained in each stage is also compounded. The rings of nozzles are fixed at the beginning of each stage and pressure remains constant during each stage. The changes in pressure and velocity are shown in Fig. 19.5.

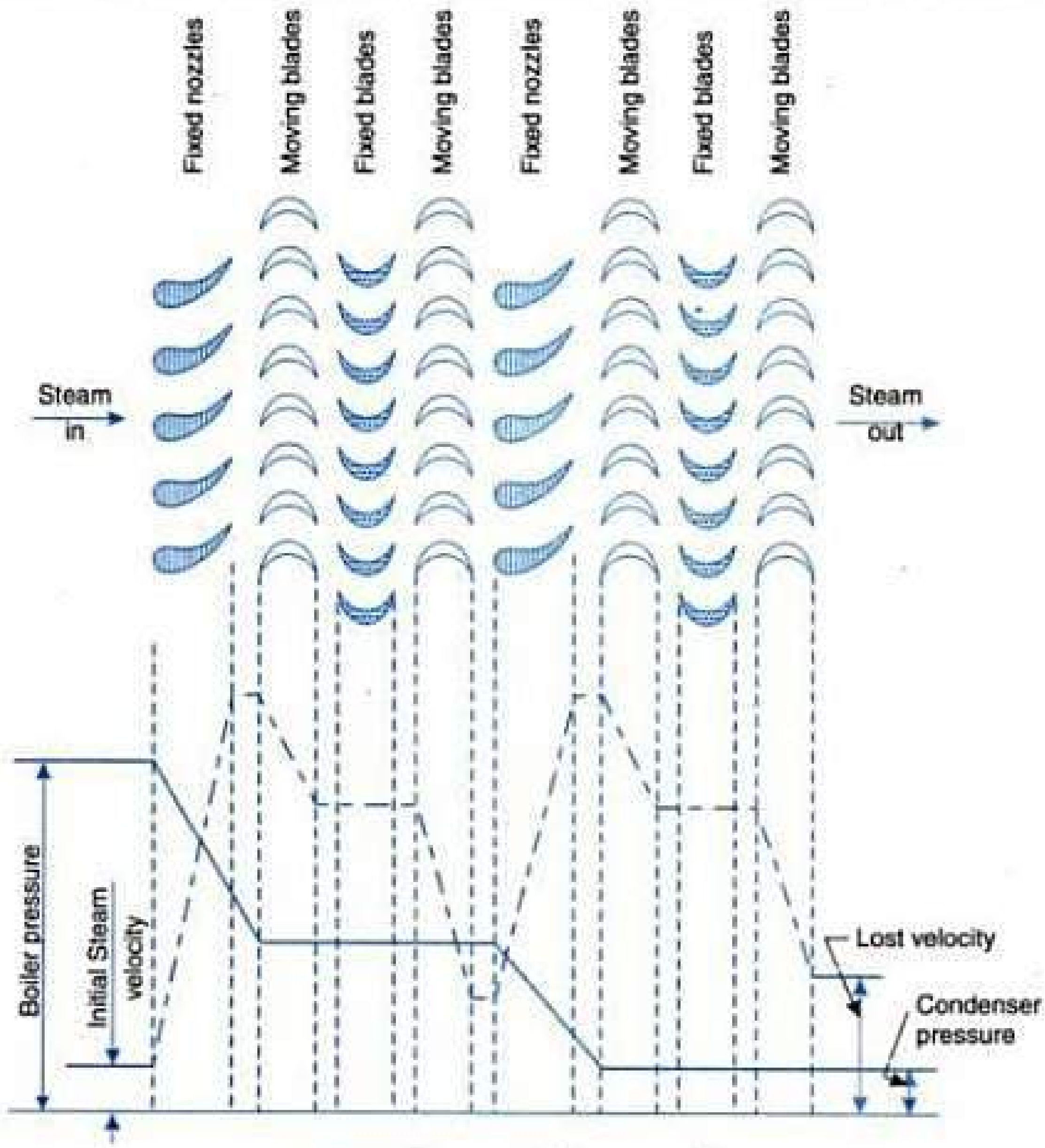


Fig. 19.5. Pressure velocity compounding.

This method of compounding is used in *Curits and Moore turbine.*

4. Reaction turbine

It has been discussed in Article 19.4.

19.6. DIFFERENCE BETWEEN IMPULSE AND REACTION TURBINES

S. No.	*Particulars*	*Impulse turbine*	*Reaction turbine*
1.	*Pressure drop*	Only in nozzles and not in moving blades.	In fixed blades (nozzles) as well as in moving blades.
2.	*Area of blade channels*	Constant.	Varying (converging type).
3.	*Blades*	Profile type.	Aerofoil type.
4.	*Admission of steam*	Not all round or complete.	All round or complete.
5.	*Nozzles/fixed blades*	Diaphram contains the nozzle.	Fixed blades similar to moving blades attached to the casing serve as nozzles and guide the steam.
6.	*Power*	Not much power can be developed.	Much power can be developed.
7.	*Space*	Requires less space for same power.	Requires more space for same power.
8.	*Efficiency*	Low.	High.
9.	*Suitability*	Suitable for small power requirements.	Suitable for medium and higher power requirements.
10.	*Blade manufacture*	Not difficult.	Difficult.

19.7. IMPULSE TURBINES

19.7.1. Velocity Diagram for Moving blade

Fig. 19.6 shows the velocity diagram of a *single stage impulse turbine*.

C_{bl} = Linear velocity of moving blade (m/s)

C_1 = Absolute velocity of steam entering moving blade (m/s)

C_0 = Absolute velocity of steam leaving moving blade (m/s)

C_{w_1} = Velocity of whirl at the entrance of moving blade.
= tangential component of C_1.

C_{w_0} = Velocity of whirl at exit of the moving blade.
= tangential component of C_0.

C_{f_1} = Velocity of flow at entrance of moving blade.
= axial component of C_1.

C_{f_0} = Velocity of flow at exit of the moving blade.
= axial component of C_0.

C_{r_1} = Relative velocity of steam to moving blade at entrance.

C_{r_0} = Relative velocity of steam to moving blade at exit.

α = Angle with the tangent of the wheel at which the steam with velocity C_1 enters. This is also called *nozzle engle*.

β = Angle which the discharging steam makes with the tangent of the wheel at the exit of moving blade.

θ = Entrance angle of moving blade.

φ = Exit angle of moving blade.

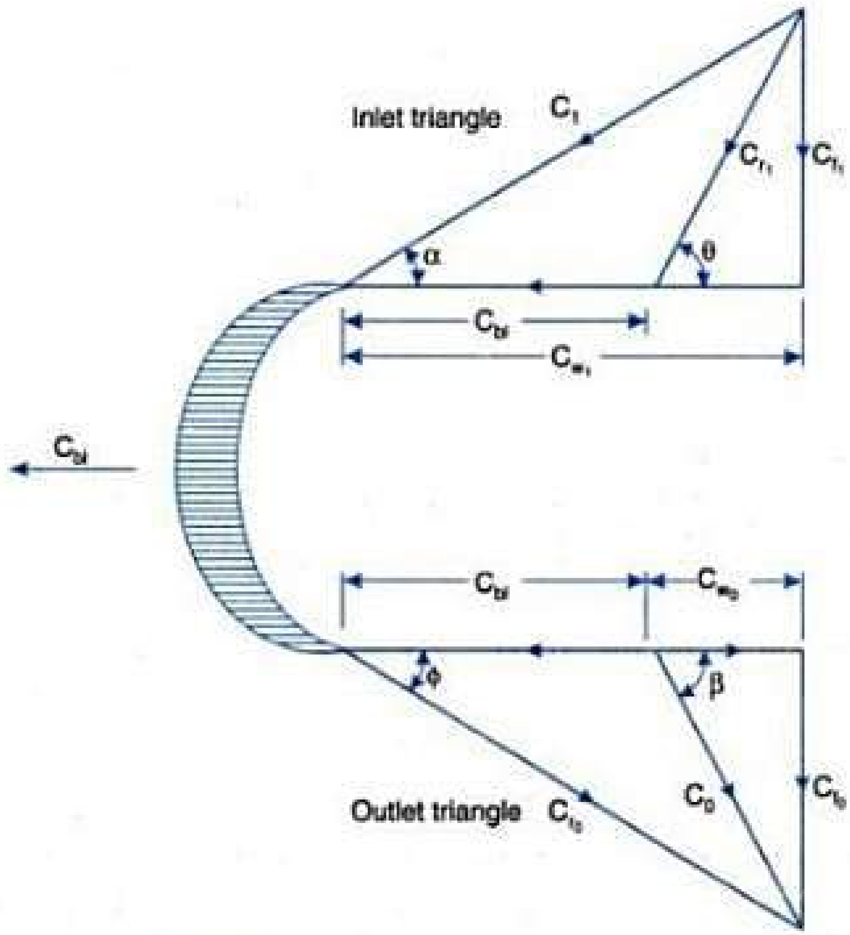


Fig. 19.6. Velocity diagram for moving blade.

The steam jet issuing from the nozzle at a velocity of C_1 impinges on the blade at an angle α. The tangential component of this jet (C_{w_1}) *performs work on the blade*, the axial component (C_{f_1}) however *does no work but causes the steam to flow through the turbine*. As the blades move with a tangential velocity of C_{bl}, the entering steam jet has a relative velocity C_{r_1} (with respect to blade) which makes an angle θ with the wheel tangent. The steam then glides over the blade without any shock and discharges at a relative velocity of C_0 at an angle φ with the tangent of the blades. The relative velocity at the inlet (C_{r_1}) is the same as the relative velocity at the outlet (C_{r_0}) if *there is no frictional loss at the blade*. The absolute velocity (C_0) of leaving steam make an angle β to the tangent at the wheel.

To have convenience in solving the problems on turbines it is a common practice to combine the two vector velocity diagrams on a common base which represents the blade velocity (C_{bl}) as shown in Fig. 19.7. This diagram has been obtained by superimposing the inlet velocity diagram on the outlet diagram in order that the blade velocity lines C_{bl} coincide.

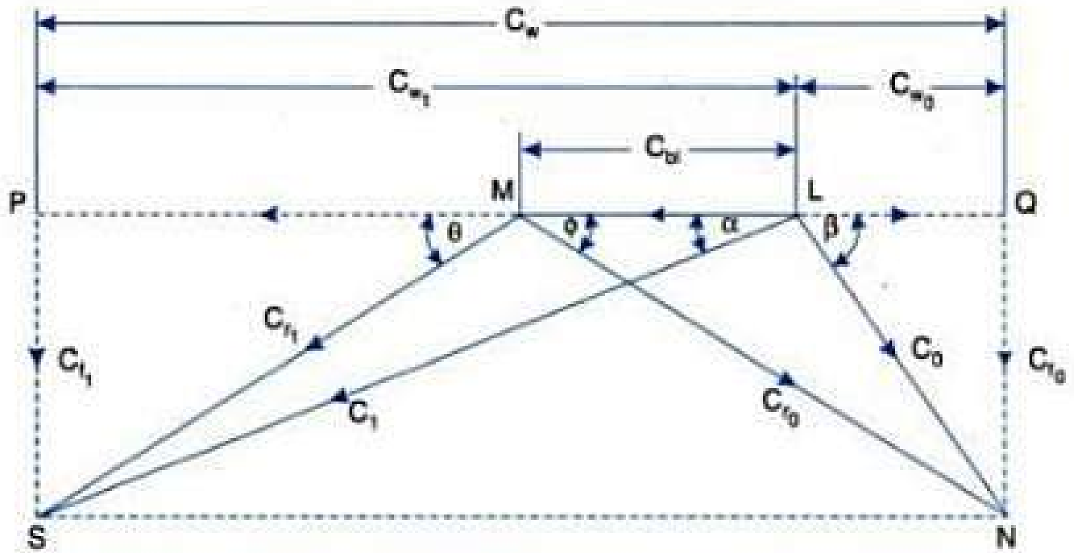


Fig. 19.7

19.7.2. Work done on the Blade

The work done on the blade may be found out from the change of momentum of the steam jet during its flow over the blade. As earlier discussed, it is only the velocity of whirl which performs work on the blade since it acts in its (blade) direction of motion.

From Newton's second law of motion,

Force (tangential) on the wheel = Mass of steam × acceleration

= Mass of steam/sec. × change of velocity

$$= \dot{m}_s(C_{w_1} - C_{w_0}) \quad ...(19.1)$$

The value of C_{w_0} is actually negative as the steam is discharged in the *opposite direction* to the blade motion, therefore due consideration should be given to the fact that *values of* C_{w_1} and C_{w_0} *are to be added while doing the solution of the problem.* (*i.e.*, when $\beta < 90°$)

Work done on blades/sec. = Force × distance travelled/sec.

$$= \dot{m}_s(C_{w_1} + C_{w_0}) \times C_{bl}$$

Power per wheel

$$= \dot{m}_s(C_{w_1} + C_{w_0}) \times C_{bl}$$

$$= \frac{\dot{m}_s C_w C_{bl}}{1000}\ \text{kW} \quad ...(19.2)$$

$$(\because\ C_w = C_{w_1} + C_{w_0})$$

Blade or diagram efficiency $= \dfrac{\text{Work done the blade}}{\text{Energy supplied to the blade}}$

$$= \frac{\dot{m}_s(C_{w_1} + C_{w_0}).C_{bl}}{\dfrac{\dot{m}\,C_1^{\,2}}{2}}$$

$$= \frac{2\,C_{bl}(C_{w_1} + C_{w_0})}{C_1^{\,2}} \quad ...(19.3)$$

If h_1 and h_2 be the total heats before and after expansion through the nozzles, then $(h_1 - h_2)$ is the heat drop through a stage of fixed blades ring and moving blades ring.

$\therefore$ **Stage efficiency,**

$$\eta_{stage} = \frac{\text{Work done on blade per kg of steam}}{\text{Total energy supplied per kg of steam}}$$

$$= \frac{C_{bl}(C_{w_1} + C_{w_2})}{(h_1 - h_2)} \quad ...(19.4)$$

Now, nozzle efficiency $= \dfrac{C_1^2}{2(h_1 - h_2)}$

Also η_{stage} = Blade efficiency × nozzle efficiency

$$= \frac{2C_{bl}(C_{w_1} + C_{w_2})}{C_1^2} \times \frac{C_1^2}{2(h_1 - h_2)} = \frac{C_{bl}(C_{w_1} + C_{w_2})}{(h_1 - h_2)}$$

The **axial thrust** on the wheel is due to *difference* between the velocities of flow at entrance and outlet.

Axial force on the wheel = Mass of steam × axial acceleration

$$= \dot{m}_s(C_{f_1} - C_{f_0}) \quad ...(19.5)$$

The *axial force on the wheel must be balanced or must be taken by a thrust bearing.*

Energy converted to heat by blade friction

= loss of kinetic energy during flow over blades

$$= \dot{m}_s(C_{r_1}^2 - C_{r_0}^2) \quad ...(19.6)$$

19.7.3. Blade Velocity Co-efficient

In an impulse turbine, if friction is neglected the relative velocity will remain unaltered as it passes over blades. In practice the flow of steam over the blades is resisted by friction. The effect of the friction is to reduce the relative velocity of steam as it passes over the blades. In general there is a loss of 10 to 15 per cent in the relative velocity. Owing to friction in the blades, C_{r_0} is less than C_{r_1} and we may write

$$C_{r_0} = K \, . \, C_{r_1} \quad ...(19.7)$$

where K is termed a **blade velocity co-efficient.**

19.7.4. Expression for optimum Value of the Ratio of Blade Speed to Steam Speed (for maximum efficiency) for a Single Stage Impulse Turbine

Refer Fig. 19.7.

$$C_w = PQ = MP + MQ = C_{r_1} \cos\theta + C_{r_0} \cos\phi$$

$$= C_{r_1} \cos\theta \left[1 + \frac{C_{r_0}\cos\phi}{C_{r_1}\cos\theta}\right]$$

$$= C_{r_1} \cos\theta\,(1 + K \, . \, Z) \text{ where } Z = \frac{\cos\phi}{\cos\theta} \quad ...(i)$$

Generally, *the angles θ and φ are nearly equal for impulse turbine* and hence it can safely be assumed that Z is a constant.

But, $C_{r_1} \cos\theta = MP = LP - LM = C_1 \cos\alpha - C_{bl}$

From eqn. (*i*), $C_w = (C_1 \cos\alpha - C_{bl})(1 + K \, . \, Z)$

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We know that, Blade efficiency, $\eta_{bl} = \dfrac{2C_{bl} \cdot C_w}{C_1^2}$ ...(*ii*)

$$\eta_{bl} = \frac{2C_{bl}(C_1 \cos\alpha - C_{bl})(1 + KZ)}{C_1^2}$$

$$= 2(\rho \cos\alpha - \rho^2)(1 + KZ)$$

$$= 2\rho(\cos\alpha - \rho)(1 + KZ) \quad ...(iii)$$

where $\rho = \dfrac{C_{bl}}{C_1}$ is the ratio of *blade speed to steam speed* and is commonly called as **"Blade speed ratio"**.

For particular impulse turbine α, K and Z may assumed to be constant and from equation (*iii*) it can be seen clearly that η_{bl} depends on the value of ρ only. Hence differentiating (*iii*),

$$\frac{d\eta_{bl}}{d\rho} = 2(\cos\alpha - 2\rho)(1 + KZ)$$

For a maximum or minimum value of η_{bl} this should be zero

$$\cos\alpha - 2\rho = 0, \quad \therefore \quad \rho = \frac{\cos\alpha}{2}$$

Now, $$\frac{d^2\eta_{bl}}{d\rho^2} = 2(-2)(1 + KZ) = -4(1 + KZ)$$

which is a *negative quantity* and thus the value so obtained is the *maximum*.

Optimum value of ratio of blade speed to steam speed is

$$\rho_{opt} = \frac{\cos\alpha}{2} \quad ...(19.8)$$

Substituting this value of ρ in eqn. (*iii*), we get

$$(\eta_{bl})_{max} = 2 \times \frac{\cos\alpha}{2}\left(\cos\alpha - \frac{\cos\alpha}{2}\right)(1 + K \cdot Z)$$

$$= \frac{\cos^2\alpha}{2}(1 + KZ) \quad ...(19.9)$$

It is sufficiently accurate to assume symmetrical blades ($\theta = \phi$) and no friction in fluid passage for the purpose of analysis.

$\therefore$ $Z = 1$ and $K = 1$

$\therefore$ $(\eta_{bl})_{max} = \cos^2\alpha$ ...(19.10)

The work done per kg of steam is given by

$$W = (C_{w_1} + C_{w_0})\, C_{bl}$$

Substituting the value of $C_{w_1} + C_{w_0}$ $(= C_w)$

$$W = (C_1 \cos\alpha - C_{bl})(1 + KZ)C_{bl} = 2C_{bl}(C_1 \cos\alpha - C_{bl}) \text{ when } K = 1 \text{ and } Z = 1$$

The maximum value of W can be obtained by substituting the value of $\cos\alpha$ from equation (19.8),

$$\cos\alpha = 2\rho = 2\frac{C_{bl}}{C_1}$$

$\therefore$ $$W_{max} = 2C_{bl}(2C_{bl} - C_{bl}) = 2C_{bl}^2 \quad ...(19.11)$$

It is obvious from the equation (19.8) that the blade velocity should be *approximately* half of absolute velocity of steam jet coming out from the nozzle (fixed blade) for *the maximum work developed per kg of steam or for maximum efficiency. For the other values of blade speed the absolute velocity at outlet from the blade will increase, consequently, more energy will be carried away by the steam and efficiency will decrease.*

For equiangular blades with no friction losses, optimum value of $\frac{C_bl}{C_1}$ corresponds to the case, when the outlet absolute velocity is *axial* as shown in Fig. 19.8.

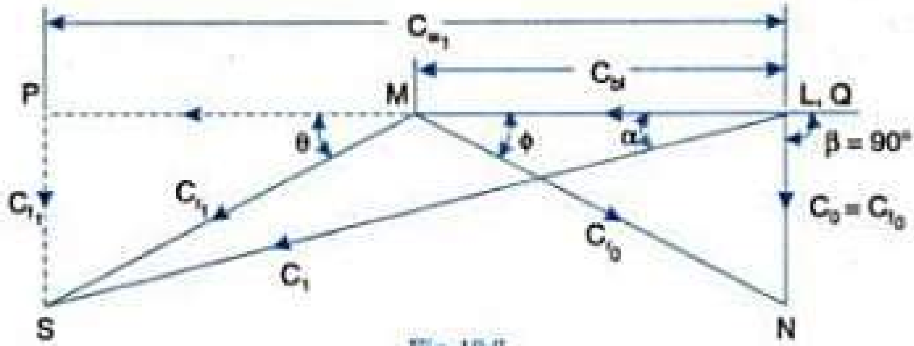

Fig. 19.8

Since the discharge is axial $\beta = 90°$, $\therefore C_0 = C_{f_0}$ and $C_{w_0} = 0$.

The variations of η_{bl} or work developed per kg of steam with $\frac{C_{bl}}{C_1}$ is shown in Fig. 19.9. This figure shows that :

(*i*) When $\frac{C_{bl}}{C_1} = 0$, the work done becomes zero as the distance travelled by the blade (C_{bl}) is zero, even though the torque on the blade is maximum.

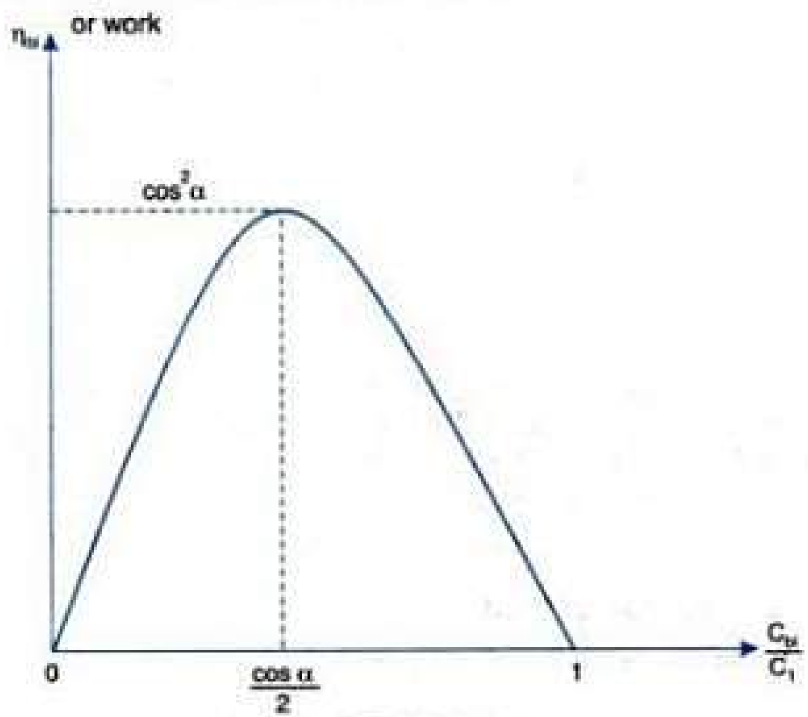

Fig. 19.9

(*ii*) The maximum efficiency is $\cos^2 \alpha$ and maximum work done per kg of steam is $2C^2_{bl}$ when $\dfrac{C_{bl}}{C_1} = \cos \alpha/2$.

(*iii*) When $\dfrac{C_{bl}}{C_1} = 1$, the work done is zero as the torque acting on the blade becomes zero even though the distance travelled by the blade is maximum.

When the high pressure steam is expanded from the boiler pressure to condenser pressure in a single stage of nozzle, the absolute velocity of steam becomes maximum and blade velocity also becomes tremendously high. In such a case, *a velocity compounded stage is used to give lower blade speed ratio* and *better utilization of the kinetic energy of the steam*. The arrangement of velocity compounding has already been dealt with.

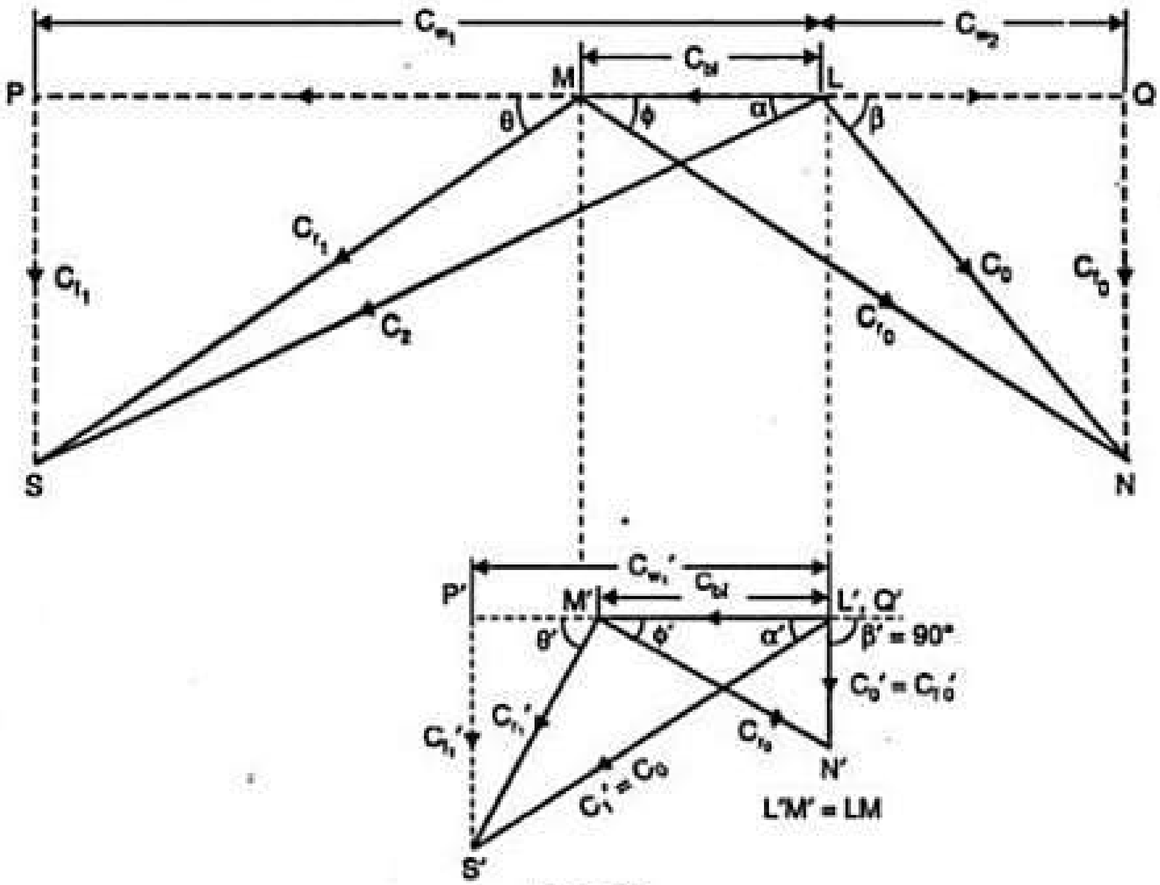

Fig. 19.10

Fig. 19.10 shows the velocity diagrams for the first and second row of moving blades of *velocity compounded unit*. The speed and angles are such that the final absolute velocity of the steam leaving the second row is *axial*. With this arrangement, *the K.E. carried by the steam is minimum*, therefore, *the efficiency becomes maximum*.

The velocity of blades (C_{bl}) is same for both the rows since they are mounted on the same shaft.

Consider first row of moving blades :

Work done per kg of steam, $\quad W_1 = C_{bl}\,(C_{w_1} + C_{w_2})$

$$= C_{bl}[C_{r_1} \cos \theta + C_{r_0} \cos \phi]$$

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If there is no friction loss and symmetrical blading is used, then

$$C_{r_1} = C_{r_0} \quad \text{and} \quad \theta = \phi$$

$\therefore$
$$W_1 = C_{bl} \times 2C_{r_1} \cos\theta = 2C_{bl}(C_1 \cos\alpha - C_{bl}) \qquad ...(19.12)$$

The magnitude of absolute velocity of steam leaving the first row and entering into the second row of moving blades is same and *its direction only is changed.*

$\therefore$
$$C_1' = C_0$$

Consider **second row of moving blades :**

Work done per kg. $\quad W_2 = C_{bl} \,.\, C'_{w_1}$ as $C'_{w_0} = 0$ because discharge is *axial* and $\beta' = 90°$

Alternately, $\quad W_2' = C_{bl}\,[C'_{r_1} \cos\theta' + C'_{r_0} \cos\phi']$

For symmetrical blades $\quad \theta' = \phi'$

and, if there is no friction loss, then $C'_{r_1} = C'_{r_0}$

$\therefore$
$$W_2 = 2C_{bl}\, C'_{r_1} \cos\theta'$$
$$= 2C_{bl}\,(C_1' \cos\alpha' - C_{bl}) \qquad ...(19.13)$$

Now α' may be equal to β.

$\therefore$
$$C_1' \cos\alpha' = C_0 \cos\beta = C_{r_0} \cos\phi - C_{bl}$$
$$= C_{r_1} \cos\theta - C_{bl} = (C_1 \cos\alpha - C_{bl}) - C_{bl}$$
$$= C_1 \cos\alpha - 2C_{bl}$$

Substituting the value of $C_1' \cos\alpha'$ in eqn. (19.13), we get

$$W_2 = 2C_{bl}\,[(C_1 \cos\alpha - 2C_{bl}) - C_{bl}]$$
$$= 2C_{bl}\,(C_1 \cos\alpha - 3C_{bl}) \qquad ...(19.14)$$

Total work done per kg of steam passing through both stages is given by

$$W_t = W_1 + W_2$$
$$= 2C_{bl}\,[C_1 \cos\alpha - C_{bl}] + 2C_{bl}\,[C_1 \cos\alpha - 3C_{bl}]$$
$$= 2C_{bl}\,(2C_1 \cos\alpha - 4C_{bl})$$
$$= 4C_{bl}\,(C_1 \cos\alpha - 2C_{bl}) \qquad ...(19.15)$$

The blade efficiency for two stage impulse turbine is given by

$$\eta_{bl} = \frac{W_t}{\frac{C_1^2}{2}} = 4C_{bl}\,[C_1 \cos\alpha_1 - 2C_{bl}] \times \frac{2}{C_1^2}$$
$$= \frac{8C_{bl}}{C_1^2}\,(C_1 \cos\alpha - 2C_{bl}) = 8\,\frac{C_{bl}}{C_1}\left(\cos\alpha - 2\,.\,\frac{C_{bl}}{C_1}\right)$$
$$= 8\rho\,(\cos\alpha - 2\rho) \qquad ...(19.16)$$

where ρ (velocity ratio) $= \dfrac{C_{bl}}{C_1}$.

The blade efficiency for two stage turbine will be maximum when $\dfrac{d\eta_{bl}}{d\rho} = 0$

$\therefore$
$$\frac{d}{d\rho}\,[8\rho \cos\alpha - 16\rho^2] = 0$$

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$$\therefore \qquad 8 \cos \alpha - 32\rho = 0$$

From which, $\rho = \dfrac{\cos \alpha}{4}$ ...(19.17)

Substituting this value in eqn. (19.16), we get

$$(\eta_{bl})_{max} = 8 \cdot \frac{\cos \alpha}{4}\left[\cos \alpha - 2 \cdot \frac{\cos \alpha}{4}\right] = \cos^2 \alpha \qquad ...(19.18)$$

The maximum work done per kg of steam is obtained bv substituting the value of

$$\rho = \frac{C_{bl}}{C_1} = \frac{\cos \alpha}{4}$$

or

$$C_1 = \frac{4C_{bl}}{\cos \alpha} \text{ in the eqn. (19.15).}$$

$$\therefore \qquad (W_b)_{max} = 4C_{bl}\left(\frac{4C_{bl}}{\cos \alpha} \cdot \cos \alpha - 2C_{bl}\right)$$

$$= 8{C_{bl}}^2 \qquad ...(19.19)$$

The present analysis is done for *two stages* only.The similar procedure is adopted for analysing the problem with three or four stages.

In general, optimum blade speed ratio for maximum blade efficiency or maximum work done is given by

$$\rho = \frac{\cos \alpha}{2 \cdot n} \qquad ...(19.20)$$

and work done in the last row $= \dfrac{1}{2^n}$ of total work ...(19.21)

where n is the number of moving/rotating blade rows in series.

As the number of rows increases, the utility of last row decreases. In practice, *more than* two rows are hardly preferred.

19.7.5. Advantages of Velocity Compounded Impulse Turbine

1. Owing to relatively large heat drop, a velocity-compounded impulse turbine requires a comparatively small number of stages.
2. Due to number of stages being small, its cost is less.
3. As the number of moving blades' rows in a wheel increases, the maximum stage efficiency and optimum value of ρ decreases.
4. Since the steam temperature is sufficiently low in a two or three row wheel, therefore, cast iron cylinder may be used. This will cause saving in material cost.

Disadvantages of velocity-compounded impulse turbine :

1. It has high steam consumption and low efficiency (Fig. 19.11 on next page).
2. In a single row wheel, the steam temperature is high so cast iron cylinder cannot be used due to phenomenon of growth ; cast steel cylinder is used which is costlier than cost iron.

Example 19.1. *A stage of a steam turbine is supplied with steam at a pressure of 50 bar and 350°C, and exhausts at a pressure of 5 bar. The isentropic efficiency of the stage is 0.82 and the steam consumption is 2270 kg/min. Determine the power output of the stage.*

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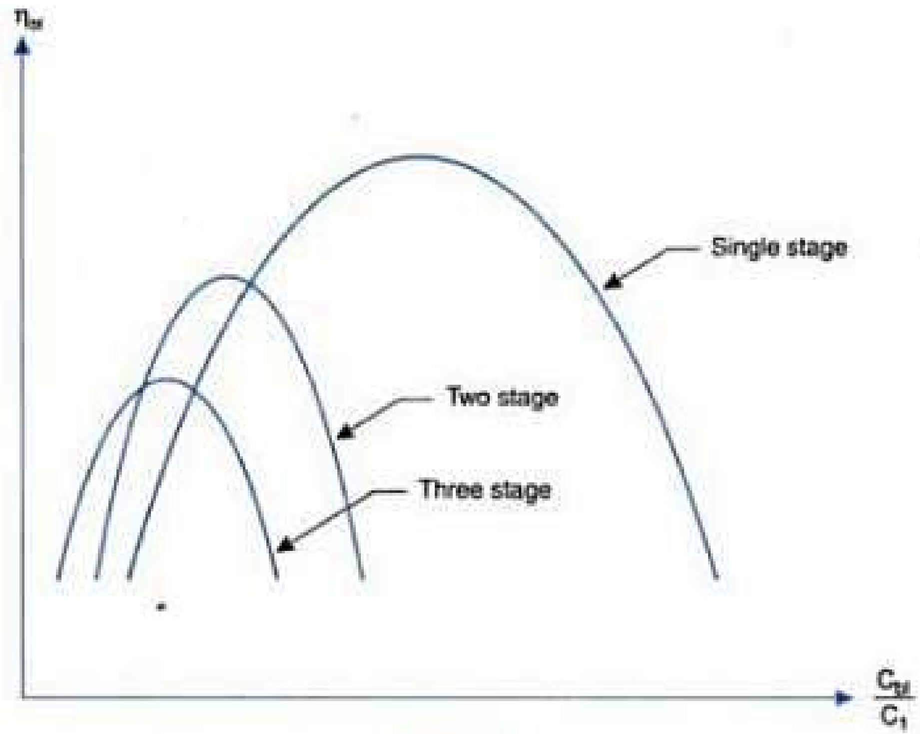

Fig. 19.11

Solution. Steam supply pressure, $p_1 = 50$ bar, 350°C

Exhaust pressure $p_2 = 5$ bar

Isentropic efficiency of the stage, $\eta_{stage} = 0.82$

Steam consumption, $m_s = 2270$ kg/min

Power output of the stage, P :

Refer Fig. 19.12.

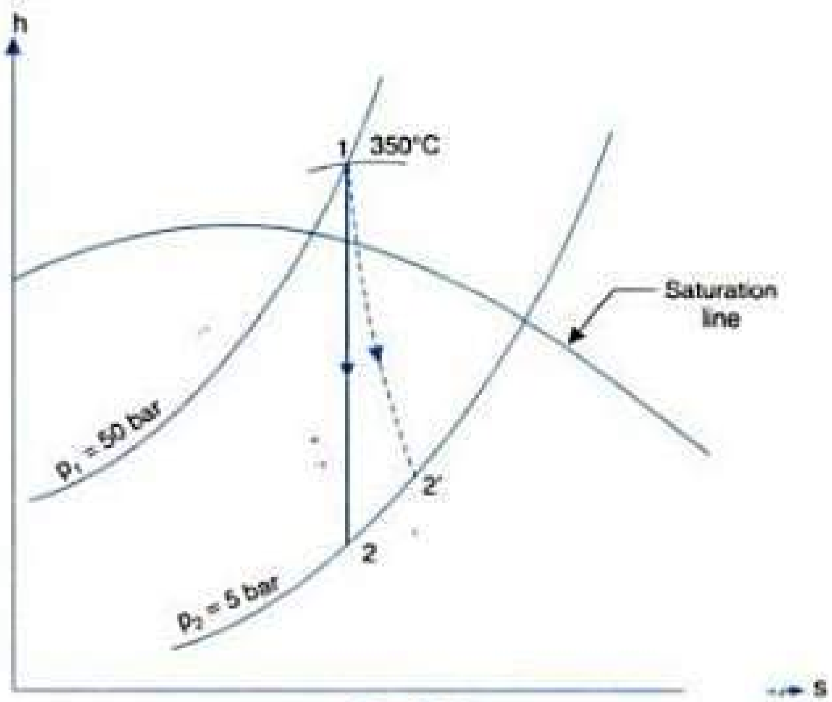

Fig. 19.12

From **Mollier chart :**

$$h_1 = 3130.7 \text{ kJ/kg of steam}$$
$$h_2 = 2640 \text{ kJ/kg of steam}$$

Isentropic heat drop $= h_1 - h_2 = 3130.7 - 2640 = 490.7$ kJ/kg

Actual heat drop $= h_1 - h_2'$

But,
$$\eta_{isen.\,(stage)} = \frac{h_1 - h_2'}{h_1 - h_2}$$

or
$$0.82 = \frac{h_1 - h_2'}{490.7} = \text{or } h_1 - h_2' = 0.82 \times 490.7 = 402.4 \text{ kJ/kg}$$

$\therefore$ *Power developed* $= m_s(h_1 - h_2')$

$$= \frac{2270}{60} \times 402.4 \text{ kW} = \mathbf{15224 \text{ kW.}} \quad \textbf{(Ans.)}$$

Example 19.2. *In a De Laval turbine steam issues from the nozzle with a velocity of 1200 m/s. The nozzle angle is 20°, the mean blade velocity is 400 m/s, and the inlet and outlet angles of blades are equal. The mass of steam flowing through the turbine per hour is 1000 kg. Calculate :*

(i) Blade angles.

(ii) Relative velocity of steam entering the blades.

(iii) Tangential force on the blades.

(iv) Power developed.

(v) Blade efficiency.

Take blade velocity co-efficient as 0.8.

Solution. Absolute velocity of steam entering the blade, $C_1 = 1200$ m/s

Nozzle blade, $\quad \alpha = 20°$

Mean blade velocity, $\quad C_{bl} = 400$ m/s

Inlet blade angle, $\quad \theta =$ Outlet blade angle, ϕ

Blade velocity co-efficient, $\quad K = 0.8$

Mass of steam flowing through the turbine, $m_s = 1000$ kg/h.

Refer Fig. 19.13. Procedure of drawing the inlet and outlet triangles (LMS and LMN) respectively is as follows :

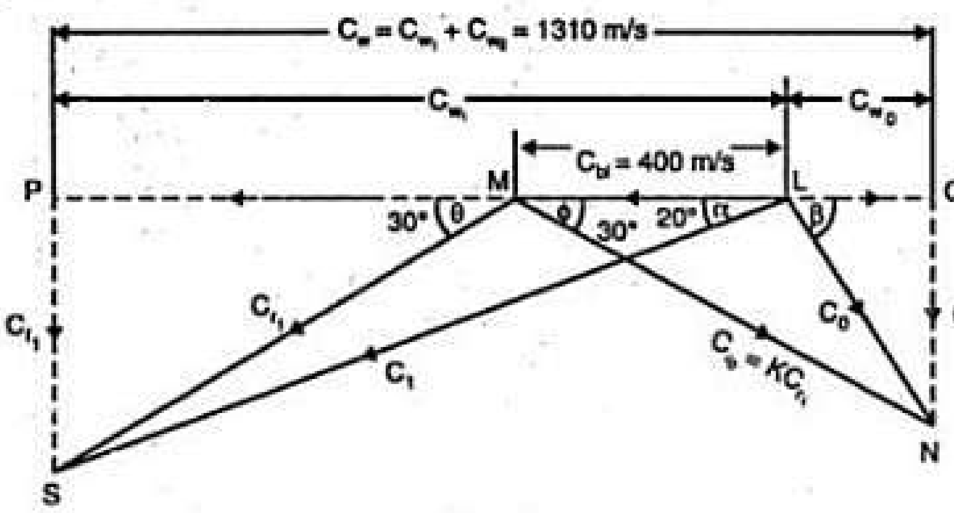

Fig. 19.13

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- Select a suitable scale and draw line LM to represent C_{bl}(= 400 m/s).
- At point L make angle of 20° (α) and cut length LS to represent velocity C_1(= 1200 m/s). Join MS. Produce M to meet the perpendicular drawn from S at P. Thus *inlet triangle* is completed.

By measurement : $\theta = 30°$, $C_{r_1} = MS = 830$ m/s

$\theta = \phi = 30°$ (given)

Now, $C_{r_0} = KC_{r_1} = 0.8 \times 830 = 664$ m/s

- At point M make an angle of 30° (ϕ) and cut the length MN to represent C_{r_0} (= 664 m/s). Join LN. Produce L to meet the perpendicular drawn from N at Q. Thus *outlet triangle* is completed.

(*i*) **Blade angles θ, ϕ :**

As the blades are symmetrical (given)

$\therefore$ $\theta = \phi =$ **30°. (Ans.)**

(*ii*) **Relative velocity of steam entering the blades, C_{r_1} :**

$C_{r_1} = MS =$ **830 m/s. (Ans.)**

(*iii*) **Tangential force on the blades :**

$$\text{Tangential force } = \dot{m}_s(C_{w_1} + C_{w_0}) = \frac{1000}{60 \times 60}(1310) = \mathbf{363.8\ N.} \quad \textbf{(Ans.)}$$

(*iv*) **Power developed, P :**

$$P = \dot{m}_s(C_{w_1} + C_{w_0})C_{bl} = \frac{1000}{60 \times 60} \times \frac{1310 \times 400}{1000} \text{ kW} = \mathbf{145.5\ kW.} \quad \textbf{(Ans.)}$$

(*v*) **Blade efficiency, η_{bl} :**

$$\eta_{bl} = \frac{2C_{bl}(C_{w_1} + C_{w_0})}{C_1^2} = \frac{2 \times 400 \times 1310}{1200^2} = \mathbf{72.8\%.} \quad \textbf{(Ans.)}$$

Example 19.3. *The velocity of steam exiting the nozzle of the impulse stage of a turbine is 400 m/s. The blades operate close to the maximum blading efficiency. The nozzle angle is 20°. Considering equiangular blades and neglecting blade friction, calculate for a steam flow of 0.6. kg/s, the diagram power and the diagram efficiency.* **(GATE)**

Solution. *Given :* $C_1 = 400$ m/s, $\alpha = 20°$, $\theta = \phi$; $\dot{m}_s = 0.6$ kg/s.

For maximum blade efficiency, $\rho = \dfrac{C_{bl}}{C_1} = \dfrac{\cos\alpha}{2}$

$$\therefore \quad \frac{C_{bl}}{400} = \frac{\cos 20°}{2} \quad \text{or} \quad C_{bl} = 187.9 \text{ m/s}$$

$$C_{w_1} = C_1 \cos\alpha = 400 \cos 20° = 375.9 \text{ m/s}$$

$$C_{f_1} = C_1 \sin\alpha = 400 \sin 20° = 136.8 \text{ m/s}$$

$$\tan\theta = \frac{C_{f_1}}{C_{w_1} - C_{bl}} = \frac{136.8}{375.9 - 187.9} = 0.727$$

$$\therefore \quad \theta = \tan^{-1}(0.727) = 36°$$

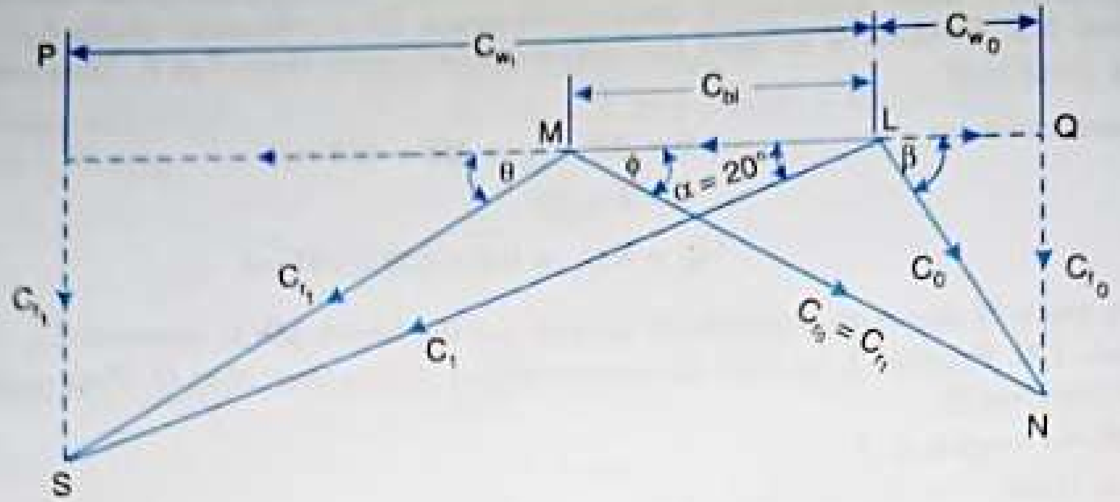

Fig. 19.14

Now, $C_{r_1} \sin\theta = C_{f_1}$ or $C_{r_1} = \dfrac{C_{f_1}}{\sin\theta}$, $\therefore$ $C_{r_1} = \dfrac{136.8}{\sin 36°} = 232.7$ m/s

Now neglecting friction, $C_{r_0} = C_{r_1} = 232.7$ m/s

Since the blades are *equiangular*, therefore,

$$\theta = \phi = 36°$$

$$C_{w_0} = C_{r_0} \cos 36° - C_{bl}$$
$$= 237.7 \cos 36° - 187.9 = 0.36 \text{ m/s}$$

$\therefore$ $$C_w = C_{w_1} + C_{w_0} = 375.9 + 0.36 = 376.26 \text{ m/s}$$

Diagram power, $$P = \dot{m}_s(C_{w_1} + C_{w_0}) \times C_{bl} \times 10^{-3} \text{ kW}$$
$$= 0.6 \times 376.26 \times 187.9 \times 10^{-3} = \mathbf{42.4\ kW.} \quad \textbf{(Ans.)}$$

Blade or diagram efficiency, $$\eta_{bl} = \frac{2C_{bl}(C_{w_1} + C_{w_0})}{C_1^2}$$
$$= \frac{2 \times 187.9 \times 376.26}{(400)^2} = 0.884 \quad \text{or} \quad \mathbf{88.4\%} \quad \textbf{(Ans.)}$$

Example 19.4. *A single stage steam turbine is supplied with steam at 5 bar, 200°C at the rate of 50 kg/min. It expands into a condenser at a pressure of 0.2 bar. The blade speed is 400 m/s. The nozzles are inclined at an angle of 20° to the plane of the wheel and the outlet blade angle is 30°. Neglecting friction losses, determine the power developed, blade efficiency, and stage efficiency.*

(U.P.S.C., 1994)

Solution. *Given* : $p_1 = 5$ bar, 200°C ; $p_2 = 0.2$ bar, $m_s = 50$ kg/min, $C_{bl} = 400$ m/s ; $\alpha = 20°$, $\phi = 30°$; $C_{r_1} = C_{r_0}$ (because friction losses are neglected)

Refer Fig. 19.15

From steam tables : ***At 5 bar, 200°C :*** $h_1 = 2855.4$ kJ/kg ; $s_1 = 7.0592$ kJ/kg K

At 0.2 bar : $h_{f_2} = 251.5$ kJ/kg, $h_{fg_2} = 2358.4$ kJ/kg

$s_{f_2} = 0.832$ kJ/kg K ; $s_{fg} = 7.0773$ kJ/kg K

Since the steam expansion takes place *isentropically*,

$\therefore \quad s_1 = s_2$

or $\quad 7.0592 = 0.8321 + x_2 \times 7.0773$

or $\quad x_2 = \dfrac{7.0592 - 0.8321}{7.0773} = 0.88$

$\therefore$ Enthalpy of steam at 0.2 bar,

$$h_2 = h_{f_2} + x_2\, h_{fg_2}$$

$$= 251.5 + 0.88 \times 2358.4 = 2326.9 \text{ kJ/kg}$$

Enthalpy drop $= h_1 - h_2 = 2855.4 - 2326.9 = 528.5$ kJ/kg

Velocity of steam entering the blades,

$$C_1 = 44.7\sqrt{h_1 - h_2} = 44.7\sqrt{528.5} \approx 1028 \text{ m/s}$$

The velocity diagram is shown in Fig. 19.15

Now, $\quad C_{w_1} = 1028 \cos 20^\circ = 966$ m/s

$$C_{f_1} = 1028 \sin 20^\circ = 351.6 \text{ m/s}$$

$$\tan\theta = \frac{C_{f_1}}{C_1 \cos 20^\circ - C_{bl}} = \frac{351.6}{1028 \cos 20^\circ - 400} = 0.6212$$

$\therefore \quad \theta = \tan^{-1}(0.6212) = 31.85^\circ$

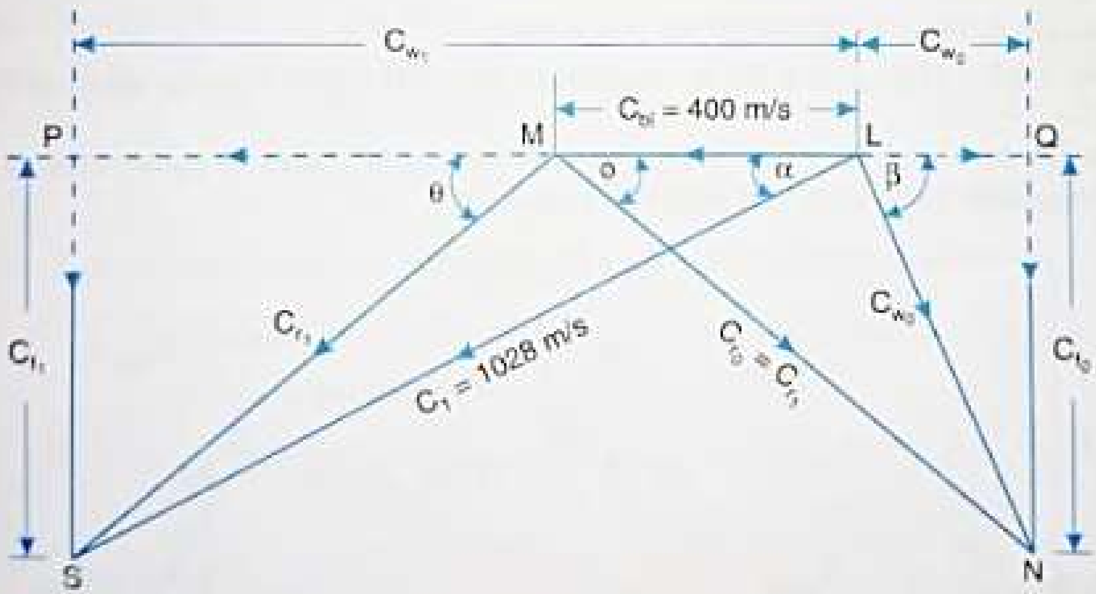

Fig. 19.15

Now, $\quad C_{r_1} \sin 31.85^\circ = C_{f_1} = 351.6$

$\therefore \quad C_{r_1} = \dfrac{351.6}{\sin 31.85^\circ} = 666$ m/s

$\therefore \quad C_{w_2} = C_{r_2} \cos 30^\circ - C_{bl} \qquad (\because C_{r_2} = C_{r_1})$

$$= 666 \cos 30^\circ - 400 = 177 \text{ m/s}$$

Power developed, $P = \dot{m}_s(C_{w_1} + C_{w_0})\ C_{bl}$

$$= \frac{50}{60}\,(966 + 177) \times 400 \times 10^{-3}\ \text{kW} = \mathbf{381\ kW.}\quad \textbf{(Ans.)}$$

Blade efficiency, $\eta_{bl} = \dfrac{2C_{bl}(C_{w_1} + C_{w_0})}{C_1^2}$

$$= \frac{2 \times 400 \times (966 + 177)}{(1028)^2} = \mathbf{0.865\ \text{or}\ 86.5\%.}\quad \textbf{(Ans.)}$$

Since there are no losses, therefore,

Stage efficiency = blade efficiency = **86.5%. (Ans.)**

Example 19.5. *The following data relate to a single stage impulse turbine :*

Steam velocity = 600 m/s ; Blade speed = 250 m/s

Nozzle angle = 20° ; Blade outlet angle = 25°.

Neglecting the effect of friction, calculate the work developed by the turbine for the steam flow rate of 20 kg/s. Also calculate the axial thrust on the bearings.

Solution. Absolute velocity of steam entering the blades, C_1 = 600 m/s

Blade speed, C_{bl} = 250 m/s ; Nozzle angle, $\alpha = 20°$

Blade outlet angle, $\phi = 25°$; Steam flow rate, $\dot{m}_s$ = 20 kg/s

Refer Fig. 19.16.

- Triangle *LMS* is drawn with the above data.
- Then angle *LMN i.e.*, $\phi = 25°$ is drawn such that $NM = MS$ (because effect of friction is to be neglected *i.e.*, $K = 1$).
- Join *LN* by vector C_0 which represents the velocity of steam at outlet from the wheel, This completes both inlet and outlet triangles.

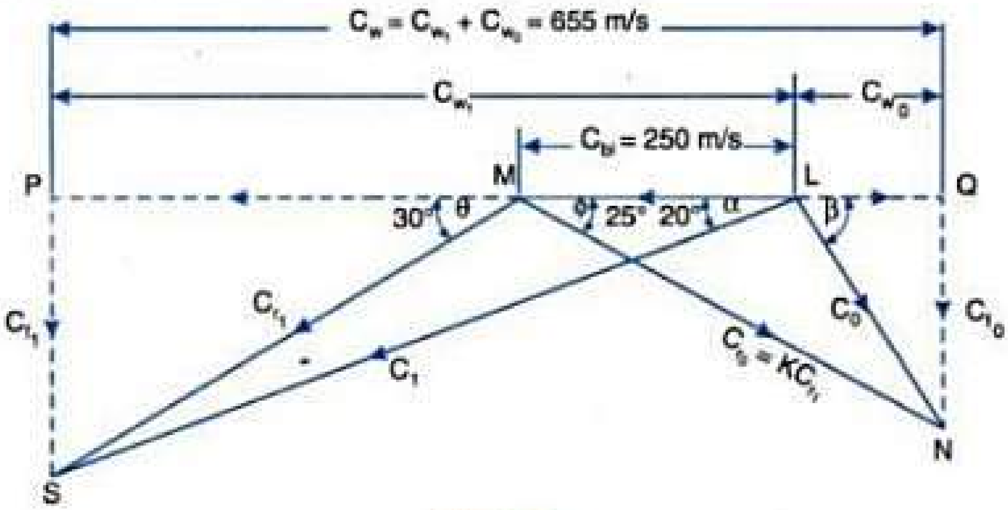

Fig. 19.16

By measurement :

$$C_w = C_{w_1} + C_{w_0} = 655\ \text{m/s} ;\ C_{f_1} = 200\ \text{m/s} ;\ C_{f_0} = 160\ \text{m/s.}$$

Work developed, W :

$$W = \dot{m}_s(C_{w_1} + C_{w_0})\ C_{bl} = 20 \times 655 \times 250$$

$$= \mathbf{3275000\ Nm/s.}\quad \textbf{(Ans.)}$$

Axial Thrust :

Axial thrust $= \dot{m}_s(C_{f_1} - C_{f_0}) = 20(200 - 160) = $ **800 N.** (Ans.)

Example 19.6. *A single row impulse turbine develops 132.4 kW at a blade speed of 175 m/s, using 2 kg of steam per sec. Steam leaves the nozzle at 400 m/s. Velocity coefficient of the blades is 0.9. Steam leaves the turbine blades axially.*

Determine nozzle angle, blade angles at entry and exit, assuming no shock.

Solution. Power developed, $P = 132.4$ kW

Blade speed, $C_{bl} = 175$ m/s

Steam used, $\dot{m}_s = 2$ kg/s

Velocity of steam leaving the nozzle, $C_1 = 400$ m/s

Blade velocity co-efficient, $K = 0.9$

Power developed, $$P = \dot{m}_s \frac{(C_{w_1} + C_{w_0}) \times C_{bl}}{1000} \text{ kW}$$

$$132.4 = \frac{2(C_{w_1} + C_{w_0}) \times 175}{1000} \quad \text{or} \quad (C_{w_1} + C_{w_0}) = \frac{132.4 \times 1000}{2 \times 175} = 378 \text{ m/s}$$

$C_{w_0} = 0$, since the *discharge is axial*.

Construct the velocity diagram as shown in Fig. 19.17.

In this diagram $\frac{C_{r_0}}{C_{r_1}} = 0.9$, $\beta = 90°$, since the *discharge is axial* ; and

$$C_{w_1} + C_{w_0} = PL = PQ = 378 \text{ m/s.}$$

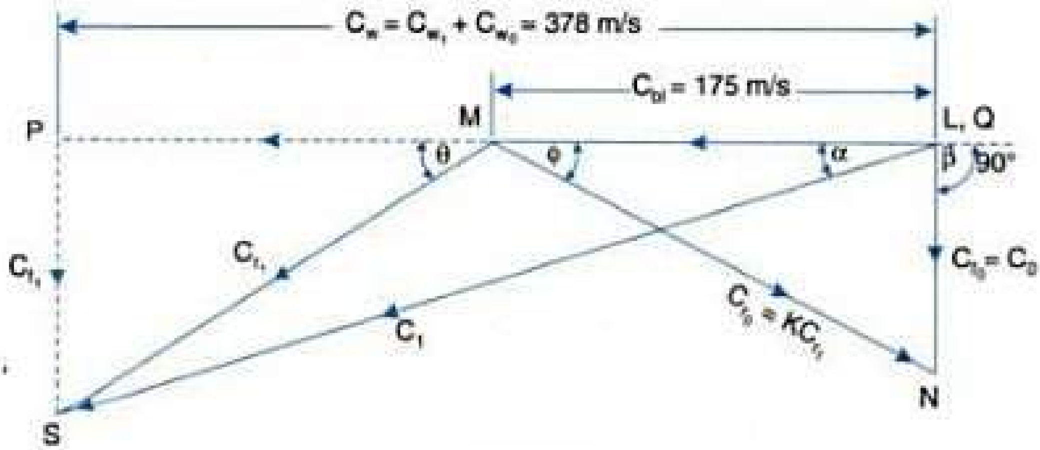

Fig. 19.17

From the diagram (by measurement) :

Nozzle angle, $\alpha = $ **21°.** (Ans.)

Blade inlet angle, $\theta = $ **36°.** (Ans.)

Blade outlet angle, $\phi = $ **32°.** (Ans.)

Example 19.7. *A simple impulse turbine has a mean blade speed of 200 m/s. The nozzles are inclined at 20° to the plane of rotation of the blades. The steam velocity from nozzles is 600 m/s. The turbine uses 3500 kg/h of steam. The absolute velocity at exit is along the axits of the turbine. Determine :*

(iii) The diagram efficiency.

(iv) The axial thrust (per kg steam per second).

Assume inlet and outlet angles to be equal. **(U.P.S.C., 1998)**

Solution. *Given* : C_{bl} = 200 m/s ; α = 20° ; C_1 = 600 m/s ; m_s = 3500 kg/h ; β = 90° ; $\theta = \phi$.

(i) Inlet and exit angles of the blades, θ, ϕ :

Refer Fig. 19.18

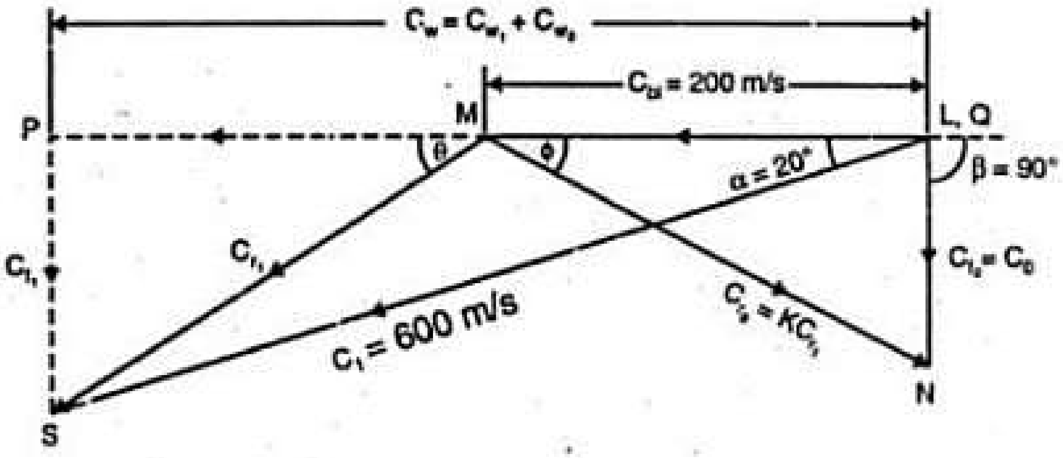

Fig. 19 18.

$$C_{f_1} = C_1 \sin 20° = 600 \sin 20° = 205.2 \text{ m/s}$$

$$\tan \theta = \frac{SP}{PM} = \frac{C_{f_1}}{C_1 \cos 20° - C_{bl}} = \frac{205.2}{600 \cos 20° - 200} = 0.564$$

$\therefore$ $\theta = \tan^{-1} (0.564) = 29.4°$. **(Ans.)**

Also $\theta = \phi$ ...(Given)

$\therefore$ $\phi = 29.4°$. **(Ans.)**

(ii) The power output of the turbine, P :

$$\mathbf{P} = \dot{m}_s (C_{w_1} + C_{w_2}) C_{bl}$$

$$= \frac{3500}{3600} (600 \cos 20° + 0) \times 200 \times 10^{-3} \text{ kW} = 109.6 \text{ kW}. \quad \textbf{(Ans.)}$$

(C_{w_2} = 0, since the discharge is axial)

(iii) The blade or diagram efficiency, η_{bl} :

$$\eta_{bl} = \frac{2C_{bl}(C_{w_1} + C_{w_2})}{C_1^2}$$

$$= \frac{2 \times 200(600 \cos 20° + 0)}{600^2} = 0.626 \text{ or } 62.6\%. \quad \textbf{(Ans.)}$$

(iv) The axial thrust (per kg steam per second) :

The axial thrust per kg per second

$$= 1 \times (C_{f_1} - C_{f_2}) \; N$$

where $C_{f_1} = C_1 \sin 20° = 600 \sin 20° = 205.2$ m/s.

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where $C_{f_1} = C_1 \sin 20° = 600 \sin 20° = 205.2$ m/s.

Now, $\dfrac{C_{f_0}}{C_{bl}} = \tan 29.4°$

$\therefore$ $C_{f_0} = 200 \times \tan 29.4° = 112.69$ m/s

Substituting the values, we get

Axial thrust (per kg steam per second)

$= 1 \times (205.2 - 112.69) =$ **92.51 N.** (Ans.)

Example 19.8. *Steam with absolute velocity of 300 m/s is supplied through a nozzle to a single stage impulse turbine. The nozzle angle is 25°. The mean diameter of the blade rotor is 1 metre and it has a speed of 2000 r.p.m. Find suitable blade angles for zero axial thrust. If the blade velocity co-efficient is 0.9 and the steam flow rate is 10 kg/s, calculate the power developed.*

Solution. Absolute velocity of steam entering the blade, $C_1 = 300$ m/s

Nozzle angle, $\alpha = 25°$

Mean diameter of the rotor blade, $D = 1$ m

Speed of the rotor, $N = 2000$ r.p.m.

Blade velocity co-efficient, $K = 0.9$

Steam flow rate, $\dot{m}_s = 10$ kg/s

Blade angles :

Blade speed, $C_{bl} = \dfrac{\pi DN}{60} = \dfrac{\pi \times 1 \times 2000}{60} = 105$ m/s.

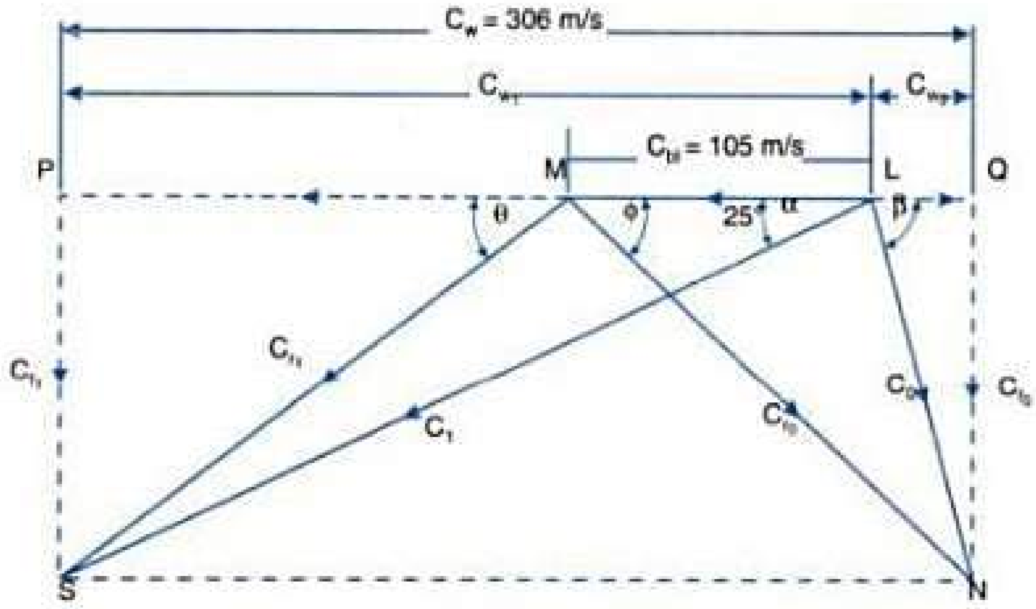

Fig. 19.19

With the above data (*i.e.*, $C_1 = 300$ m/s, $C_{bl} = 105$ m/s and $\alpha = 25°$) draw triangle LMS (Fig. 19.19). From S draw perpendicular SP on LM produced. Measure C_{r_1}.

— From S draw a line *parallel to LP* ($\because\ C_{f_1} = C_{f_2}$) and from point M draw an arc equal to $C_{r_2} (= 0.9\, C_{r_1})$ to get the point of intersection N. Complete the triangle LMN. From N draw perpendicular NQ on PL produced to get C_{f_2}.

Measure θ and ϕ (the blade angles) from the velocity diagram.

$$\theta = 37° \text{ and } \phi = 42°. \quad \text{(Ans.)}$$

Power developed, P :

$$P = \frac{\dot{m}_s (C_{w_1} + C_{w_2}) \times C_{bl}}{1000} = \frac{10 \times 306 \times 105}{1000} = \mathbf{321.3\ kW.} \quad \text{(Ans.)}$$

Example 19.9. *In an impulse turbine (with a single row wheel) the mean diameter of the blades is 1.05 m and the speed is 3000 r.p.m. The nozzle angle is 18°, the ratio of blade speed to steam speed is 0.42 and the ratio of the relative velocity at outlet from the blades to that at inlet is 0.84. The outlet angle of the blade is to be made 3° less than the inlet angle. The steam flow is 10 kg/s. Draw the velocity diagram for the blades and derive the following :*

(i) Tangential thrust on the blades *(ii) Axial thrust on the blades*

(iii) Resultant thrust on the blades *(iv) Power developed in the blades*

(v) Blading efficiency. **(P.U.)**

Solution. Mean diameter of the blades, $D = 1.05$ m

Speed of the turbine, $N = 3000$ r.p.m.

Nozzle angle, $\alpha = 18°$

Ratio of blade speed to steam speed, $\rho = 0.42$

Ratio, $\dfrac{C_{r_2}}{C_{r_1}} = 0.84$

Outlet blade angle, $\phi = \theta - 3°$

Steam flow rate $\dot{m}_s = 10$ kg/s

Blade speed, $C_{bl} = \dfrac{\pi DN}{60} = \dfrac{\pi \times 1.05 \times 3000}{60} = 164.5$ m/s

But $\rho = \dfrac{C_{bl}}{C_1} = 0.42$ (given)

$\therefore$ $C_1 = \dfrac{C_{bl}}{0.42} = \dfrac{164.5}{0.42} = 392$ m/s

With the data, $C_1 = 392$ m/s ;

$\alpha = 18°$, complete ΔLMS

$\theta = 30°$ (on measurement)

$\therefore$ $\phi = 30° - 3 = 27°$.

Now complete the ΔLMN by taking $\phi = 27°$ and $C_{r_2} = 0.84\ C_{r_1}$.

Finally complete the whole diagram as shown in Fig. 19.20.

(i) Tangential thrust on the blades :

Tangential thrust $= \dot{m}_s (C_{w_1} + C_{w_2}) = 10 \times 390 = \mathbf{3900\ N.}$ **(Ans.)**

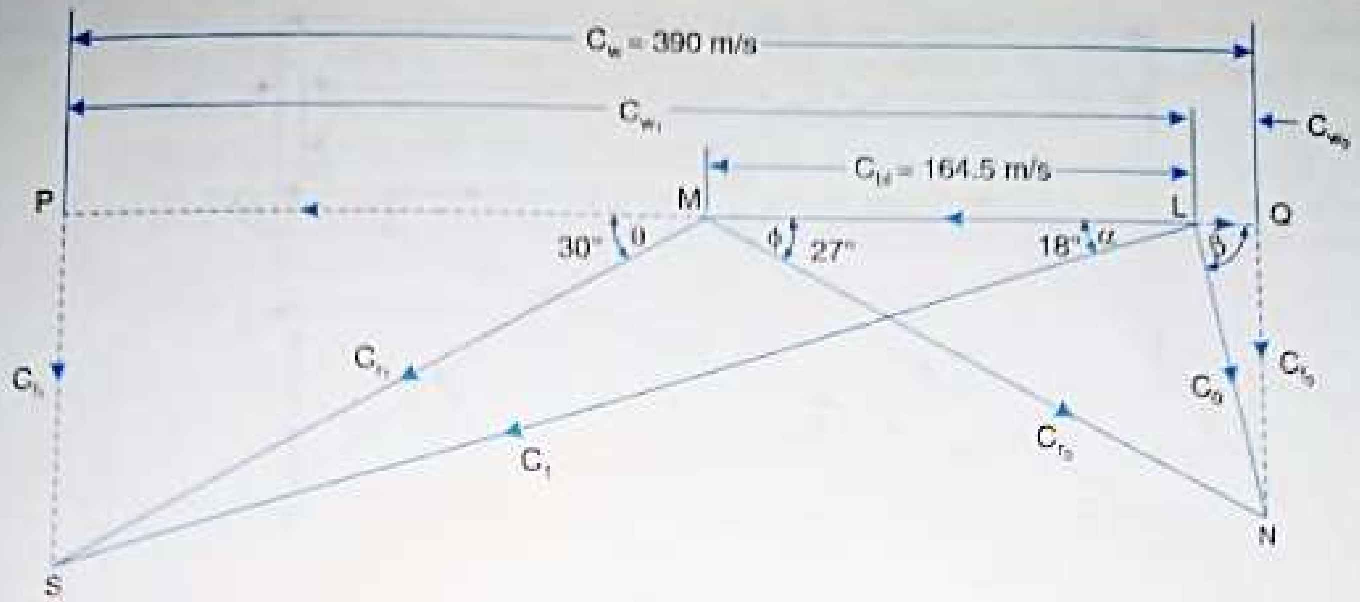

Fig. 19.20

(*ii*) **Axial thrust :**

Axial thrust $= \dot{m}_s(C_{f_1} - C_{f_0}) = 10\,(120 - 95) = \mathbf{250\ N.}$ (**Ans.**)

(*iii*) **Resultant thrust :**

Resultant thrust $= \sqrt{(3900)^2 + (250)^2} = \mathbf{3908\ N.}$ (**Ans.**)

(*iv*) **Power developed, P :**

$$P = \frac{\dot{m}_s(C_{w_1} + C_{w_0}) \times C_{bl}}{1000} = \frac{10 \times 390 \times 164.5}{1000} = \mathbf{641.55\ kW.}\ \textbf{(Ans.)}$$

(*v*) **Blading efficiency,** η_{bl} **:**

$$\eta_{bl} = \frac{2\,C_{bl}\,(C_{w_1} + C_{w_0})}{C_1^2} = \frac{2 \times 164.5 \times 390}{392^2} = \mathbf{83.5\%.}\ \textbf{(Ans.)}$$

Example 19.10. *In a stage of impulse reaction turbine provided with single row wheel, the mean diameter of the blades in 1 m. It runs at 3000 r.p.m. The steam issues from the nozzle at a velocity of 350 m/s and the nozzle angle is 20°. The rotor blades are equiangular. The blade friction factor is 0.86. Determine the power developed if the axial thrust on the end bearing of a rotor is 118 N.*

Solution. Mean diameter of the blades, $D = 1$ m

Speed of the turbine, $N = 3000$ r.p.m.

Velocity of steam issuing from the nozzle, $C_1 = 350$ m/s

Nozzle angle, $\alpha = 20°$

Blade angles, $\theta = \phi$

Blade friction factor, $K = 0.86$

Axial thrust $= 118\ N$

Power developed, P :

Blade, velocity, $$C_{bl} = \frac{\pi DN}{60} = \frac{\pi \times 1 \times 3000}{60} = 157\ \text{m/s}$$

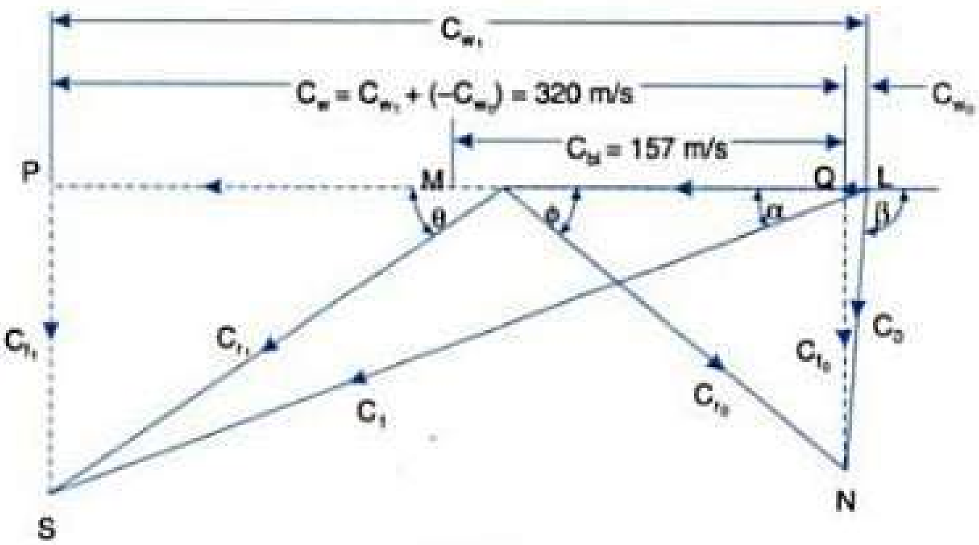

Fig. 19.21

— With the data, C_{bl} = 157 m/s, C_1 = 350 m/s, α = 20°, draw the Δ*LMS* (Fig. 19.21).

By measurement, $\theta = 35°$

Since the blades are equiangular, $\theta = \phi = 35°$

— Now with $\phi = 35°$ and $C_{r_0} = 0.86\ C_{r_1}$, complete the Δ*LMN*.

On measurement ; C_{f_1} = 120 m/s, C_{f_0} = 102.5 m/s

Also, axial thrust $\dot{m}_s(C_{f_1} - C_{f_0}) = 118$

$$\therefore \qquad \dot{m}_s = \frac{118}{C_{f_1} - C_{f_0}} = \frac{118}{(120 - 102.5)} = 6.74 \text{ kg/s}$$

Further in this case, $C_w = C_{w_1} + C_{w_0} = C_{w_1} + {}^*(-C_{w_0}) = 320$ m/s (*β > 90°)

Now, *power developed*,
$$P = \frac{\dot{m}_s(C_{w_1} + C_{w_0}) \times C_{bl}}{1000} \text{ kW}$$
$$= \frac{6.74 \times 320 \times 157}{1000} = \textbf{338.6 kW.} \quad \textbf{(Ans.)}$$

Example 19.11. *A simple impulse turbine has one ring of moving blades running at 150 m/s. The absolute velocity of steam at exit from the stage is 85 m/s at an angle of 80° from the tangential direction. Blade velocity co-efficient is 0.82 and the rate of steam flowing through the stage is 2.5 kg/s. If the blades are equiangular, determine :*

(i) Blade angles ; *(ii) Nozzle angle ;*

(iii) Absolute velocity of steam issuing from the nozzle ;

(iv) Axial thrust.

Solution. Blade velocity, C_{bl} = 150 m/s

Absolute velocity of steam at exit from the stage, C_0 = 85 m/s

Angle, $\beta = 80°$

Blade velocity co-efficient, $K = \dfrac{C_{r_0}}{C_{r_1}} = 0.82$

Rate of steam flowing through the stage, $\dot{m}_s$ = 2.5 kg/s

Blades are equiangular, *i.e.*, $\theta = \phi$.

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— With the above given data velocity triangle for *exit* can be drawn to a suitable scale. From that, value $\phi = \theta$ can be obtained. Also the value of C_{r_0} can be obtained which helps to get the value of C_{r_1} with the help of given value of 'K'. With these values having being known the inlet velocity triangle of the velocity diagram can be completed to get the value of C_1, the absolute velocity of steam issuing from the nozzle and value of axial thrust can also be calculated. The Fig. 19.22 gives the velocity diagram of the turbine stage to a suitable scale.

— From the outlet velocity ΔLMN

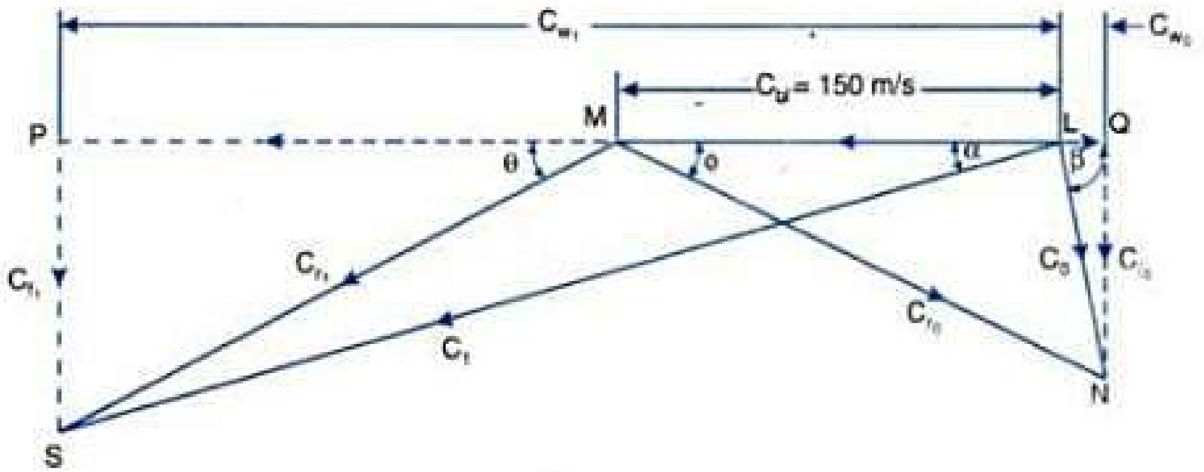

Fig. 19.22

By measurement, $\quad C_{r_0} = 186$ m/s

$$\therefore \quad C_{r_1} = \frac{C_{r_0}}{K} = \frac{186}{0.82} = 226.8 \text{ m/s}$$

(*i*) **Blades angles θ, ϕ :**

By measurement $\quad \theta = \phi =$ blade angles $=$ **27°. (Ans.)**

(*ii*) **Nozzle angle, α :**

By measurement ; nozzle angle, α = **16°. (Ans.)**

(*iii*) **Absolute velocity, C_1 :**

Absolute velocity of steam issuing from the nozzle,

$\quad C_1$ = **366 m/s (by measurement). (Ans.)**

(*iv*) **Axial thrust :**

Also, $\left.\begin{aligned} C_{f_0} &= 84 \text{ m/s} \\ C_{f_1} &= 102 \text{ m/s}\end{aligned}\right]$ By measurement.

$\therefore$ Axial thrust $= \dot{m}_s(C_{f_1} - C_{f_0})$

$= 2.5\,(102 - 84) =$ **45 N. (Ans.)**

Example 19.12. *One stage of an impulse turbine consists of a converging nozzle ring and one ring of moving blades. The nozzles are inclined at 22° to the blades whose tip angles are both 35°. If the velocity of steam at exit from nozzle is 660 m/s, find the blade speed so that the steam passes on without shock. Find the diagram efficiency neglecting losses if the blades are run at this speed.* **(U.P.S.C.)**

Solution. *Given :* $\alpha = 22°$; $\theta = \phi = 35°$; $C_1 = 660$ m/s.

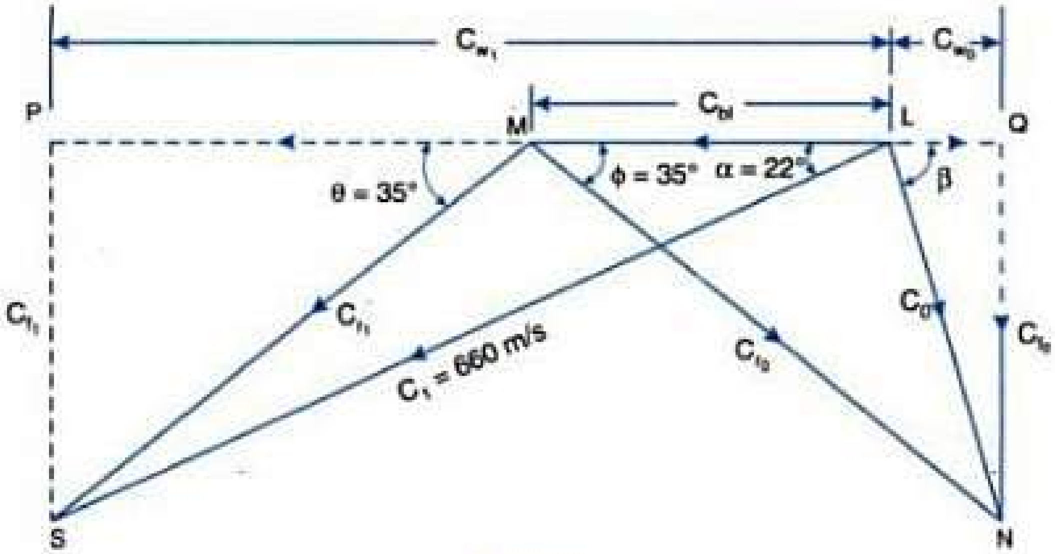

Fig. 19.23

In case of impulse turbine, *maximum blade efficiency*,

$$(\eta_{bl})_{max} = \frac{\cos^2 \alpha}{2}(1 + KZ) \quad \text{...[Eqn. (19.19)]}$$

where K (= blade velocity co-efficient) = 1, (∵ Losses are neglected)

$$Z = \frac{\cos \phi}{\cos \theta} = 1 \quad \text{(∵ Blades are } \textit{equiangular})$$

∴ $$\eta_{bl} = \frac{\cos^2 \alpha}{2}(1 + 1) = \cos^2 \alpha = (\cos 22°)^2 = \mathbf{0.86} \text{ or } \mathbf{86\%}. \text{ (Ans.)}$$

Also, $$\rho_{opt.} = \frac{\cos \alpha}{2} \quad \text{...[Eqn. (19.8)]}$$

or $$\frac{C_{bl}}{C_1} = \frac{\cos 22°}{2} = 0.4636$$

∴ $$C_{bl} = C_1 \times 0.4636 = 660 \times 0.4636 \simeq \mathbf{306 \text{ m/s.}} \text{ (Ans.)}$$

Example 19.13. *In a single stage impulse turbine the mean diameter of the blade ring is 1 metre and the rotational speed is 3000 r.p.m. The steam is issued from the nozzle at 300 m/s and nozzle angle is 20°. The blades are equiangular. If the friction loss in the blade channel is 19% of the kinetic energy corresponding to the relative velocity at the inlet to the blades, what is the power developed in the blading when the axial thrust on the blades is 98 N ?*

Solution. Mean diameter of the blade ring, D = 1 m

Speed of the turbine, N = 3000 r.p.m.

Absolute velocity of steam issuing from the nozzle, C_1 = 300 m/s

Nozzle angle, $\alpha = 20°$

Blade angles are equiangular, $\theta = \phi$

Friction loss in the blade channel = 19%

i.e., $C_{r_0} = (1 - 0.19)\, C_{r_1} = 0.81\, C_{r_1}$

Axial thrust on the blades = 98 N

Power developed, P :

Blade speed, $$C_{bl} = \frac{\pi DN}{60} = \frac{\pi \times 1 \times 3000}{60} = 157.1 \text{ m/s}$$

Also $\theta = \phi$ (given)

Now, velocity diagram is drawn to a suitable scale as shown in Fig. 19.24.

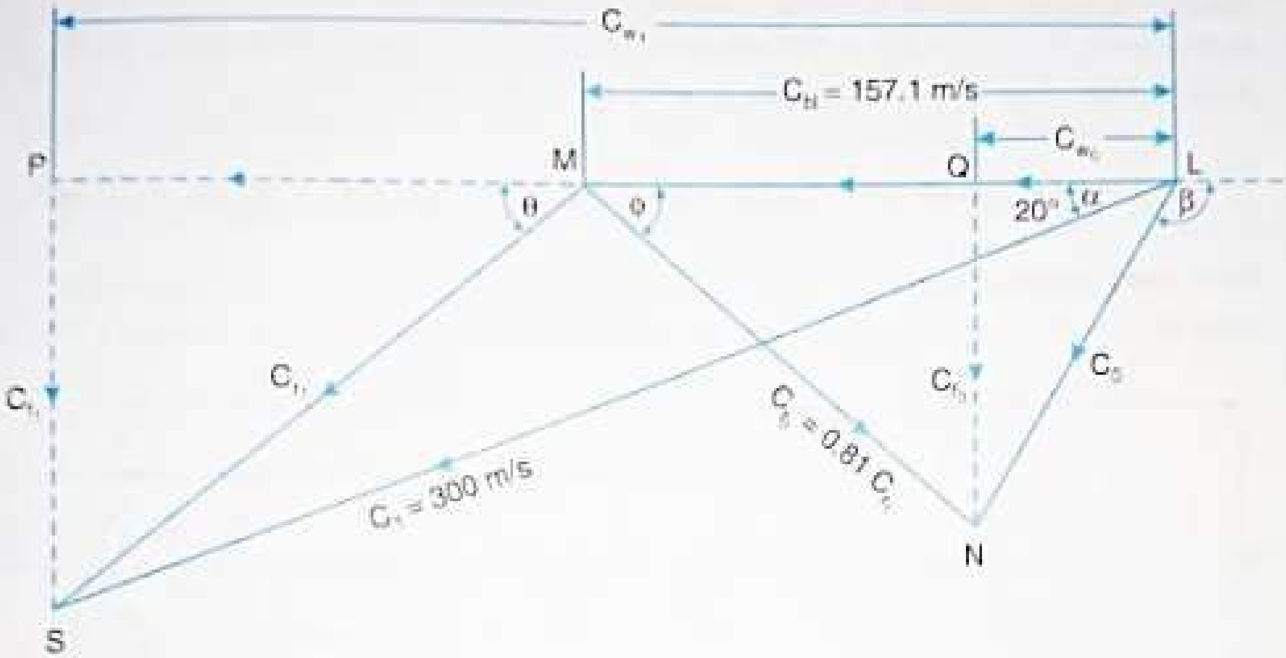

Fig. 19.24

By measurement (from diagram) ;

$$C_{w_1} = 283.5 \text{ m/s} ; \; C_{w_0} = 54 \text{ m/s}$$

$$C_{f_1} = 100.5 \text{ m/s}$$

$$C_{f_0} = 81 \text{ m/s.}$$

Axial thrust $$= \dot{m}_s (C_{f_1} - C_{f_0})$$

$$98 = \dot{m}_s\, (100.5 - 81) \text{ or } \dot{m}_s = \frac{98}{100.5 - 81} = 5.025 \text{ kg}$$

Power developed, $$P = \frac{\dot{m}_s [C_{w_1} + {}^*(-C_{w_0})]}{1000}\, C_{bl} \qquad (^*\beta > 90°)$$

$$= \frac{5.025\,(283.5 - 54) \times 157.1}{1000} = \mathbf{181.2 \text{ kW.}} \quad \textbf{(Ans.)}$$

Example 19.14. *Show that the maximum possible efficiency of a De Laval steam turbine is 88.3% when nozzle angle is 20°. Deduce the formula used.*

Solution. Maximum possible efficiency, η_{max} = 88.3%

Nozzle angle, α = 20°

Maximum possible efficiency of a De-Laval turbine (impulse turbine) = $\cos^2 \alpha$ where α is the nozzle angle.

$\therefore \quad \eta_{max} = \cos^2 20° = (0.9396)^2 = 0.883 =$ **88.3%.** **(Proved).**

For derivation of the formula used refer Article 19.7.

In a simple impulse turbine the nozzles are inclined at 20° to the direction of motion of the moving blades. The steam leaves the nozzle at 375 m/s. The blade velocity is 165 m/s. Calculate suitable inlet and outlet angles for the blades in order that the axial thrust is zero. The relative velocity of steam as it flows over the blades is reduced by 15% by friction. Also, determine the power developed for a flow rate of 10 kg/s.

Solution. Nozzles, $\alpha = 20°$

Velocity of steam issuing from the nozzles, $C_1 = 375$ m/s

Blade speed $C_{bl} = 165$ m/s

Axial thrust = zero *i.e.*, $C_{f_1} = C_{f_0}$

$\frac{C_{r_0}}{C_{r_1}} = (1 - 0.15) = 0.85$, *i.e.*, 15% loss due to friction, steam flow rate, $\dot{m}_s = 10$ kg/s.

Inlet and outlet angles :

With the above given data, draw velocity diagram to a suitable scale as shown in Fig. 19.25.

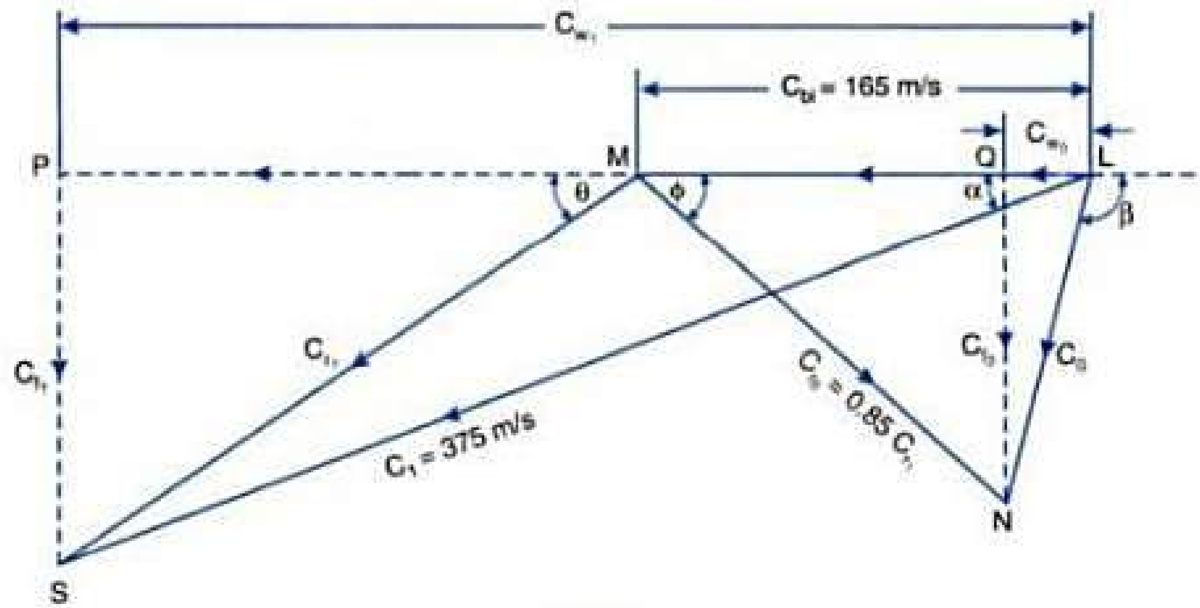

Fig. 19.25

By measurement (from velocity diagram),

$$\left.\begin{aligned} \theta &= \mathbf{35°} \\ \phi &= \mathbf{42°} \\ \beta &= \mathbf{100°} \end{aligned}\right\}. \quad \textbf{(Ans.)}$$

Power developed, P :

Also, $C_{w_1} = 354$ m/s ; $C_{w_0} = 24$ m/s (By measurement)

$\therefore$ *Power developed,* $P = \frac{\dot{m}_s [C_{w_1} + C_{w_0}] \times C_{bl}}{1000}$

$$= \frac{\dot{m}_s [C_{w_1} + (-C_{w_0})] \times C_{bl}}{1000} = \frac{10[354 + (-24)] \times 165}{1000} = \mathbf{544.5\ kW.} \quad \textbf{(Ans.)}$$

Example 19.16. *In a single stage impulse turbine nozzle angle is 20° and blade angles are equal. The velocity co-efficient for blade is 0.85. Find maximum blade efficiency possible. If the actual blade efficiency is 92% of the maximum blade efficiency, find the possible ratio of blade speed to steam speed.*

Solution. Nozzle angle, $\alpha = 20°$

Blade angles are equal *i.e.,* $\theta = \phi$

Blade velocity co-efficient, $K = 0.85 \left(= \dfrac{C_{r_0}}{C_{r_1}}\right)$

Actual blade efficiency = 92% of maximum blade efficiency

Ratio of blade speed to steam speed, $\rho = \dfrac{C_{bl}}{C_1}$.

Maximum blade efficiency is given by :

$$(\eta_{bl})_{max} = \frac{\cos^2 \alpha}{2}(1 + KZ) \quad \text{...[Eqn. (19.19)]}$$

$$= \frac{\cos^2 \alpha}{2}(1 + K) \text{ as } Z = \frac{\cos \phi}{\cos \theta} = 1$$

$$(\eta_{bl})_{max} = \frac{\cos^2 20°}{2}(1 + 0.85) = 0.816 \text{ or } 81.6\%$$

The actual efficiency of the turbine

$$= 0.92 \times 0.816 = 0.75$$

The blade efficiency of a single stage impulse turbine is given by be relation,

$$\eta_{bl} = 2(1 + K)(\rho \times \cos \alpha - \rho^2)$$

$$0.75 = 2(1 + 0.85)(\rho \times \cos 20° - \rho^2)$$

$$0.75 = 2 \times 1.85(0.94\,\rho - \rho^2)$$

$$0.203 = 0.94\,\rho - \rho^2$$

$$\rho^2 - 0.94\,\rho + 0.203 = 0$$

$$\therefore \quad \rho = \frac{0.94 \pm \sqrt{(0.94)^2 - 4 \times 0.203}}{2} = \frac{0.94 \pm 0.267}{2} \text{ or } \rho = 0.603 \text{ or } 0.336$$

Hence possible ratio, ρ = **0.603 or 0.336. (Ans.)**

Example 19.17. *The following data refer to a single stage impulse turbine :*

Isentropic nozzle heat drop = 251 kJ/kg ; nozzle efficiency = 90% ; nozzle angle = 20° ; ratio of blade speed to whirl component of steam speed = 0.5 ; blade velocity co-efficient = 0.9 ; the velocity of steam entering the nozzle = 20 m/s.

Determine : (i) The blade angles at inlet and outlet if the steam enters into the blades without shock and leaves the blades in an axial direction.

(ii) Blade efficiency.

(iii) Power developed and axial thrust if the steam flow is 8 kg/s.

Solution. Isentropic heat drop = 251 kJ/kg

Nozzle efficiency, $\eta_{nozzle} = 90\%$

Nozzle angle, $\alpha = 20°$

Ratio of blade speed to whirl component of steam speed = 0.5

Blade velocity co-efficient, $K = 0.9$

Velocity of steam entering the nozzle $= 20$ m/s

(*i*) **Blade angles :**

Nozzle efficiency is given by : $\eta_{nozzle} = \dfrac{\text{Useful heat drop}}{\text{Isentropic heat drop}}$

or $0.9 = \dfrac{\text{Useful heat drop}}{251}$

$\therefore$ Useful heat drop $= 0.9 \times 251 = 225.9$ kJ/kg

Applying the energy equation to the nozzle, we get

$$\frac{C_1^2 - 20^2}{2} = 225.9 \times 1000$$

$\therefore$ $C_1^2 = 2 \times 225.9 \times 1000 + 400 = 452200$

i.e., $C_1 = 672.4$ m/s

Other data given : $\alpha = 20°, \dfrac{C_{bl}}{C_w} = 0.5, K = \dfrac{C_{r_0}}{C_{r_1}} = 0.9$

and $C_{w_0} = 0$, *as the steam leaves the blades axially.*

$\therefore$ $C_0 = C_{f_0}$

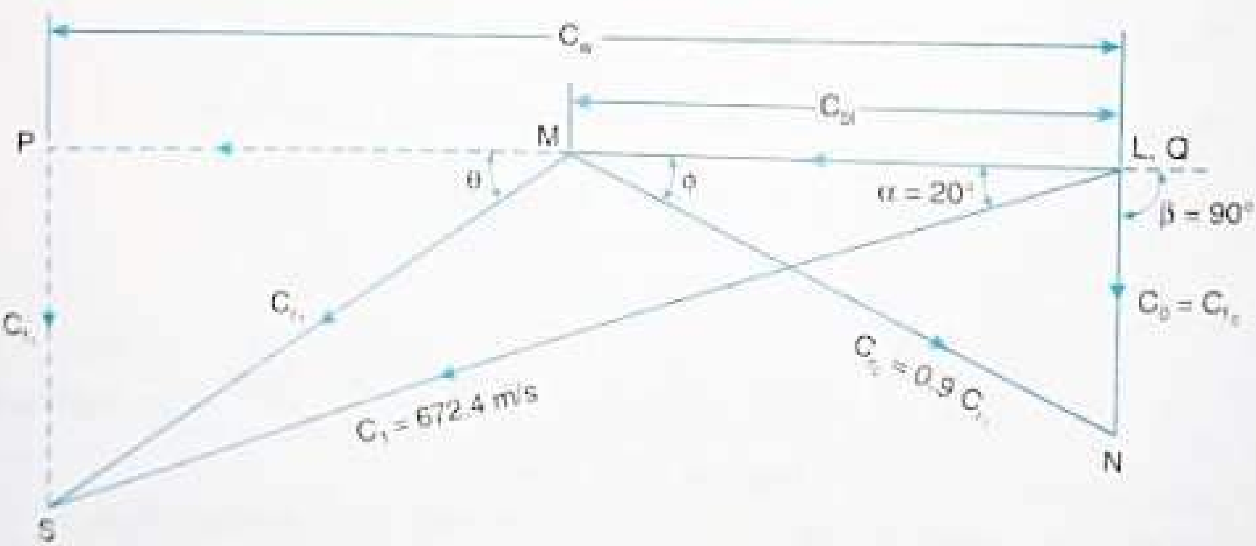

Fig. 19.26

With the above data construct the velocity triangles as follows :

- Select a suitable scale, say 1 cm = 50 m/s.
- Draw a horizontal line through a point L and angle $\alpha = 20°$. Mark the point along LS as

$$LS = C_1 = 672.4 \text{ m} = \frac{672.4}{50} = 13.5 \text{ cm.}$$

- Draw a line through S which is perpendicular to the horizontal line through L and it cuts at the point P. Measure the distance $LP = 12.7$ cm.

$\therefore$ $C_{w_1} = C_w = 12.7$ cm.

and $C_{bl} = 0.5\ C_w = 0.5 \times 12.7 = 6.35$ cm.

- Mark the point M as $LM = C_{bl} = 6.35$ cm
- Join point MS and complete the inlet velocity triangle LMS.
- Measure MS $(C_{r_1}) = 7.7$ cm. $\qquad C_{r_0} = 0.9 \times 7.7 = 6.93$ cm.
- Draw a perpendicular line through point L to the line LM. From M cut an arc of radius 6.93 cm to cut the vertical line through L and mark the point N and join MN which completes the outlet triangle LMN.

Now find out velocities converting lengths into velocities :

$$C_1 = 672.4 \text{ m/s}$$
$$C_w = 12.7 \times 50 = 635 \text{ m/s}$$
$$C_{bl} = 0.5\, C_w = 0.5 \times 635 = 317.5 \text{ m/s}$$
$$C_{r_1} = 7.7 \text{ cm} = 7.7 \times 50 = 385 \text{ m/s}$$
$$C_{r_0} = 0.9\, C_{r_1} = 0.9 \times 385 = 346.5 \text{ m/s}$$
$$C_{f_1} = 4.45 \text{ cm} = 4.45 \times 50 = 222.5 \text{ m/s}$$
$$C_{f_0} = 2.6 \text{ cm} = 2.6 \times 50 = 130 \text{ m/s.}$$

Blade angles measured from the diagram :

$$\theta = 35°, \phi = 22°. \quad \text{(Ans.)}$$

(*ii*) **Blade efficiency, η_{bl} :**

$$\eta_{bl} = \frac{2C_{bl}C_w}{C_1^2} = \frac{2 \times 317.5 \times 635}{(672.4)^2} = \mathbf{0.89 \text{ or } 89\%}. \quad \text{(Ans.)}$$

(*iii*) **Power developed, P and axial thrust :**

$$P = \frac{\dot{m}_s(C_{w_1} + C_{w_0}) \times C_{bl}}{1000} = \frac{8 \times (635 + 0) \times 317.5}{1000} = \mathbf{1612.9 \text{ kW}}. \quad \text{(Ans.)}$$

Axial thrust $= \dot{m}_s(C_{f_1} - C_{f_0}) = 8(222.5 - 130) = \mathbf{740 \text{ N}}$. **(Ans.)**

Example 19.18. *In a single stage steam turbine saturated steam at 10 bar abs. is supplied through a convergent-divergent steam nozzle. The nozzle angle is 20° and the mean blade speed is 400 m/s. The steam pressure leaving the nozzle is 1 bar abs. Find :*

(*i*) *The best blade angles if blades are equiangular.*

(*ii*) *The maximum power developed by the turbine if a number of nozzles used are 5 and area at the throat of each nozzle is 0.6 cm².*

Assume nozzle efficiency 88% and blade friction co-efficient of 0.87.

Solution. Supply steam pressure (to nozzles) = 10 bar abs.

Nozzle angle, $\alpha = 20°$

Mean blade speed, $C_{bl} = 400$ m/s

Steam pressure leaving the nozzle = 1 bar abs.

Number of nozzles used = 5

Area of throat at each nozzle = 0.6 cm²

Nozzle efficiency, $\eta_{nozzle} = 88\%$

Blade friction co-efficient, $K = \dfrac{C_{r_0}}{C_{r_1}} = 0.87.$

The velocity of steam at the outlet of nozzle is found representing the expansion through nozzle on h-s chart as shown in Fig. 19.27.

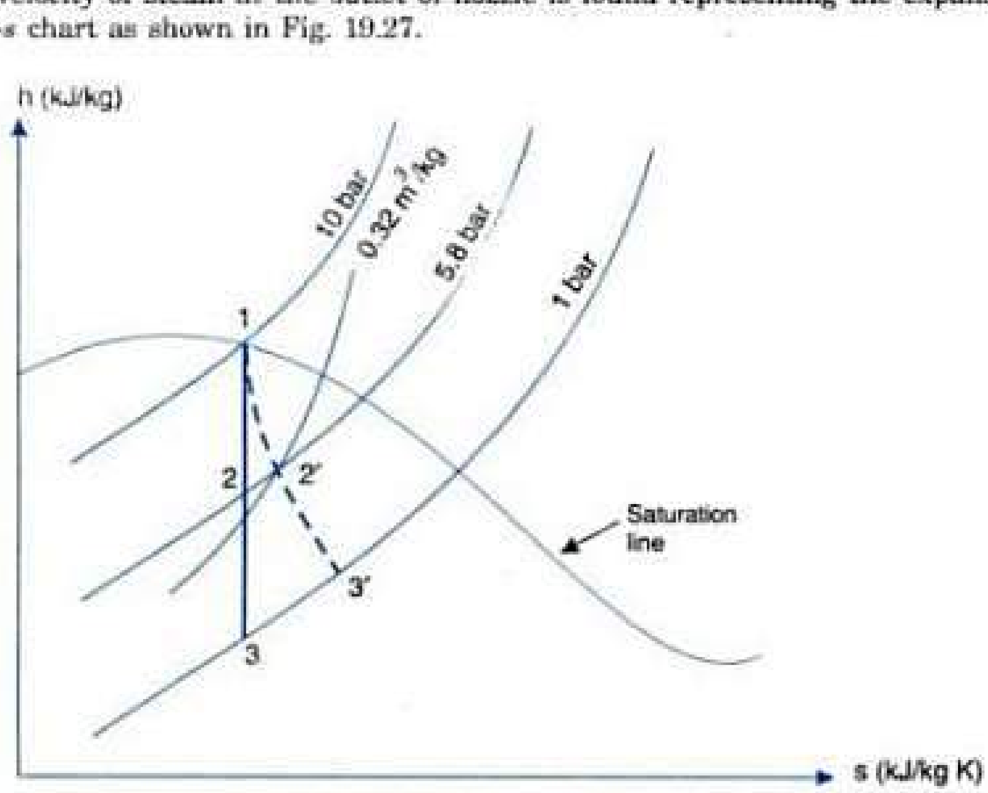

Fig. 19.27

From h-s chart,

$$h_1 - h_3 = 402 \text{ kJ/kg}$$

$$\eta_{nozzle} = \frac{h_1 - h_3'}{h_1 - h_3} = 0.88$$

$\therefore$
$$h_1 - h_3' = 0.88 \times 402 = 353.76 \text{ kJ/kg}$$

Also
$$\frac{C_3'^2}{2} = h_1 - h_3'$$

or
$$C_3' = \sqrt{2(h_1 - h_3')} = \sqrt{2 \times 353.76 \times 1000} = 841.14 \text{ m/s}$$

(*i*) **Blade angles :**

Construct the velocity triangles as per data given as shown in Fig. 19.27.

By measurement, θ (= ϕ) = **35.5°. (Ans.)**

(*ii*) **Maximum power developed, P :**

For finding out the maximum power developed by the turbine let us first find out the maximum mass of steam passing through the nozzle.

The required condition for the maximum mass flow through the nozzle is given by

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}}$$

where, p_1 = Pressure of steam at inlet of the nozzle,

p_2 = Pressure of steam at the throat of the nozzle, and

n (index of expansion) = 1.135 as steam is saturated.

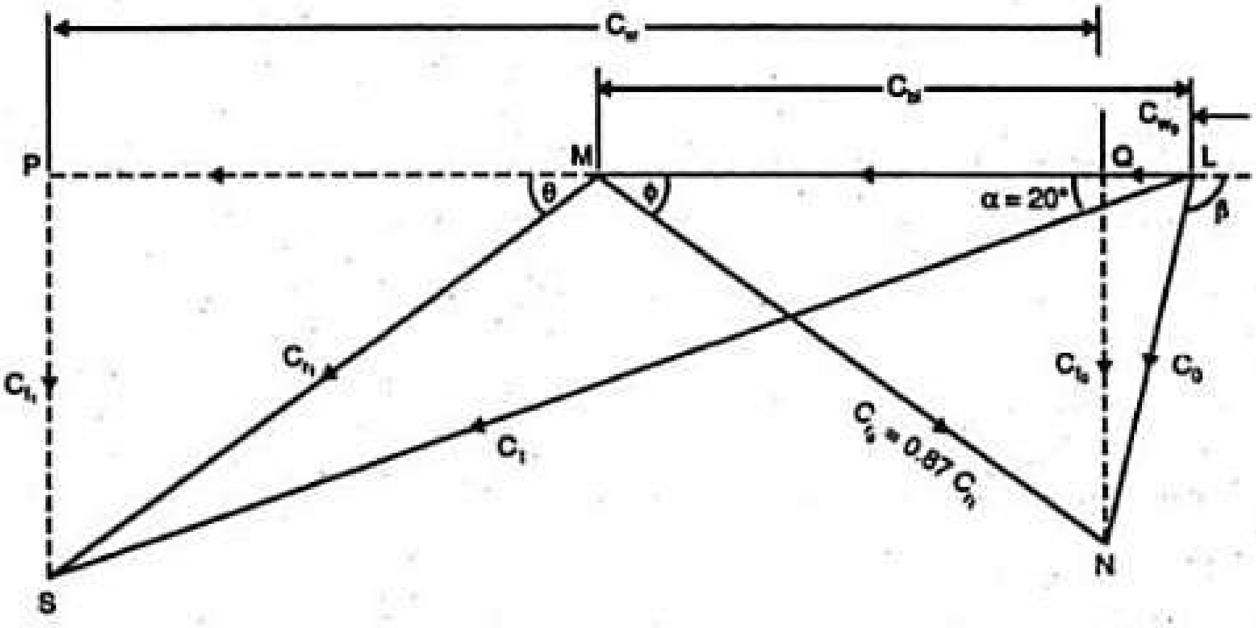

Fig. 19.28

$$\therefore \qquad \frac{p_2}{p_1} = \left(\frac{2}{1.135+1}\right)^{\frac{1.135}{0.135}} = 0.58$$

$$\therefore \qquad p_2 = 10 \times 0.58 = 5.8 \text{ bar}$$

From h-s chart (Fig. 19.27)

$$h_1 - h_2 = 105 \text{ kJ/kg}$$

and $$h_1 - h_2' = 0.88 \times 105 = 92.5 \text{ kJ/kg}$$

v_2' (specific volume at point 2') = 0.32 m³/kg

The maximum velocity of steam at the throat of the nozzle is given by

$$C = \sqrt{2(h_1 - h_2')} = \sqrt{2 \times 92.4 \times 1000} = 429.88 \text{ m/s}$$

Using the continuity equation at the throat of the nozzle, we can write

$$m \,.\, v_2' = A \times C \text{ where } A \text{ is the area of the nozzle.}$$

$$\therefore \qquad m \times 0.32 = 0.6 \times 10^{-4} \times 429.88$$

$$\therefore \qquad m = \frac{0.6 \times 10^{-4} \times 429.88}{0.32} = 0.0806 \text{ kg/s.}$$

Total mass of steam passing through 5 nozzles per second is given by

$$m_t = 0.0806 \times 5 = 0.403 \text{ kg/s}$$

$\therefore$ Power developed by the turbine $= \dfrac{m_t \times C_w C_{bl}}{1000}$ kW

From velocity diagram, C_w = 750 m/s (by measurement)

$\therefore$ *Power developed* $= \dfrac{0.403 \times 750 \times 400}{1000} = $ **120.9 kW. (Ans.)**

Example 19.19. *The first stage of an impulse turbine is compounded for velocity and has two rows of moving blades and one ring of fixed blades. The nozzle angle is 15° and the leaving angles of blades are respectively, first-moving 30°, fixed 20° ; second-moving 30°. The*

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velocity of steam leaving the nozzle is 540 m/s. The friction loss in each blade row is 10% of the relative velocity. Steam leaves the second row of moving blades axially.

Find : *(i) Blade velocity ;* *(ii) Blade efficiency ;*

(iii) Specific steam consumption.

Solution. Refer Fig. 19.29.

Nozzle angle, $\alpha = 15°$; $\alpha' = 20°$

$\beta' = 90°$ [since the steam leaves the blades *axially*]

$\phi = \phi' = 30°$

Velocity of steam leaving the nozzle, $C_1 = 540$ m/s and $\dfrac{C_{r_0}}{C_{r_1}} = 0.9$

$$\frac{C_1'}{C_0} = 0.9 \text{ and } \frac{C_{r_0}'}{C_{r_1}'} = 0.9.$$

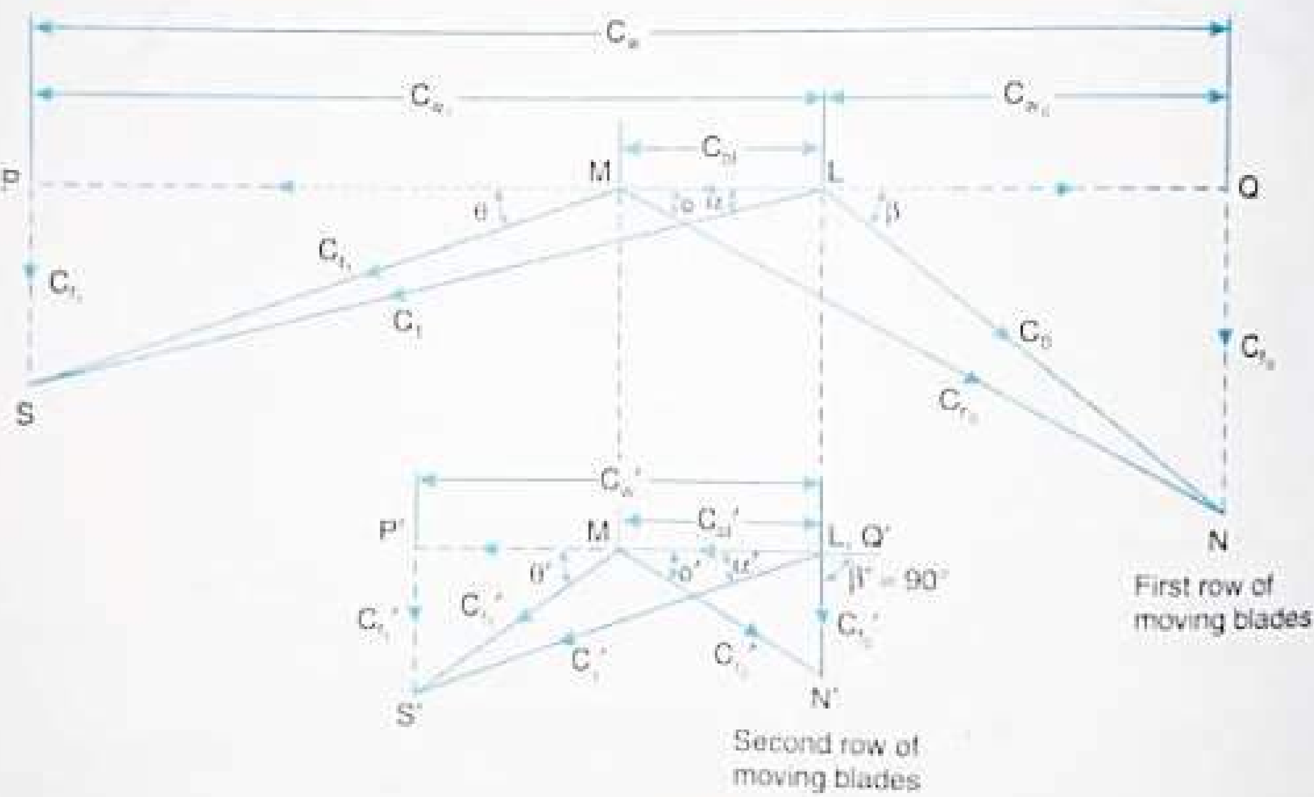

Fig. 19.29

Second row of moving blades :

The velocity triangles should be drawn *starting from the second row of moving **blades***.

The procedure is as follows : Refer Fig. 19.29.

- Draw LM to any convenient scale (say 3 cm) as C_{bl} is *not* known.
- Draw $\phi' = 30°$ and draw perpendicular through the point L (or Q') to LM as $\beta' = 90°$. This meets the line MN' at N'. This completes the outlet triangle LMN'.
- Measure $C_{r_0}' = MN' = 3.5$ cm

$$C_{r_1}' = \frac{C_{r_0}'}{0.9} = \frac{3.5}{0.9} = 3.9 \text{ cm.}$$

- Draw an $\angle\alpha' = 20°$ and draw an arc of radius 3.9 cm with centre at M to cut the line LS' at S'. Join MS'. This completes the inlet velocity $\Delta LMS'$. Measure $LS' = C_1' = 6.5$ cm.

First row of moving blades :

The following steps are involved in drawing velocity triangle for *first row of moving blades*.

- Draw $$LM = C_{bl}\ (= 3 \text{ cm})$$

$$C_0 = \frac{C_1'}{0.9} = \frac{6.5}{0.9} = 7.22 \text{ cm}.$$

- Draw $\angle\phi = 30°$, through the point M to the line LM.
- Draw an arc of radius 7.22 cm with centre L. This arc cuts the line LN at point N. Join MN. This completes the outlet ΔLMN.
- Measure $$C_{r_0} = MN = 9.7 \text{ cm}$$

$$MS = C_{r_1} = \frac{C_{r_0}}{0.9} = \frac{9.7}{0.9} = 10.8 \text{ cm}$$

— Draw an $\angle\alpha = 15°$, through a point L. Draw an arc of radius of 10.8 cm with centre at M. This arc cuts the line LS at S. Join MS. This completes the inlet velocity triangle.

Measure LS from the velocity triangle

$$LS = 13.8 \text{ cm} = C_1 = 540 \text{ m/s}.$$

The scale is now calculated from the above.

$$\therefore \qquad \text{Scale 1 cm} = \frac{540}{13.8} = 39.1 \text{ m/s}$$

(*i*) **Blade velocity, C_{bl} :**

Measure the following distances from the velocity diagram and convert into velocities :

$$C_{bl} = LM = 3 \text{ cm} = 3 \times 39.1 = \mathbf{117.3\ m/s.} \quad \text{(Ans.)}$$

(*ii*) **Blade efficiency, η_{bl}**

$$C_w = PQ = 18.8 \text{ cm} = 18.8 \times 39.1 = 735.1 \text{ m/s}$$
$$C_w' = P'Q' = 6.2 \text{ cm} = 6.2 \times 39.1 = 242.4 \text{ m/s}$$
$$= \frac{2C_{bl}(C_w + C_w')}{C_1^2}$$
$$= \frac{2 \times 117.3(735.1 + 242.4)}{(540)^2} = \mathbf{0.786 \text{ or } 78.6\%.} \quad \text{(Ans.)}$$

(*iii*) **Specific steam consumption, m_s :**

$$1 = \frac{m_s(C_w + C_w')C_{bl}}{3600 \times 1000} = \frac{m_s(735.1 + 242.4) \times 117.3}{3600 \times 1000}$$

$$\therefore \qquad m_s = \frac{3600 \times 1000}{(735.1 + 242.4) \times 117.3} = \mathbf{31.39\ kg/kWh.} \quad \text{(Ans.)}$$

Example 19.20. *The following particulars relate to a two-row velocity compounded impulse wheel :*

Steam velocity at nozzle outlet	*= 650 m/s*
Mean blade velocity	*= 125 m/s*
Nozzle outlet angle	*= 16°*
Outlet angle, first row of moving blades	*= 18°*

Outlet angle, fixed guide blades = 22°

Outlet angle, second row of moving blades = 36°

Steam flow = 2.5 *kg/s*

The ratio of the relative velocity at outlet to that at inlet is 0.84 for all blades.

Determine the following :

(*i*) *The axial thrust on the blades ;* (*ii*) *The power developed,*

(*iii*) *The efficiency of the wheel.* **(AMIE Winter, 2001)**

Solution. With given data, C_{bl} = 125 m/s, C_1 = 650 m/s, α = 16°,

first row inlet velocity diagram is drawn.

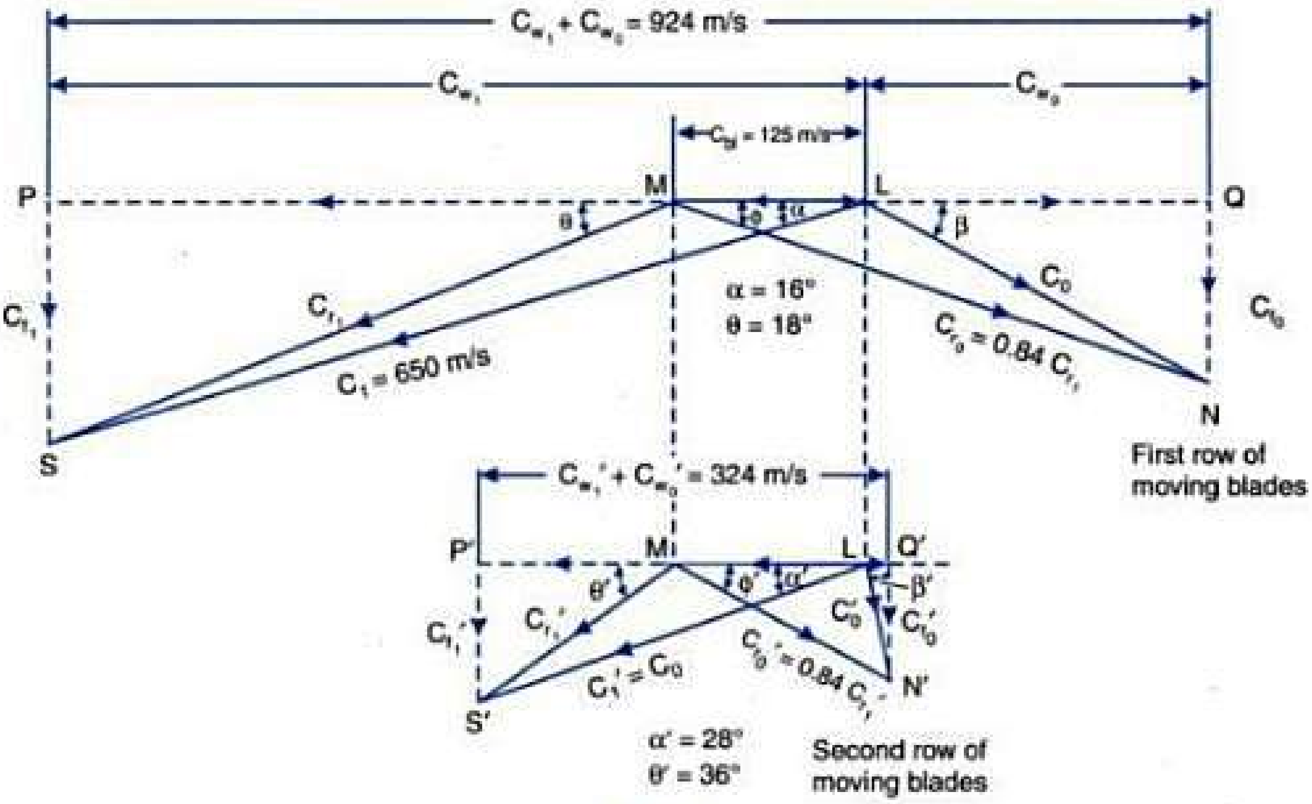

Fig. 19.30

Now with given, $C_{r_0} = 0.84\, C_{r_1}$, ϕ = 18°,

first row exit diagram is drawn.

With $C_1' = C_0$; α' = 22°,

second row inlet velocity diagram is drawn

With $C_{r_0}' = 0.84\ C_{r_1}'$; ϕ' = 36°,

second row exit diagram is drawn.

The values read form the diagram are as follows :

$$C_{f_1} = 180 \text{ m/s}, \qquad C_{f_0} = 138 \text{ m/s},$$
$$C_{f_1}' = 122 \text{ m/s}, \qquad C_{f_0}' = 107 \text{ m/s},$$
$$C_{w_1} + C_{w_0} = 924 \text{ m/s}, \qquad C_{w_1}' + C_{w_0}' = 324 \text{ m/s}.$$

(*i*) **Axial thrust on the blades**

$$= \dot{m}_s[(C_{f_1} - C_{f_0}) + (C_{f_1}' - C_{f_0}')]$$

$$= 2.5\ [(180 - 138) + (122 - 107)] = \mathbf{142.5\ N.}\quad \textbf{(Ans.)}$$

(*ii*) **Power developed**

$$= \frac{\dot{m}_s[(C_{w_1} + C_{w_0}) + (C_{w_1}' + C_{w_0}')]C_{bl}}{1000}$$

$$= \frac{2.5(924 + 324) \times 125}{1000} = \mathbf{390\ kW.}\quad \textbf{(Ans.)}$$

(*iii*) **Efficiency**

$$= \frac{(924 + 324) \times 125}{C_1^2/2} = \frac{(924 + 324) \times 125}{650^2/2}$$

$$= \mathbf{0.738\ or\ 73.8\%.}\quad \textbf{(Ans.)}$$

Example 19.21. *The first stage of an impulse turbine is compounded for velocity and has two rings of moving blades and one ring of fixed blades. The nozzle angle is 20° and the leaving angles of the blades are respectively as follows :*

First moving 20°, fixed 25° and second moving 30°. Velocity of steam leaving the nozzles is 600 m/sec and the steam velocity relative to the blade is reduced by 10% during the passage through each ring. Find the diagram efficiency and power developed for a steam flow of 4 kg per second. Blade speed may be taken as 125 m/sec. **(M.U.)**

Solution. $C_{bl} = 125$ m/s, $C_1 = 600$ m/s

$\alpha = 20°,\ \phi = 20°$

$\alpha' = 25°,\ \phi' = 30°$

$$K = \left(1 - \frac{10}{100}\right) = 0.9$$

$\dot{m}_s = 4$ kg/s

With these values velocity triangles can be drawn (Fig. 19.31).

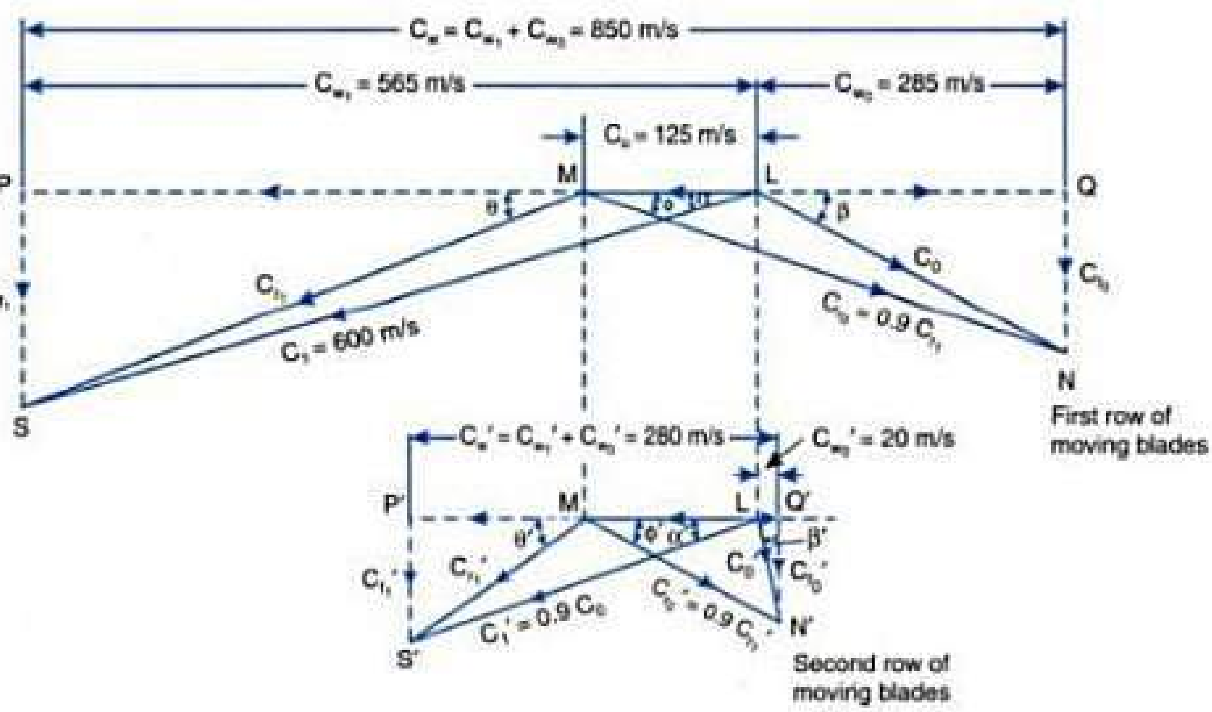

Fig. 19.31

From diagram (By measurement) :

$$C_{w_1} = 565 \text{ m/s}, \qquad C_{w_2} = 285 \text{ m/s}$$

$$C_{w_1}' = 260 \text{ m/s}, \qquad C_{w_2}' = 20 \text{ m/s}$$

Now,
$$C_w = C_{w_1} + C_{w_2} = 565 + 285 = 850 \text{ m/s}$$

$$C_w' = C_{w_1}' + C_{w_2}' = 260 + 20 = 280 \text{ m/s}$$

Power developed
$$= \frac{m_s(C_w + C_w')C_bl}{1000}$$

$$= \frac{4 \times (850 + 280) \times 125}{1000} = \textbf{565 kW.} \quad \textbf{(Ans.)}$$

Diagram efficiency
$$= \frac{C_{bl}(C_w + C'_w)}{C_1^2/2}$$

$$= \frac{2 \times 125(850 + 280)}{600^2} = \textbf{0.7847 or 78.47\%.} \quad \textbf{(Ans.)}$$

Example 19.22. *The following data relate to a compound impulse turbine having two rows of moving blades and one row of fixed blades in between them.*

The velocity of steam leaving the nozzle	*= 600 m/s*
Blade speed	*= 125 m/s*
Nozzle angle	*= 20°*
First moving blade discharge angle	*= 20°*
First fixed blade discharge angle	*= 25°*
Second moving blade discharge angle	*= 30°*
Friction loss in each ring	*= 10% of relative velocity.*

Find : (i) Diagram efficiency ;

(ii) Power developed for a steam flow of 6 kg/s.

Solution. Refer Fig. 19.32.

First row of moving blades :

To draw velocity triangles for first row of moving blades the following procedure may be followed :

Select a suitable scale.

- Draw LM = blade velocity (C_{bl}) = 125 m/s.
- Make $\angle MLS$ = nozzle angle, α = 20°.
- Draw LS = velocity of steam leaving the nozzle = 600 m/s.
- Join MS to complete the inlet triangle LMS.
- Make $\angle LMN$ = outlet angle of first moving blades = 20°.

and cut $MN = 0.9\ MS$, since $K = 0.9$.

— Join LN to complete the outlet ΔLMN.

Second row of moving blades :

The velocity triangles for second row of moving blades may be drawn as follows :

- Draw LM = blade velocity (C_{bl}) = 125 m/s.
- Make $\angle MLS'$ = outlet angle of fixed blade = 25°

and cut $LS' = 0.9\ LN$. $(\because\ K = 0.9)$

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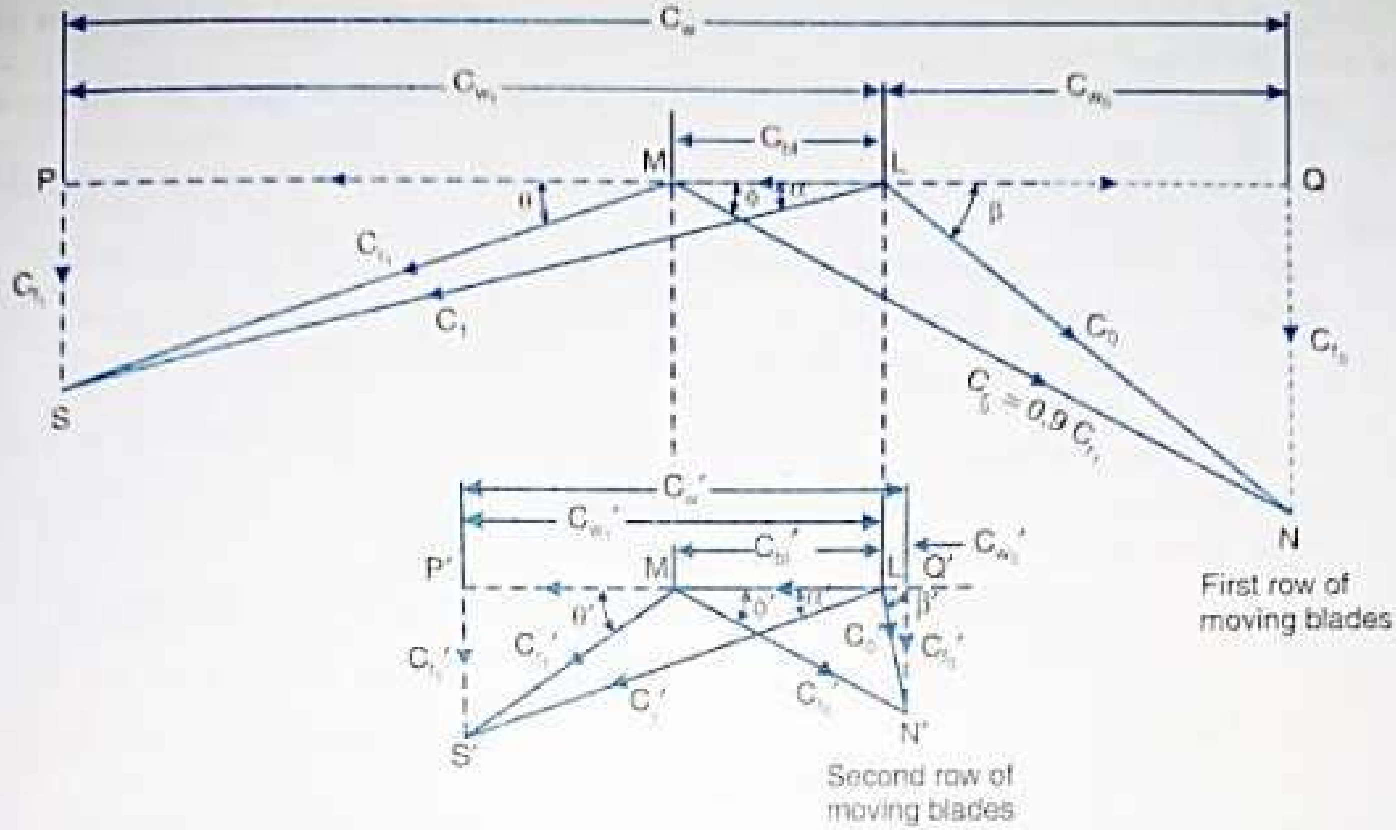

Fig. 19.32

- Join MS'. The inlet velocity triangle LMS' is completed.
- Make $\angle LMN'$ = outer angle of second moving blades = 30°

and cut $MN' = 0.9\ MS'$ $(\because\ K = 0.9)$

- Join LN'. The outlet velocity triangle is completed.

The following required data may now be scaled off from the diagram :

$$C_w = C_{w_1} + C_{w_0} = PQ = 845 \text{ m/s}$$

$$C_w' = P'Q' = 280 \text{ m/s.}$$

(i) **Diagram efficiency,** $\eta_{bl} = \dfrac{2C_{bl}(C_w + C_w')}{C_1^2}$

$$= \frac{2 \times 125(845 + 280)}{(600)^2} = \mathbf{0.781 \text{ or } 78.1\%.} \quad \textbf{(Ans.)}$$

(ii) **Power developed,** $P = \dfrac{m_s(C_w + C_w')}{1000} C_{bl}$

$$= \frac{6(845 + 280) \times 125}{1000} = \mathbf{843.75 \text{ kW.}} \quad \textbf{(Ans.)}$$

Example 19.23. *The first stage of a turbine is a two-row velocity compounded impulse wheel. The steam velocity at inlet is 600 m/s and the mean blade velocity is 120 m/s. The nozzle angle is 16° and the exit angles for the first-row of moving blades, the fixed blades, and the second row of moving blades are 18°, 21° and 35° respectively.*

(i) Calculate the blade inlet angles for each row.

***(ii)* Calculate also for each row of moving blades, the driving force and the axial thrust on the wheel for a mass flow of 1 kg/s.**

***(iii)* Calculate the diagram efficiency for the wheel and the diagram power per kg/s steam flow.**

***(iv)* What would be the maximum possible diagram efficiency for the given steam inlet velocity and nozzle angle ?**

Take the blade velocity co-efficient as 0.9 for all blades.

Solution. Refer Fig. 19.33.

$$\alpha = 16°,\ \phi = 18°,\ C_1 = 600 \text{ m/s},\ C_{bl} = 120 \text{ m/s}$$
$$\alpha' = 21°,\ \phi' = 35°,\ \dot{m}_s = 1 \text{ kg/s},$$

Blade velocity co-efficient, $K = 0.9$

With the above data velocity triangles can be drawn.

From the diagram (by measurement)

$$C_w = C_{w_1} + C_{w_2} = 875 \text{ m/s} \ ;\ C_w' = C_{w_1}' + C_{w_2}' = 294 \text{ m/s}$$
$$C_{f_1} = 168 \text{ m/s},\ C_{f_2} = 135 \text{ m/s} \ ;\ C_{f_1}' = 106 \text{ m/s},\ C_{f_2}' = 97 \text{ m/s}.$$

(i) **Blade inlet angles :**

First row : $\theta = 20°$ (moving blade)

$\beta = 24.5°$ (fixed blade)

Second row : $\theta' = 34.5°$ (moving blade).

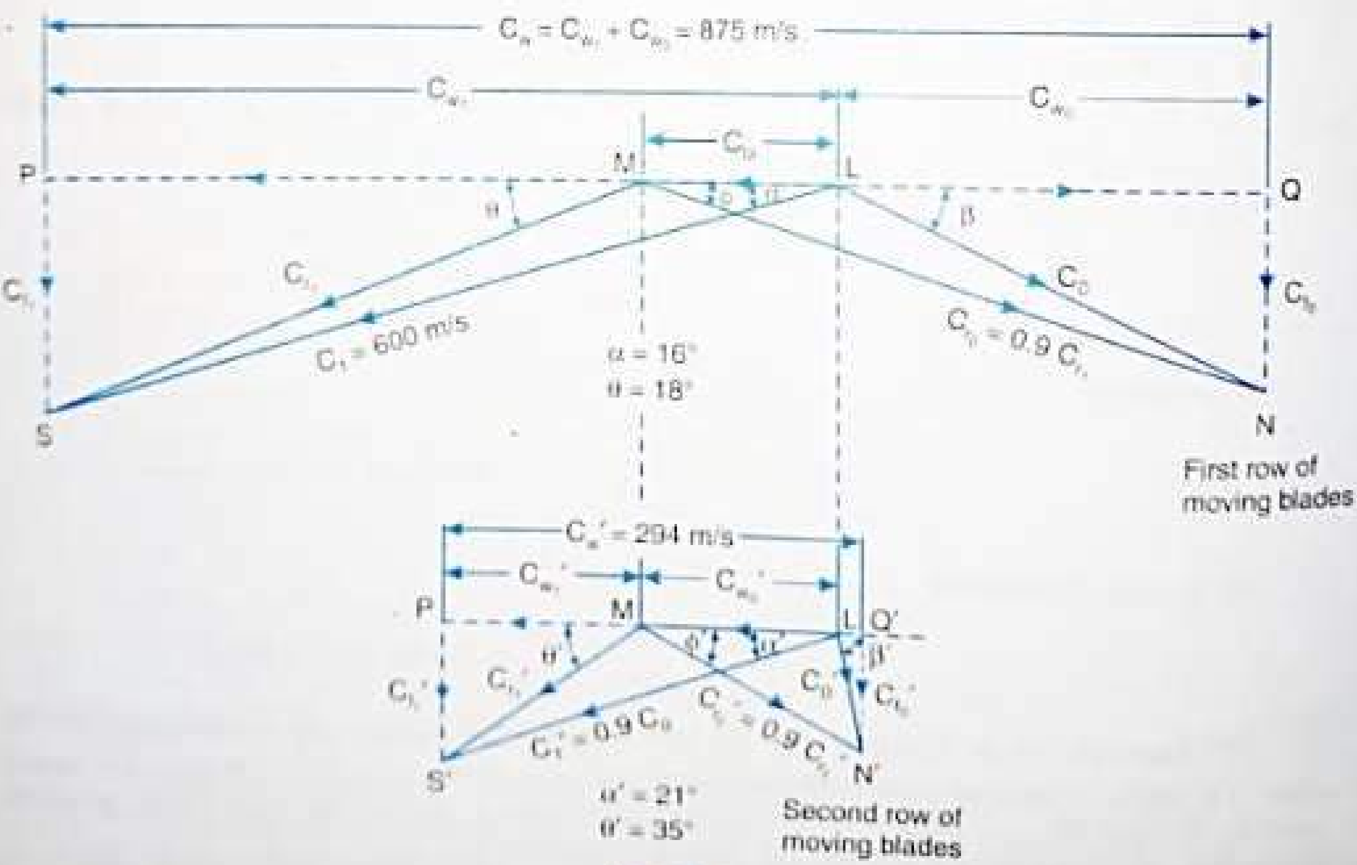

Fig. 19.33

(*ii*) **Driving force :**

First row of moving blades $= \dot{m}_s(C_{w_1} + C_{w_0}) = \dot{m}_s C_w = 1 \times 875 =$ **875 N. (Ans.)**

Second row of moving blades $= \dot{m}_s(C_{w_1}' + C_{w_0}') = \dot{m}_s C_w' = 1 \times 294 =$ **294 N. (Ans.)**

Axial thrust :

First row of moving blades $= \dot{m}_s(C_{f_1}' + C_{f_0}') = 1 \times (168 - 135) = 33$ N

Second row of moving blades $= \dot{m}_s(C_{f_1}' - C_{f_0}') = 1 \times (106 - 97) = 9$ N

Total axial thrust $= 33 + 9 =$ **42 N per kg/s. (Ans.)**

(*iii*) **Power developed** $= \dfrac{\dot{m}_s(C_w + C_w')C_{bl}}{1000}$

$= \dfrac{1 \times (875 + 294) \times 120}{1000} =$ **140.28 kW per kg/s. (Ans.)**

Diagram efficiency $= \dfrac{2C_{bl}(C_w + C_w')}{C_1^2} = \dfrac{2 \times 120\,(875 + 294)}{(600)^2}$

$=$ **0.7793 or 77.93%. (Ans.)**

(*iv*) **Maximum diagram efficiency**

$= \cos^2 \alpha = \cos^2 16° =$ **0.924 or 92.4%. (Ans.)**

Example 19.24. *An impulse stage of a turbine has two rows of moving blades separated by fixed blades. The steam leaves the nozzles at an angle of 20° with the direction of motion of the blades. The blade exit angles are : 1st moving 30° ; fixed 22° ; 2nd moving 30°.*

If the adiabatic heat drop for the nozzle is 186.2 kJ/kg and the nozzle efficiency 90%, find the blade speed necessary if the final velocity of the steam is to be axial. Assume a loss of 15% in relative velocity for all blade passages. Final also the blade efficiency and the stage efficiency. **(P.U.)**

Solution. Steam velocity, $C_1 = 44.72\sqrt{\eta_n h_d} = 44.72\sqrt{0.9 \times 186.2} = 579$ m/s.

The velocity diagram *for axial discharge turbine is drawn in reverse direction* (see Fig. 19.34).

- The blade velocity (C_{bl}) LM is drawn to any convenient scale.
- As discharge is axial LN' is drawn perpendicular to LM.
- Knowing the outlet angle of the second moving ring ($\phi' = 30°$), N' is located.

  MN' represents relative velocity at outlet (C_{r_0}').
- Relative velocity at inlet to the second moving blade is

$$C_{r_0}' = MS' = \frac{MN'}{0.85}\left(= \frac{C_{r_0}'}{K}\right).$$

- The triangle at inlet to the second moving blades ring LMS' is obtained by drawing the discharge angle $\angle MLS'$ ($\alpha' = 22°$), where LS' (C_1') is the exit velocity of the second blade ring.
- Now, $C_0 = LN = \dfrac{C_1'}{0.85} = \dfrac{LS'}{0.85}$ assuming a loss of 15% (given).

- **With compass at centre *L* and radius *LN* arc is drawn and the velocity triangle at the exit of the first moving blade ring *LMN* is completed, knowing the exit angle of the first moving blade ring (ϕ = 30°).**

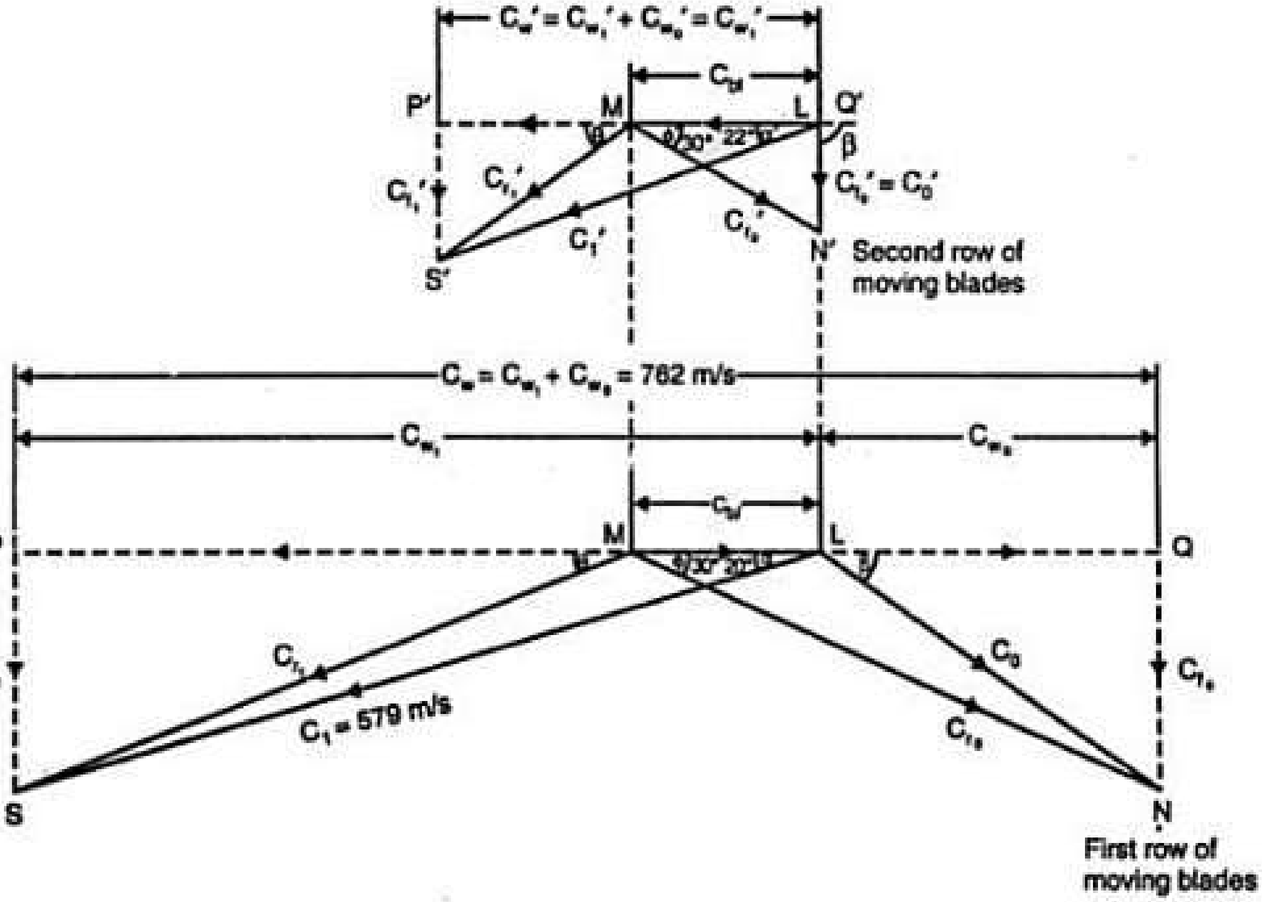

Fig. 19.34

- **With α = 20° and $C_{r_1} = MS = \dfrac{C_{r_2}}{0.85} = \dfrac{MN}{0.85}$ the inlet velocity triangle is completed.**

Now *LS* is the absolute velocity from the nozzle. Since this velocity is known, scale can be calculated. It is

$$= \frac{\text{Absolute velocity at exit from the nozzle}}{\text{Length } LS}$$

and then the blade velocity, etc. can be calculated.

From the diagram :

$$C_{bl} \ (= LM) = 117 \text{ m/s}$$

$$C_{w_1} + C_{w_2} = 762 \text{ m/s}$$

$$C_{w_1}' + C_{w_2}' = C_{w_1}' = 234 \text{ m/s} \qquad (\because \ C_{w_2}' = 0)$$

Blade efficiency, $$\eta_{bl} = \frac{(C_w + C_w') \times C_{bl}}{C_1^2/2} = \frac{(762 + 234) \times 117}{579^2/2}$$

$$= 0.6952 \text{ or } 69.52\%. \quad \textbf{(Ans.)}$$

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Stage efficiency, $$\eta_{stage} = \frac{(C_w + C_w') \times C_{bl}}{h_d} = \frac{(762 + 234) \times 117}{186.2 \times 1000}$$
$$= \mathbf{0.626 \text{ or } 62.6\%.} \quad \textbf{(Ans.)}$$

19.8. REACTION TURBINES

The reaction turbines which are used these days are really **impulse-reaction** turbine. Pure reaction turbines are *not* in general use. The expansion of steam and heat drop occur both in fixed and moving blades.

19.8.1. Velocity Diagram for Reaction Turbine Blade

Fig. 19.35 shows the velocity diagram for reaction turbine blade. In case of an impulse turbine blade the relative velocity of steam either remains constant as the steam glides over the blades or is reduced slightly due to friction. In reaction turbine blades, the steam continuously expands at it flows over the blades. *The effect of the continuous expansion of steam during the flow over the blade is to increase the relative velocity of steam.*

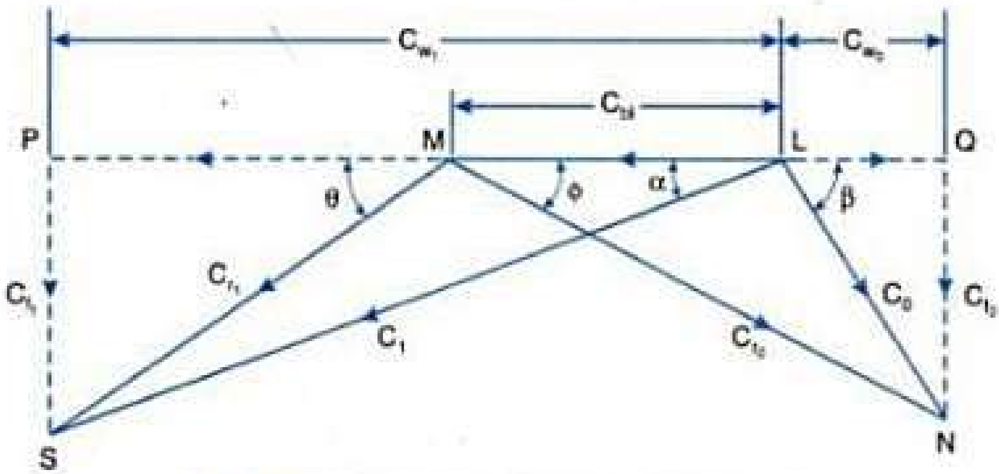


Fig. 19.35. Velocity diagram for reaction turbine blade.

$\therefore \quad C_{r_0} > C_{r_1}$ for *reaction turbine blade.*

($C_{r_0} \le C_{r_1}$ for *impulse turbine blade*).

19.8.2. Degree of Reaction (R_d)

The ***degree of reaction*** of reaction turbine stage is defined as the *ratio of heat drop over moving blades to the total heat drop in the stage.*

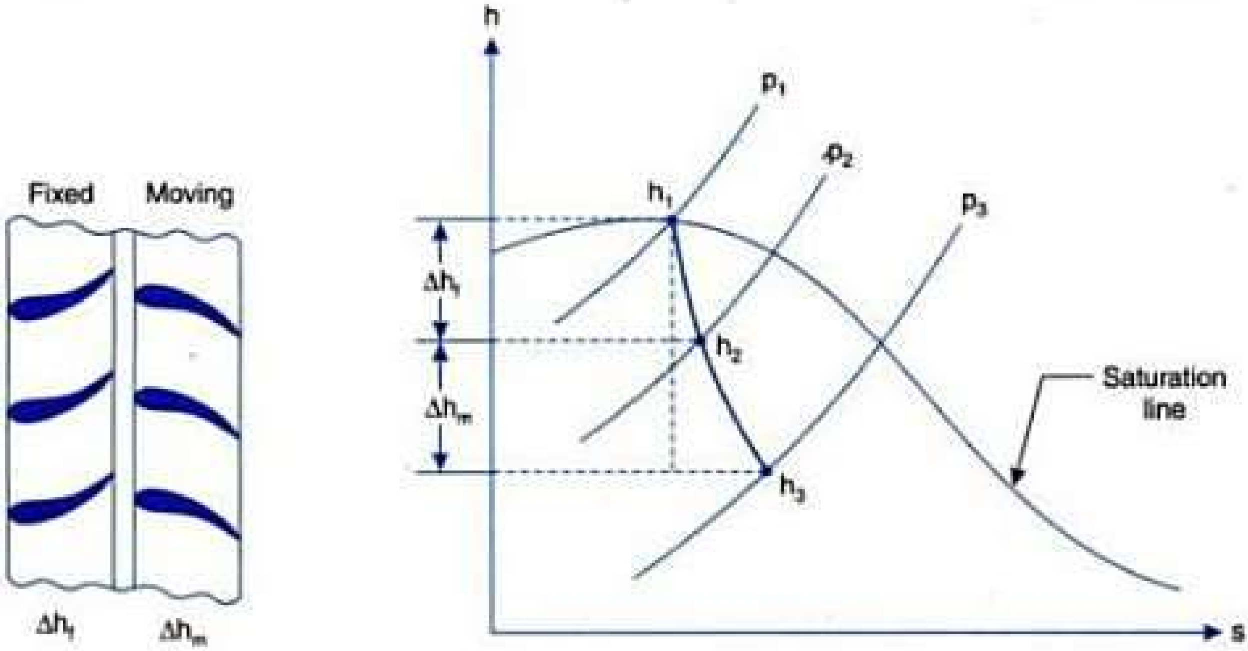
Thus the degree of reaction of reaction turbine is given by,

$$R_d = \frac{\text{Heat drop in moving blades}}{\text{Heat drop in the stage}}$$

$$= \frac{\Delta h_m}{\Delta h_f + \Delta h_m} \text{ as shown in Fig. 19.35.}$$

The heat drop in moving blades is equal to increase in relative velocity of steam passing through the blade.

$$\therefore \qquad \Delta h_m = \frac{C_{r_0}^{\ 2} - C_{r_1}^{\ 2}}{2}$$



(a) (b)

Fig. 19.36

The total heat drop in the stage ($\Delta h_f + \Delta h_m$) is equal to the work done by the steam in the stage and it is given by

$$\Delta h_f + \Delta h_m = C_{bl}\,(C_{w_1} + C_{w_0})$$

$$\therefore \qquad R_d = \frac{C_{r_0}^{\ 2} - C_{r_1}^{\ 2}}{2C_{bl}(C_{w_1} + C_{w_0})} \qquad \text{...(19.22)}$$

Referring to Fig. 19.36,

$$C_{r_0} = C_{f_0}\ \text{cosec}\ \phi \text{ and } C_{r_1} = C_{f_1}\ \text{cosec}\ \theta$$

and
$$(C_{w_1} + C_{w_0}) = C_{f_1}\cot\theta + C_{f_0}\cot\phi$$

The velocity of flow generally remains constant through the blades.

$$\therefore \qquad C_{f_1} = C_{f_0} = C_f\,.$$

Substituting the values of C_{r_1}, C_{r_0} and $(C_{w_1} + C_{w_0})$ in eqn. (19.22), we get

$$R_d = \frac{C_f^{\ 2}\,(\text{cosec}^2\,\phi - \text{cosec}^2\,\theta)}{2\,C_{bl}\,C_f\,(\cot\theta + \cot\phi)} = \frac{C_f}{2C_{bl}}\left[\frac{(\cot^2\phi + 1) - (\cot^2\theta + 1)}{\cot\theta + \cot\phi}\right]$$

$$= \frac{C_f}{2C_{bl}}\left[\frac{\cot^2\phi - \cot^2\theta}{\cot\phi + \cot\theta}\right]$$

$$= \frac{C_f}{2C_{bl}}\,(\cot\phi - \cot\theta) \qquad \text{...(19.23)}$$

If the turbine is designed for 50% reaction ($\Delta h_f = \Delta h_m$), then the eqn. (19.23) can be written as

$$\frac{1}{2} = \frac{C_f}{2C_{bl}}\,(\cot\phi - \cot\theta)$$

$$\therefore \qquad C_{bl} = C_f (\cot \phi - \cot \theta) \qquad ...(19.24)$$

Also C_{bl} can be written as

$$C_{bl} = C_f (\cot \phi - \cot \beta) \qquad ...(19.25)$$

and

$$C_{bl} = C_f (\cot \alpha - \cot \theta) \qquad ...(19.26)$$

$C_{f_1} = C_{f_0} = C_f$ is assumed in writing the above equations.

Comparing the eqns. (19.24), (19.25), (19.26)

$$\theta = \beta \text{ and } \phi = \alpha$$

which *means that moving blade and fixed blade* must have the same shape if the degree of reaction is 50%. This condition gives symmetrical velocity diagrams. This type of turbine is known as **"Parson's reaction turbine"**. Velocity diagram for the blades of this turbine is given in Fig. 19.37.

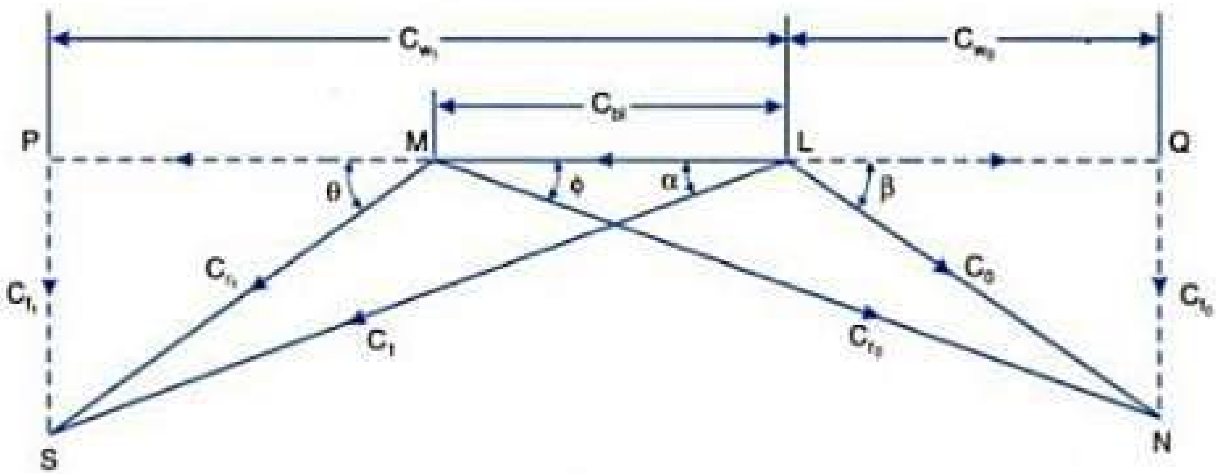


Fig. 19.37

Example 19.25. *Define the term 'degree of reaction' as applied to a steam turbine. Show that for Parson's reaction turbine the degree of reaction is 50%.* **(AMIE Summer, 1998)**

Solution. Refer Fig. 19.38.

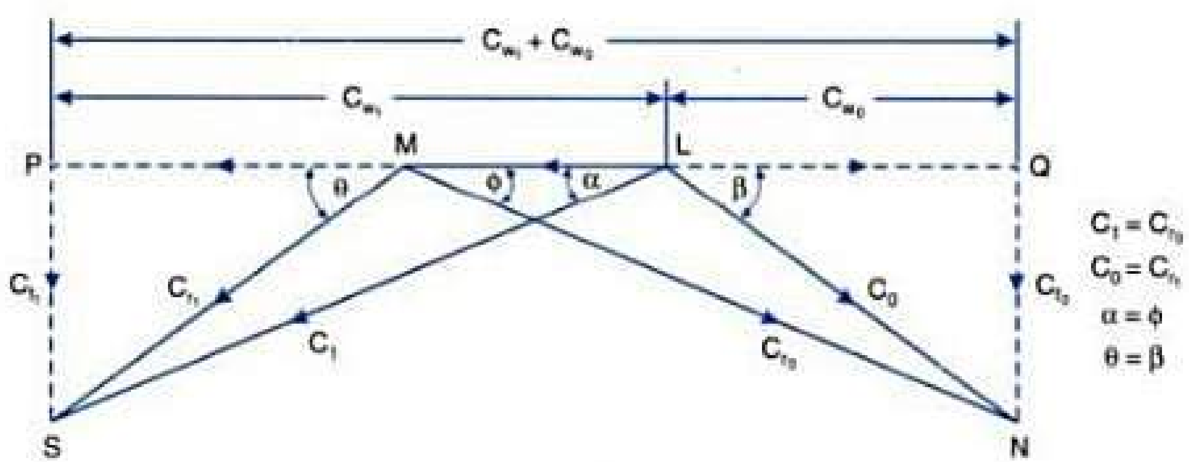


Fig. 19.38

The pressure drop in reaction turbines takes place in both fixed and moving blades. The division generally is given in terms of enthalpy drops. The criterion used is the *degree of reaction*. It is defined as

$$\frac{\text{Enthalpy drop in rotor blades}}{\text{Total enthalpy drop in stage}} = \frac{\Delta h_m}{\Delta h_f + \Delta h_m} \qquad \text{(Refer Fig. 19.36)}$$

A special case is when the degree of reaction is zero ; it means no heat drop in the moving blades. This becomes a case of impulse stage. Other common case is of Parson's turbine which has the same reason for both the fixed and moving blades. The blades are *symmetrical*, *i.e.*, the exit angle of moving blade is equal to the exit angle of the fixed blade and the inlet angle of the moving blade is equal to the inlet angle of the fixed blade. Since the blades are symmetrical the velocity diagram is also symmetrical. In such a case the degree of reaction is 50%.

Applying the steady flow energy equation to the fixed blades and assuming that the velocity of steam entering the fixed blade is equal to the absolute velocity of steam leaving the previous moving row, we have

$$\Delta h_f = \frac{C_1^2 - C_0^2}{2}$$

Similarly, for the moving blades

$$\Delta h_m = \frac{C_{r_0}^2 - C_{r_1}^2}{2}$$

But $\quad C_1 = C_{r_0}$ and $C_0 = C_{r_1}$

$\therefore \quad \Delta h_f = \Delta h_m$

Hence degree of reaction $= \dfrac{\Delta h_m}{\Delta h_m + \Delta h_m} = \dfrac{1}{2}$

This is a proof that **Parson's reaction turbine is a 50% reaction turbine.**

Example 19.26. (*a*) *Explain the functions of the blading of a reaction turbine.*

(*b*) *A certain stage of a Parson's turbine consists of one row of fixed blades and one row of moving blades. The details of the turbine are as below :*

The mean diameter of the blades	*= 68 cm*
R.P.M. of the turbine	*= 3,000*
The mass of steam passing per sec	*= 13.5 kg*
Steam velocity at exit from fixed blades	*= 143.7 m/s*
The blade outlet angle	*= 20°*

Calculate the power developed in the stage and gross efficiency, assuming carry over co-efficient as 0.74 and the efficiency of conversion of heat energy into kinetic energy in the blade channel as 0.92. (M.U.)

Solution. (*a*) The blades of reaction turbine has to perform two functions :

1. They change the direction of motion of steam causing change of momentum, responsible for motive force.

2. The blades also act as nozzles causing pressure drop as steam moves in the blade passage.

(*b*) $\quad D = 0.68$ m, $N = 3000$ r.p.m., $\dot{m} = 13.5$ kg/s

$$C_{r_0} = 143.7 \text{ m/s}, \ \phi = 20°, \ \psi = 0.74, \ \eta = 0.92$$

Blade velocity, $$C_{bl} = \frac{\pi DN}{60} = \frac{\pi \times 0.68 \times 3000}{60} = 106.8 \text{ m/s}$$

$$C_{w_1} + C_{w_0} = 165 \text{ m/s (Fig. 19.39)}$$

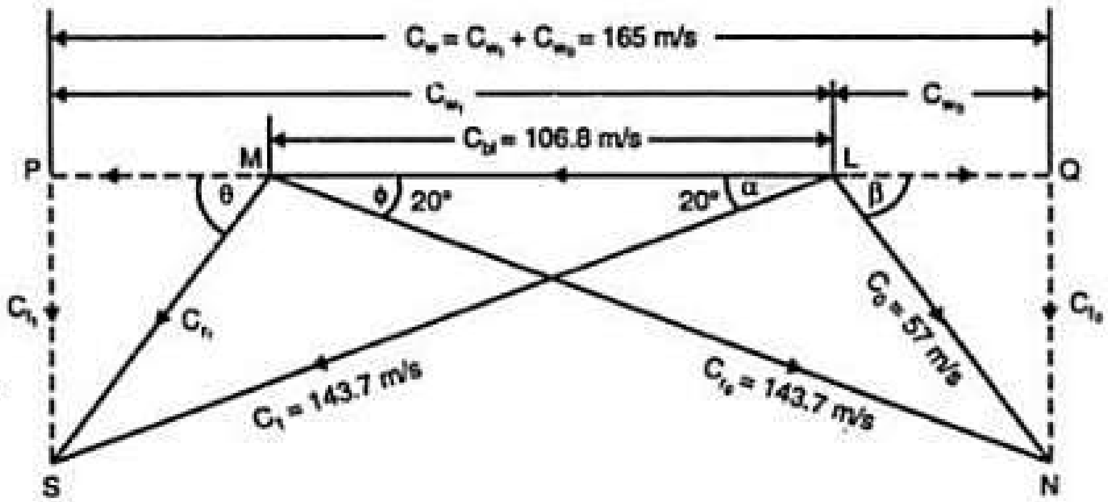


Fig. 19.39

Power developed $$\dot{m}_s \times \frac{(C_{w_1} + C_{w_0}) C_{bl}}{1000}$$

$$= 13.5 \times \frac{165 \times 106.8}{1000} = \mathbf{237.89 \text{ kW.}} \quad \textbf{(Ans.)}$$

In Parson turbine blades are symmetrical, *i.e.*,

$$\alpha = \phi, \theta = \beta$$

$$C_1 = C_{r_0}, \; C_{r_1} = C_0$$

Enthalpy drop $$= 2 \times \frac{C_1^2 - \psi C_0^2}{2\eta} = 2 \times \left(\frac{143.7^2 - 0.74 \times 57^2}{2 \times 0.92 \times 1000} \right) = 19.83 \text{ kJ/kg}$$

$\therefore$ **Gross efficiency** $$= \frac{\text{Work done/kg}}{\text{Enthalpy drop/kg}} = \frac{(C_{w_1} + C_{w_0}) C_{bl}}{19.83 \times 1000}$$

$$= \frac{165 \times 106.8}{19.83 \times 1000} = \mathbf{0.888 \text{ or } 88.8\%.} \quad \textbf{(Ans.)}$$

Example 19.27. (*a*) *Discuss the factors that influence the erosion of turbine blades. On a sketch mark the portions of the blades more likely to be eroded. Sketch the methods used to prevent erosion of steam turbines blades.*

(*b*) *A reaction turbine running at 360 r.p.m. consumes 5 kg of steam per second. Tip leakage is 10%. Discharge blade tip angle for both moving and fixed blades is 20°. Axial velocity of flow is 0.75 times blade velocity. The power developed by a certain pair is 4.8 kW where the pressure is 2 bar and dryness fraction is 0.95. Find the drum diameter and blades height.*

(U.P.S.C.)

Solution. (*a*) In the *high pressure* and *intermediate pressure stages* of turbine the pressures and temperatures are high and the blade material should be such that it stands high pressures and temperatures. In the intermediate pressure stages steam is wet therefore, the material

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be able to *withstand both corrosion and erosion* due to the presence of water particles. In addition to corrosion and erosion the blades *are also subjected to high centrifugal stresses* as the low pressure stages are longer, therefore, the blade material and its *design* should be such that *it stands corrosion, erosion* and *high centrifugal stresses.*

When the speed is high and moisture exceeds 10 per cent the effect of moisture is most prominent. The most effected portion is the back of the inlet edge of the blade, where either grooves are formed or even some portion breaks away. Due to centrifugal force the water particles tend to concentrate in the outer annulus and their tip speed is greater than the root speed, hence erosion effect is most on tips (Fig. 19.40).

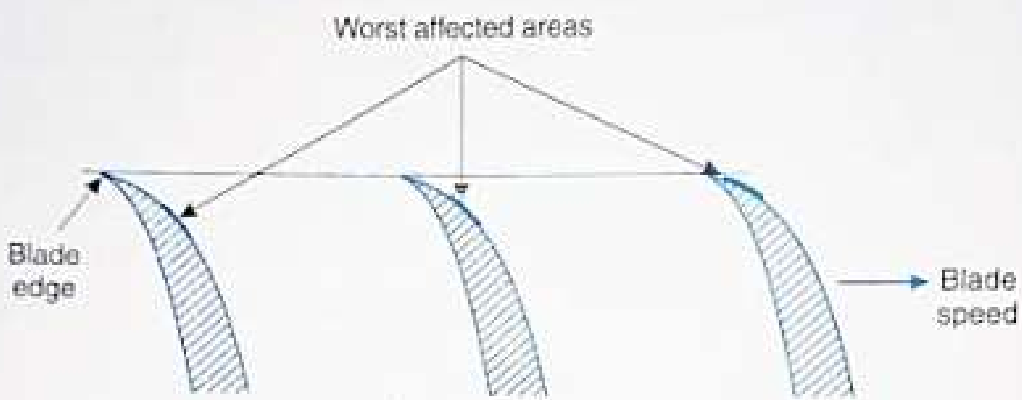

Fig. 19.40

Methods adopted to prevent erosion :

(*i*) By raising the temperature of steam at inlet, so that at exit of turbine the wetness does not exceed 10 per cent.

(*ii*) By adopting reheat cycle ; so that the wetness at exit remains in limit.

(*iii*) Drainage belts are provided on the turbine, so that the water droplets which are on outer periphery, due to centrifugal force are drained. The drained amount is about 25 per cent of total water particles present.

(*iv*) The leading edge of the turbine is provided with a shield of hard material.

— In the method (*i*) difficulties are the limits of temperature a material can withstand.

— Reheat cycle [method (*ii*)] has its own advantages and disadvantages.

— Drainage belts [method (*iii*)] cause structural changes in the turbine casing design, however, bleeding may help.

The most satisfactory solution is *providing tungsten shield. This prolongs the blade life, however, it does not remove the resistance which the water droplets impose on the rotation of the rotor.*

(*b*) Speed, $N = 360$ r.p.m.

$$m_s = 5\left[1 - \frac{10}{100}(\text{tip leakage})\right] = 4.5 \text{ kg/s}$$

$$\alpha = \phi = 20°,\ C_f = 0.75\ C_{bl}$$

Power developed in a certain pair

$$= 4.8 \text{ kW at 2 bar } (x = 0.95)$$

Blade velocity, $$C_{bl}\ (LM) = \frac{\pi DN}{60} = \frac{\pi \times D \times 360}{60} = 18.85\ D \text{ m/s.}$$

$$C_{f_i} = 0.75 \times 18.85\ D = 14.138\ D \text{ m/s}$$

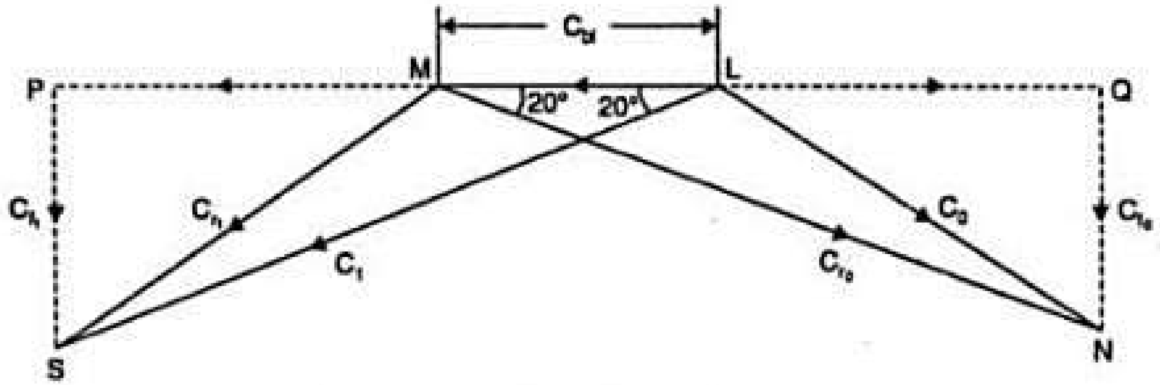

Fig. 19.41

From, ΔLSP, $$C_1 = \frac{C_{f_1}}{\sin 20°} = \frac{14.138\,D}{\sin 20°} \text{ m/s}$$

and $$LP = C_1 \cos 20° = 14.13\,D \times \frac{\cos 20°}{\sin 20°} = 14.138\,D \cot 20°$$

$$PQ = 2LP - LM = (2 \times 14.138\,D \cot 20° - 18.85\,D) \text{ m/s}$$

Power developed $$= \frac{\dot{m}_s \times PQ \times LM}{1000}$$

$$4.8 = \frac{4.5 \times (2 \times 14.138\,D \cot 20° - 18.85\,D) \times 18.85\,D}{1000}$$

Solving this equation, we get

Drum diameter, $D = 0.98$ **m. (Ans.)**

From steam tables at 2 bar, $v_g = 0.885 \text{ m}^3\text{/kg}$

Mass flow rate $$= \frac{\text{Flow area} \times \text{flow velocity}}{\text{Specific volume}}$$

$$4.5 = \frac{\pi D h \times C_{f_1}}{x v_g}$$

$$4.5 = \frac{\pi \times 0.98 \times h \times (14.138 \times 0.98)}{0.95 \times 0.885}$$

$$h = \frac{4.5 \times 0.95 \times 0.885}{\pi \times 0.98 \times 14.138 \times 0.98} = 0.0887 \text{ m}$$

$\therefore$ **Blade height** **= 0.0887 m. (Ans.)**

Example 19.28. *(a) List the advantages of steam turbines over gas turbines.*

(b) Determine the isentropic enthalpy drop in the stage of Parson's reaction turbine which has the following particulars :

Speed = 1500 rpm ; mean diameter of rotor = 1 m ;
stage efficiency = 80% ; speed ratio = 0.7 ;
blade outlet angle = 20°. **(B.U.)**

Solution. (*a*) **Advantages of steam turbines over gas turbines :**

1. The load control in steam turbines is easy simply by throttle governing or cut-off governing. In gas turbines the air-fuel ratio becomes too high, 100 to 150 at part loads. This causes problems to sustain the flame.

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2. The steam turbine works on Rankine cycle. In this cycle most of the heat is supplied at constant temperature in the form of latent heat of evaporation. Also the heat is rejected in the condenser isothermally. Hence the cycle is more efficient, and its efficiency is close to that of Carnot cycle. On the other hand, the gas turbine works on Brayton cycle whose efficiency is must less than that of Carnot cycle working between the same maximum and minimum limits of temperatures.

3. The efficiency of steam turbine at part load is not very much reduced. In gas turbines the maximum cycle temperature decreases considerably at part load ; therefore its part load efficiency is considerably low.

4. The blade material for steam turbines is cheap. For gas-turbines the blade material is costly as it is required to sustain considerably high temperatures.

(*b*) For Parson's reaction turbine, the velocity triangles are symmetrical, as shown in Fig. 19.37.

Given : $N = 1500$ r.p.m., $D = 1$ m, $\eta_{stage} = 80\%$;

Speed ratio, $\dfrac{C_{bl}}{C_1} = 0.7,\ \phi = \alpha = 20°$

$$C_1 = C_{r_0} \text{ and } C_{r_1} = C_0$$

$$C_{bl} = \frac{\pi DN}{60} = \frac{\pi \times 1 \times 1500}{60} = 78.54 \text{ m/s}$$

Speed ratio $= 0.7 = \dfrac{C_{bl}}{C_1}$

$\therefore$ $$C_1 = \frac{78.54}{0.7} = 112.2 \text{ m/s}$$

$$C_{r_1}^{\ 2} = C_1^{\ 2} + C_{bl}^{\ 2} - 2C_1C_{bl} \cos \alpha$$
$$= (112.2)^2 + (78.54)^2 - 2 \times 112.2 \times 78.54 \cos 20° = 2195.84$$

or $$C_{r_1} = 46.86 \text{ m/s}$$

$$\Delta h = \text{Actual enthalpy drop for the stage}$$
$$= \tfrac{1}{2}[(C_1^{\ 2} - C_0^{\ 2}) + (C_{r_0}^{\ 2} - C_{r_1}^{\ 2})]$$
$$= \tfrac{1}{2}[(C_1^{\ 2} - C_{r_1}^{\ 2}) + (C_1^{\ 2} - C_{r_1}^{\ 2})] = C_1^{\ 2} - C_{r_1}^{\ 2}$$
$$(\because\ C_0 = C_{r_1}\ ; C_{r_0} = C_1)$$

or $$\Delta h = [(112.2)^2 - (46.84)^2] \times 1/1000 \text{ kJ/kg} = 10.39 \text{ kJ/kg}$$

Isentropic enthalpy drop, $(\Delta h') = \dfrac{(\Delta h)}{\eta_{stage}} = \dfrac{10.39}{0.8} = $ **12.99 kJ/kg. (Ans.)**

19.8.3. Condition for Maximum Efficiency

The condition for *maximum efficiency* is derived by making the following *assumptions* :

(*i*) The degree of reaction is 50%.

(*ii*) The moving and fixed blades are symmetrical.

(*iii*) The velocity of steam at exit from the preceding stage is same as velocity of steam at the entrance to the succeeding stage.

Refer Fig. 19.37 (velocity diagram for reaction blade).

Work done per kg of steam,

$$W = C_{bl}\ (C_{w_1} + C_{w_0}) = C_{bl}[C_1 \cos \alpha + (C_{r_0} \cos \phi - C_{bl})]$$

as $\phi = \alpha$ and $C_{r_0} = C_{r_1}$ as per the assumptions

$$\therefore \quad W = C_{bl}\,[2C_1 \cos\alpha - C_{bl}]$$

or

$$W = C_1^2\left[\frac{2C_{bl}\,C_1 \cos\alpha}{C_1^2} - \frac{C_{bl}^2}{C_1^2}\right]$$

$$= C_1^2\,[2\rho\,.\cos\alpha - \rho^2] \qquad \text{...(19.27)}$$

where $\rho = \dfrac{C_{bl}}{C_1}$.

The K.E. supplied to the fixed blade $= \dfrac{C_1^2}{2g}$.

The K.E. supplied to the moving blade $= \dfrac{C_{r_0}^2 - C_{r_1}^2}{2}$.

$\therefore$ Total energy supplied to the stage,

$$\Delta h = \frac{C_1^2}{2} + \frac{C_{r_0}^2 - C_{r_1}^2}{2}$$

as $C_{r_0} = C_1$ for symmetrical triangles.

$$\therefore \quad \Delta h = \frac{C_1^2}{2} + \frac{C_1^2 - C_{r_1}^2}{2}$$

$$= C_1^2 - \frac{C_{r_1}^2}{2} \qquad \text{...(19.28)}$$

Considering the ΔLMS (Fig. 19.35)

$$C_{r_1}^2 = C_1^2 + C_{bl}^2 - 2C_1\,.\,C_{bl}\,.\cos\alpha$$

Substituting this value of $C_{r_1}^2$ is eqn. (19.35), we have

Total energy supplied to the stage

$$\Delta h = C_1^2 - (C_1^2 + C_{bl}^2 - 2C_1\,.\,C_{bl}\,.\cos\alpha)/2$$

$$= (C_1^2 + 2C_1\,C_{bl}\cos\alpha - C_{bl}^2)/2$$

$$= \frac{C_1^2}{2}\left[1 + \frac{2C_{bl}}{C_1}\,.\cos\alpha - \left(\frac{C_{bl}}{C_1}\right)^2\right]$$

$$= \frac{C_1^2}{2}\left[1 + 2\rho\cos\alpha - \rho^2\right] \qquad \text{...(19.29)}$$

The blade efficiency of the reaction turbine is given by,

$$\eta_{bl} = \frac{W}{\Delta h}$$

Substituting the value of W and Δh from eqns. (19.27) and (19.29), we get

$$\eta_{bl} = \frac{C_1^2\,[2\rho\cos\alpha - \rho^2]}{\dfrac{C_1^2}{2}(1 + 2\rho\cos\alpha - \rho^2)}$$

$$= \frac{2(2\rho \cos \alpha - \rho^2)}{(1 + 2\rho \cos \alpha - \rho^2)} = \frac{2\rho(2 \cos \alpha - \rho)}{(1 + 2\rho \cos \alpha - \rho^2)} = \frac{2(1 + 2\rho \cos \alpha - \rho^2) - 2}{(1 + 2\rho \cos \alpha - \rho^2)}$$

$$= 2 - \frac{2}{1 + 2\rho \cos \alpha - \rho^2} \qquad \text{...(19.30)}$$

The η_{bl} becomes maximum when the value of $(1 + 2\rho \cos \alpha - \rho^2)$ becomes maximum.

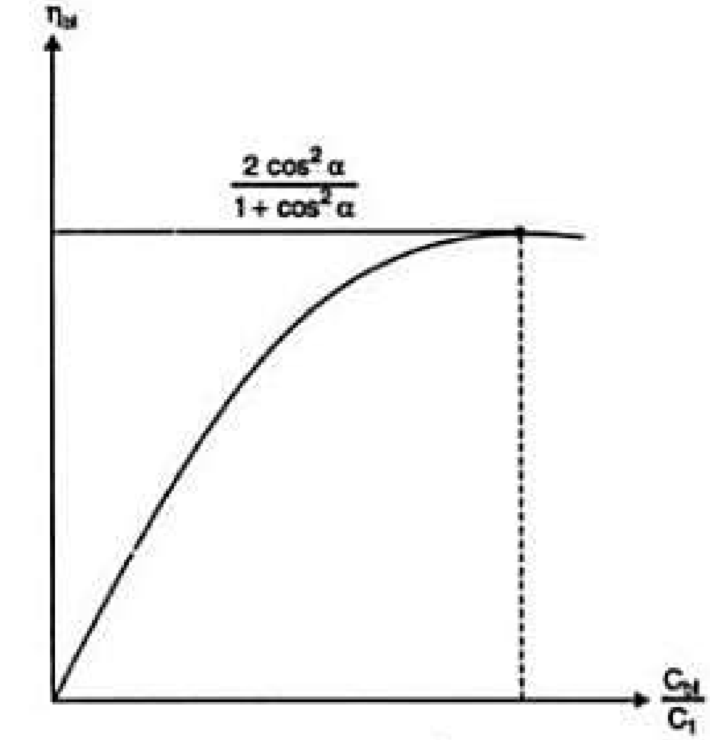

Fig. 19.42

∴ The required equation is

$$\frac{d}{d\rho}(1 + 2\rho \cos \alpha - \rho^2) = 0$$

$$2 \cos \alpha - 2\rho = 0$$

∴ $$\rho = \cos \alpha \qquad \text{...(19.31)}$$

Substituting the value of ρ from eqn. (19.31) into the eqn. (19.30), the value of *maximum efficiency* is given by,

$$(\eta_{bl})_{max} = 2 - \frac{2}{1 + 2 \cos^2 \alpha - \cos^2 \alpha} = 2\left(1 - \frac{1}{1 + \cos^2 \alpha}\right) = \frac{2 \cos^2 \alpha}{1 + \cos^2 \alpha}$$

Hence $$(\eta_b)_{max} = \frac{2 \cos^2 \alpha}{1 + \cos^2 \alpha} \qquad \text{...(19.32)}$$

The variation of η_{bl} with blade speed ratio $\left(\frac{C_{bl}}{C_1}\right)$ for the reaction stage is shown in Fig. 19.42.

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19.9. TURBINES EFFICIENCIES

1. **Blade or diagram efficiency** (η_{bl}). *It is the ratio of work done on the blade per second to the energy entering the blade per second.*

2. **Stage efficiency** (η_{stage}). The stage efficiency covers all the losses in the nozzles, blades, diaphragms and discs that are associated with that stage.

$$\eta_{stage} = \frac{\text{Network done on shaft per stage per kg of steam flowing}}{\text{Adiabatic heat drop per stage}}$$

$$= \frac{\text{Network done on blades} - \text{Disc friction and windage}}{\text{Adiabatic heat drop per stage}}.$$

3. **Internal efficiency** ($\eta_{internal}$). This is equivalent to the stage efficiency when applied to the whole turbine, and is given by :

$$\eta_{internal} = \frac{\text{Heat coverted into useful work}}{\text{Total adiabatic heat drop}}.$$

4. **Overall or turbine efficiency** ($\eta_{overall}$). This efficiency covers internal and external losses ; for example, bearings and steam friction, leakage, radiation etc.

$$\eta_{overall} = \frac{\text{Work delivered at the turbine coupling in heat units per kg of steam}}{\text{Total adiabatic heat drop}}.$$

5. **Net efficiency or efficiency ratio** (η_{net}). It is the ratio

$$\frac{\text{Brake thermal efficiency}}{\text{Thermal efficiency on the Rankine cycle}}$$

Also the actual thermal efficiency

$$= \frac{\text{Heat converted into useful work per kg of steam}}{\text{Total heat in steam at stop valve} - \text{Water heat in exhaust}}.$$

Again, Rankine efficiency

$$= \frac{\text{Adiabatic heat drop}}{\text{Total heat in steam at stop valve} - \text{Water heat in exhaust}}.$$

$$\eta_{net} = \frac{\text{Heat converted into useful work}}{\text{Total adiabatic heat drop}}.$$

Hence $\eta_{net} = \eta_{overall}$.

It is the overall or net efficiency that is meant when the efficiency of a turbine is spoken of without qualification.

19.10. TYPES OF POWER IN STEAM TURBINE PRACTICE

In steam turbine performance the following types of power are generally used :

1. **Adiabatic power (A.P.).** It is the power based on the total internal steam flow and adiabatic heat drop.

2. **Shaft power (S.P.).** It is the actual power transmitted by the turbine.

3. **Rim power (R.P.).** It is the power developed at the rim. It is also called ***blade power.***

Power losses are usually expressed as follows :

(*i*) $(P)_{fD}$ = Power lost in overcoming disc friction.

(*ii*) $(P)_{bw}$ = Power lost in blade windage losses.

Let us consider the case of an impulse turbine. Let $\dot{m}_s$ be the total internal steam flow in kg/s.

Refer Fig. 19.43. The line (1-2) represents the adiabatic or isentropic expansion of steam in the nozzle from pressure p_1 to p_2. But the actual path of the stage point during expansion in nozzles is shown by (1-3) which takes into account the effect of 'nozzles losses'

Then,
$$\text{A.P.} = \dot{m}_s\ (h_1 - h_2)\ \text{kW} \quad \ldots(19.33)$$

After expansion in the nozzle the steam enters the blades where the R.P. is developed. Due to blade friction the steam is somewhat reheated and this reheating is shown by (3-4) along the constant pressure p_2 line just for convenience. But in actual practice though the pressure at outlet of the blade is equal to that at the inlet, the pressure in the blade channels is not constant. However, with this simplification ;

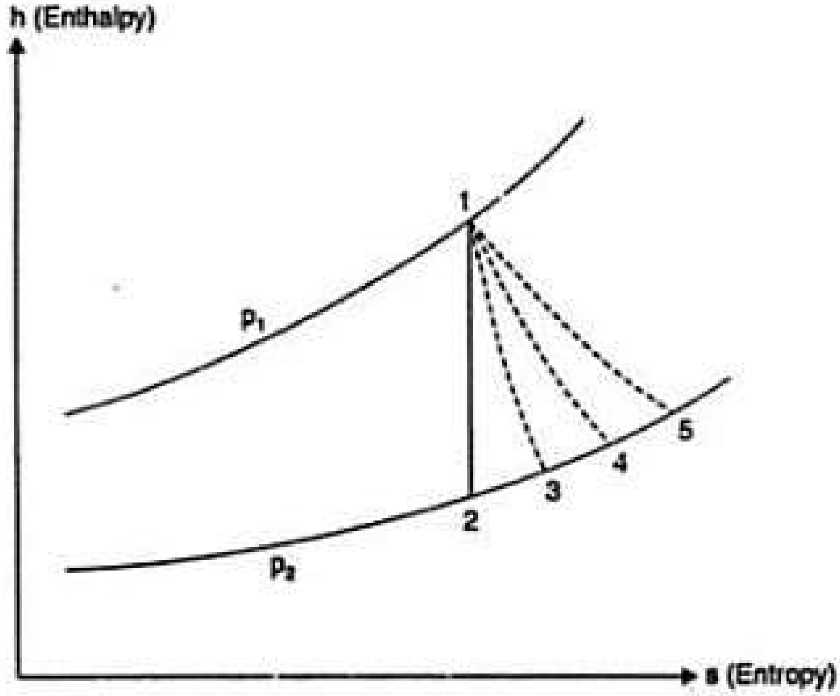

Fig. 19.43

$$\text{R.P.} = \dot{m}_s\ (h_1 - h_4)\ \text{kW} \quad \ldots(19.34)$$

4-5 shows the further reheating due to friction and blade windage and these losses are given as

$$(P)_{w.f.} = \dot{m}_s\ (h_5 - h_4)\ \text{kW} \quad \ldots(19.35)$$

Now points 1 and 5 are the initial and final stage points respectively for a single stage impulse turbine. It, therefore, follows that

$$\text{S.P.} = m_s(h_1 - h_5)\ \text{kW}. \quad \ldots(19.36)$$

REACTION TURBINES

Example 19.29. *The following data refer to a particular stage of a Parson's reaction turbine :*

Speed of the turbine	= *1500 r.p.m.*
Mean diameter of the rotor	= *1 metre*
Stage efficiency	= *80 per cent*

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Blade outlet angle = 20°

Speed ratio = 0.7

Determine the available isentropic enthalpy drop in the stage.

Solution. Mean diameter of the rotor, $D = 1$ m

Turbine speed, $N = 1500$ r.p.m.

Blade outlet angle, $\phi = 20°$

Speed ratio, $\rho = \dfrac{C_b}{C_1} = 0.7$

Stage efficiency, $\eta_{stage} = 80\%$

Isentropic enthalpy drop :

Blade speed, $$C_{bl} = \frac{\pi DN}{60} = \frac{\pi \times 1 \times 1500}{60} = 78.54 \text{ m/s}$$

But $$\rho = \frac{C_{bl}}{C_1} = 0.7 \text{ (given)}$$

$\therefore$ $$C_1 = \frac{C_{bl}}{0.7} = \frac{75.54}{0.7} = 112.2 \text{ m/s}$$

In Parson's turbine $\alpha = \phi$.

With the above data known, the velocity diagram for the turbine can be drawn to a suitable scale as shown in Fig. 19.44.

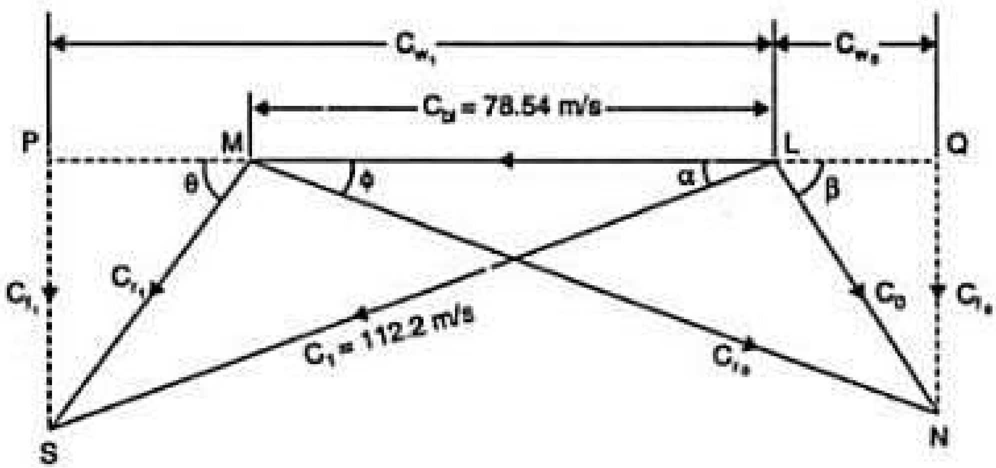

Fig. 19.44

By measurement (from the diagram)

$$C_{w_1} = 106.5 \text{ m/s} \; ; \; C_{w_2} = 27 \text{ m/s}$$

$$\eta_{stage} = \frac{C_{bl}(C_{w_1} + C_{w_2})}{h_d}, \text{ where } h_d = \text{isentropic enthalpy drop.}$$

i.e., $$0.8 = \frac{78.54(106.25 + 27)}{h_d \times 1000}$$

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$$\therefore \qquad h_d = \frac{78.54(106.25 + 27)}{0.8 \times 1000} = 13.08 \text{ kJ}$$

Hence, *isentropic enthalpy drop* = **13.08 kJ/kg. (Ans.)**

Example 19.30. *In a reaction turbine, the blade tips are inclined at 35° and 20° in the direction of motion. The guide blades are of the same shape as the moving blades, but reversed in direction. At a certain place in the turbine, the drum diameter is 1 metre and the blades are 10 cm high. At this place, the steam has a pressure of 1.75 bar and dryness 0.935. If the speed of this turbine is 250 r.p.m. and the steam passes through the blades without shock, find the mass of steam flow and power developed in the ring of moving blades.*

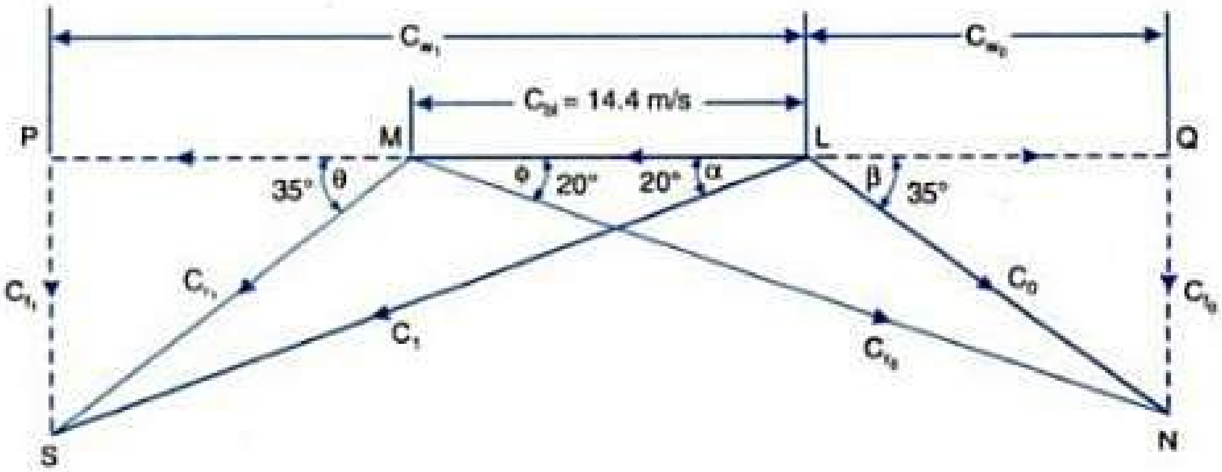


Fig. 19.45

Solution. Refer Fig. 19.45.

Angles, $\alpha = \phi = 20°$, and $\theta = \beta = 35°$

Mean drum diameter, $D_m = 1 + 0.1 = 1.1$ m

Area of flow $= \pi D_m h$, where h is the height of blade

$= \pi \times 1.1 \times 0.1 = 0.3456 \text{ m}^2$

Steam pressure $= 1.75$ bar

Dryness fraction of steam, $x = 0.935$

Speed of the turbine, $N = 250$ r.p.m.

Rate of steam flow, $\dot{m}_s$:

Blade speed, $$C_{bl} = \frac{\pi DN}{60} = \frac{\pi \times 1.1 \times 250}{60} = 14.4 \text{ m/s}$$

With the above given data the velocity diagram can be drawn to a suitable scale as shown in Fig. 19.45.

By measurement (from diagram) :

$$C_{w_1} = 30 \text{ m/s} ; C_{w_0} = 15.45 \text{ m/s} ; C_{f_1} = C_{f_0} = 10.8 \text{ m/s}$$

From steam tables corresponding to 1.75 bar pressure.

v_g = Specific volume of dry saturated steam

$= 1.004 \text{ m}^3/\text{kg}$

$x = 0.935$ (given)

$\therefore$ Specific volume of wet steam $= xv_g = 0.935 \times 1.004 = 0.938 \text{ m}^3/\text{kg}$

Mean flow rate is given by :

$$\dot{m}_s = \frac{\text{Area of flow} \times \text{Velocity of flow}}{\text{Specific volume of steam}} = \frac{0.3456 \times 10.8}{0.938} = \mathbf{3.98\ kg/s.}$$

Power developed, P :

$$P = \frac{\dot{m}_s(C_{w_1} + C_{w_2})C_{bl}}{1000}\ \text{kW}$$

$$= \frac{3.98(30 + 15.45) \times 14.4}{1000} = \mathbf{2.6\ kW.}\ \ \textbf{(Ans.)}$$

Example 19.31. *In a reaction turbine, the fixed blades and moving blades are of the same shape but reversed in direction. The angles of the receiving tips are 35° and of the discharging tips 20°. Find the power developed per pair of blades for a steam consumption of 2.5 kg/s, when the blade speed is 50 m/s. If the heat drop per pair is 10.04 kJ/kg, find the efficiency of the pair.*

Solution. Angles of receiving tips, $\theta = \beta = 35°$

Angles of discharging tips, $\alpha = \phi = 20°$

Steam consumption, $\dot{m}_s = 2.5$ kg/s

Blade speed, $C_{bl} = 50$ m/s

Heat drop per pair, $h_d = 10.04$ kJ/kg

Power developed per pair of blades :

Refer Fig. 19.46.

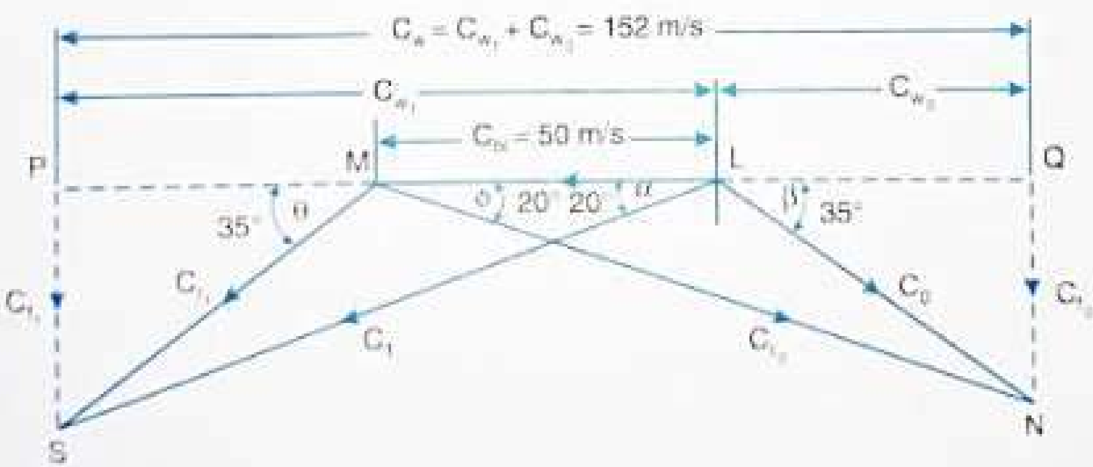


Fig. 19.46

$$NS = PQ = 152 \text{ m/s}$$

Work done per pair per kg of steam

$$= (C_{w_1} + C_{w_2})\ C_{bl} = 152 \times 50 = 7600 \text{ Nm/kg of steam.}$$

$$\textit{Power / pair} = \frac{\dot{m}_s(C_{w_1} + C_{w_2})C_{bl}}{1000} = \frac{2.5 \times 7600}{1000} = \mathbf{19\ kW.}\ \ \textbf{(Ans.)}$$

Efficiency of the pair :

$$\textit{Efficiency} = \frac{\text{Work done per pair per kg of steam}}{h_d}$$

$$= \frac{7600}{10.04 \times 1000} = 0.757 = \mathbf{75.7\%.}\ \ \textbf{(Ans.)}$$

Example 19.32. *A stage of a turbine with Parson's blading delivers dry saturated steam at 2.7 bar from the fixed blades at 90 m/s. The mean blade height is 40 mm, and the moving blade exit angle is 20°. The axial velocity of steam is 3/4 of the blade velocity at the mean radius. Steam is supplied to the stage at the rate of 9000 kg/h. The effect of the blade tip thickness on the annulus area can be neglected. Calculate :*

(i) *The wheel speed in r.p.m. ;* (ii) *The diagram power ;*

(iii) *The diagram efficiency ;* (iv) *The enthalpy drop of the steam in this stage.*

Solution. The velocity diagram is shown in Fig. 19.47 (*a*) and the blade wheel annulus is represented in Fig. 19.47 (*b*).

Pressure = 2.7 bar, $x = 1$, $C_1 = 90$ m/s, $h = 40$ mm = 0.04 m

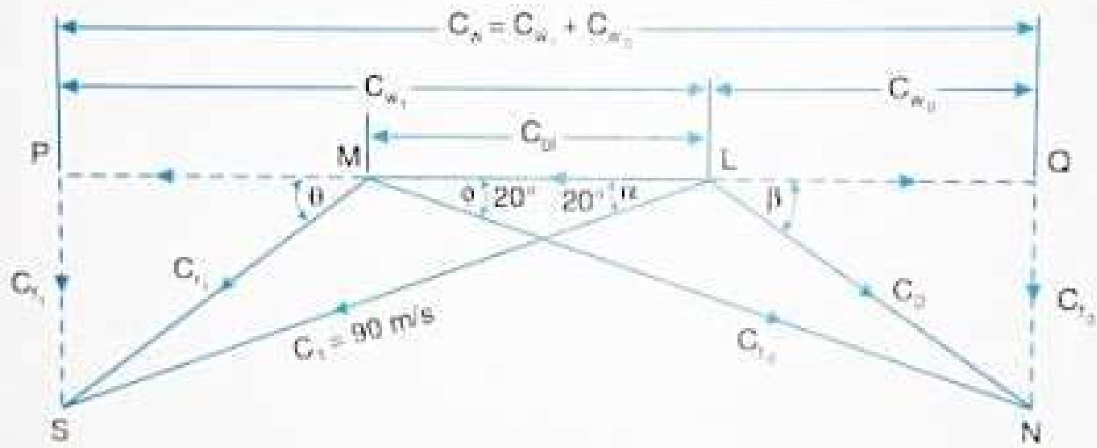
$$\alpha = \phi = 20°, \; C_{f_i} = C_{f_o} = 3/4 \; C_{bl}$$

Rate of steam supply $= 9000$ kg/h.

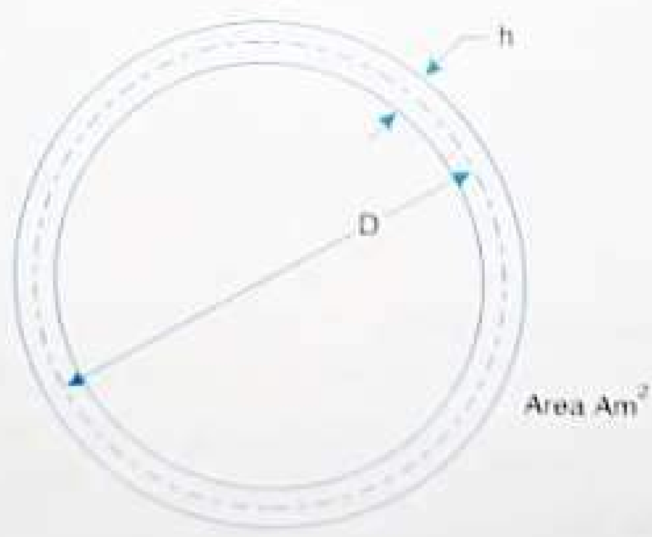
(i) **Wheel speed, N :**

$$C_f = 3/4 \; C_{bl} = C_1 \sin 20° = 90 \sin 20° = 30.78 \text{ m/s}$$

$$\therefore \quad C_{bl} = 30.78 \times 4/3 = 41.04 \text{ m/s}$$



(*a*)



(*b*)

Fig. 19.47

The mass flow of steam is given by : $\dot{m}_s = \frac{C_f A}{v}$

(where A is the annulus area, and v is the specific volume of the steam).

In this case, $v = v_g$ at 2.7 bar = 0.6686 m³/kg

$$\therefore \quad \dot{m}_s = \frac{9000}{3600} = \frac{30.78}{0.6686} \quad \text{or} \quad A = \frac{9000 \times 0.6686}{3600 \times 30.78} = 0.054 \text{ m}^2$$

Now, annulus area, $A = \pi D h$

(where D is the mean diameter, and h is the mean blade height)

$$\therefore \quad 0.054 = \pi D \times 0.04 \quad \text{or} \quad D = \frac{0.054}{\pi \times 0.04} = 0.43 \text{ m}$$

Also,
$$C_{bl} = \frac{\pi D N}{60} \quad \text{or} \quad 41.04 = \frac{\pi \times 0.43 \times N}{60}$$

or
$$N = \frac{41.04 \times 60}{\pi \times 0.43} = \mathbf{1823\ r.p.m.} \quad \textbf{(Ans.)}$$

(*ii*) **The diagram power :**

Diagram power $= \dot{m}_s\ C_w\ C_{bl}$

Now,
$$C_w = 2C_1 \cos \alpha - C_{bl}$$
$$= 2 \times 90 \times \cos 20° - 41.04 = 128.1 \text{ m/s}$$

$$\therefore \quad \textit{Diagram power} = \frac{9000 \times 128.1 \times 41.04}{3600 \times 1000} = \mathbf{13.14\ kW.} \quad \textbf{(Ans.)}$$

(*iii*) **The diagram efficiency :**

Rate of doing work per kg/s = $C_w\ C_{bl}$ = 128.1 × 41.04 N m/s

Also, energy input to the moving blades per stage

$$= \frac{C_1^2}{2} + \frac{C_{r_0}^2 - C_{r_1}^2}{2} = \frac{C_1^2}{2} + \frac{C_1^2 - C_{r_1}^2}{2} = C_1^2 - \frac{C_{r_1}^2}{2} \quad (\because\ C_{r_0} = C_1)$$

Referring to Fig. 19.47 (*a*), we have

$$C_{r_1}^2 = C_1^2 + C_{bl}^2 - 2C_1\ C_{bl} \cos \alpha$$
$$= 90^2 + 41.04^2 - 2 \times 90 \times 41.04 \times \cos 20°$$
$$= 8100 + 1684.28 - 6941.69$$

$$\therefore \quad C_{r_1} = 53.3 \text{ m/s}$$

Energy input
$$= 90^2 - \frac{53.3^2}{2} = 6679.5 \text{ Nm per kg/s}$$

$$\therefore \quad \textit{Diagram efficiency} = \frac{128.1 \times 41.04}{6679.5} = \mathbf{0.787 \quad or \quad 78.7\%.} \quad \textbf{(Ans.)}$$

(*iv*) **Enthalpy drop in the stage :**

Enthalpy drop in the moving blades

$$= \frac{C_{r_0}^2 - C_{r_1}^2}{2} = \frac{90^2 - 53.3^2}{2 \times 1000} = 2.63 \text{ kJ/kg} \qquad (\because\ C_{r_0} = C_{r_1})$$

$\therefore$ *Total enthalpy drop per stage* = 2 × 2.63 = **5.26 kJ/kg.** **(Ans.)**

Example 19.33. *The outlet angle of the blade of a Parson's turbine is 20° and the axial velocity of flow of steam is 0.5 times the mean blade velocity. If the diameter of the ring is 1.25 m and the rotational speed is 3000 r.p.m. determine :*

(*i*) *Inlet angles of blades.*

(*ii*) *Power developed if dry saturated steam at 5 bar passes through the blade whose height may be assumed as 6 cm. Neglect the effect of blade thickness.*

Solution. Refer Fig. 19.48.

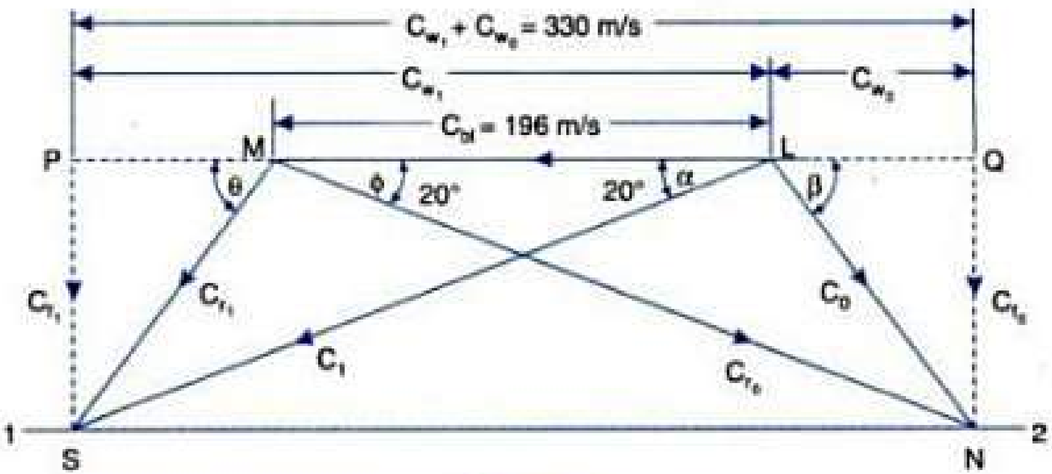


Fig. 19.48

Angles, $\alpha = \phi = 20°$

Axial velocity of flow of steam,

$$C_{f_1} = C_{f_0} = 0.5\ C_{bl} \text{ (blade speed)}$$

Diameter of the ring, $D = 1.25$ m

Rotational speed, $N = 3000$ r.p.m.

Blade speed, $$C_{bl} = \frac{\pi DN}{60} = \frac{\pi \times 1.25 \times 3000}{60} = 196 \text{ m/s}$$

$$\therefore \quad C_{f_1} = C_{f_0} = 0.5 \times 196 = 98 \text{ m/s}$$

Velocity diagram is drawn as follows :

- Takes LM (C_{bl}) = 196 m/s, and $\alpha = \phi = 20°$.
- Draw line 1–2 parallel to LM at a value of 98 m/s (according to scale). The points S and N are thus located on the line 1–2.
- Complete the rest of the diagram as shown in Fig. 19.48.

(*i*) **Inlet angles of blades :**

The *inlet angles* (by measurement) are :

$$\beta = \theta = 55° \quad \textbf{(Ans.)}$$

(*ii*) **Power developed, P :**

Area of flow is given by, $A = \pi \times D$ (mean diameter) $\times h$ (height of blade)

Mean flow rate is given by,

$$m_s = \frac{\text{Area of flow} \times \text{Velocity of flow}}{\text{Specific volume of steam}} = \frac{\pi D h \times C_f}{v}$$

From steam tables, $\qquad v_g = 0.375 \text{ m}^3/\text{kg at 5 bar}$

$$\therefore \qquad \dot{m}_s = \frac{\pi \times 1.25 \times \left(\dfrac{6}{100}\right) \times 98}{0.375} = 61.57 \text{ kg/s}$$

Power developed, $\qquad P = \dfrac{\dot{m}_s \times C_w \times C_{bl}}{1000} = \dfrac{61.57 \times 330 \times 196}{1000} = \mathbf{3982.3 \text{ kW.}}$ **(Ans.)**

Example 19.34. *A 50% reaction turbine (with symmetrical velocity triangles) running at 400 r.p.m. has the exit angle of the blades as 20° and the velocity of steam relative to the blades at the exit is 1.35 times the mean blade speed. The steam flow rate is 8.33 kg/s and at a particular stage the specific volume is 1.381 m³/kg. Calculate for this stage :*

(i) A suitable blade height, assuming the rotor mean diameter 12 times the blade height, and

(ii) The diagram work. **(N.U.)**

Solution. Speed, $\qquad N = 400 \text{ r.p.m.} \; ; \; \alpha = 20°$

$$C_{r_0} = C_1 = 1.35\, C_{bl} \; ; \; \dot{m}_s = 8.33 \text{ kg/s}$$

$$v = 1.381 \text{ m}^3/\text{kg} \; ; \; D = 12\, h$$

(i) Blade height, h :

Refer Fig. 19.49.

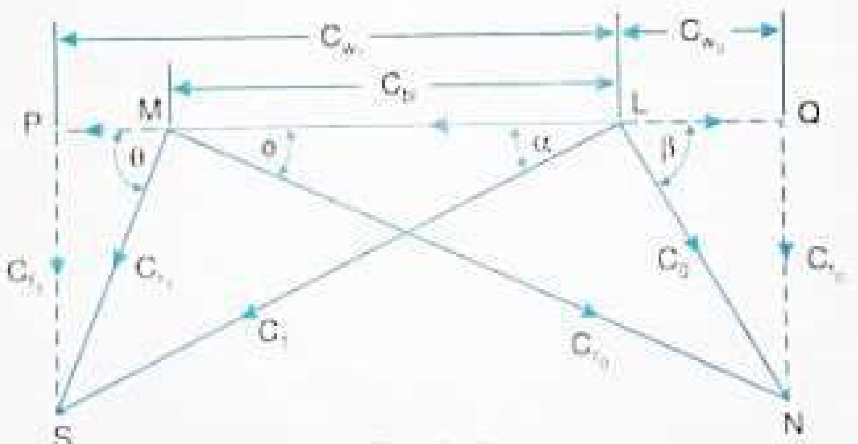


Fig. 19.49

Axial flow velocity, $\qquad C_{f_0} = C_{f_1} = C_f = C_1 \sin \alpha$

$$= 1.35\, C_{bl} \sin 20°$$

$$= 0.4617\, C_{bl}$$

Area of flow, $\qquad A = \pi D h = \pi D \times \dfrac{D}{12} = \dfrac{\pi D^2}{12}$

Mass flow rate, $\qquad \dot{m} = \dfrac{A C_f}{v} \quad \text{or} \quad 8.33 = \dfrac{A \times 0.4617\, C_{bl}}{1.381}$

or $\qquad \dfrac{8.33 \times 1.381}{0.4917} = A \times C_{bl} = \dfrac{\pi D^2}{12} \times \dfrac{\pi D N}{60}$

or $\qquad 24.916 = \dfrac{\pi^2 D^3 \times 400}{720} \quad \text{or} \quad D^3 = \dfrac{24.919 \times 720}{\pi^2 \times 400} \quad \text{or} \quad D = 1.656 \text{ m}$

$\therefore$ Blade height, $h = \dfrac{D}{12} = \dfrac{1.656}{12} = 0.138$ m or **138 mm. (Ans.)**

(ii) The diagram work :

Diagram work $= \dot{m} \times C_{bl}\,(C_{w_1} + C_{w_0})$

$= \dot{m} \times C_{bl}\,(2\,C_1 \cos\alpha - C_{bl})$

$= 8.33 \times C_{bl}\,(2 \times 1.35\,C_{bl} \cos 20° - C_{bl})$

$= 8.33 \times C_{bl}^2\,(2 \times 1.35 \cos 20° - 1)$

$= 8.33 \times \left(\dfrac{\pi DN}{60}\right)^2 \times 1.537 = 8.33 \times \left(\dfrac{\pi \times 1.656 \times 400}{60}\right)^2 \times 1.537$

$= 15401.2$ W or **15.4 kW. (Ans.)**

Example 19.35. *300 kg/min of steam (2 bar, 0.98 dry) flows through a given stage of a reaction turbine. The exit angle of fixed blades as well as moving blades is 20° and 3.68 kW of power is developed. If the rotor speed is 360 r.p.m. and tip leakage is 5 per cent, calculate the mean drum diameter and the blade height. The axial flow velocity is 0.8 times the blade velocity.*

(Roorkee University)

Solution. Rate of flow of steam through the turbine, $\dot{m}_s = \dfrac{300}{60} = 5$ kg/s

Pressure and condition of steam, $p = 2$ bar, $x = 0.98$.

The exit angles of fixed blades as well as moving blades, $\alpha = \phi = 20°$

Power developed, $P = 3.68$ kW

Speed of the rotor, $N = 360$ r.p.m.

Tip leakage $= 5$ per cent

Axial flow velocity, $C_f = 0.8\,C_{bl}$ (blade velocity)

Refer Fig. 19.49.

Mean drum diameter, D :

Mean blade velocity, $C_{bl} = \dfrac{\pi DN}{60} = \dfrac{\pi D \times 360}{60} = 18.85\,D$ m/s

Power developed, $P = \dfrac{\dot{m}_s C_{bl} C_w}{1000}$

or $3.67 = \dfrac{(5 \times 0.95) \times 18.85\,D \times C_w}{1000}$

$\therefore$ $C_w = \dfrac{3.67 \times 1000}{(5 \times 0.95) \times 18.85 D} = \dfrac{40.988}{D}$

Assuming Parson's reaction turbine, we have

$$C_{f_1} = C_1 \sin\alpha \quad \text{or} \quad C_1 = \frac{C_{f_1}}{\sin\alpha} = \frac{0.8\,C_{bl}}{\sin\alpha} = \frac{0.8 \times 18.85\,D}{\sin 20°} = 44.091\,D$$

$(C_{f_1} = C_{f_0} = C_f)$

Also, $C_w = 2\,C_1 \cos\alpha - C_{bl}$ or $\dfrac{40.988}{D} = 2 \times 44.091\,D \cos 20° - 18.85\,D = 64.01\,D$

or $40.988 = 64.01\,D^2$ or $D = 0.8$ m or **800 mm. (Ans.)**

Blade height, h :

Mean steam flow rate, $\dot{m}_s = \dfrac{\pi D h C_f}{x v_g}$

or $$(5 \times 0.95) = \frac{\pi \times 0.8 \times h \times C_1 \sin \alpha}{0.98 \times 0.885} = \frac{\pi \times 0.8 \times h \times (44.091 D \times \sin 20°)}{0.98 \times 0.885}$$

(At 2 bar : v_g = 0.885 kg/m³)

or $$h = \frac{(5 \times 0.95) \times 0.98 \times 0.885}{\pi \times 0.8 \times (44.091 \times 0.80 \times 0.3420)} = 0.1359 \text{ m} \quad \text{or} \quad \mathbf{135.9 \text{ mm.}} \quad \textbf{(Ans.)}$$

Example 19.36. *(a) Why is drum type construction preferred to disc type construction in reaction turbine ?*

(b) Why is partial admission of steam adopted for H.P. imoulse stages while full admission is essential for any stage of a reaction turbine ?

(c) In a 50% reaction turbine, the speed of rotation of a blade group is 3000 r.p.m. with mean blade velocity of 120 m/s. The velocity ratio is 0.8 and the exit angle of the blades is 20°. If the mean blade height is 30 mm, calculate the total steam flow rate through the turbine. Neglect the effect of blade edge thickness of the annular area but consider 10% of the total steam flow rate as the tip leakage loss. The mean condition of steam in that blade group is found to be 2.7 bar and 0.95 dry.

(d) What do you mean by once through boilder ? **(AMIE Summer, 1998)**

Solution. (a) The rotor of the turbine can be of drum type or disc type. *Disc type construction is difficult (complicated) to make, but lighter in weight.* Hence the centrifugal stresses are lower at a particular speed. On the other hand drum type construction is simple in construction, and it is easy to attach aerofoil shape blades. Further it is easier to design for tip lea age reduction which is a major problem in reaction turbines. Moreover due to small pressure drop per stage (larger number of stages) in reaction turbines, their rotational speeds are lower and so the *centrifugal stresses are not very high* (even the reaction blades are lighter). Therefore *drum type construction is preferred to disc type in reaction turbines.*

To accommodate increase in specific volume at lower pressures the drum diameter is stepped up which allows greater area without unduly increasing blade height. The *increased drum diameter also increases the torque due to steam pressure.*

(b) In **impulse turbines** there is no expansion of steam in moving blades, and the pressure of steam remains constant while flowing over the moving blades. The expansion takes place only in the nozzles at the inlet to the turbine in H.P. stages, or through the fixed blades in the subsequent stages. The nozzles need not occupy the complete circumference. Therefore partial admission of steam is feasible and adopted for H.P. impulse stages.

In **reaction turbines,** *pressure drop is required in the moving blades also. This is not possible with partial admission. Hence full admission is essen·al for all stages of a reaction turbine.*

(c) Refer ¨g. 19.37.

Given : $N = 3000$ r.p.m. ; $\phi = \alpha = 20°$; $C_{bl} = 120$ m/s ; $\dfrac{C_{bl}}{C_1} = 0.8$;

$\therefore$ $$C_1 = \frac{C_{bl}}{0.8} = \frac{120}{0.8} = 150 \text{ m/s}$$

Also $$C_{bl} = \frac{\pi D N}{60}$$

or $$120 = \frac{\pi DN}{60}$$

$\therefore$ $$D = \frac{120 \times 60}{\pi \times 3000} = 0.764 \text{ m.}$$

From steam tables, v_g (at 2.7 bar) = 0.668 m³/kg

$$v = 0.95 \times 0.668 = 0.6346 \text{ m}^3/\text{kg}$$

Flow area $$A = \pi Dh = \pi \times 0.764 \times \frac{30}{1000} = 0.072 \text{ m}^2$$

Flow velocity $$C_f = C_1 \sin \alpha = 150 \sin 20° = 51.3 \text{ m/s} \quad (C_{f_1} = C_{f_0} = C_f)$$

Mass flow rate $$\dot{m} = \frac{AC_f}{v} = \frac{0.072 \times 51.3}{0.6346} = 5.82 \text{ kg/s}$$

Accounting for 10 per cent leakage (of total steam flow), the total steam flow rate is

$$\frac{5.82}{0.9} = \mathbf{6.467 \text{ kg/s.}} \quad \text{(Ans.)}$$

(*d*) *Once through* boiler is a boiler which *does not require any water or steam drum.* It is a *monotube boiler* using about 1.5 kg long tube arranged in the combustion chamber and the furnace. The *economizer, boiler and superheater are in series with no fixed surfaces as separators between the steam and water.*

Benson boiler is an example of once through boiler, operating at supercritical pressure. The tube length to diameter ratio of such a boiler is about 2500. Due to large frictional resistance the feed pressure should be about 1.4 times the boiler pressure.

Example 19.37. *A twenty-stage Parson turbine receives steam at 15 bar at 300°C. The steam leaves the turbine at 0.1 bar pressure. The turbine has a stage efficiency of 80% and the reheat factor 1.06. The total power developed by the turbine is 10665 kW. Find the steam flow rate through the turbine assuming all stages develop equal power.*

The pressure of steam, at certain stage of the turbine is 1 bar abs., and is dry and saturated. The blade exit angle is 25° and the blade speed ratio is 0.75. Find the mean diameter of the rotor of this stage and also the rotor speed. Take blade height as 1/12th of the mean diameter. The thickness of the blades may be neglected.

Solution. Number of stage = 20

Steam supply pressure = 15 bar, 300°C

Exhaust pressure = 0.1 bar

Stage efficiency of turbine, η_{stage} = 80%

Reheat factor = 1.06

Total power developed = 10665 kW

Steam pressure at a certain stage = 1 bar abs., $x = 1$

Blade exit angle = 25°

Blade speed ratio, $$\rho = \frac{C_{bl}}{C_1} = 0.75$$

Height of the blade, $$h = \frac{1}{12} D \text{ (mean dia. of rotor)}$$

(*i*) **Steam flow rate, $\dot{m}_s$:**

Refer Fig. 19.50.

Isentropic drop, $(\Delta h)_{isentropic} = h_1 - h_2 = 3040 - 2195 = 845$ **kJ/kg**

$\eta_{overall} = \eta_{stage} \times$ Reheat factor $= 0.8 \times 1.06 = 0.848$

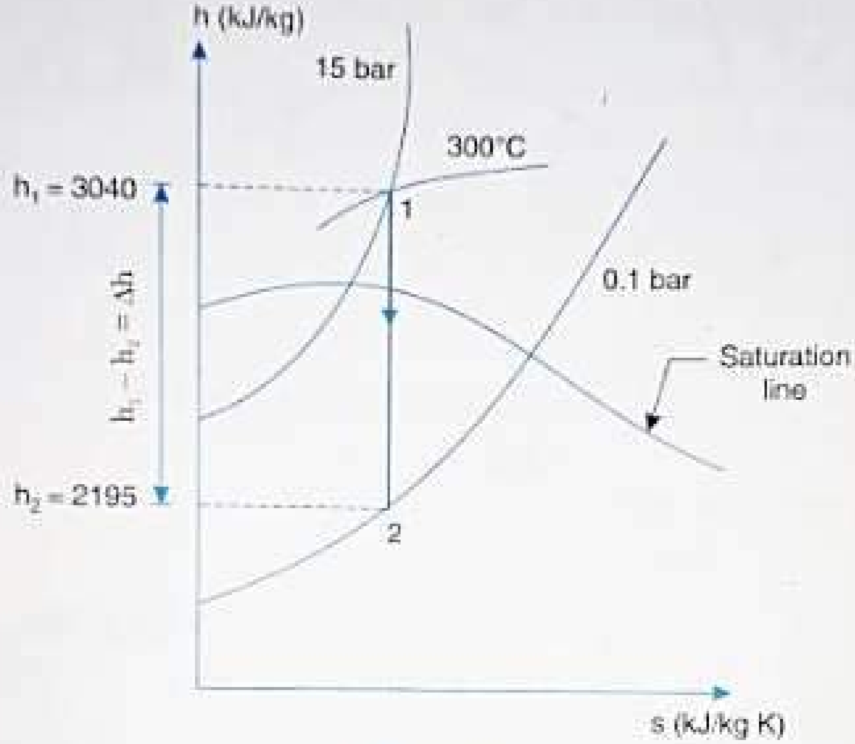


Fig. 19.50

Work done = Actual enthalpy drop

$= (\Delta h)_{isentropic} \times \eta_{overall}$

$= 845 \times 0.848 = 716.56$ kJ/kg

Work done per stage per kg $= \dfrac{716.56}{20} = 35.83$ kJ ...(*i*)

Also, total power = No. of stages $\times \dot{m}_s \times$ work done/kg stage

$\therefore \quad 10665 = 20 \times \dot{m}_s \times 35.83$

$\therefore \quad \dot{m}_s = \dfrac{10665}{20 \times 35.83} =$ **14.88 kg/s. (Ans.)**

(*ii*) **Mean diameter of rotor, D :**

Rotor speed, N :

Refer Fig. 19.51.

Work done per kg per stage $= C_{bl} \times C_w = C_{bl}(2C_1 \cos 25° - C_{bl})$

Also, $\dfrac{C_{bl}}{C_1} = 0.75$ ...(Given)

$\therefore \quad C_1 = \dfrac{C_{bl}}{0.75} = 1.33C_{bl}$

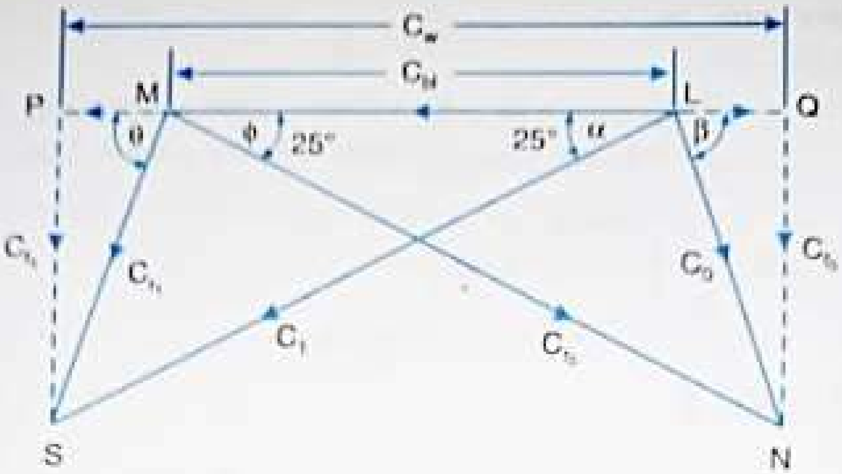


Fig. 19.51

i.e., Work done per kg per stage

$$= C_{bl}\,(2 \times 1.33 C_{bl} \times 0.906 - C_{bl})$$

$$= 1.41\ C_{bl}^2\ \text{Nm} \qquad \ldots(ii)$$

Equating (*i*) and (*ii*), we get

$$1.41\ C_{bl}^2 = 35.83 \times 1000$$

$$\therefore \quad C_{bl}^2 = \frac{35.83 \times 1000}{1.41} \quad \text{or} \quad C_{bl} = 159.41 \text{ m/s}$$

$$\therefore \quad C_1 = 1.33 \times 159.41 = 212 \text{ m/s}$$

From Fig. 19.51

$$C_{f_1} = C_1 \sin \alpha = 212 \sin 25° = 89.59 \text{ m/s}$$

v_g = Specific volume at 1 bar when steam is dry and saturated = 1.694 m^3/kg (from steam tables)

Mass flow rate, $$\dot{m}_s = \frac{\pi D h C_{f_1}}{v}$$

$$\therefore \quad 14.88 = \frac{\pi \times D \times \left(\frac{D}{12}\right) 89.59}{1.694} \quad \text{or} \quad D^2 = \frac{14.88 \times 1.694 \times 12}{\pi \times 89.59}$$

$$\therefore \quad D = \mathbf{1.036\ m.} \quad \text{(Ans.)}$$

Now, $$h = \frac{D}{12} = \frac{1.036}{12} = 0.086 \text{ m} = \mathbf{8.6\ cm.} \quad \text{(Ans.)}$$

Also, $$C_{bl} = \frac{\pi D N}{60}$$

$$\therefore \quad N = \frac{C_{bl} \times 60}{\pi D} = \frac{159.41 \times 60}{\pi \times 1.036} = \mathbf{2938.7\ r.p.m.} \quad \text{(Ans.)}$$

Example 19.38. *The following data relate to a stage of reaction turbine :*

Mean rotor diameter = 1.5 m ; speed ratio = 0.72 ; blade outlet angle = 20° ; rotor speed = 3000 r.p.m.

(i) Determine the diagram efficiency.

(ii) Determine the percentage increase in diagram efficiency and rotor speed if the rotor is designed to run at the best theoretical speed, the exit angle being 20°.

Solution. Mean rotor diameter, $D = 1.5$ m

Speed ratio, $\rho = \dfrac{C_{bl}}{C_1} = 0.72$

Blade outlet angle $= 20°$

Rotor speed, $N = 3000$ r.p.m.

(This example solved purely by calculations (Fig. 19.52) is not drawn to scale.)

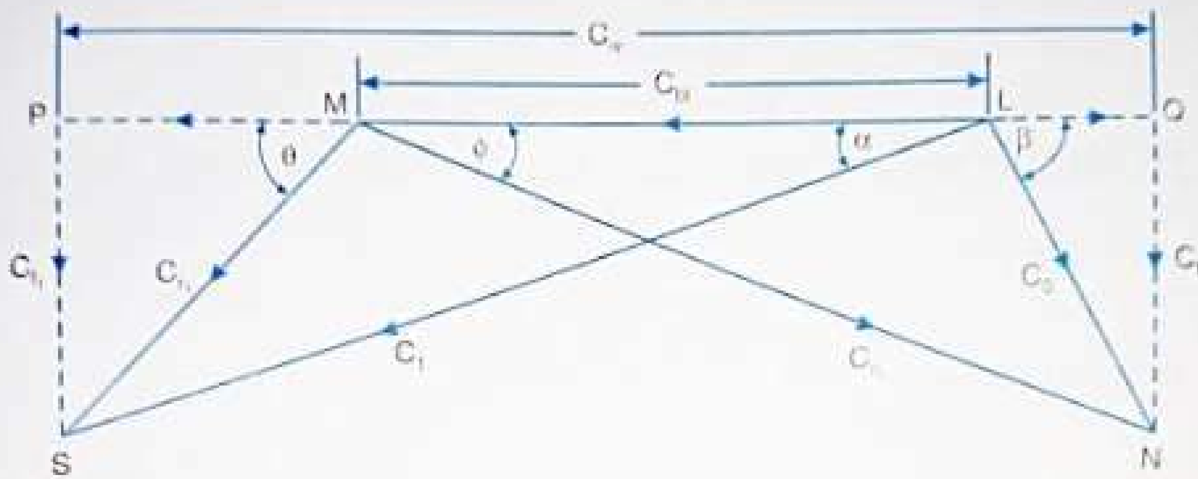


Fig. 19.52

(i) **Diagram efficiency :**

Blade velocity, $C_{bl} = \dfrac{\pi DN}{60} = \dfrac{\pi \times 1.5 \times 3000}{60} = 235.6$ m/s

Speed ratio, $\rho = \dfrac{C_{bl}}{C_1} = 0.72$

$\therefore \quad C_1 = \dfrac{C_{bl}}{0.72} = \dfrac{235.6}{0.72} = 327.2$ m/s

Assuming that velocity triangles are symmetrical

$$\alpha = \phi = 20°$$

From the velocity ΔLMS

$$C_{r_1}^{\ 2} = C_1^{\ 2} + C_{bl}^{\ 2} - 2C_1C_{bl} \cos \alpha$$

$$C_{r_1} = \sqrt{(327.2)^2 + (235.6)^2 - 2 \times 327.2 \times 235.6 \cos 20°}$$

$$= 100\sqrt{10.7 + 5.55 - 14.48} = 133 \text{ m/s}$$

i.e., $\quad C_{r_1} = 133$ m/s

Work done per kg of steam $= C_{bl}C_w = C_{bl}(2C_1 \cos \alpha - C_{bl})$

$= 235.6\,(2 \times 327.2 \cos 20° - 235.6) =$ **89371.3 Nm.**

Energy supplied per kg of steam

$$= \frac{C_1^{\ 2} + C_{r_0}^{\ 2} - C_{r_1}^{\ 2}}{2}$$

$$= \frac{2C_1^{\ 2} - C_{r_1}^{\ 2}}{2} \qquad (\because \; C_1 = C_{r_0})$$

$$= \frac{2 \times (327.2)^2 - (133)^2}{2} = 98215.3 \text{ Nm}$$

$\therefore$ *Diagram efficiency* $= \frac{89371.3}{98215.3} = 0.91 = \textbf{91\%}.$ **(Ans.)**

(*a*) **Percentage increase in diagram efficiency :**

For the best diagram efficiency (*maximum*), the required condition is

$$\rho = \frac{C_{bl}}{C_1} = \cos \alpha$$

$$\therefore \quad C_{bl} = C_1 \cos \alpha = 372.2 \cos 20° = 307.46 \text{ m/s}$$

For this blade speed, the value of C_{r_1} is again calculated by using eqn. (*i*),

$$C_{r_1} = \sqrt{(327.2)^2 + (307.46)^2 - 2 \times 327.2 \times 307.46 \times \cos 20°}$$

$$= 100 \sqrt{10.7 + 9.45 - 18.906} = 111.5 \text{ m/s}$$

Diagram efficiency
$$= \frac{2C_{bl}(2C_1 \cos \alpha - C_{bl})}{(C_1^2 + C_{r_0}^2 - C_{r_1}^2)}$$

$$= \frac{2 \times 307.46(2 \times 327.2 \cos 20° - 307.46)}{(327.2)^2 + (327.2)^2 - (111.5)^2} = 0.937 \text{ or } 93.7\%$$

Percentage increase in diagram efficiency

$$= \frac{0.937 - 0.91}{0.91} = \textbf{0.0296} \text{ or } \textbf{2.96\%}. \text{ (Ans.)}$$

[The diagram efficiency for the best speed can also be calculated by using relation

$$\eta_{blade} = \frac{2\cos^2 \alpha}{1 + \cos^2 \alpha} = \frac{2 \times \cos^2 20°}{1 + \cos^2 20°} = \frac{1.766}{1 + 0.883} = 0.937 \text{ or } 93.7\%. \text{ (Ans.)}$$]

The **best theoretical speed of the rotor** is given by,

$$C_{bl} = \frac{\pi DN}{60} \quad \text{or} \quad N = \frac{60\, C_{bl}}{\pi D} = \frac{60 \times 307.46}{\pi \times 1.5} = \textbf{3914.7 r.p.m.} \text{ (Ans.)}$$

Example 19.39. (Impulse reaction turbine). *The following data relate to a stage of an impulse reaction turbine :*

Steam velocity coming out of nozzle = 245 m/s ; nozzle angle = 20° ; blade mean speed = 145 m/s ; speed of the rotor = 300 r.p.m. ; blade height = 10 cm ; specific volume of steam at nozzle outlet and blade outlet respectively = 3.45 m³/kg and 3.95 m³/kg ; Power developed by the turbine = 287 kW ; efficiency of nozzle and blades combinedly = 90% ; carry over co-efficient = 0.82.

Find : (i) The heat drop in each stage ; (ii) Degree of reaction ; (iii) Stage efficiency.

Solution. Steam velocity coming out of nozzle, $C_1 = 245$ m/s

Nozzle angle, $\alpha = 20°$

Blade mean speed, $C_{bl} = 145$ m/s

Speed of the rotor, $N = 3000$ r.p.m.

Blade height, $h = 10 \text{ cm} = 0.1 \text{ m}$

Specific volume of steam at nozzle outlet or blade inlet, v_1 = 3.45 m³/kg

Specific volume of steam at blade outlet, v_0 = 3.95 m³/kg

Power developed by the turbine = 287 kW

Efficiency of nozzle and blades combinedly = 90%

Carry over co-efficient, ψ = 0.82

Blade speed, $$C_{bl} = \frac{\pi DN}{60}$$

∴ $$D = \frac{60\,C_{bl}}{\pi N} = \frac{60 \times 145}{\pi \times 3000} = 0.923 \text{ m}$$

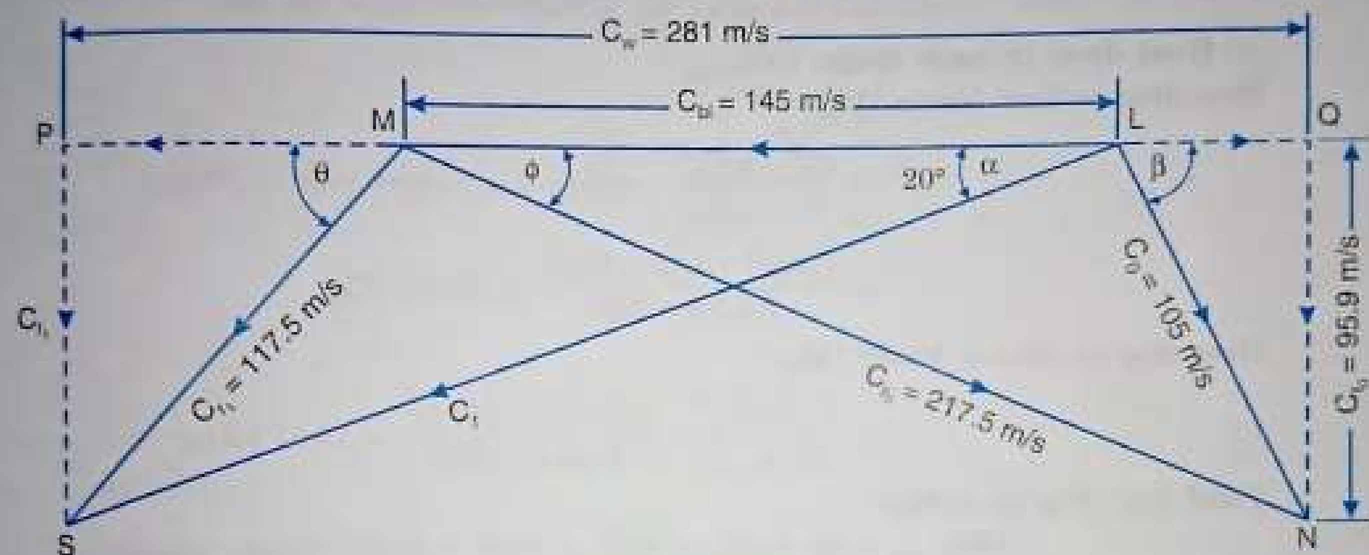


Fig. 19.53

Mass flow rate, $$\dot{m}_s = \frac{C_{f_1} \times \pi Dh}{v_1} = \frac{C_1 \sin\alpha \times \pi Dh}{v_1}$$

$$= \frac{245 \times \sin 20° \times \pi \times 0.923 \times 0.1}{3.45} = 7.04 \text{ kg/s}$$

Also $$\dot{m}_s = \frac{C_{f_0}\,\pi Dh}{v_0}$$

∴ $$C_{f_0} = \frac{\dot{m}_s v_0}{\pi Dh} = \frac{7.04 \times 3.95}{\pi \times 0.923 \times 0.1} = 95.9 \text{ m/s}$$

The power is given by, $$P = \frac{\dot{m}_s \times C_{bl} \times C_w}{1000}$$

$$287 = \frac{7.04 \times 145 \times C_w}{1000}$$

∴ $$C_w = \frac{287 \times 1000}{7.04 \times 145} = 281 \text{ m/s}$$

Now draw velocity triangles as follows :

Select a suitable scale (say 1 cm = 25 m/s)

- Draw LM = blade velocity = 145 m/s ; $\angle MLS$ = nozzle angle = 20°
  Join MS to complete the inlet ΔLMS.
- Draw a perpendicular from S which cuts the line through LM at point P.
  Mark the point Q such that $PQ = C_u$ = 281 m/s.
- Draw a perpendicular through point Q and the point N as QN = 95.9 m/s.
  Join LN and MN to complete the *outlet* velocity triangle.

From the velocity triangles :

$$C_{r_i} = 117.5 \text{ m/s} \; ; \; C_{r_o} = 217.5 \text{ m/s} \; ; \; C_0 = 105 \text{ m/s}.$$

(*i*) **Heat drop in each stage, $(\Delta h)_{stage}$:**

Heat drop in *fixed blades* (Δh_f)

$$= \frac{C_1^2 - \psi C_0^2}{2 \times \eta_{nozzle}}, \text{ where } \psi \text{ is a carry over co-efficient}$$

$$= \frac{(245)^2 - 0.82 \times (105)^2}{2 \times 0.9 \times 1000} = 28.32 \text{ kJ/kg}$$

Heat drop in *moving blades* (Δh_m)

$$= \frac{C_{r_o}^2 - C_{r_i}^2}{2 \times \eta_{nozzle}} = \frac{(217.5)^2 - (117.5)^2}{2 \times 0.9 \times 1000} = 18.61 \text{ kJ/kg}$$

Total heat drop in a stage,

$$(\Delta h)_{stage} = \Delta h_f + \Delta h_m = 28.32 + 18.61 = \mathbf{46.93 \text{ kJ/kg.}} \quad \text{(Ans.)}$$

(*ii*) **Degree of reaction, R_d :**

$$R_d = \frac{\Delta h_m}{\Delta h_m + \Delta h_f} = \frac{18.61}{18.61 + 28.32} = \mathbf{0.396.} \quad \text{(Ans.)}$$

(*iii*) **Stage efficiency, η_{stage} :**

Work done per kg of steam

$$= \frac{C_{bl} \times C_u}{1000} = \frac{145 \times 281}{1000} = 40.74 \text{ kJ/kg of steam}$$

$$\therefore \quad \eta_{stage} = \frac{\text{Work done per kg of steam}}{\text{Total heat drop in a stage}} = \frac{40.74}{46.93} = \mathbf{0.868 \text{ or } 86.8\%.} \quad \text{(Ans.)}$$

19.11. "STATE POINT LOCUS" AND "REHEAT FACTOR"

The terms *state point locus* and *reheat factor* are discussed below :

State Point Locus

The **state point** *may be defined as that point on h-s diagram which represents the condition of steam at that instant.* Thus knowing the initial condition of steam entering the nozzle of a turbine the initial state point of Fig. 19.43 may be located on h-s diagram. If stage efficiency $\left[\eta_{stage} = \frac{\text{S.P.}}{\text{A.P.}} = \frac{h_1 - h_5}{h_1 - h_2}\right]$ be known or assumed the position of the end state point 5 for a stage be readily obtained. The point 5 now becomes the initial state point for the succeeding stage of the turbine.

Let us now consider a *multistage turbine having four stages*. Refer Fig. 19.54.

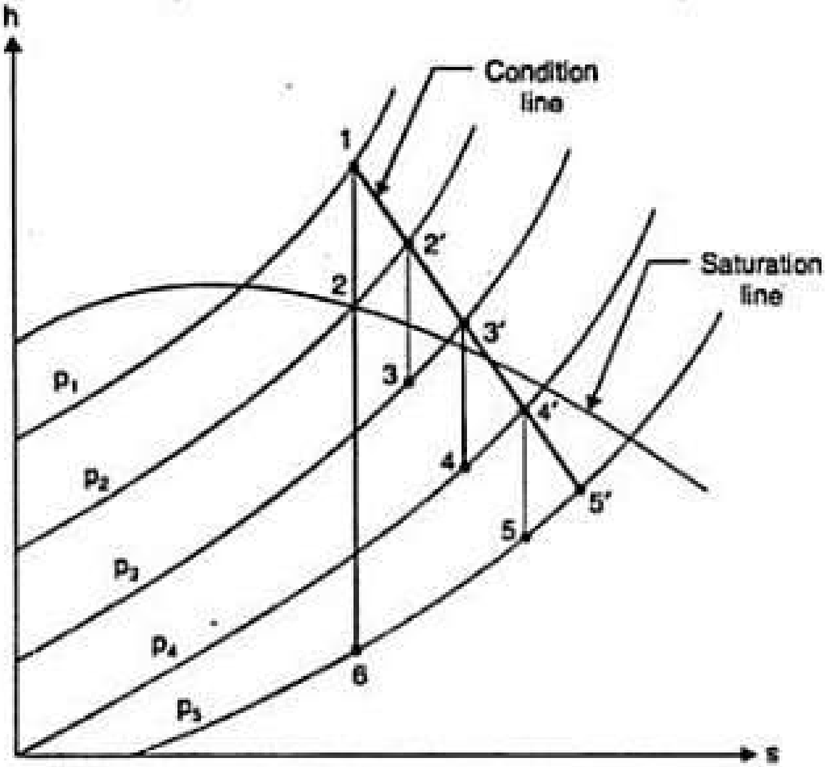


Fig. 19.54

The initial point 1 is located according to the given initial condition. (1-2) is adiabatic expansion in the first stage. h_2' may be calculated from the following relation, $h_1 - h_2' = \eta_{stage}(h_1 - h_2)$. Point 2' is then located with the value h_2' on p_2 line. Then (2'-3) is drawn showing adiabatic expansion. Point 3' may be located by finding h_3' from $h_2' - h_3' = \eta_{stage}(h_2' - h_3)$. Proceeding in this way all stage points 3', 4' and 5' may be fixed. The locus passing through these points is called "*State point locus*". The sum of the adiabatic heat drops (1-2) + (2'-3) + (3'-4) + (4'-5) is generally called "*cumulative heat drop*" and is represented as $h_{cum.}$ For the purpose of design the various quantities obtained from the Mollier diagram are set out in some curves form, and these curves are termed **"condition curves"**.

Reheat factor

Referring to Fig. 19.54, the *adiabatic heat drop* (h_{adi}) from pressure p_1 to final pressure p_5, considering all the stages as one unit, is $(h_1 - h_6)$. It will be found that h_{adi} is less that $h_{cum.}$ The ratio $\frac{h_{cum}}{h_{adi}}$ is termed as ***Reheat factor***.

The value of reheat factor depends on the following factors :

(*i*) Stage efficiency ;

(*ii*) Initial pressure and condition of steam ;

(*iii*) Final pressure.

Example 19.40. *In a three-stage steam turbine steam enters at 35 bar and 400°C and exhausts at 0.05 bar, 0.9 dry. If the work developed per stage is equal, determine :*

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(i) Condition of steam at entry to each stage.

(ii) The stage efficiencies.

(iii) The reheat factor. *(iv) Internal turbine efficiency.*

Assume condition line to be straight.

Solution. Initial condition of steam = 35 bar, 400°C

Exhaust condition of steam = 0.05 bar, 0.9 dry

(i) Condition of steam at entry to each stage :

Refer Fig. 19.55.

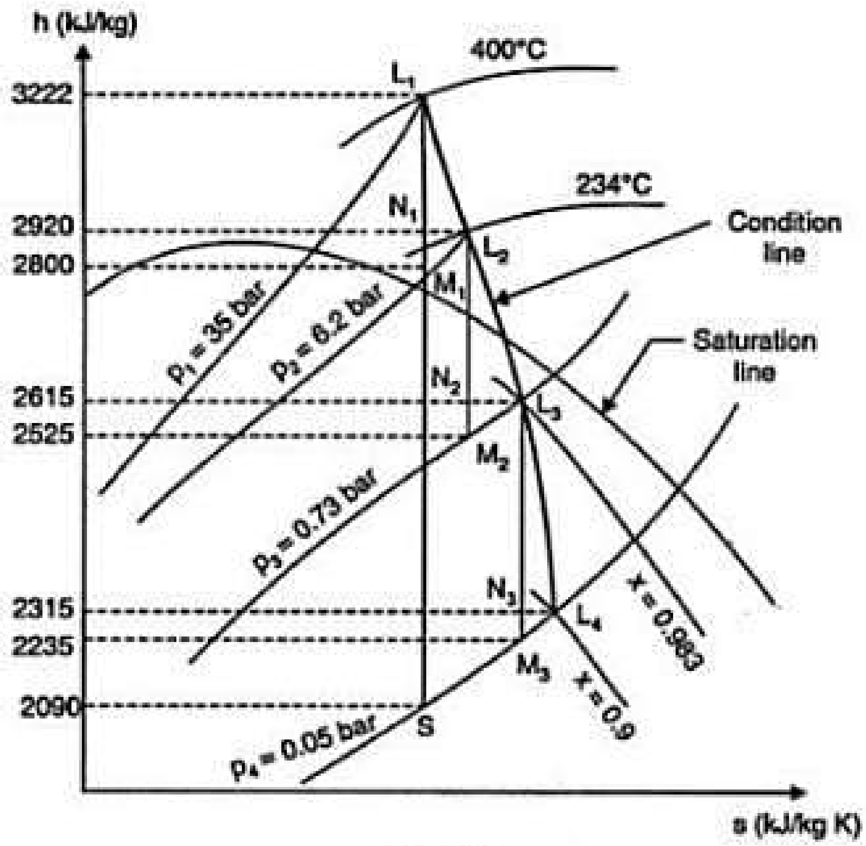

Fig. 19.55

- Locate points L_1 and L_4 corresponding to entry and exhaust conditions of steam.
- Since the condition line is straight (given), points L_1 and L_4 are joined by a straight line.

Heat drop due to expansion from L_1 to L_4 = 3222 – 2315 = 907 kJ/kg

- Since the work developed per stage is equal,

$\therefore$ Useful work/stage $= \dfrac{907}{3} = 302.3$ kJ/kg *i.e.*, $(h_{L_1} - h_{N_1}) = 302.3$ kJ/kg

Produce horizontal to cut condition line at L_2. L_1N_1 produced cuts the pressure line through L_2 at M_1. Thus L_2, N_1 and M_1 are located.

Proceeding for other stages likewise, we get the following results :

Points	*Pressure*	*Temp. or quality of steam*	
L_2	6.2 bar	234°C	**(Ans.)**
L_3	0.73 bar	0.98 dry	**(Ans.)**

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$$h_{L_1M_1} = h_{L_1} - h_{M_1} = 3222 - 2800 = 422 \text{ kJ/kg}$$
$$h_{L_2M_2} = h_{L_2} - h_{M_2} = 2920 - 2525 = 395 \text{ kJ/kg}$$
$$h_{L_3M_3} = h_{L_3} - h_{M_3} = 2615 - 2235 = 380 \text{ kJ/kg.}$$

(*ii*) **Stage efficiencies :**

Efficiency of stage 1, $\eta_1 = \dfrac{h_{L_1} - h_{N_1}}{h_{L_1} - h_{M_1}} = \dfrac{302.3}{422} = \mathbf{0.7163}$ **or 71.63%. (Ans.)**

Efficiency of stage 2, $\eta_2 = \dfrac{h_{L_2} - h_{N_2}}{h_{L_2} - h_{M_2}} = \dfrac{302.3}{395} = \mathbf{0.7653}$ **or 76.53%. (Ans.)**

Efficiency of stage 3, $\eta_3 = \dfrac{h_{L_3} - h_{N_3}}{h_{L_3} - h_{M_3}} = \dfrac{302.3}{380} = 0.7955 =$ **79.55%. (Ans.)**

(*iii*) **Reheat factor :**

$$\text{Reheat factor} = \frac{\text{Cumulative drop}}{\text{Isentropic enthalpy drop}} = \frac{h_{L_1M_1} + h_{L_2M_2} + h_{L_3M_3}}{h_{L_1} - h_S}$$

$$= \frac{422 + 395 + 380}{3222 - 2090} = \mathbf{1.057.} \quad \textbf{(Ans.)}$$

(*iv*) **Internal turbine efficiency :**

$$\text{Internal turbine efficiency} = \frac{h_{L_1} - h_{L_4}}{h_{L_1} - h_S} = \frac{3222 - 2315}{3222 - 2090} = \mathbf{0.801} \quad \textbf{or} \quad \mathbf{80.1\%.} \quad \textbf{(Ans.)}$$

19.12. REHEATING STEAM

Please refers Art. 15.5 (Reheat cycle)

19.13. BLEEDING

***Bleeding** is the process of draining steam from the turbine, at certain points during its expansion, and using this steam for heating the feed water supplied to the boiler.* In this process a small quantity of steam, at certain sections of the turbine, is drained from the turbine and is then circulated around the feed water pipe leading from hot well to the boiler. The steam is thus condensed due to relatively cold water, the heat so lost by steam is transferred to the feed water. The condensed steam then finds its way to hot well.

There is a usual practice in bleeding installations to allow the bled steam to mix with the feed water. The mixture of steam and water then proceeds to the boiler.

By bleeding process hotter water is supplied to the boiler of course at the cost of loss of small amount of turbine work. *Due to this process efficiency is slightly increased but at the same time power developed is also decreased.*

The bleeding process in steam turbines approximates to cascade heating and tends to modify the Rankine cycle to a reversible cycle, thus increasing the efficiency ; but any increase in efficiency due to an approach to the condition of thermodynamic reversibility is accompanied by a decrease in power. Hence it follows that the thermodynamic benefits derived from the process of bleeding are of a *limited character. The ideal Rankine cycle, modified to take into account the effect of bleeding is known as the* ***regenerative cycle.***

Note. Please refer Art. 15.4 (Regenerative cycle) also.

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19.14. ENERGY LOSSES IN STEAM TURBINES

The increase in heat energy required for doing mechanical work in actual practice as compared to the theoretical value, in which the process of expansion takes place strictly according to the adiabatic process, is termed as energy loss in a steam turbine.

The losses which appear in an actual turbine may be divided into two following groups :

1. Internal losses. Losses directly connected with the steam conditions while in its flow through the turbine. They may be further classified as :

(*i*) Losses in regulating valves.

(*ii*) Losses in nozzles (guide blades).

(*iii*) Losses in moving blades :

(*a*) losses due to trailing edge wake ;

(*b*) impingement losses ;

(*c*) losses due to leakage of steam through the angular space ;

(*d*) frictional losses ;

(*e*) losses due to turning of the steam jet in the blades ;

(*f*) losses due to shrouding.

(*iv*) Leaving velocity losses (exit velocity).

(*v*) Losses due to friction of disc carrying the blades and windage losses.

(*vi*) Losses due to clearance between the rotor and guide blade discs.

(*vii*) Losses due to wetness of steam.

(*viii*) Losses in exhaust piping etc.

2. External losses. Losses which do not influence the steam conditions. They may be further classified as :

(*i*) Mechanical losses.

(*ii*) Losses due to leakage of steam from the labyrinth gland seals.

19.15. STEAM TURBINE GOVERNING AND CONTROL

The objective of governing is to keep the turbine speed fairly constant irrespective of load.

The principal methods of steam turbine governing are as follows :

1. Throttle governing
2. Nozzle governing
3. By-pass governing
4. Combination of 1 and 2 and 1 and 3.

1. Throttle governing

Throttle governing is the most widely used particularly on *small turbines*, because *its initial cost is less* and *the mechanism is simple*. The object of throttle governing is to *throttle the steam* whenever *there is a reduction of load* compared to economic or design load for maintaining speed and *vice versa*.

Fig. 19.56 (*a*) shows a simple throttle arrangement. To start the turbine for full load running valve *A* is opened. The operation of double beat valve *B* is carried out by an oil servo motor which is controlled by a centrifugal governor. As the steam turbine gains speed the valve *B* closes to throttle the steam and reduces the supply to the nozzle.

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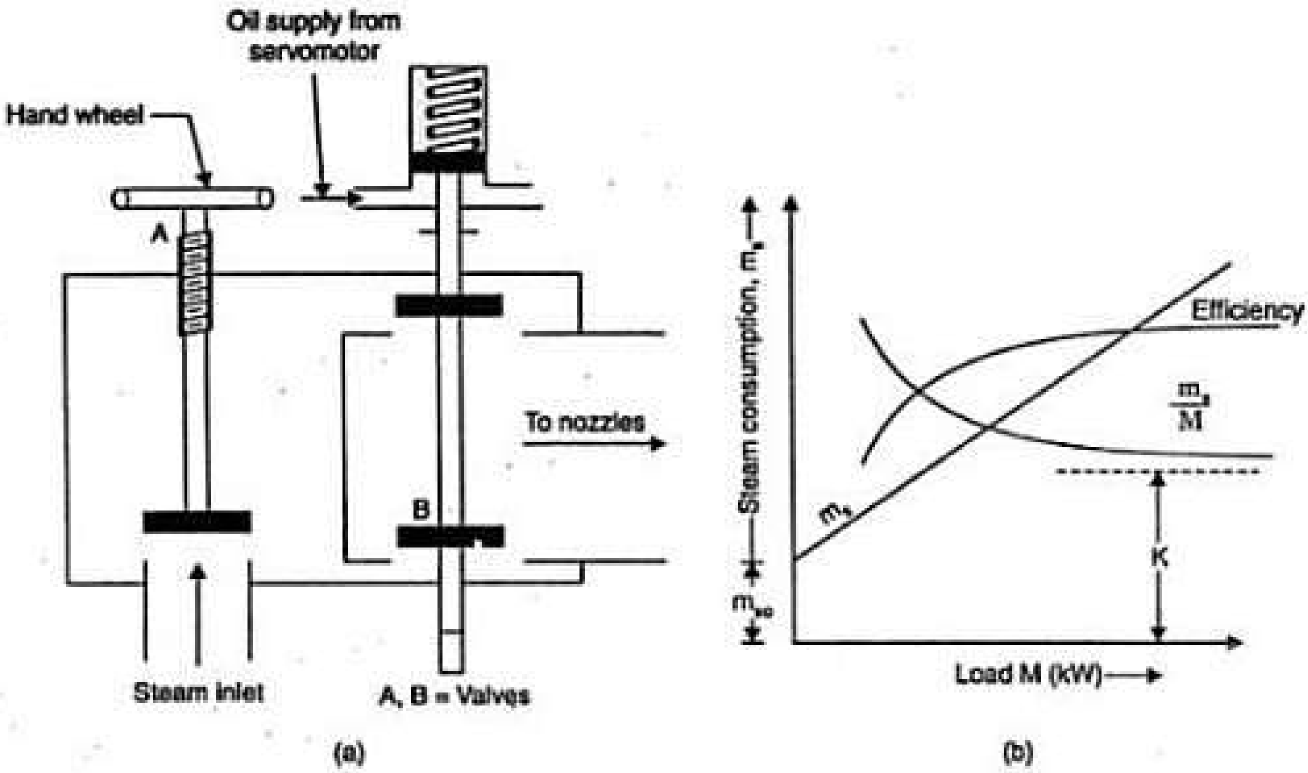

Fig. 19.56. Throttle governing.

For a turbine governed by throttling the relationship between steam consumption and load is given by the well known *Willan's line* as shown in Fig. 19.56 (*b*). Several tests have shown that when a turbine is governed by throttling, the Willan's line is straight. It is expressed as :

$$m_s = KM + m_{s_0} \qquad \text{...(19.37)}$$

where, m_s = Steam consumption in kg/h at any load M,

m_{s_0} = Steam consumption in kg/h at no load,

m_{s_1} = Steam consumption in kg/h at full load,

M = Any other load in kW,

M_1 = Full load in kW, and

K = Constant.

m_{s_0} varies from about 0.1 to 0.14 times the full load consumption. The eqns. (19.37) can also be written as :

$$\frac{m_s}{M} = K + \frac{m_{s_0}}{M}, \text{ where } \frac{m_s}{M} \text{ is called the steam consumption per kWh.}$$

2. Nozzle governing

The efficiency of a steam turbine is considerably reduced if throttle governing is carried out at low loads. An alternative, and more efficient form of governing is by means of nozzle control. Fig. 19.57 shows a diagrammatic arrangement of typical nozzle control governing. In this method of governing, the nozzles are grouped together 3 to 5 or more groups and supply of steam to each group is controlled by regulating valves. Under full load conditions the valves remain fully open.

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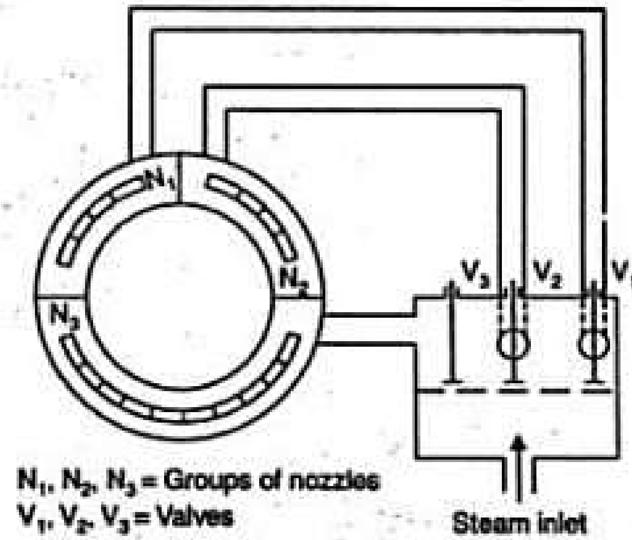

Fig. 19.57. Nozzle governing.

When the load on the turbine becomes more or less than the design value, the supply of steam to a group of nozzles may be varied accordingly so as to restore the original speed.

Nozzle control can only be applied to the first stage of a turbine. It is suitable for *simple impulse turbine* and *larger units which have an impulse stage followed* by an *impulse-reaction turbine*. In pressure compounded impulse turbines, there will be some drop in pressure at entry to second stage when some of the first stage nozzles are cut out.

Comparison of Throttle and Nozzle control governing

S. No.	*Aspects*	*Throttle Control*	*Nozzle Control*
1.	*Throttling losses*	Severe	No throttling losses (Actually there are a little throttling losses in nozzles valves which are partially open).
2.	*Partial admission losses*	Low	High.
3.	*Heat drop available*	Lesser	Larger
4.	*Use*	Used in impulse and reaction turbines both.	Used in impulse and also in reaction (if initial stage impulse) turbines.
5.	*Suitability*	Small turbines	Medium and larger turbines.

3. By-pass governing

The steam turbines which are designed to work at economic load it is desirable to have full admission of steam in the high pressure stages. At the maximum load, which is greater than the economic load, the additional steam required could not pass through the first stage since additional nozzles are not available. By-pass regulation allows for this in a turbine which is throttle governed, by means of a second by-pass valve in the first stage nozzle (Fig. 19.58). This valve opens when throttle valve has opened a definite amount. Steam is by-passed through the second valve to a lower stage in the turbine. When by-pass valve operates it is under the control of the turbine governor. The secondary and tertiary supplies of steam in the lower stages increase the work output in these stages, but there is a *loss in efficiency* and a curving of the Willian's line.

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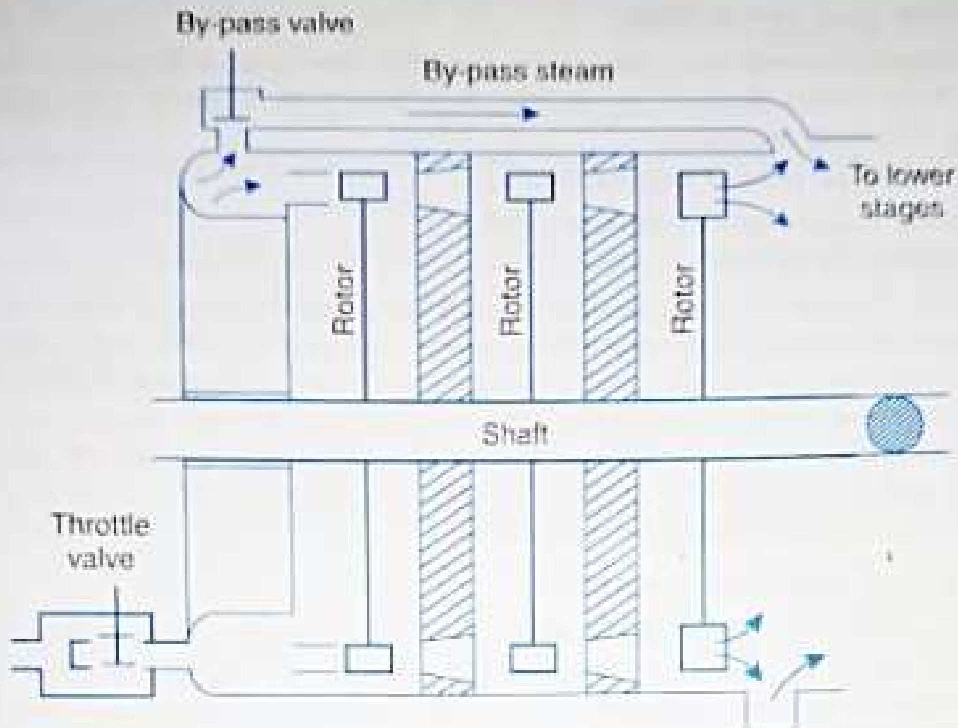

Fig. 19.58. By-pass governing.

In reaction turbines, because of the pressure drop required in the moving blades, ***nozzles control governing*** *is not possible,* and *throttle governing plus by-pass governing,* is used.

19.16. SPECIAL FORMS OF STEAM TURBINES

In many industries such as chemical, sugar refining, paper making, textile etc., where combined use of power and heating and process work is required it is wasteful to generate steam for power and process purposes separately, because about 70 per cent of heat supplied for power purposes will normally be carried away by the cooling water. On the other hand, if the engine or turbine is operated with a normal exhaust pressure then the temperature of the exhaust steam is too low to be of any use for heating process. It would be possible to generate the required power and still have available for process work a large quantity of heat in the exhaust steam, if suitable modification of the initial steam pressure and exhaust pressure is made. Thus in combined power and process plants following type of steam turbines are used ; (1) *Back pressure turbines,* and (2) *Steam extraction or pass-out turbines.*

1. Back pressure turbine

In this type of turbine steam at boiler pressure enters the turbine and is exhausted into a pipe. This pipe leads to process plant or other turbine. The back pressure turbine may be used in cases where the power generated (by expanding steam) from economical initial pressure down to the heating pressure is equal to, or greater than, the power requirements. The steam exhausted from the turbine is *usually superheated* and in most cases it is not suitable for process work due to the reasons :

(*i*) It is impossible to control its temperature, and

(*ii*) Rate of the heat transfer from superheated steam to the heating surface is lower than that of saturated steam. Consequently a desuperheater is invariably used. To enhance the power capacity of the existing installation, a high pressure boiler and a back-pressure turbine are added to it. This added high pressure boiler supplies steam to the back pressure turbine which exhausts into the old low pressure turbine.

2. Extraction pass out turbine

It is found that in several cases the power available from a back pressure turbine (through which the whole of the steam flows) is appreciably less than that required in the factory and this may be due to the following reasons :

(i) Small heating or process requirements ;

(ii) A relatively high exhaust pressure ; and

(iii) A combination of the both.

In such a case it would be possible to install a back-pressure turbine to provide the heating steam and a condensing turbine to generate extra power, but it is possible, and useful, to combine functions of both machines in a single turbine. Such a machine is called **extraction or pass out turbine** and here at some point intermediate between inlet and exhaust some steam is extracted or passed out for process or heating purposes. In this type of turbine a sensitive governor is used which controls the admission of steam to the high pressure section so that regardless of power or process requirements, constant speed is maintained.

Exhaust or low pressure turbine

If an uninterrupted supply of low pressure steam is available (such as from reciprocating steam engines exhaust) it is possible to improve the efficiency of the whole plant by fitting an exhaust or low pressure turbine. The use of exhaust turbine is chiefly made where there are several reciprocating steam engines which work intermittently ; and are non-condensing (*e.g.*, rolling mill and colliery engines). The exhaust steam from these engines is expanded in an exhaust turbine and then condensed. In this turbine some form of heat accumulator is needed to collect the more or less irregular supply of low pressure steam from the non-condensing steam engines and deliver it to the turbine at the rate required. In some cases when the supply of low pressure steam falls below the demand, live steam from the boiler, with its pressure and temperature reduced ; is used to make up the deficiency.

The necessary drop in pressure may be obtained by the use of a reducing valve, or for large flows, more economically by expansion through another turbine. The high pressure and low pressure turbines are sometimes combined on a common spindle and because of two supply pressures this combined unit is known as '**mixed pressure turbine**'.

HIGHLIGHTS

1. The **steam turbine** is a prime mover in which the potential energy of the steam is transformed into **kinetic energy**, and latter in its turn is transformed into the mechanical energy of rotation of the turbine shaft.
2. The most important **classification** of steam turbines is as follows :
   (i) Impulse turbines (ii) Reaction turbines
   (iii) Combination of impulse and reaction turbines.
3. The main difference between **Impulse** and **Reaction turbines** lies in the way in which steam in expanded while its moves through them. In the former type, steam expands in the nozzle and its pressure does not change as it moves over the blades while in the latter type the steam expands continuously as it passes over the blades and thus there is a gradual fall in pressure during expansion.
4. The different methods of compounding are :
   (i) Velocity compounding (ii) Pressure compounding
   (iii) Pressure velocity compounding (iv) Reaction turbine.
5. Force (tangential) on the wheel $= \dot{m}_s(C_{w_1} + C_{w_2})$ N-m

   Power per wheel $= \dfrac{\dot{m}_s(C_{w_1} + C_{w_2}) \times C_{bl}}{1000}$ kW

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Blade or diagram efficiency, $\eta_{bl} = \dfrac{2C_{bl}(C_{w_1} + C_{w_0})}{C_1^2}$.

Stage efficiency, $\eta_{stage} = \dfrac{C_{bl}(C_{w_1} + C_{w_0})}{(h_1 - h_2)}$.

6. The axial thrust on the wheel due to *difference* between the velocities of flow at entrance and outlet.
Axial force on the wheel $= \dot{m}_s(C_{f_1} - C_{f_0})$.

7. Energy converted to heat by blade friction = Loss of kinetic energy during flow over blades
$$= \dot{m}_s(C_{r_1}^{\ 2} - C_{r_0}^{\ 2}).$$

8. Optimum value of ratio of blade speed to steam speed is, $\rho_{opt} = \dfrac{\cos\alpha}{2}$.

9. The blade efficiency for *two-stage turbine* will be maximum when, $\rho_{opt} = \dfrac{\cos\alpha}{4}$.
In general optimum blade speed ratio for maximum blade efficiency or maximum work done is given by
$$\rho_{opt} = \frac{\cos\alpha}{2n}$$
and the work done in the last row $= \dfrac{1}{2^n}$ of total work,
where n is the number of moving/rotating blade rows in series.
In practice more than *two rows are hardly preferred.*

10. The **degree of reaction** of reaction turbine stage is defined as the ratio of heat drop over moving blades to the total heat drop in the stage.

11. The blade efficiency of the reaction turbine is given by $\eta_{bl} = 2 - \dfrac{2}{1 + 2\rho\cos\alpha - \rho^2}$;
η_{bl} becomes maximum when, $\rho = \cos\alpha$
and hence $(\eta_{bl})_{max} = \dfrac{2\cos^2\alpha}{1 + \cos^2\alpha}$.

12. The **state point** may be defined as that point on h-s diagram which represents the condition of steam at that instant.

13. Theoretical efficiency of reheat cycle is given by $\eta_{thermal} = \dfrac{(h_1 - h_2) + (h_3' - h_4)}{(h_1 - h_{f_4}) + (h_3 - h_2)}$, neglecting pump work.

14. The principal methods of **steam governing** are as follows :
(*i*) Throttle governing (*ii*) Nozzle governing
(*iii*) By-pass governing (*iv*) Combination of (*i*), (*ii*) and (*iii*).

OBJECTIVE TYPE QUESTIONS

Choose the Correct Answer :

1. In case of impulse steam turbine
(*a*) there is enthalpy drop in fixed and moving blades
(*b*) there is enthalpy drop only in moving blades
(*c*) there is enthalpy drop in nozzles (*d*) none of the above.

2. De-Laval turbine is
(*a*) pressure compounded impulse turbine (*b*) velocity compounded impulse turbine
(*c*) simple single wheel impulse turbine (*d*) simple single wheel reaction turbine.

3. The pressure on the two sides of the impulse wheel of a steam turbine
   (a) is same
   (b) is different
   (c) increases from one side to the other side
   (d) decreases from one side to the other side.
4. In De Laval steam turbine
   (a) the pressure in the turbine rotor is approximately same as in condenser
   (b) the pressure in the turbine rotor is higher than pressure in the condenser
   (c) the pressure in the turbine rotor gradually decreases from inlet to exit to condenser
   (d) none of the above.
5. In case of reaction steam turbine
   (a) there is enthalpy drop both in fixed and moving blades
   (b) there is enthalpy drop only in fixed blades
   (c) there is enthalpy drop only in moving blades
   (d) none of the above.
6. Curtis turbine is
   (a) reaction steam turbine
   (b) pressure velocity compounded steam turbine
   (c) pressure compounded impulse steam turbine
   (d) velocity compounded impulse steam turbine.
7. Rateau steam turbine is
   (a) reaction steam turbine
   (b) velocity compounded impulse steam turbine
   (c) pressure compounded impulse steam turbine
   (d) pressure velocity compounded steam turbine.
8. Parson's turbine is
   (a) pressure compounded steam turbine
   (b) simple single wheel, impulse steam turbine
   (c) simple single wheel reaction steam turbine
   (d) multiwheel reaction steam turbine.
9. Blade or diagram efficiency is given by
   (a) $\dfrac{(C_{w_1} \pm C_{w_0})C_{bl}}{C_1}$
   (b) $\dfrac{2C_{bl}(C_{w_1} \pm C_{w_0})}{C_1^2}$
   (c) $\dfrac{C_{bl}^2}{C_1^2}$
   (d) $\dfrac{C_1^2 - C_0^2}{C_1^2}$.
10. Axial thrust on rotor of steam turbine is
    (a) $\dot{m}_s(C_{f_1} - C_{f_0})$
    (b) $\dot{m}_s(C_{f_1} - 2C_{f_0})$
    (c) $\dot{m}_s(C_{f_1} + C_{f_0})$
    (d) $\dot{m}_s(2C_{f_1} - C_{f_0})$.
11. Stage efficiency of steam turbine is
    (a) $\eta_{blade}/\eta_{nozzle}$
    (b) $\eta_{nozzle}/\eta_{blade}$
    (c) $\eta_{nozzle} \times \eta_{blade}$
    (d) none of the above.
12. For maximum blade efficiency for single stage impulse turbine
    (a) $\rho\left(= \dfrac{C_{bl}}{C_1}\right) = \cos^2 \alpha$
    (b) $\rho = \cos \alpha$
    (c) $\rho = \dfrac{\cos \alpha}{2}$
    (d) $\rho = \dfrac{\cos^2 \alpha}{2}$.
13. Degree of reaction as referred to steam turbine is defined as
    (a) $\dfrac{\Delta h_f}{\Delta h_m}$
    (b) $\dfrac{\Delta h_m}{\Delta h_f}$
    (c) $\dfrac{\Delta h_m}{\Delta h_m + \Delta h_f}$
    (d) $\dfrac{\Delta h_f}{\Delta h_f + \Delta h_m}$.

14. For Parson's reaction steam turbine, degree of reaction is
(*a*) 75% (*b*) 100%
(*c*) 50% (*d*) 60%.

15. The maximum efficiency for Parson's reaction turbine is given by

(*a*) $\eta_{max} = \dfrac{\cos \alpha}{1 + \cos \alpha}$ (*b*) $\eta_{max} = \dfrac{2 \cos \alpha}{1 + \cos \alpha}$

(*c*) $\eta_{max} = \dfrac{2 \cos^2 \alpha}{1 + \cos^2 \alpha}$ (*d*) $\eta_{max} = \dfrac{1 + \cos^2 \alpha}{2 \cos^2 \alpha}$.

16. Reheat factor in steam turbines depends on
(*a*) exit pressure only (*b*) stage efficiency only
(*c*) initial pressure and temperature only (*d*) all of the above.

17. For multistage steam turbine reheat factor is defined as
(*a*) stage efficiency × nozzle efficiency (*b*) commulative enthalpy drop × η_{nozzle}

(*c*) $\dfrac{\text{commulative enthalpy drop}}{\text{isentropic enthalpy drop}}$ (*d*) $\dfrac{\text{isentropic enthalpy drop}}{\text{cumulative actual enthalpy drop}}$.

18. The value of reheat factor normally varies from
(*a*) 0.5 to 0.6 (*b*) 0.9 to 0.95
(*c*) 1.02 to 1.06 (*d*) 1.2 to 1.6.

19. Steam turbines are governed by the following methods
(*a*) Throttle governing (*b*) Nozzle control governing
(*c*) By-pass governing (*d*) All of the above.

20. In steam turbines the reheat factor
(*a*) increases with the increase in number of stages
(*b*) decreases with the increase in number of stages
(*c*) remains same irrespective of number of stages
(*d*) none of the above.

ANSWERS

1. (*c*)	**2.** (*c*)	**3.** (*a*)	**4.** (*a*)	**5.** (*a*)	**6.** (*d*)	**7.** (*c*)
8. (*d*)	**9.** (*b*)	**10.** (*a*)	**11.** (*c*)	**12.** (*c*)	**13.** (*c*)	**14.** (*c*)
15. (*c*)	**16.** (*d*)	**17.** (*c*)	**18.** (*c*)	**19.** (*d*)	**20.** (*a*).	

THEORETICAL QUESTIONS

1. Define a steam turbine and state its fields of application.
2. How are the steam turbines classified ?
3. Discuss the advantages of a steam turbine over the steam engines.
4. Explain the difference between an impulse turbine and a reaction turbine.
5. What do you mean by compounding of steam turbines ? Discuss various methods of compounding steam turbines.
6. What methods are used in reducing the speed of the turbine rotor ?
7. Explain with the help of neat sketch a single-stage impulse turbine. Also explain the pressure and velocity variations along the axial direction.
8. Define the following as related to steam turbines
(*i*) Speed ratio (*ii*) Blade velocity co-efficient
(*iii*) Diagram efficiency (*iv*) Stage efficiency.

9. In case of steam turbines derive expressions for the following :
   (*i*) Force (*ii*) Work done
   (*iii*) Diagram efficiency (*iv*) Stage efficiency
   (*v*) Axial thrust.
10. Derive the expression for maximum blade efficiency in a single-stage impulse turbine.
11. Explain the pressure compounded impulse steam turbine showing pressure and velocity variations along the axis of the turbine.
12. Explain velocity compounded impulse steam turbine showing pressure and velocity variations along the axis of the turbine.
13. Define the term "degree of reaction" used in reaction turbines and prove that it is given by

$$R_d = \frac{C_f}{2C_{bl}} (\cot \phi - \cot \theta) \text{ when } C_{f_i} = C_{f_o} = C_f$$

Further prove that the moving and fixed blades should have the same shape for a 50% reaction.

14. Prove that the diagram or blade efficiency of a single stage reaction turbine is given by

$$\eta_{bl} = 2 - \frac{2}{1 + 2\rho \cos \alpha - \rho^2} \text{ where } R_d = 50\% \text{ and } C_{f_i} = C_{f_o}$$

Further prove that maximum blade efficiency is given by $(\eta_{bl})_{max} = \dfrac{2 \cos^2 \alpha}{1 + \cos^2 \alpha}$.

15. Explain 'reheat factor'. Why is its magnitude always greater than unity ?
16. Describe the process and purpose of reheating as applicable to steam flowing through a turbine.
17. State the advantages and disadvantages of reheating steam.
18. Write a short note on 'bleeding of steam turbines'.
19. Enumerate the energy losses in steam turbines.
20. Describe briefly the various methods of 'steam turbine governing'.

UNSOLVED EXAMPLES

IMPULSE TURBINES

1. A steam jet enters the row of blades with a velocity of 380 m/s at an angle of 22° with the direction of motion of the moving blades. If the blade speed is 180 m/s and there is no thrust on the blades, determine the inlet and outlet blade angles. Velocity of steam while passing over the blade is reduced by 10%. Also determine the power developed by turbine when the rate of flow of steam is 1000 kg per minute. [Ans. 679 kW]
2. In a simple impulse turbine, the nozzles are inclined at 20° to the direction of motion of moving blades. The steam leaves the nozzles at 375 m/s. The blade speed is 165 m/s. Find suitable inlet and outlet angles for the blades in order that the axial thrust is zero. The relative velocity of steam as it flows over the blades is reduced by 15% by friction. Determine also the power developed for a flow rate of 10 kg/s. [Ans. 34°, 41°, 532 kW]
3. In an impulse turbine the nozzles are inclined at 24° to the plane of rotation of the blades. The steam speed is 1000 m/s and blade speed is 400 m/s. Assuming equiangular blades, determine :
   (*i*) Blade angles (*ii*) Force on the blades in the direction of motion
   (*iii*) Axial thrust (*iv*) Power developed for a flow rate of 1000 kg/h.
   [Ans. (*i*) 39°, (*ii*) 1.135 kN, (*iii*) 113.5 kW]
4. In a De Laval turbine, the steam issues from the nozzles with a velocity of 850 m/s. The nozzle angle is 20°. Mean blade velocity is 350 m/s. The blades are equiangular. The mass flow rate is 1000 kg/min. Friction factor is 0.8. Determine : (*i*) Blade angles (*ii*) axial thrust on the end bearing (*iii*) power developed in kW (*iv*) blade efficiency (*v*) stage efficiency, if nozzle efficiency is 93%.
   [Ans. (*i*) 33°, 33°, (*ii*) 500 N, (*iii*) 4666.7 kW, (*iv*) 77.5%, (*v*) 72.1%]

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5. In a single stage impulse turbine the nozzles discharge the steam on to the blades at an angle of 25° to the plane of rotation and the fluid leaves the blades with an absolute velocity of 300 m/s at an angle of 120° to the direction of motion of the blades. If the blades have equal inlet and outlet angles and there is no axial thrust, estimate :

(i) Blade angle (ii) Power produced per kg/s flow of steam

(iii) Diagram efficiency. [**Ans.** (i) 36.3°, (ii) 144 kW, (iii) 0.762]

6. Steam enters the blade row of an impulse turbine with a velocity of 600 m/s at an angle of 25° to the plane of rotation of the blades. The mean blade speed is 255 m/s. The blade angle on the exit side is 30°. The blade friction co-efficient is 10%. Determine :

(i) Work done per kg of steam (ii) Diagram efficiency

(iii) Axial thrust per kg of steam/s. [**Ans.** (i) 150.45 kW, (ii) 83.6%, (iii) 90 N]

7. The nozzles of an impulse turbine are inclined at 22° to the plane of rotation. The blade angles both at inlet and outlet are 36°. The mean diameter of the blade ring is 1.25 m and the steam velocity is 680 m/s. Assuming shockless entry determine : (i) The speed of the turbine rotor in r.p.m., (ii) the absolute velocity of steam leaving the blades, and (iii) The torque on the rotor for a flow rate of 2500 kg/h.

[**Ans.** (i) 4580 r.p.m., (ii) 225 m/s, (iii) 290.5 Nm]

8. A single-stage steam turbine is provided with nozzles from which steam is released at a velocity of 1000 m/s at an angle of 24° to the direction of motion of blades. The speed of the blades is 400 m/s. The blade angles at inlet and outlet are equal. Find : (i) Inlet blade angle, (ii) Force exerted on the blades in the direction of their motion, (iii) Power developed in kW for steam flow rate of 40000 kg/h.

Assume that the steam enters and leaves the blades without shock.

[**Ans.** (i) 39°, (ii) 1135 N, (iii) 4540 kW]

9. In a single row impulse turbine the nozzle angle is 30° and the blade speed is 215 m/s. The steam speed is 550 m/s. The blade friction co-efficient is 0.85. Assuming axial exit and a flow rate of 700 kg/h, determine :

(i) Blade angles. (ii) Absolute velocity of steam at exit.

(iii) The power output of the turbine. [**Ans.** (i) 46°, 49° ; (ii) 243 m/s ; (iii) 19.8 kW]

10. In a steam turbine, steam expands from an inlet condition of 7 bar and 300°C with an isentropic efficiency of 0.9. The nozzle angle is 20°. The stage operates at optimum blade speed ratio. The blade inlet angle is equal to the outlet angle. Determine :

(i) Blade angles. (ii) Power developed if the steam flow rate is 0.472 kg/s.

[**Ans.** (i) 36°, (ii) 75 kW]

11. Steam at 7 bar and 300°C expands to 3 bar in an impulse stage. The nozzle angle is 20°, the rotor blades have equal inlet and outlet angles and the stage operates with the optimum blade speed ratio. Assuming that isentropic efficiency of nozzles is 90% and velocity at entry to the stage is negligible, deduce the blade angles used and the mass flow required for this stage to produce 50 kW. [**Ans.** 36°, 0.317 kg/s]

12. In a two-stage velocity compounded steam turbine, the mean blade speed is 150 m/s while the steam velocity as it is issued from the nozzle is 675 m/s. The nozzle angle is 20°. The exit angle of first row moving blade, fixed blade and the second row moving blades are 25°, 25° and 30° respectively. The blade friction co-efficient is 0.9. If the steam flow rate is 4.5 kg/s, determine :

(i) Power output. (ii) Diagram efficiency. [**Ans.** (i) 807 kW, (ii) 78.5%]

REACTION TURBINES

13. At a particular stage of reaction turbine, the mean blade speed is 60 m/s and the steam pressure is 3.5 bar with a temperature of 175°C. The identical fixed and moving blades have inlet angles of 30° and outlet angles of 20°. Determine :

(i) The blade height if it is $\frac{1}{10}$ th of the blade ring diameter, for flow rate of 13.5 kg/s.

(ii) The power developed by a pair.

(iii) Specific enthalpy drop if the stage efficiency is 85%. [**Ans.** (i) 64 mm, (ii) 218 kW, (iii) 19.1 kJ/kg]

14. In a stage of impulse reaction turbine operating with 50% degree of reaction, the blades are identical in shape. The outlet angle of the moving blades is 19° and the absolute discharge velocity of steam is 100 m/s in the direction at 100° to the motion of the blades. If the rate of flow of steam through the turbine is 15000 kg/h, calculate the power developed by the turbine in kW. **[Ans.** 327.5 kW**]**
15. At a stage in a reaction turbine the pressure of steam is 0.34 bar and the dryness 0.95. For a flow rate of 36000 kg/h, the stage develops 950 kW. The turbine runs at 3600 r.p.m. and the velocity of flow is 0.72 times the blade velocity. The outlet angle of both stator and rotor blades is 20°. Determine at this stage :
(*i*) Mean rotor diameter. (*ii*) Height of blades. **[Ans.** (*i*) 0.951 m, (*ii*) 115 mm**]**
16. In a multi-stage reaction turbine at one of the stages the rotor diameter is 1250 mm and speed ratio 0.72. The speed of the rotor is 3000 r.p.m. Determine : (*i*) The blade inlet angle if the outlet blade angle is 22°, (*ii*) Diagram efficiency, (*iii*) The percentage increase in diagram efficiency and rotor speed if turbine is designed to run at the best theoretical speed. **[Ans.** (*i*) 61.5°, (*ii*) 82.2%, (*iii*) 30.47%**]**
17. In a 50 per cent reaction turbine stage running at 3000 r.p.m., the exit angles are 30° and the inlet angles are 50°. The mean diameter is 1 m. The steam flow rate is 10000 kg/min and the stage efficiency is 85%. Determine :
(*i*) Power output of the stage.
(*ii*) The specific enthalpy drop in the stage.
(*iii*) The percentage increase in the relative velocity of steam when it flows over the moving blades.
[Ans. (*i*) 11.6 MW, (*ii*) 82 kJ/kg, (*iii*) 52.2%**]**
18. Twelve successive stages of a reaction turbine have blades with effective inlet and outlet angles of 80° and 20° respectively. The mean diameter of the blade row is 1.2 m and the speed or rotation is 3000 r.p.m. Assuming constant velocity of flow throughout, estimate the enthalpy drop per stage.
For a steam inlet condition of 10 bar and 250°C and an outlet condition of 0.2 bar, estimate the stage efficiency.
Assume a reheat factor of 1.04, determine the blade height at a stage where the specific volume is 1.02 m^3/kg. **[Ans.** 40.4 kJ/kg, 70.3%, 57 mm**]**

Steam Condensers

Definition:

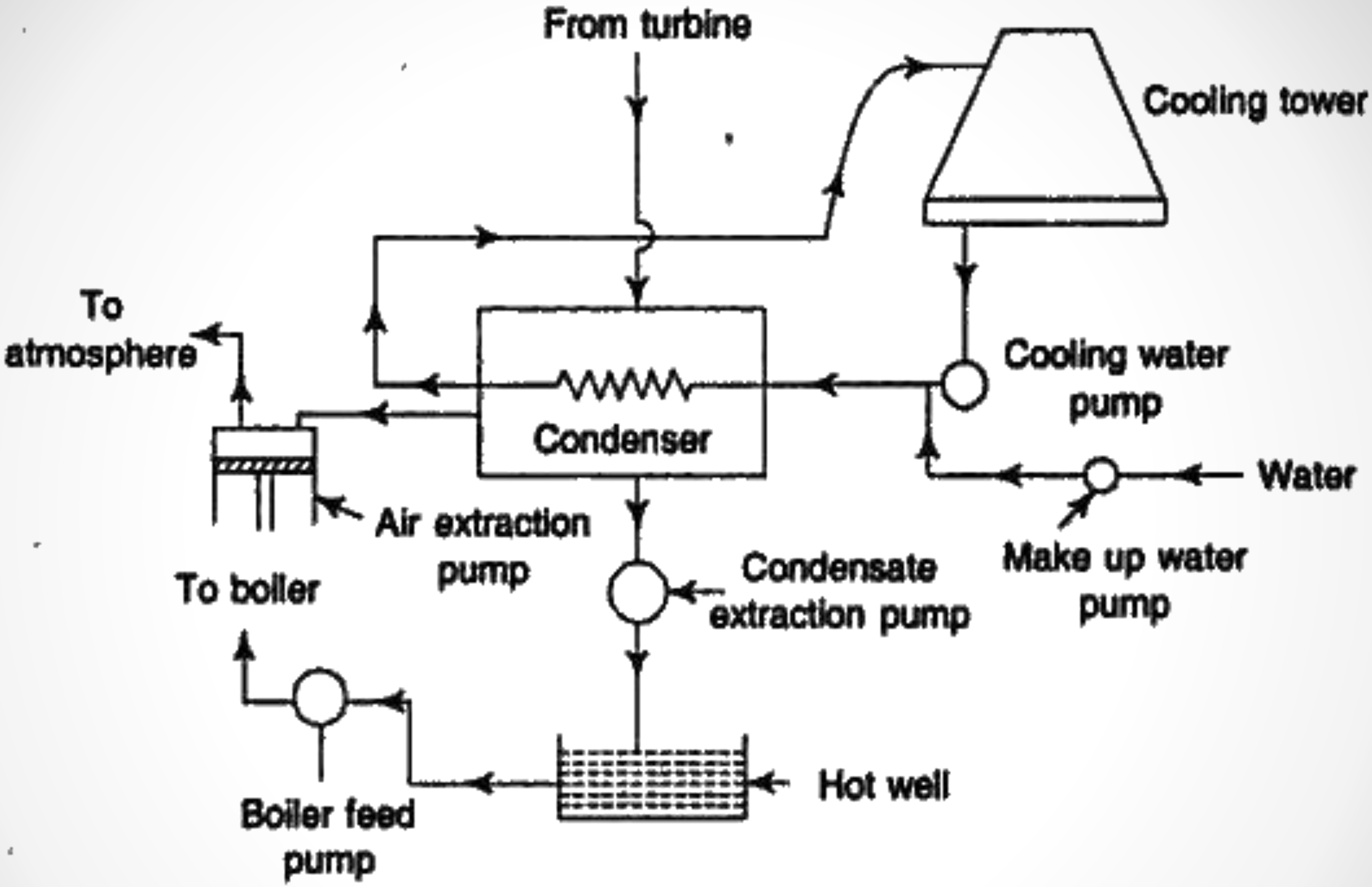
- Condenser is a device in which steam is condensed to water at a pressure less than atmosphere.
- Condensation can be done by removing heat from exhaust steam using circulating cooling water.
- During condensation, the working substance changes its phase from vapour to liquid and rejects latent heat.
- The exhaust pressure in the condenser is maintaned nearly 7 to 8 kpa which corresponds to condensate temperature of nearly 313 kelvin.

Functions of Condenser:

- To reduce the turbine exhaust pressure so as to increase the specific output and hence increase the plant efficiency and decrease the specific steam consumption.
- To condense the exhaust steam from the turbine and reuse it as pure feed water in the boiler. Thus only make up water is required to compensate loss of water
- Enables removal of air and other non condensable gases from steam. Hence improved heat transfer.

Elements of Condensing Plant:

- Condenser
- Air Extraction Pump
- Condensate Extraction Pump
- Cooling Water Circulating Pump
- Hot Well
- Cooling Tower
- Make up Water Pump
- Boiler Feed Pump



Elements of steam condensing plant

Classification of Condensers:

❑ *According to the type of flow:*

- Parallel flow , Counter flow & Cross flow

❑ *According to the Cooling Action:*

- Jet Condensers:
- Low Level Parallel Flow Jet Condenser
- Low Level Counter Flow Jet Condenser
- High Level Jet Condenser
- Ejector Jet Condenser

- Surface Condensers:
- Shell and Tube type

1. Down Flow 2. Central Flow 3. Inverted Flow

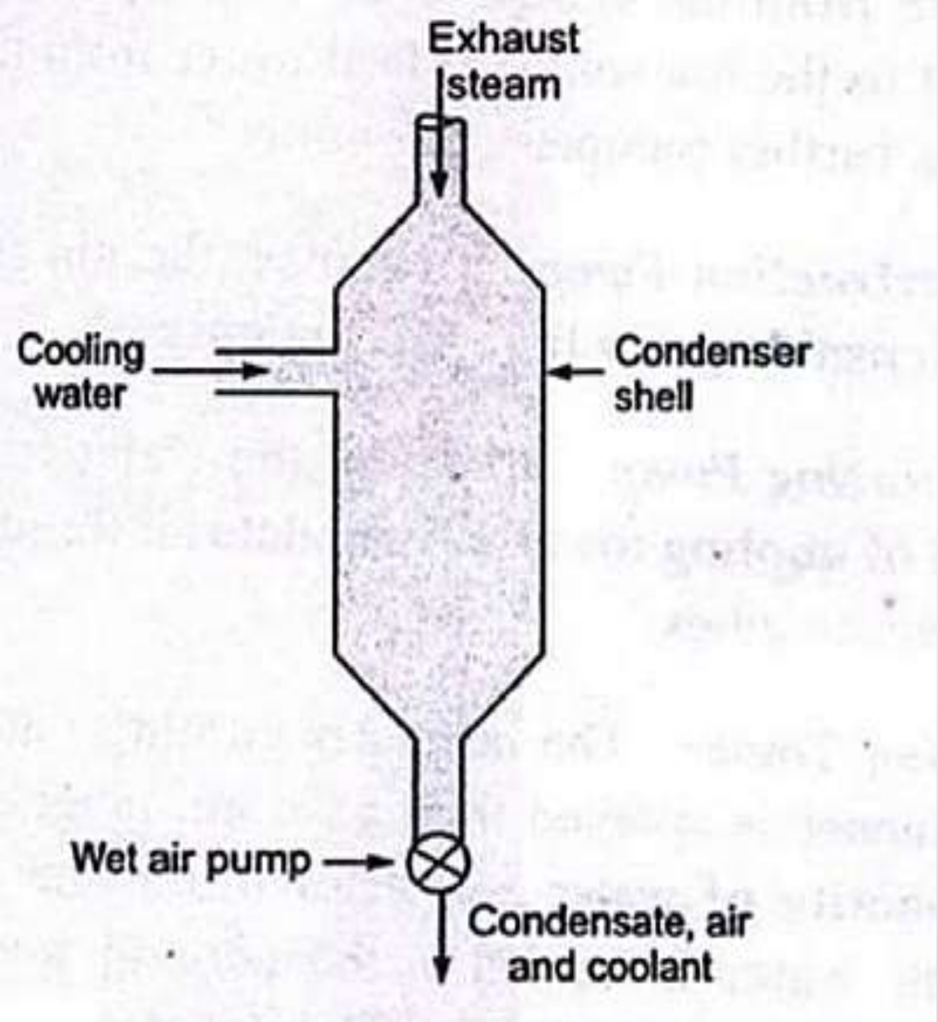
- Evaporative type

Jet Condensers:

- In jet condensers exhaust steam and cooling water come in direct contact and mix up together. Thus, the final temperature of condensate and cooling water leaving the condenser is same.
- A jet condenser is very simple in design and cheaper.
- It can be used when cooling water is cheaply and easily available.
- Condensate can not be reused in boiler, because it contains impurities like dust, oil, metal particles etc.

Low Level Parallel Flow Jet Condenser:

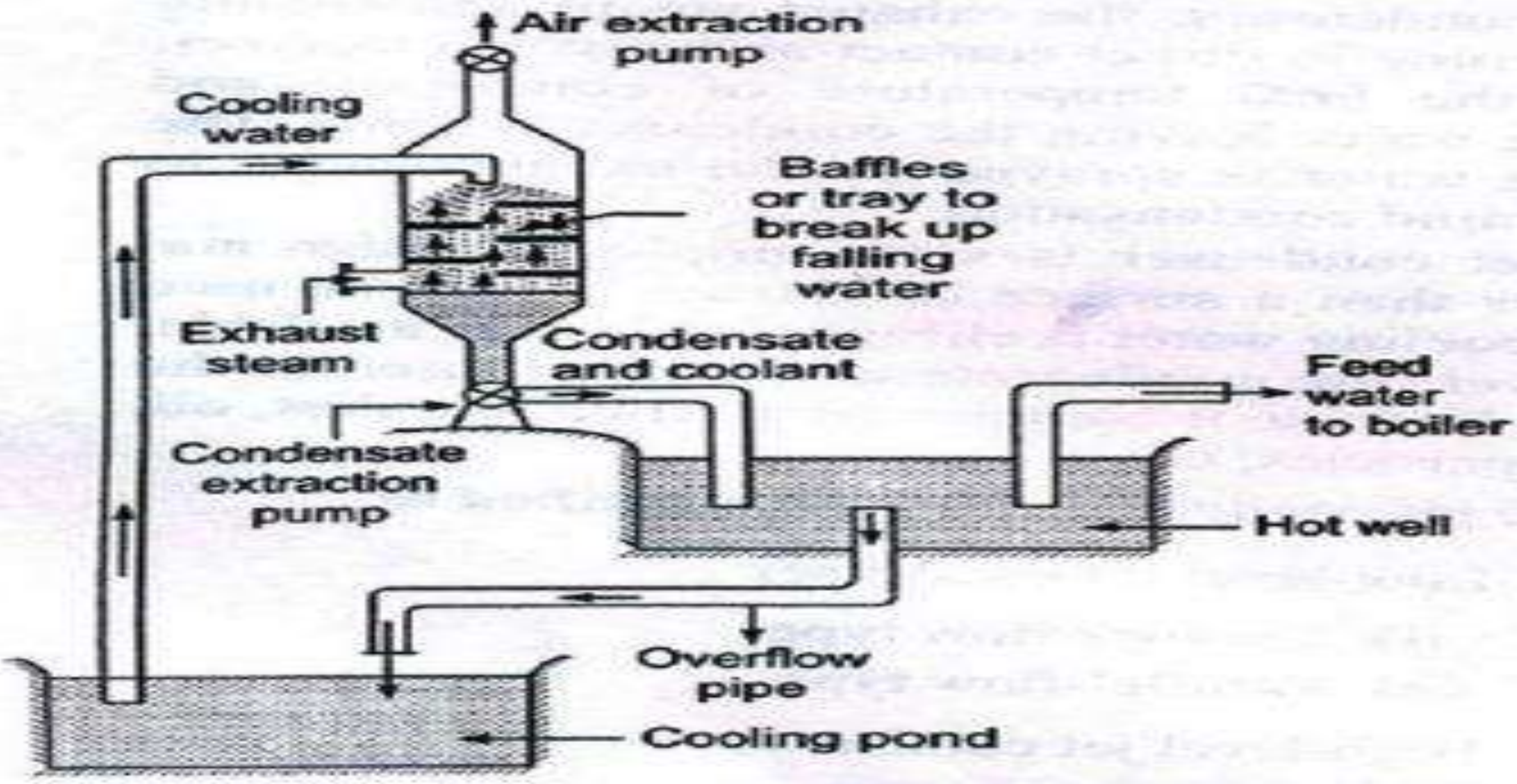
- Wet air pump is used to extract the mixture of condensate, air & coolant.
- Vacuum created is up to 6 kpa.



Low-level parallel-flow jet condenser

Low Level Counter Flow Jet Condenser:

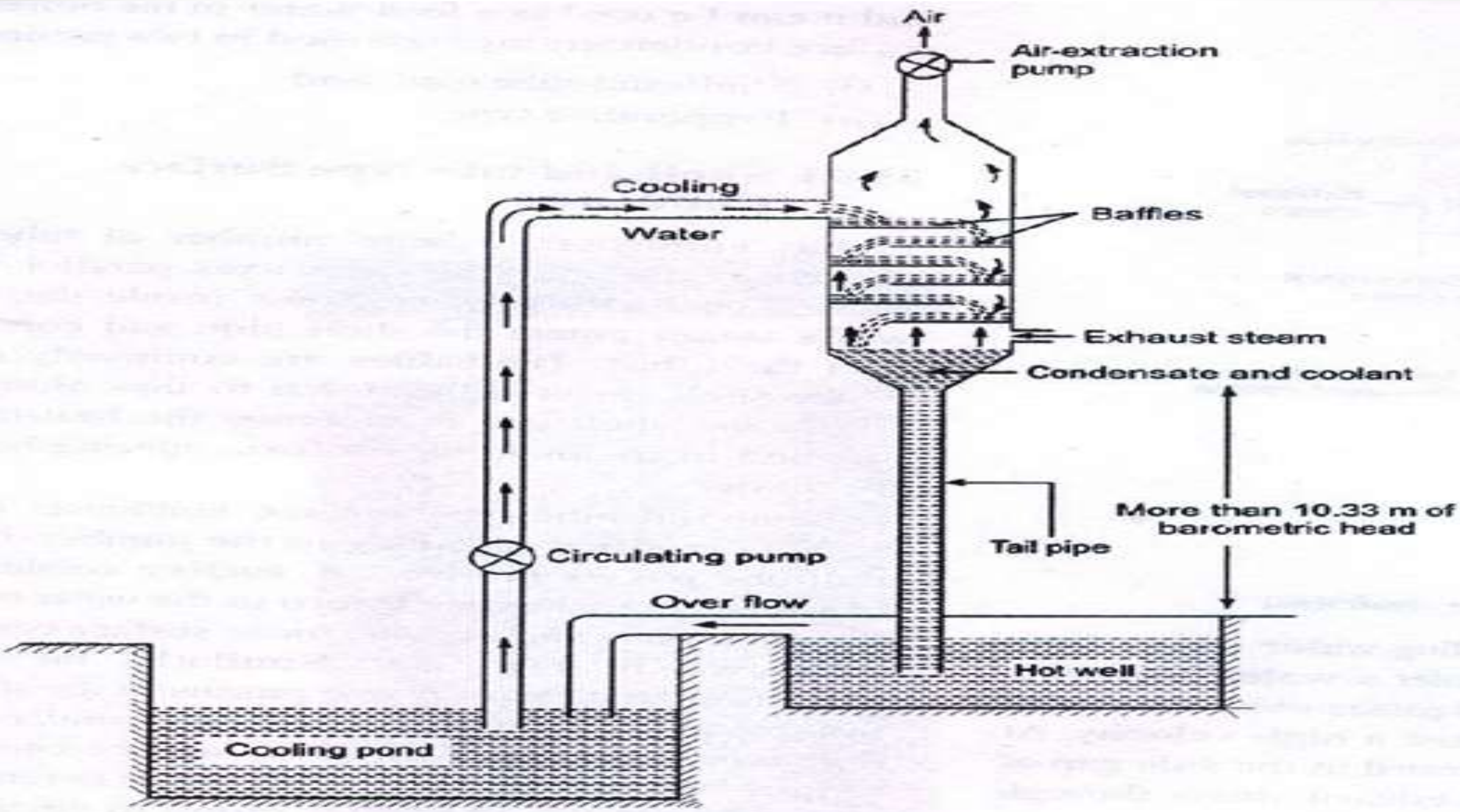
A pump for water supply is required if it is to be lifted more than 5.5 m in height.



Low-level counter-flow jet condenser

High Level Jet Condenser:

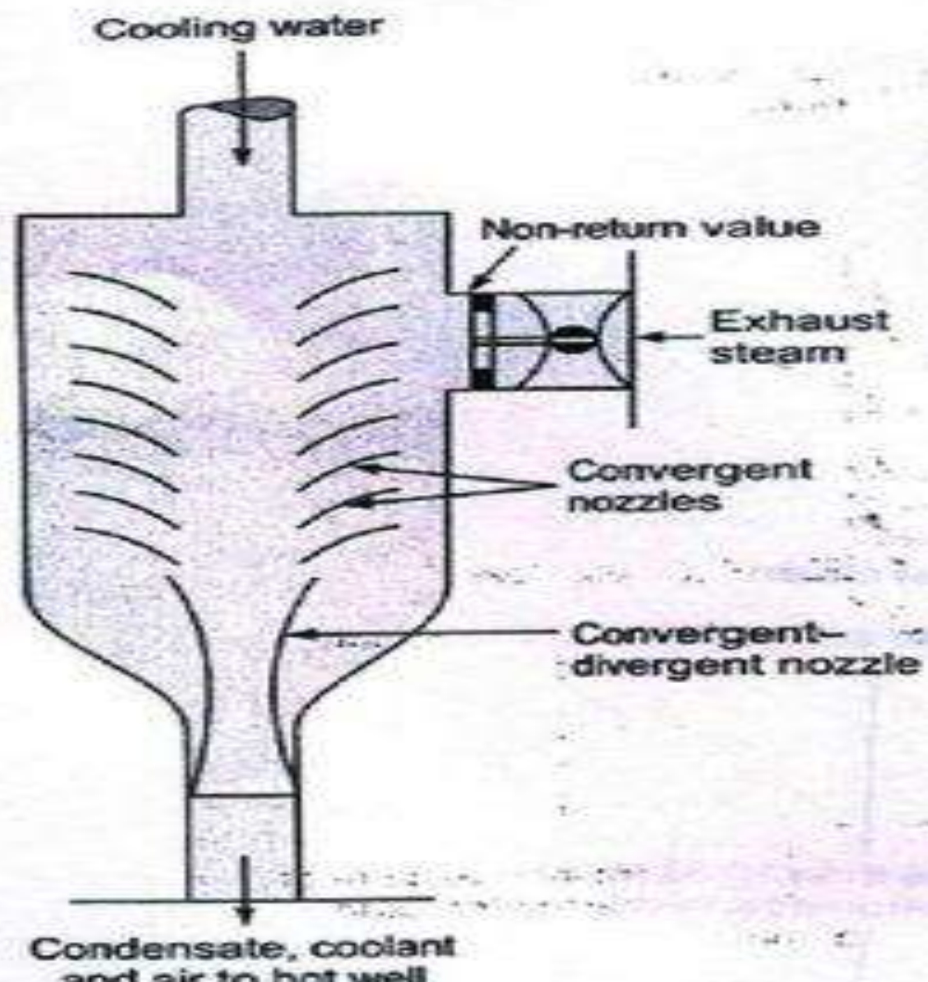
Condenser shell is installed at height greater than that of atmospheric pressure in water column i.e. 10.33 m .



High-level jet condenser

Ejector Condenser:

- Momentum of flowing water is used to remove the mixture of condensate & coolant from condenser without the use of any extraction pump.



Ejector condenser

Advantages & Disadvantages of Jet Condensers:

Advantages:

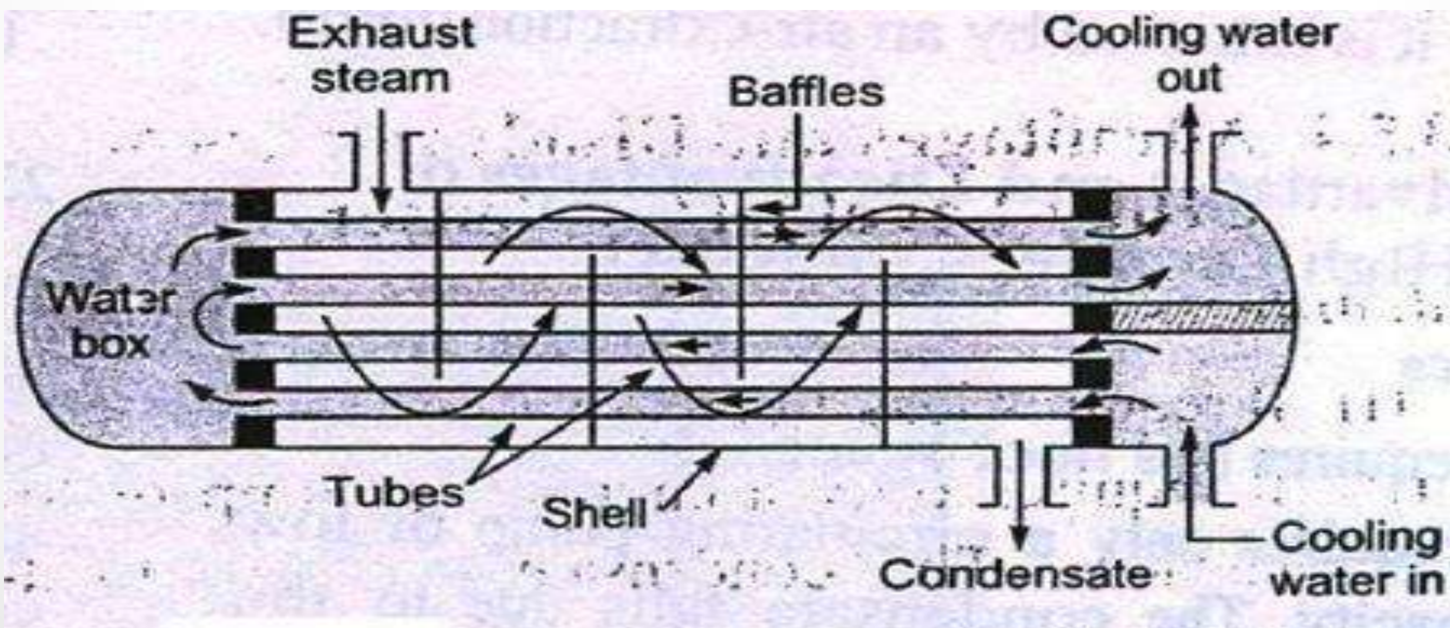
- Simple in design & cheaper.
- Less floor area is required.

Disadvantages:

- Condensate is not pure hence can not be reused.
- Low vacuum efficiency.

Surface Condensers:

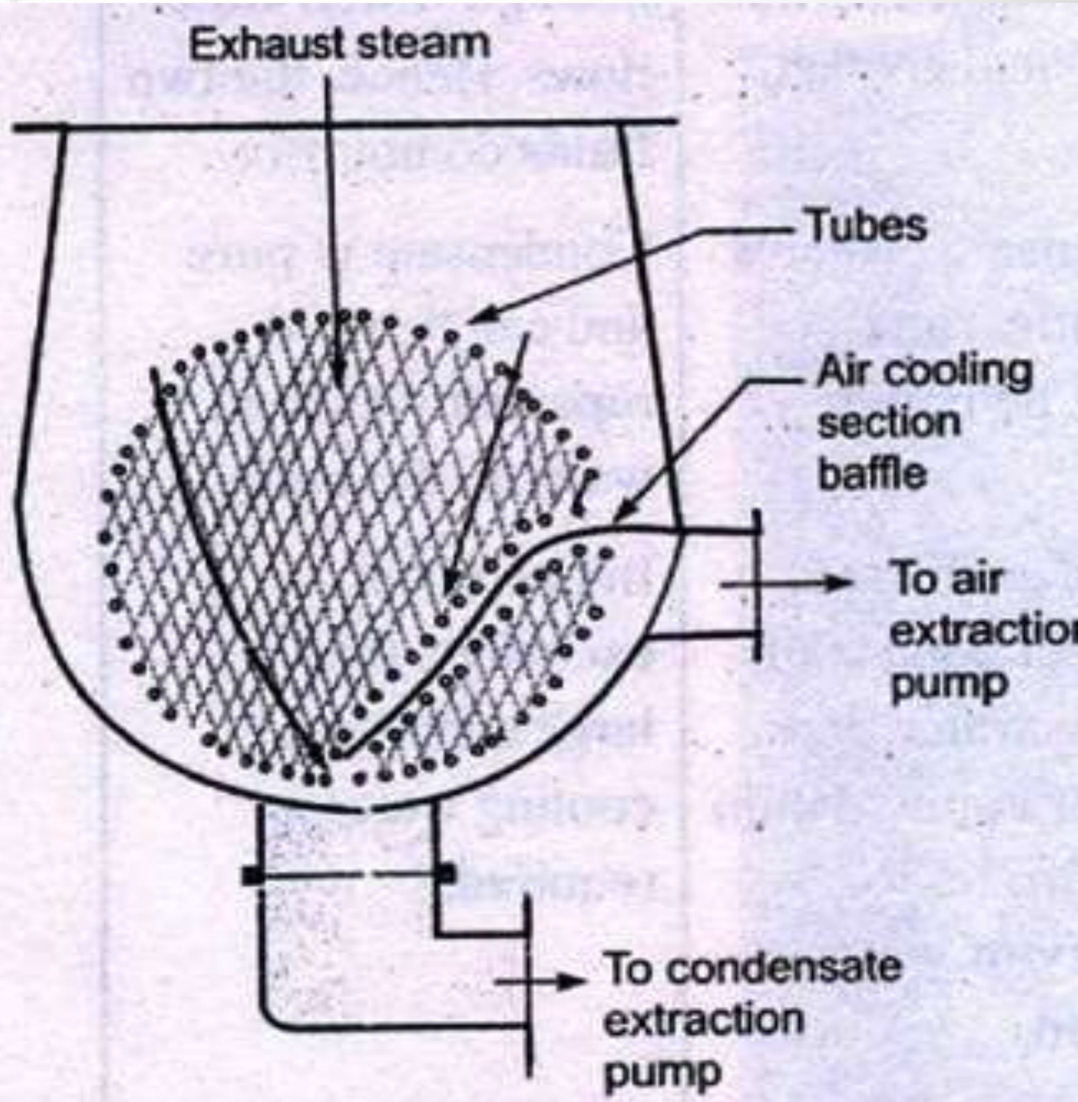
- In surface condenser, the exhaust steam and cooling water do not come in physical contact, rather they are separated by heat transfer wall. Hence condensate remains pure & can be reused.



Shell and tube type condenser

Down Flow Surface Condenser:

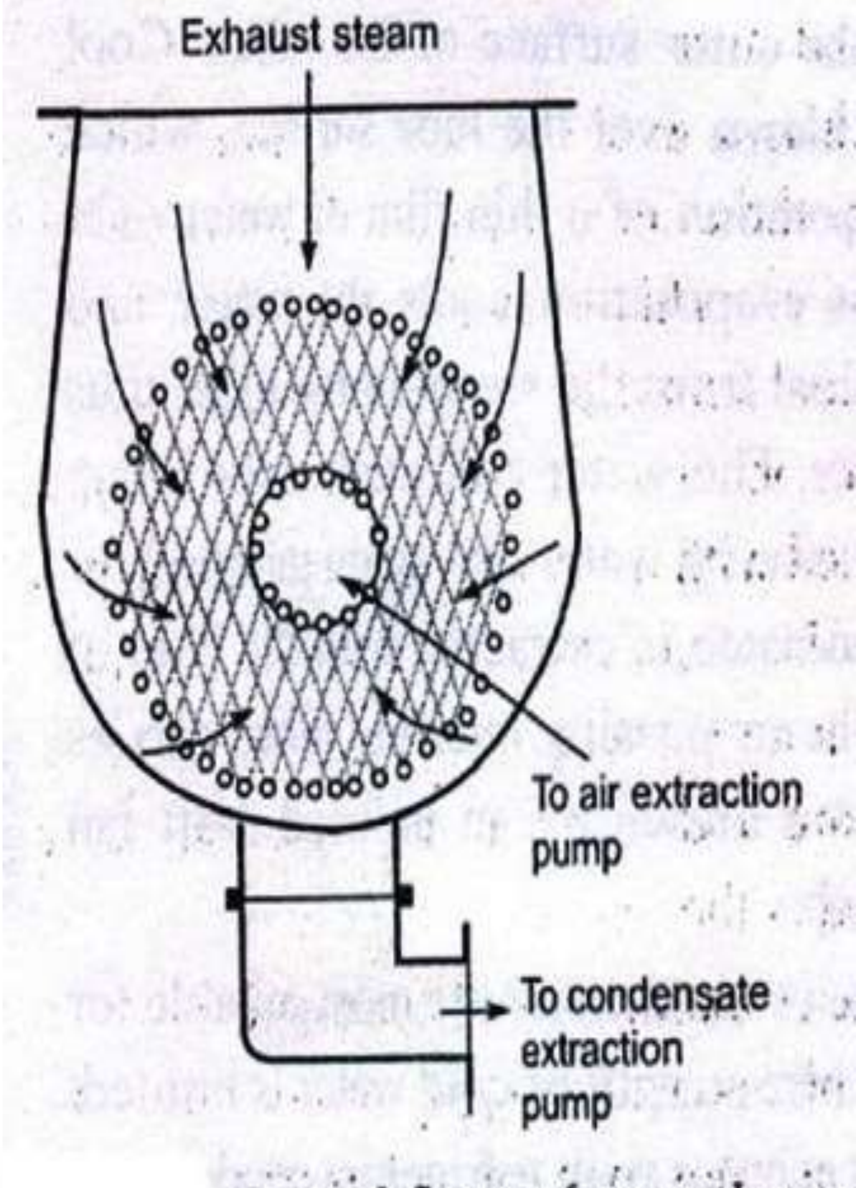
- Exhaust steam enters the top of condenser shell & flows downward over water tubes.
- Water tubes are double passed. The cold water flows in lower side first & then in upper side in the reverse direction, which enables the maximum heat transfer.



Down-flow surface condenser

Central Flow Surface Condenser:

- The steam flows radially inward
- The condensate is collected at the bottom of the shell from where it is taken out by the condensate extraction pump.
- The steam gets access to the entire periphery of tubes, and thus a large surface area for the hear transfer is available as compared to the down flow.



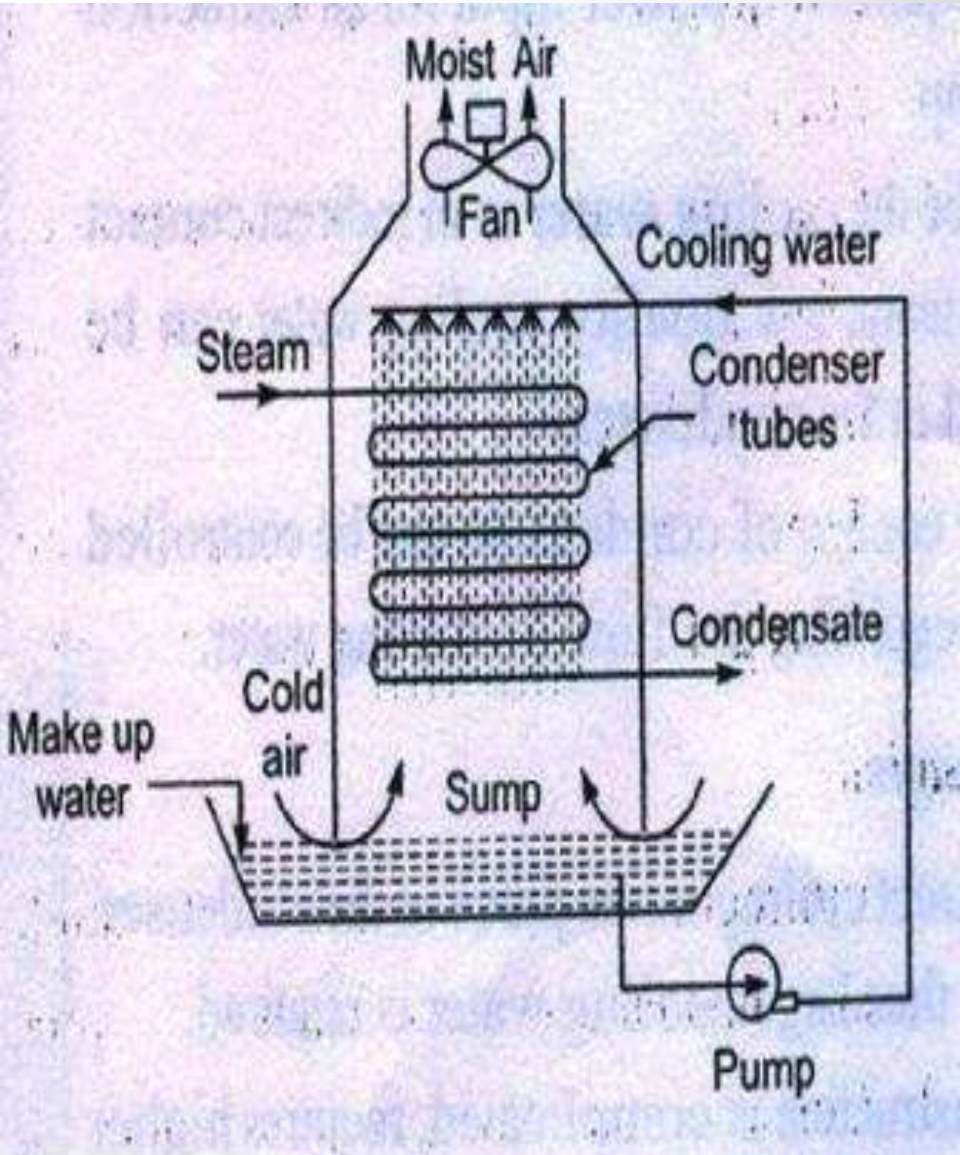
Central-flow surface condenser

Inverted Flow Condenser:

- The steam enters the bottom of the shell and air extraction pump connected at the top.
- Steam flows upward first and subsequently, returns to the bottom of the condenser.
- The condensate extraction pump is connected at the bottom of the shell to extract the condensate.

Evaporative Condenser:

- The evaporation of some cooling water provides the cooling effect, thereby steam condenses.
- Steam to be condensed is passed through grilled tubes & cooling water is sprayed over outer surface of tubes.
- The evaporative condensers are most suitable for small plants, where supply of cold water is limited.



Evaporative steam condenser

Advantages & Disadvantages of Surface Condensers:

Advantages:

- High vacuum efficiency.
- Pure condensate.
- Low quality cooling water can be used.
- It allows the expansion of steam through a higher pressure ratio.

Disadvantages:

- Large amount of water is required.
- Construction is complicated.
- Costly maintenance and skilled workers.
- Large floor area.

Comparison of Jet & Surface Condensers:

Jet Condensers	Surface Condensers
1) Cooling water and steam are mixed up	1) Cooling water & steam aren't mixed up
2) Requires small floor space	2) Requires large floor space
3) The condensate cannot be used as feed water to boiler unless it is free from impurities	3) The condensate can be used as feed water to boiler as it is not mixed with cooling water
4) More power is required for air pump	4) Less power is required for air pump
5) Less power is required for water pump	5) More power is required for water pump
6) Requires less quantity of cooling water	6) Requires large quantity of cooling water
7) The condensing plant is simple	7) The condensing plant is complicated
8) Less suitable for high capacity plants due to low vacuum efficiency	8) More suitable for high capacity plants as vacuum efficiency is high

Effect of Condenser Pressure on Rankine Efficiency:

- Lowering the condenser pressure will increase the area enclosed by the cycle on a T-s diagram which indicates that the net work will increase. Thus, the thermal efficiency of the cycle will be increased
- Lowering the back pressure causes an increase in moisture content of steam leaving the turbine.
- Increase in moisture content of steam in low pressure stages, there is decrease in efficiency & erosion of blade may be a very serious problem and also the pump work required will be high.

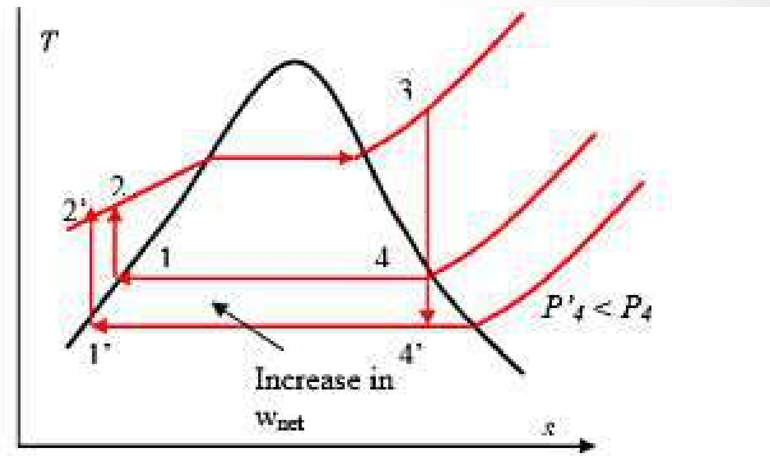


Fig. 4: Effect of lowering the condenser pressure on ideal Rankine cy

Vacuum Creation in Condenser:

- When the steam condenses in a closed vessel, the vapour phase of working substance changes to liquid phase, and thus its specific volume reduces to more than one thousand times.
- Due to change in specific volume, the absolute pressure in the condenser falls below atmospheric pressure and a high vacuum is created.
- This minimum pressure that can be attained depends on the temperature of condensate and air present in the condenser.

The absolute pressure in the condenser = Atmospheric pressure – Vacuum Gauge Pressure

Sources of Air in the Condenser:

- The ambient air leaks to the condenser chamber at the joints & glands which are internally under pressure lower than that of ambient.
- Another source of air is the dissolved air with feed water. The dissolved air in feed water enters into boiler and it travels with steam into condenser.

Effects of Air Leakage:

- The presence of air lowers vacuum in the condenser. Thus back pressure of the plant increases, and consequently, the work output decreases.
- Air has very poor thermal conductivity. Hence, the rate of heat transfer from vapour to cooling medium is reduced.
- The presence of air in the condenser corrodes to the metal surfaces. Therefore, the life of condenser is reduced.

Artistic to Scientific Design of Cooling Towers

- The art of evaporative cooling is quite ancient, although it is only relatively recently that it has been studied scientifically.
- Merkel developed the theory for the thermal evaluation of cooling towers in 1925.
- This work was largely neglected until 1941 when the paper was translated into English.
- Since then, the model has been widely applied.
- The Merkel theory relies on several critical assumptions to reduce the solution to a simple hand calculation.
- Because of these assumptions, the Merkel method does not accurately represent the physics of heat and mass transfer process in the cooling tower fill.

What Does A Cooling Tower Do?

- Cooling Towers are used to transfer heat from <u>cooling water</u> to the atmosphere.
  - Promotes efficient water usage
  - Prevents environmental damage

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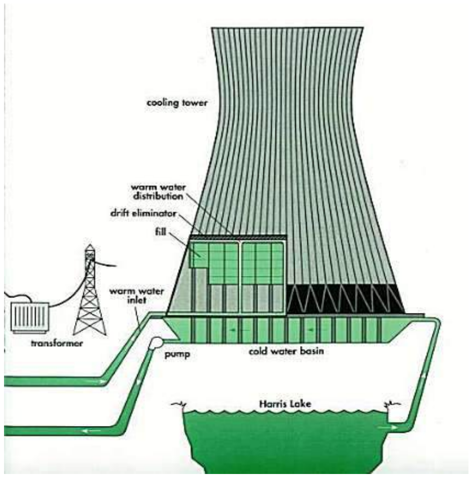
- Cooling water is continuously circulated through heat exchangers to absorb heat from process material and machinery.
- Because it's cost efficient to reuse water and plants can't dump excessive amounts of hot water into rivers and lakes, cooling towers are used to remove the heat from the water, so it can be recirculated.
- Used in power stations, oil refineries, petrochemical plants and natural gas plants.

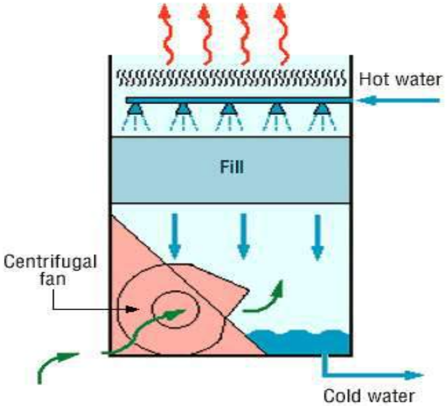
Heat Transfer Methods

- Wet Cooling Tower
  - Uses evaporation to transfer heat
  - Water can be cooled to a temperature lower than the ambient air "dry-bulb" temperature
- Dry Cooling Tower
  - Uses convection to transfer heat
  - Heat is transferred through a surface that separates the water from ambient air, such as in a heat exchanger.

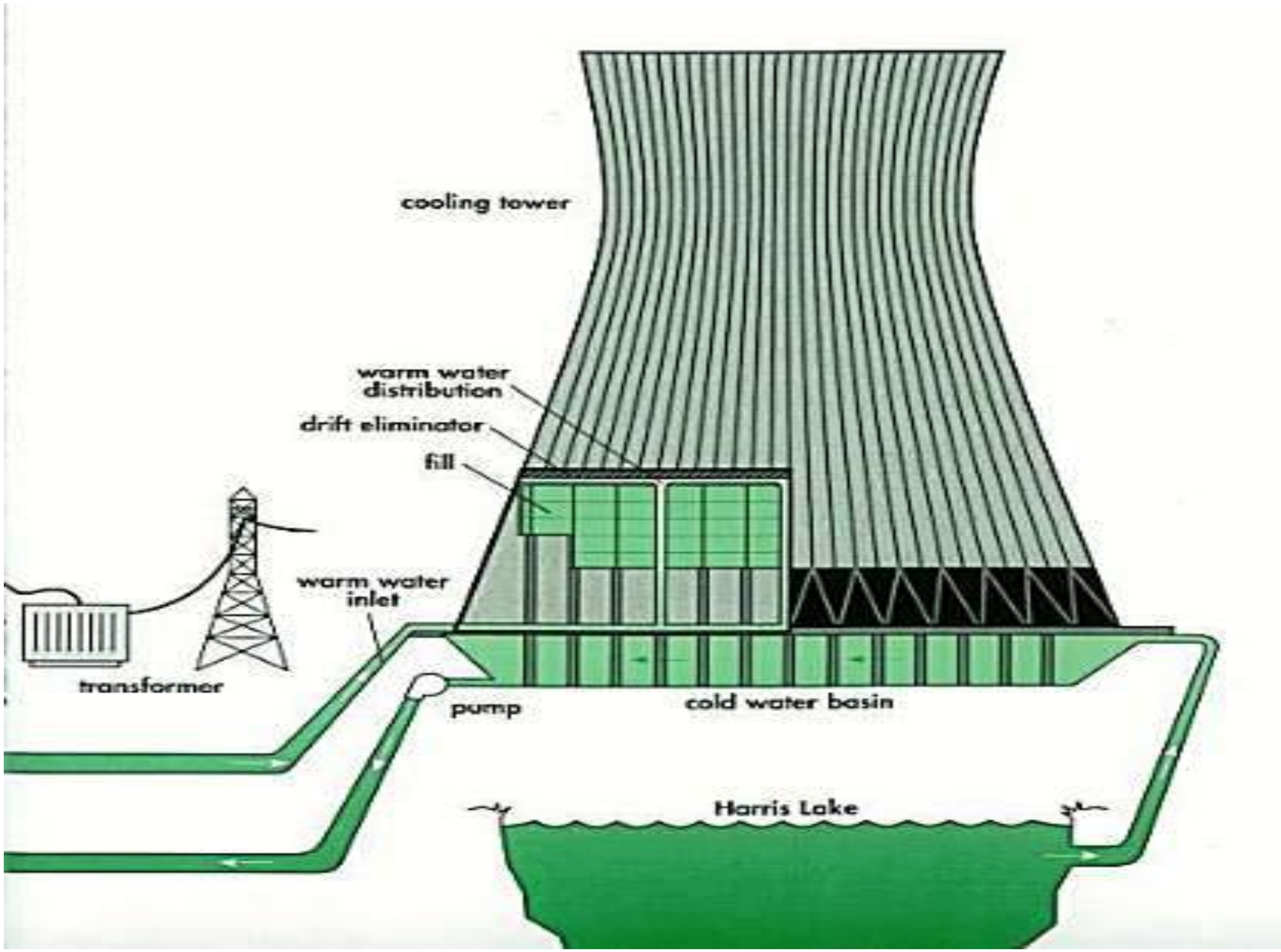
Air Flow Generation Methods

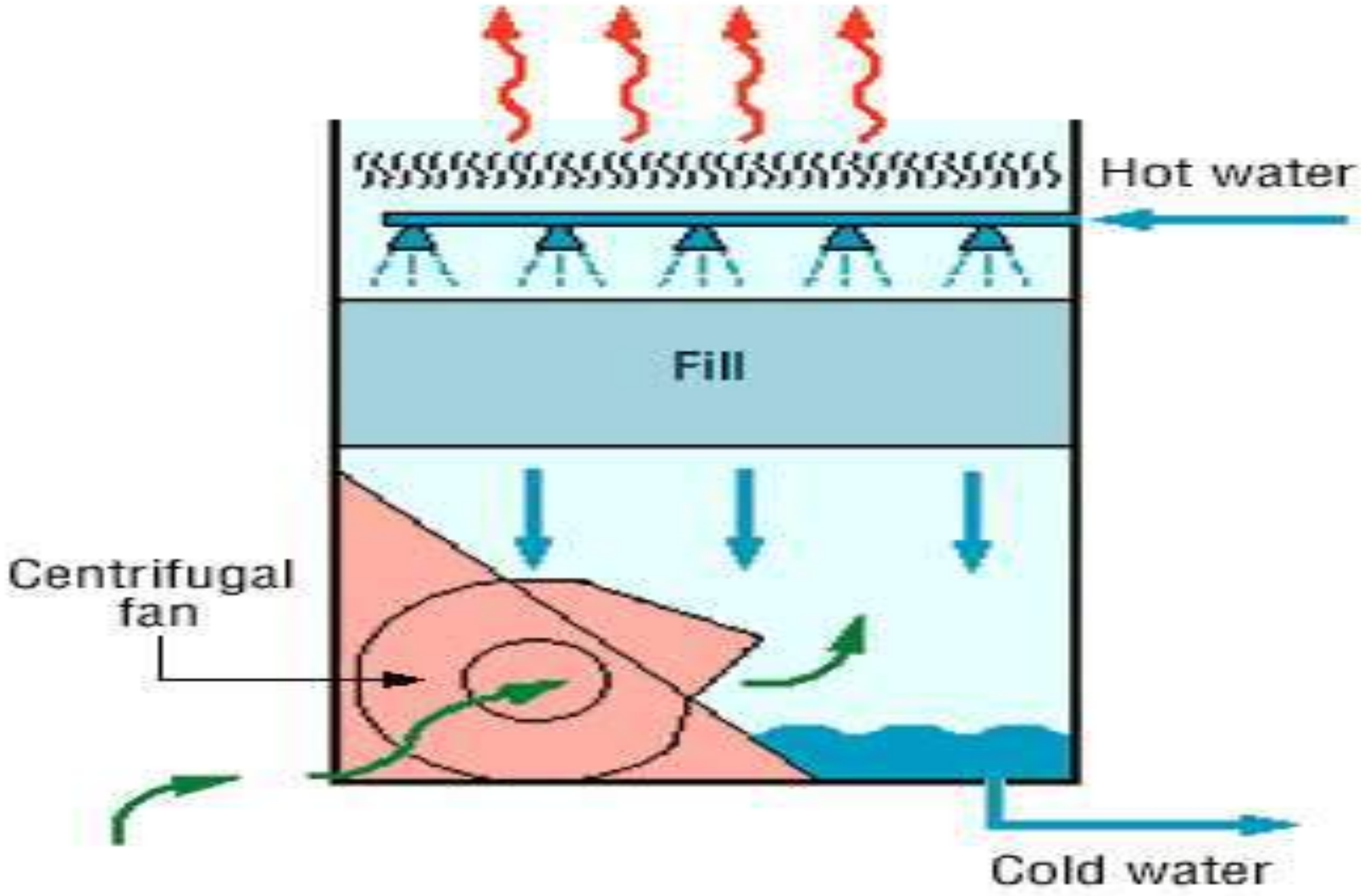
- Natural Draft
  - Warm air naturally rises due to the density differential to the dry, cooler outside air. This moist air buoyancy produces an airflow through the tower.
- Mechanical Draft
  - A fan induces airflow through a tower.



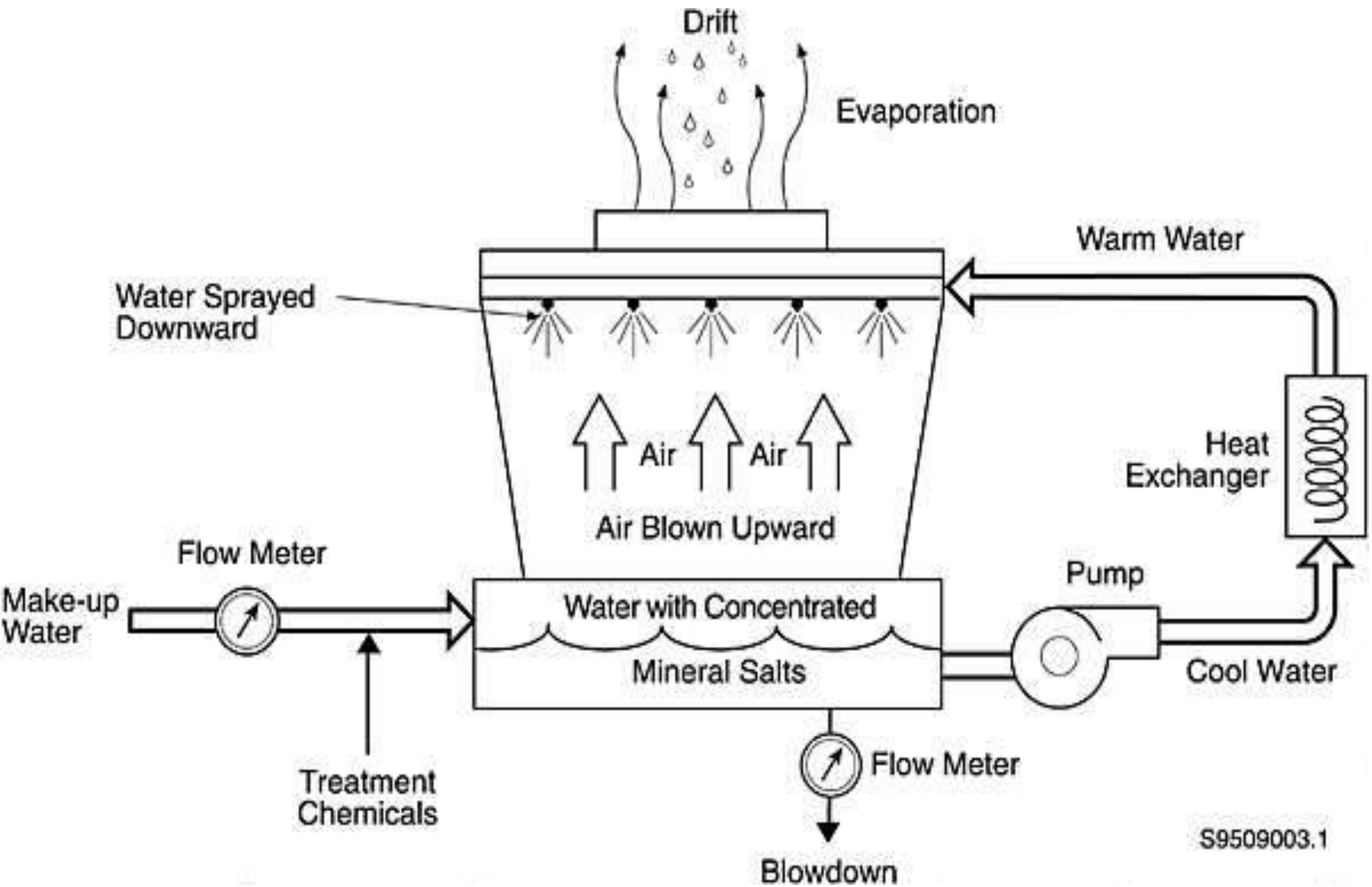


NATURAL DRAFT





MECHANICAL DRAFT

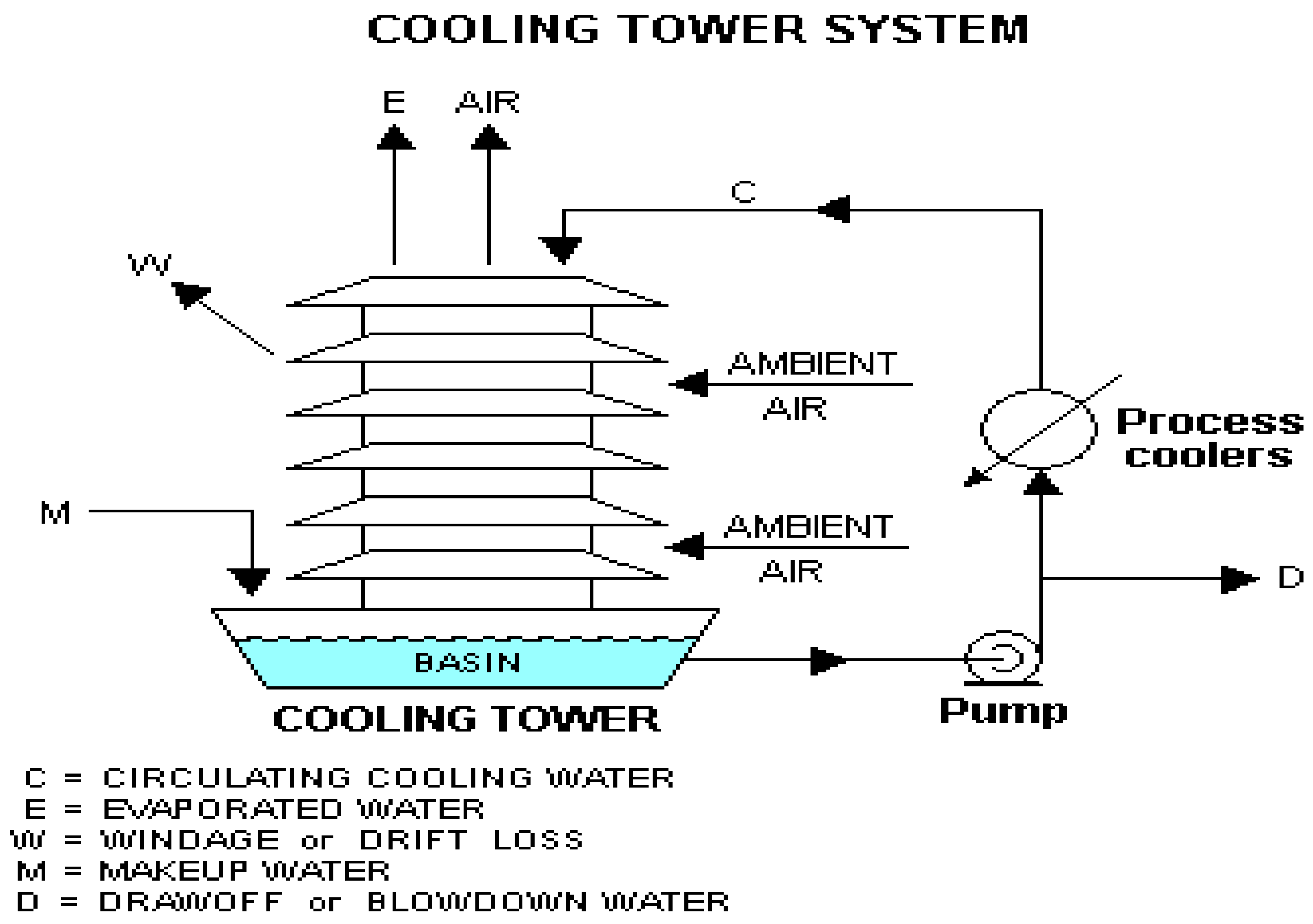

Drift
Evaporation
Warm Water
Water Sprayed
Downward
Air
Air
Air Blown Upward
Heat
Exchanger
Flow Meter
Make-up
Water
Water with Concentrated
Mineral Salts
Pump
Cool Water
Treatment
Chemicals
Flow Meter
Blowdown
S9509003.1

Parameters of Cooling Towers

- A number of parameters describe the performance of a cooling tower.
- **Range** is the temperature difference between the hot water entering the cooling tower and the cold water leaving.
- The range is virtually identical with the condenser rise.
- Note that the range is not determined by performance of the tower, but is determined by the heat loading.

- **Approach** is the difference between the temperature of the water leaving the tower and the wet bulb temperature of the entering air.
- The approach is affected by the cooling tower capability.
- For a given heat loading, water flow rate, and entering air conditions, a larger tower will produce a smaller approach; i.e., the water leaving the tower will be colder.
- **Water/Air Ratio (m_w/m_a)** is the mass ratio of water (Liquid) flowing through the tower to the air (Gas) flow.
- Each tower will have a design water/air ratio.
- An increase in this ratio will result in an increase of the approach, that is, warmer water will be leaving the tower.
- A test ratio is calculated when the cooling tower performance is evaluated.

Cooling Tower Mass Balances



Water and Salt Balances

A water balance around the entire system is:

$$\mathbf{M = E + D + W}$$

Since the evaporated water (E) has no salts, a chloride balance around the system is:

$$M\ (X_M) = D\ (X_C) + W\ (X_C) = X_C\ (D + W)$$

and, therefore:

$$\mathbf{X_C/X_M = \text{Cycles of concentration}}$$

$$\mathbf{= M/(D + W)}$$

$$\mathbf{= M/(M - E) = 1 + [E \div (D + W)]}$$

Heat Balance

From a simplified heat balance around the cooling tower:

$$\mathbf{E\ H_v = C \cdot \Delta T \cdot c_p}$$

H_v = Latent heat of water evaporation
= 2260 kJ/kg

ΔT = Temperature difference top to bottom

C_p = Specific heat of water = 4.184 $kJkg^{-1}C^{-1}$

Thermodynamics of Air Water Systems

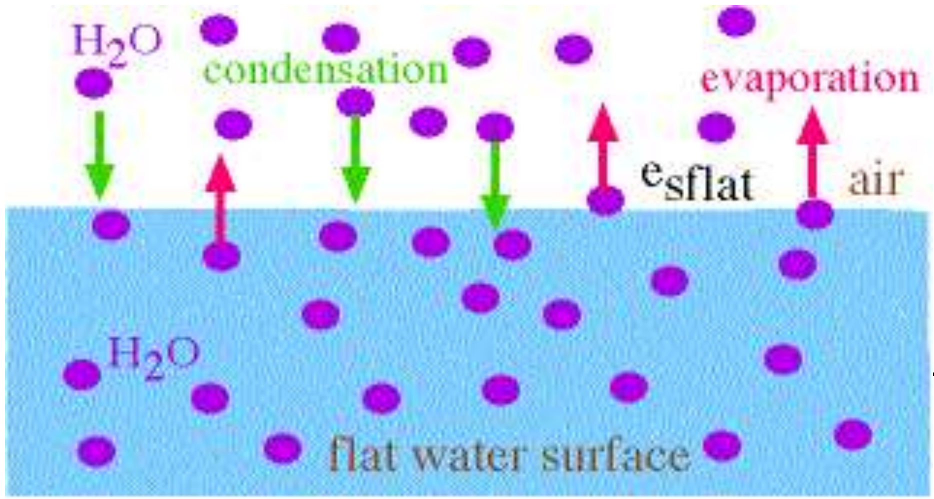
Humidity Ratio:

$$\omega = \frac{\text{Mass Flow of Water Vapour}}{\text{Mass Flow of Dry Air}} = \frac{\dot{m}_v}{\dot{m}_a}$$

$$p_a\dot{V} = \dot{m}_a R_a T = \frac{\dot{m}_a R_u T}{M_a} \qquad p_v\dot{V} = \dot{m}_v R_v T = \frac{\dot{m}_v R_u T}{M_v}$$

$$\omega = \frac{\dfrac{M_v p_v \dot{V}}{R_u T}}{\dfrac{M_a p_a \dot{V}}{R_u T}} = \left(\frac{M_v}{M_a}\right) \times \left(\frac{p_v}{p_a}\right) = 0.622\left(\frac{p_v}{p_a}\right)$$

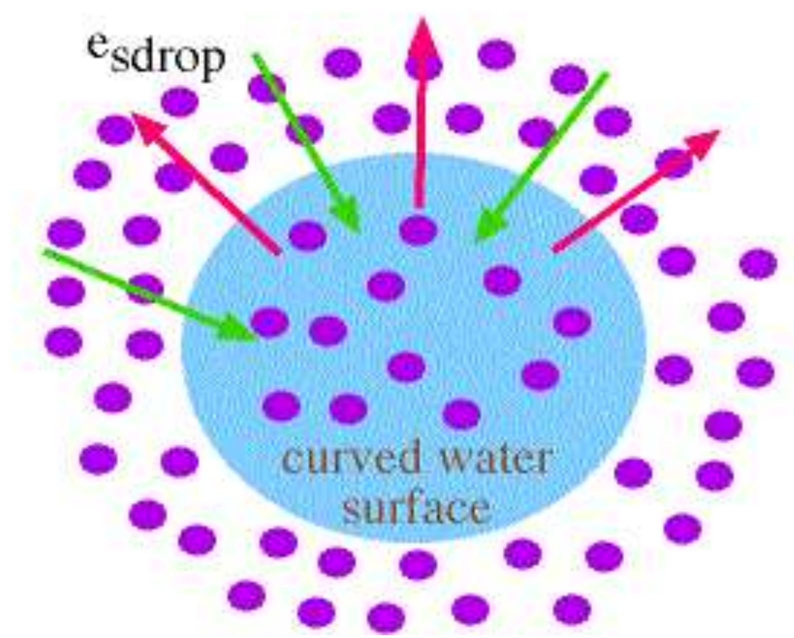
Local Cooling Tower Theory



Air at Temperature, T_A

$T_A < T_B$

.alpy < Water enthalpy



Hea

Heat is transferred fror sfer of sensible and latent heat

Mechanism of Heat Transfer in Cooling Towers

- Heat transfer in cooling towers occurs by two major mechanisms:
- Sensible heat from water to air (convection) and
- transfer of latent heat by the evaporation of water (diffusion).
- Both of these mechanisms operate at air-water boundary layer.
- The total heat transfer is the sum of these two boundary layer mechanisms.
- The total heat transfer can also be expressed in terms of the change in enthalpy of each bulk phase.
- A fundamental equation o f heat transfer in cooling towers (the Merkel equation) is obtained.

$$\dot{m}_{CW} C_W dT_W = KA(h_{sa} - h_a)dV = \dot{m}_{air} dh_{air}$$

The Merkel Method

- The Merkel method, developed in the 1920s, relies on several critical assumptions to reduce the solution to a simple manual iteration.
- These assumptions are:
- The resistance for heat transfer in the water film is negligible,
- The effect of water loss by evaporation on energy balance or air process state is neglected,
- The specific heat of air-stream mixture at constant pressure is same as that of the dry air, and
- The ratio of h_{conv}/h_{diff} (Lewis factor) for humid air is unity.
- Merkel combined equations for heat and water vapor transfer into a single equation similar as

$$Me_M = \frac{kAV}{\dot{m}_w} = \int_{T_2}^{T_1} \frac{dT}{h_{sa} - h_a}$$

where:

kAV/mw = tower characteristic

k= mass transfer coefficient

A = contact area/tower volume

V = active cooling volume/plan area

mw = water flow rate

T_1 = hot water temperature

T_2 = cold water temperature

T = bulk water temperature

h_{sa} = enthalpy of saturated air-water vapor mixture at bulk water temperature (J/kg dry air)

h_a = enthalpy of air-water vapor mixture (J/kg dry air)

Tower Characteristics

- **Tower Characteristic (*Me_M or NTU*)** is a characteristic of the tower that relates tower design and operating characteristics to the amount of heat that can be transferred.
- For a given set of operating conditions, the design constants that depend on the tower fill.
- For a tower that is to be evaluated using the characteristic curve method, the manufacturer will provide a tower characteristic curve.

$$NTU = C\left\{\frac{\dot{m}_w}{\dot{m}_a}\right\}^n$$

SUPPLY TOWER CHARACTERISTIC

- The supply tower characteristic of the cooling tower can be evaluated with the help of cooling tower fill characteristics curves provided by manufacturer which takes into account the effect of rain and spray zones as well as fill fouling.
- These curves are certified by the cooling tower institute.

Generalized Equation for Cooling Tower Supply

- A generalized equation for cooling tower supply can be developed from the manufacturer curves (known as the supply equation) and is of the form:

$$\frac{KAV}{L} = C \times u_{air}^{n} \times \left\{ \frac{\dot{m}_w}{\dot{m}_a} \right\}^{m}$$

BHP OF THE FAN

- The total pressure drop (PD) across the cooling tower which is the summation of the pressure drops across the drift eliminators, inlet louvers and the fill packing (constituting the static pressure drop) and also the velocity pressure drop is calculated.
- Now, the total fan power required is calculated as

$$BHP = (CFM * PD)/ (n * 6356)$$

where n is the efficiency of the fan.

Loss of Water

- **Evaporation Rate** is the fraction of the circulating water that is evaporated in the cooling process.
- A typical design evaporation rate is about 1% for every 12.5°C range at typical design conditions.
- It will vary with the season, since in colder weather there is more sensible heat transfer from the water to the air, and therefore less evaporation.
- The evaporation rate has a direct impact on the cooling tower makeup water requirements.

- **Drift** is water that is carried away from the tower in the form of droplets with the air discharged from the tower.
- Most towers are equipped with drift eliminators to minimize the amount of drift to a small fraction of a percent of the water circulation rate.
- Drift has a direct impact on the cooling tower makeup water requirements.
- **Recirculation** is warm, moist air discharged from the tower that mixes with the incoming air and re-enters the tower.
- This increases the wet bulb temperature of the entering air and reduces the cooling capability of the tower.
- During cold weather operation, recirculation may also lead to icing of the air intake areas.

EXERCISE

15.1 What do you understand by condenser? Discuss its significance.

15.2 How does condenser improve performance of steam power plant?

15.3 Discuss different types of condenser briefly.

15.4 Differentiate between surface condenser and jet condenser.

15.5 Give a sketch of barometric jet condenser and explain its working.

15.6 Discuss the effect of air leakage upon the performance of condenser.

15.7 How the air leaking into condenser is extracted out? Explain.

15.8 Describe the factors affecting the efficiency of condensing plant.

15.9 Discuss the relevance of Dalton's law of partial pressures in condenser calculations.

15.10 What do you understand by cooling towers? Explain their utility.

15.11 Determine the vacuum efficiency of a surface condenser having vacuum of 715 mm of Hg and temperature of 32°C. The barometer reading is 765 mm of Hg. [98%]

15.12 A surface condenser having vacuum of 715 mm Hg and temperature of 32°C has cooling water circulated at 800 kg/min. The cooling water entering condenser becomes warmer by 14°C. The condensate is available from condenser at 25 kg/min. The hot well temperature is 30°C. Barometer reading is 765 mm of Hg. Determine the mass of air in kg/m^3 of condenser volume and dryness fraction of steam entering. [0.022 kg/m^3, 0.84]

15.13 A surface condenser has vacuum of 71 cm Hg and mean temperature of 35°C. The barometer reading is 76.5 cm Hg. The hot well temperature is 28°C. Steam enters condenser at 2000 kg/hr and requires cooling water at 8°C at the rate of 1000 kg/min. Cooling water leaves condenser at 24°C. Determine

(*i*) the vacuum efficiency of condenser,
(*ii*) the undercooling in condenser
(*iii*) corrected vacuum in reference to standard barometer reading, (iv) the condenser efficiency.

[0.982, 7°C, 70.5 cm Hg, 0.505]

15.14 In a surface condenser steam enters at 40°C and dryness fraction of 0.85. Air leaks into it at 0.25 kg/min. An air pump is provided upon the condenser for extracting out air. Temperature at suction of air pump is 32°C while condensate temperature is 35°C. Determine.

(*i*) the reading of vacuum gauge
(*ii*) the volume handling capacity of air pump in m^3/hr
(*iii*) the loss of condensate in kg/hr.

[705 mm Hg, 500 m^3/hr, 16.9 kg/hr]

15.15 A steam turbine discharges steam into a surface condenser having vacuum of 700 mm Hg. The barometer reading is 760 mm Hg. Leakage into condenser is seen to be 1.4 kg/min. The air pump is employed for extracting out air leaking in. Temperature at the inlet of air pump is 20°C. The air pump is of reciprocating type running at 300 rpm and has L : D ratio of 2 : 1. Determine,

(*i*) the capacity of air pump is m^3/hr
(*ii*) the dimensions of air pump
(*iii*) the mass of vapour going out with air in air pump, kg/hr.

[1250 m^3/hr, bore: 35.36 cm, stroke: 70.72 cm, 21.5 kg/hr]

15.16 A surface condenser handles condensate at 70.15 cm Hg when barometer reads 76 cm Hg. Steam entering at 2360 kg/hr requires cooling water at 6.81×10^2 kg/hr, 10°C. Cooling water leaves condenser at 27.8°C while condenser has mean temperature of 37°C. Air leaks into condenser at 0.3 kg/min. Determine,

(*i*) the mass of vapour going out with air per hour
(*ii*) the state of steam entering.

[119 kg/hr, 0.89]

15.17 A steam condenses working at 71.5 cm Hg vacuum and temperature of 32.28°C. Condenser has steam entering at 13750 kg/hr. The leakage of air into condenser is 0.4×10^{-3} kg/kg steam entering. Considering volumetric capacity of air pump as 80% determine the capacity of air pump, handling mixture of air and water vapour. [10.3 m^3/min]

15.18 A jet condenser has pressure at inlet as 0.07 bar. It is supplied steam at 4000 kg/hr. Water is supplied for condensation at 16×10^4 kg/hr. The volume of air dissolved in water at 1 bar, 15°C is 0.05 of that water.

Air leaks in with steam at 0.05 kg/min. Temperature at suction of air pump is 30°C and volumetric efficiency is 80%. Determine the capacity of air pump in m^3/min for extracting out air.

[519 kg/hr]

15.19 A surface condenser has steam entering at 0.09 bar, 0.88 dry at the rate of 16000 kg/hr. The air leakage into condenser is 8 kg/hr. Temperature at condensate and air extraction pipe is 36°C. For the average heat transfer rate of 133760 kJ/m^2·hr, determine the surface required. [254 m^2]

15.20 A surface condenser has vacuum of 581 mm of Hg and steam enters into it at 57.4°C, 3100 kg/hr. The barometer reads 726 mm of Hg. The temperature at suction of air pump is 50°C. The cooling water is supplied at 5.6×10^4 kg/hr, 15°C and leaves at 43.6°C. Determine,

(*i*) the mass of air entering condenser per kg of steam

(*ii*) the vacuum efficiency.

[0.136 kg/kg steam, 0.981]